A NOTE ON $\mathscr{I}$-ULTRAFILTERS AND P-POINTS

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Abstract. We consider the question whether P-points can be characterized as $\mathscr{I}$-ultrafilters for $\mathscr{I}$ an ideal on ω and show that (consistently) it is not possible if $\mathscr{I}$ is an F_σ-ideal or a P-ideal.

1. Introduction

Definition 1.1 (Baumgartner [2])**.** Let $\mathscr{I}$ be a family of subsets of a set X such that $\mathscr{I}$ contains all singletons and is closed under subsets. An ultrafilter $\mathscr{U}$ on ω is called an *$\mathscr{I}$-ultrafilter* if for any $F : \omega \to X$ there is $A \in \mathscr{U}$ such that $F[A] \in \mathscr{I}$.

Several classes of $\mathscr{I}$-ultrafilters for $X = 2^\omega$ were defined by Baumgartner [2], e.g. discrete or nowhere dense ultrafilters, and some other classes were defined by Brendle [4] and Barney [1]. All those ultrafilters exist under some additional set-theoretic assumptions, but they cannot be constructed in ZFC because they are nowhere dense and Shelah proved in [10] that it is consistent with ZFC that there are no nowhere dense ultrafilters. For $X = \omega_1$ ordinal ultrafilters were introduced by Baumgartner [2] as $\mathscr{I}$-ultrafilters for $\mathscr{I} = \{A \subseteq \omega_1 : \text{order type of } A \leq \alpha\}$ for some indecomposable ordinal $\alpha < \omega_1$.

In this paper, we consider $\mathscr{I}$-ultrafilters for $X = \omega$ and the situation is slightly different here since $\mathscr{I}$-ultrafilters exist for some particular families $\mathscr{I}$ in ZFC (see Proposition 2.1). Though most of the results in the paper remain consistency results.

Throughout the article we assume that the family $\mathscr{I}$ is an ideal on ω which contains all finite subsets of ω. We can do this without loss of generality because if we replace an arbitrary family $\mathscr{I}$ in the

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definition of $\mathscr{I}$-ultrafilter by the ideal generated by $\mathscr{I}$, we get the same concept (first noticed in [1]).

An ideal $\mathscr{I} \subseteq \mathscr{P}(\omega)$ is called *tall* if every $A \notin \mathscr{I}$ contains an infinite subset that belongs to the ideal $\mathscr{I}$. (Some authors call ideals with this property *dense*.)

For $A, B \subseteq \omega$ we say that A is *almost contained* in B and we write $A \subseteq^* B$ if $A \setminus B$ is finite. Let us also recall that an ideal $\mathscr{I}$ is called a *P-ideal* if whenever $A_n \in \mathscr{I}$, $n \in \omega$, then there is $A \in \mathscr{I}$ such that $A_n \subseteq^* A$ for every n.

As additional set-theoretic assumptions we will use two instances of Martin's Axiom — Martin's Axiom for countable posets and Martin's Axiom for σ-centered posets which is equivalent to the assumption $\mathfrak{p} = \mathfrak{c}$. Let us recall that the *pseudointersection number* $\mathfrak{p}$ is defined by:

$$\mathfrak{p} = \min\{|\mathscr{F}| : \mathscr{F} \subseteq [\omega]^\omega \text{centered}, \neg(\exists A \in [\omega]^\omega)(\forall F \in \mathscr{F}) A \subseteq^* F\}$$

2. The existence of $\mathscr{I}$-ultrafilters

For some ideals on ω the existence of $\mathscr{I}$-ultrafilters can be established in ZFC in contrast to the above mentioned result of Shelah. We shall recall that the *character of* $\mathscr{I}$, $\chi(\mathscr{I})$, is the minimal cardinality of a base for $\mathscr{I}$, i.e.

$$\chi(\mathscr{I}) = \min\{|\mathcal{B}| : \mathcal{B} \subseteq \mathscr{I} \wedge (\forall I \in \mathscr{I})(\exists B \in \mathcal{B})\ I \subseteq^* B\}.$$

Proposition 2.1. *If $\mathscr{I}$ is a maximal ideal on ω such that $\chi(\mathscr{I}) = \mathfrak{c}$ then $\mathscr{I}$-ultrafilters exist.*

Proof. It is an immediate consequence of Theorem in [5]. □

There are of course many interesting ideals on ω to which we cannot apply Proposition 2.1. It seems that in general some additional set-theoretic assumptions are again necessary to construct the corresponding $\mathscr{I}$-ultrafilters. The next proposition states that for some ideals there are no $\mathscr{I}$-ultrafilters at all.

