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COMPUTERISED MUSCLE MODELLING

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Abstract

Musculoskeletal modelling has emerged as a powerful tool for simulating complex human movements. The dissertation focuses on using the Radial Basis Function (RBF) approximation technique to enhance the precision of such models. The aim is to demonstrate the efficacy of RBF in capturing the shape and motion of a muscle model by considering a broader spectrum of features in the input function. RBF can also be used to reconstruct surfaces from sets of attachment points in the context of muscle attachment estimation.

The doctoral dissertation aims to describe the current state-of-the-art muscle modelling field, describing all of the currently known approaches. The dissertation then transitions to propose a novel RBF mathematical model to describe a muscle using a set of RBF, which introduces a technique of finding an optimal centre point using multiple groups, such as vertices at borders, local extrema, points of inflexion, strategic pseudorandom positions, greedy MSE placement, and more, finding suitable shape parameters, and, ultimately, describing the mathematical model for the movement of the geometry. Therefore, the outcome of the dissertation is a mathematical model for the dynamic muscle model

Keywords

Muscle modelling, approximation, Radial Basis Functions, RBF, centre placement, curvature, Position-Based Dynamics, PBD, As-rigid-as-possible, ARAP, interpolation, Via-points, Mass-spring systems, Finite Element Method

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Chapter 1

Introduction

The dissertation explores a new muscle representation using Radial Basis Function (RBF) approximation for more precise biomechanics modelling, crossing computer science, mathematics, and biomechanics. It addresses the challenge of accurately simulating muscle movements and shapes, a gap in current methods due to human anatomy's complexity and dynamics. RBF is proposed for its flexibility in modelling complex shapes, aiming to enhance musculoskeletal models used in medical diagnostics, prosthetics design, sports science, and animation.

The research critiques existing models, introduces RBF-based muscle modelling with theoretical and practical insights, and discusses its implications through computational simulations. It combines empirical visualisation of muscle movement, generalisation of muscle structures into models, and analytical approaches to develop a novel muscle model using RBF. This work aims to advance musculoskeletal modelling, improving applications in medical research, ergonomic design, and interdisciplinary fields by merging biomechanics with computational techniques.

Chapter 2

Acquiring data

This section outlines data acquisition for computer muscle modelling, distinguishing between personalised data for specific individual applications and general data, which lacks customisation due to variability in muscle attachments ([60], [39], [61]). It stresses the need for simplifications in musculoskeletal modelling due to its complexity, focusing on muscle-tendon units (MTUs) approximated by triangular meshes and the importance of bone attachment knowledge. Challenges in model resolution and acquisition are also discussed.