Proposition 2.2. *If $\mathscr{I}$ is not tall then $\mathscr{I}$-ultrafilters do not exist.*

Proof. Suppose that for $A \in [\omega]^\omega \setminus \mathscr{I}$ we have $\mathscr{I} \cap \mathscr{P}(A) = [A]^{<\omega}$ and let $e_A : \omega \to A$ be an increasing enumeration of the set A.

Now assume for the contrary that there exists an $\mathscr{I}$-ultrafilter $\mathscr{U} \in \omega^*$. According to the definition of an $\mathscr{I}$-ultrafilter there exists

$U \in \mathscr{U}$ such that $e_A[U] \in \mathscr{I}$. Since $e_A[U] \subseteq A$ the set $e_A[U]$ is finite. It follows that U is finite because e_A is one-to-one — a contradiction to the assumption that $\mathscr{U}$ is a free ultrafilter. □

It turns out that under Martin's Axiom for σ-centered posets the necessary condition from Proposition 2.2 is also sufficient.

Proposition 2.3. *($\mathfrak{p} = \mathfrak{c}$) If $\mathscr{I}$ is tall then $\mathscr{I}$-ultrafilters exist.*

Proof. Enumerate all functions from ω to ω as $\{f_\alpha : \alpha < \mathfrak{c}\}$. By transfinite induction on $\alpha < \mathfrak{c}$ we will construct filter bases $\mathscr{F}_\alpha$ satisfying:

(i) $\mathscr{F}_0$ is the Fréchet filter
(ii) $\mathscr{F}_\alpha \subseteq \mathscr{F}_\beta$ whenever $\alpha \leq \beta$
(iii) $\mathscr{F}_\gamma = \bigcup_{\alpha<\gamma} \mathscr{F}_\alpha$ for γ limit
(iv) $(\forall \alpha)$ $|\mathscr{F}_\alpha| \leq |\alpha| \cdot \omega$
(v) $(\forall \alpha)$ $(\exists F \in \mathscr{F}_{\alpha+1})$ $f_\alpha[F] \in \mathscr{I}$

Suppose we already know $\mathscr{F}_\alpha$. If there is a set $F \in \mathscr{F}_\alpha$ such that $f_\alpha[F] \in \mathscr{I}$ then put $\mathscr{F}_{\alpha+1} = \mathscr{F}_\alpha$. Hence we may assume that $f_\alpha[F] \notin \mathscr{I}$, in particular $f_\alpha[F]$ is infinite, for every $F \in \mathscr{F}_\alpha$.

Since $|\mathscr{F}_\alpha| < \mathfrak{c} = \mathfrak{p}$ there exists $M \in [\omega]^\omega$ such that $M \subseteq^* f_\alpha[F]$ for every $F \in \mathscr{F}_\alpha$. The ideal $\mathscr{I}$ is tall, so there is $A \in \mathscr{I}$ which is an infinite subset of M, hence $A \subseteq^* f_\alpha[F]$, in particular $f_\alpha^{-1}[A] \cap F$ is infinite for every $F \in \mathscr{F}_\alpha$. It follows that $f_\alpha^{-1}[A]$ is compatible with $\mathscr{F}_\alpha$. To complete the induction step let $\mathscr{F}_{\alpha+1}$ be the filter base generated by $\mathscr{F}_\alpha$ and $f_\alpha^{-1}[A]$.

It is easy to see that every ultrafilter that extends the filter base $\mathscr{F} = \bigcup_{\alpha<\mathfrak{c}} \mathscr{F}_\alpha$ is an $\mathscr{I}$-ultrafilter. □

3. $\mathscr{I}$-ULTRAFILTERS AND P-POINTS

A free ultrafilter $\mathscr{U}$ is called a *P-point* if for all partitions of ω, $\{R_i : i \in \omega\}$, either for some i, $R_i \in \mathscr{U}$, or $(\exists U \in \mathscr{U})$ $(\forall i \in \omega)$ $|U \cap R_i| < \omega$.

There exist two characterizations of P-points as $\mathscr{I}$-ultrafilters: If $X = 2^\omega$ then P-points are precisely the $\mathscr{I}$-ultrafilters for the family $\mathscr{I}$ consisting of all finite and converging sequences; if $X = \omega_1$ then P-points are precisely the $\mathscr{I}$-ultrafilters for $\mathscr{I} = \{A \subseteq \omega_1 : A \text{ has order type } \leq \omega\}$ (see [2]).

Is there an ideal $\mathscr{I} \subseteq \mathscr{P}(\omega)$ such that P-points are precisely the $\mathscr{I}$-ultrafilters? In the next two propositions we prove (under additional set-theoretic assumptions) that such an ideal can be neither an F_σ-ideal nor a P-ideal.

The following description of F_σ-ideals is due to Mazur [9]: For every F_σ-ideal $\mathscr{I}$ there exists a lower semicontinuous submeasure $\varphi : \mathscr{P}(\omega) \to [0, \infty]$ such that $\mathscr{I} = \mathrm{Fin}(\varphi) = \{A \subseteq \omega : \varphi(A) < \infty\}$. Remember that a submeasure φ is called *lower semicontinuous* (*lsc* in short) if $\varphi(A) = \lim\limits_{n\to\infty} \varphi(A \cap n)$.

Theorem 3.1. *(MA_{ctble}) For every F_σ-ideal $\mathscr{I} \subseteq \mathscr{P}(\omega)$ there exists a P-point that is not an $\mathscr{I}$-ultrafilter.*

Proof. Let φ be the lsc submeasure for which $\mathscr{I} = \mathrm{Fin}(\varphi)$. Enumerate all partitions of ω (into infinite sets) as $\{\mathscr{R}_\alpha : \alpha < \mathfrak{c}\}$. By transfinite induction on $\alpha < \mathfrak{c}$ we will construct filter bases $\mathscr{F}_\alpha$, $\alpha < \mathfrak{c}$, so that the following conditions are satisfied:

(i) $\mathscr{F}_0$ is the Fréchet filter
(ii) $\mathscr{F}_\alpha \subseteq \mathscr{F}_\beta$ whenever $\alpha \le \beta$
(iii) $\mathscr{F}_\gamma = \bigcup_{\alpha<\gamma} \mathscr{F}_\alpha$ for γ limit
(iv) $(\forall\alpha)\ |\mathscr{F}_\alpha| \le |\alpha| \cdot \omega$
(v) $(\forall\alpha)\ (\forall F \in \mathscr{F}_\alpha)\ \varphi(F) = \infty$
(vi) $(\forall\alpha)\ (\exists F \in \mathscr{F}_{\alpha+1})$ either $(\exists R \in \mathscr{R}_\alpha)\ F \subseteq R$
or $(\forall R \in \mathscr{R}_\alpha)\ |F \cap R| < \omega$

Assume we already know $\mathscr{F}_\alpha$ and we should define $\mathscr{F}_{\alpha+1}$.

<u>Case A.</u> $(\exists R \in \mathscr{R}_\alpha)\ (\forall F \in \mathscr{F}_\alpha)\ \varphi(F \cap R) = \infty$

Let $\mathscr{F}_{\alpha+1}$ be the filter base generated by $\mathscr{F}_\alpha$ and such a set R.

<u>Case B.</u> $(\forall R \in \mathscr{R}_\alpha)\ (\exists F \in \mathscr{F}_\alpha)\ \varphi(F \cap R) < \infty$

Enumerate $\mathscr{R}_\alpha$ as $\{R_n : n \in \omega\}$. The assumption of Case B. implies that $(\forall K \in [\omega]^{<\omega})\ (\exists F_K \in \mathscr{F}_\alpha)\ \varphi(F_K \cap \bigcup_{n\in K} R_n) < \infty$.

Consider $P = \{\langle K, n\rangle \in [\omega]^{<\omega} \times \omega : K \subseteq \bigcup_{i\le n} R_i,\ K \cap R_n \neq \emptyset\}$ and define order $\le_P$ by $\langle K, n\rangle \le_P \langle L, m\rangle$ if $\langle K, n\rangle = \langle L, m\rangle$ or $K \supset L$, $\min(K \setminus L) > \max L$, $n > m$ and $(K \setminus L) \cap \bigcup_{i\le m} R_i = \emptyset$. Obviously, $(P, \le_P)$ is a countable poset. Now, for $F \in \mathscr{F}_\alpha$ and $k, j \in \omega$ define $D_{F,k} = \{\langle K, n\rangle \in P : \varphi(K \cap F) \ge k\}$ and $D_j = \{\langle K, n\rangle \in P : n \ge j\}$.

Claim: $D_{F,k}$ is dense in $(P, \le_P)$ for every $F \in \mathscr{F}_\alpha$ and $k \in \omega$; D_j is dense in $(P, \le_P)$ for every $j \in \omega$.

Proof of the claim. Consider $\langle L, m\rangle \in P$ arbitrary. Since L is finite there exists $p \geq m$ such that $[0, \max L] \subseteq \bigcup_{i\leq p} R_i$. According to the assumption there is $F_p \in \mathscr{F}_\alpha$ such that $\varphi(F_p \cap \bigcup_{i\leq p} R_i) < \infty$. It follows that $\varphi((F_p \cap F) \setminus \bigcup_{i\leq p} R_i) = \infty$. We can choose a finite set $L' \subseteq (F_p \cap F) \setminus \bigcup_{i\leq p} R_i$ such that $\varphi(L') \geq k$ because φ is lower semicontinuous. Let $n = \max\{i : L' \cap R_i \neq \emptyset\}$ and $K = L \cup L'$. Note that the choice of p implies $\min L' > \max L$. It follows that $\langle K, n\rangle \leq_P \langle L, m\rangle$ and $\langle K, n\rangle \in D_{F,k}$. So $D_{F,k}$ is dense. For $j \leq m$ we have $\langle L, m\rangle \in D_j$ and for any $j > m$ we can choose arbitrary $r \in R_j$ such that $r > \max L$. Let $K' = L \cup \{r\}$. Of course, $\langle K', j\rangle \leq_P \langle L, m\rangle$ and $\langle K', j\rangle \in D_j$. So D_j is dense. □

The family $\mathscr{D} = \{D_{F,k} : F \in \mathscr{F}_\alpha, k \in \omega\} \cup \{D_j : j \in \omega\}$ consists of dense subsets in P and $|\mathscr{D}| < \mathfrak{c}$. Therefore there is a $\mathscr{D}$-generic filter $\mathscr{G}$. Let $U = \bigcup\{K : (\exists n)\langle K, n\rangle \in \mathscr{G}\}$. It remains to check that:

- $(\forall F \in \mathscr{F}_\alpha)\ \varphi(U \cap F) = \infty$

Take $k \in \omega$ arbitrary. For every $\langle K, n\rangle \in \mathscr{G} \cap D_{F,k}$ we have $U \supseteq K$ and $k \leq \varphi(K \cap F) \leq \varphi(U \cap F)$ (submeasure φ is monotone). Hence $\varphi(U \cap F) = \infty$.

- $(\forall R_n \in \mathscr{R}_\alpha)\ |U \cap R_n| < \omega$

Fix $\langle K_n, j_n\rangle \in \mathscr{G} \cap D_n$. Observe that $j_n \geq n$ and for $\langle K, m\rangle \in \mathscr{G}$ we have $K \cap R_n = \emptyset$ if $m < n$ and that $K \cap R_n = K_n \cap R_n$ if $m \geq n$. To see the latter consider $\langle L, m'\rangle \in \mathscr{G}$ such that $\langle L, m'\rangle \leq_P \langle K, m\rangle$ and $\langle L, m'\rangle \leq_P \langle K_n, j_n\rangle$ (such a condition exists because $\mathscr{G}$ is a filter) for which we get $L \cap R_n = K \cap R_n$ and $L \cap R_n = K_n \cap R_n$. It follows that $U \cap R_n = K_n \cap R_n$ is finite.

To complete the induction step let $\mathscr{F}_{\alpha+1}$ be the filter base generated by $\mathscr{F}_\alpha$ and the set U.

It follows from condition (vi) that every ultrafilter which extends the filter base $\mathscr{F} = \bigcup_{\alpha<\mathfrak{c}} \mathscr{F}_\alpha$ is a P-point. Because of condition (v) there exists an ultrafilter extending $\mathscr{F}$ which extends also the dual filter to $\mathrm{Fin}(\varphi) = \mathscr{I}$, in particular it is not an $\mathscr{I}$-ultrafilter. □

Theorem 3.2. *($\mathfrak{p}=\mathfrak{c}$) If $\mathscr{I}$ is a tall P-ideal on ω then there is an $\mathscr{I}$-ultrafilter which is not a P-point.*

Proof. It was proved in Proposition 2.3 that assuming $\mathfrak{p} = \mathfrak{c}$ there exist $\mathscr{I}$-ultrafilters for every tall ideal $\mathscr{I}$. We will show that if $\mathscr{I}$ is a tall P-ideal then the square of an $\mathscr{I}$-ultrafilter is again an $\mathscr{I}$-ultrafilter and it is not a P-point.

So let us first recall the definition of the product of ultrafilters (see [6]): If $\mathscr{U}$ and $\mathscr{V}$ are ultrafilters on ω then $\mathscr{U} \cdot \mathscr{V} = \{A \subseteq \omega \times \omega : \{n : \{m : \langle n, m\rangle \in A\} \in \mathscr{V}\} \in \mathscr{U}\}$ is an ultrafilter on $\omega \times \omega$ which is isomorphic to (and can be identified with) an ultrafilter on ω. By the square of ultrafilter $\mathscr{U}$ we mean the ultrafilter $\mathscr{U} \cdot \mathscr{U}$.

Notice that the partition $\{\{n\} \times \omega : n \in \omega\}$ of $\omega \times \omega$ witnesses the fact that no product of free ultrafilters on ω is a P-point. Hence to complete the proof it remains to check that if $\mathscr{U}$ is an $\mathscr{I}$-ultrafilter then $\mathscr{U} \cdot \mathscr{U}$ is again an $\mathscr{I}$-ultrafilter, i. e. for every $f : \omega \times \omega \to \omega$ there is $U \in \mathscr{U} \cdot \mathscr{U}$ such that $f[U] \in \mathscr{I}$.

To this end define for arbitrary function $f : \omega \times \omega \to \omega$ and for every $n \in \omega$ a function $f_n : \omega \to \omega$ by $f_n(m) = f(\langle n, m\rangle)$. If $\mathscr{U}$ is an $\mathscr{I}$-ultrafilter then there exists $V_n \in \mathscr{U}$ such that $f_n[V_n] \in \mathscr{I}$ for every n. Now we can find a set $A \in \mathscr{I}$ such that $f_n[V_n] \subseteq^* A$ for every n because $\mathscr{I}$ is a P-ideal. It is obvious that $f_n^{-1}[f_n[V_n]] \in \mathscr{U}$ for every $n \in \omega$. Hence either $f_n^{-1}[f_n[V_n] \cap A]$ or $f_n^{-1}[f_n[V_n] \setminus A]$ belongs to $\mathscr{U}$. Let $I_0 = \{n \in \omega : f_n^{-1}[f_n[V_n] \cap A] \in \mathscr{U}\}$ and $I_1 = \{n \in \omega : f_n^{-1}[f_n[V_n] \setminus A] \in \mathscr{U}\}$. Since $\mathscr{U}$ is an ultrafilter it contains one of the sets I_0, I_1.

Case A. $I_0 \in \mathscr{U}$

Put $U = \{\{n\} \times f_n^{-1}[f_n[V_n] \cap A] : n \in I_0\}$. It is easy to see that $U \in \mathscr{U} \cdot \mathscr{U}$ and $f[U] = \bigcup_{n \in I_0} f_n[V_n] \cap A \subseteq A \in \mathscr{I}$.

Case B. $I_1 \in \mathscr{U}$

Since $f_n[V_n] \setminus A$ is finite and $\mathscr{U}$ is an ultrafilter, there exists $k_n \in f_n[V_n] \setminus A$ such that $f_n^{-1}\{k_n\} \in \mathscr{U}$ for every $n \in I_1$. Fix arbitrary $g : \omega \to \omega$ such that $g(n) = k_n$ for each $n \in I_1$. Since $\mathscr{U}$ is an $\mathscr{I}$-ultrafilter there exists $V \in \mathscr{U}$ such that $g[V] \in \mathscr{I}$. Now put $U = \{\{n\} \times f_n^{-1}\{k_n\} : n \in I_1 \cap V\}$. It is easy to check that $U \in \mathscr{U} \cdot \mathscr{U}$ and $f[U] \subseteq g[V] \in \mathscr{I}$. □

For ideals which are neither (analytic) P-ideals nor F_σ-ideals there is no 'nice' description. So it is rather difficult to prove any general statements about $\mathscr{I}$-ultrafilters and P-points in this case. We will conclude by one example of such an ideal and show that it cannot be used to characterize P-points via the corresponding $\mathscr{I}$-ultrafilters.

Definition 3.3. A set $A \in [\omega]^\omega$ with an (increasing) enumeration $A = \{a_n : n \in \mathbb{N}\}$ is called *thin* (see [3]) if $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = 0$.

Obviously, thin sets do not form an ideal (consider for example the sets $\{n! : n \in \omega\}$ and $\{n! + 1 : n \in \omega\}$), but they generate an ideal which we denote by $\mathscr{T}$. We refer to $\mathscr{T}$-ultrafilters as thin ultrafilters. Borel complexity of the ideal $\mathscr{T}$ is $F_{\sigma\delta\sigma}$ since $\mathscr{T} = \bigcup_{r\in\mathbb{N}}\bigcap_{k\in\mathbb{N}}\bigcup_{m\in\mathbb{N}}\bigcap_{n\geq m}\{A \subseteq \omega : \frac{a_n}{a_{n+r}} < \frac{1}{k}\}$ and the ideal $\mathscr{T}$ is not a P-ideal (if $A_k = \{n! + k : n \in \omega\}$ then there is no set $A \in \mathscr{T}$ such that $A_k \subseteq^* A$ for each $k \in \omega$). Thus Theorems 3.1 and 3.2 do not apply to thin ultrafilters. However, assuming Martin's Axiom for countable posets it is possible to prove that thin ultrafilters and P-points do not coincide.

Theorem 3.4. *(MA_{ctble})*

(1) *There is a P-point that is not a thin ultrafilter.*
(2) *There exists a thin ultrafilter which is not a P-point.*

Proof. The ideal generated by thin sets is a part of the F_σ-ideal $\mathcal{I}_{1/n} = \{A \subseteq \mathbb{N} : \sum_{n\in A}\frac{1}{n} < \infty\}$. Statement (1) follows from the obvious fact that for $\mathscr{I} \subseteq \mathscr{J}$ every $\mathscr{I}$-ultrafilter is a $\mathscr{J}$-ultrafiler and from Theorem 3.1.

As for (2), a thin ultrafilter which is not a P-point was constructed in [8] assuming Martin's Axiom for countable posets (published in [7] as Proposition 4 assuming Continuum Hypothesis). □

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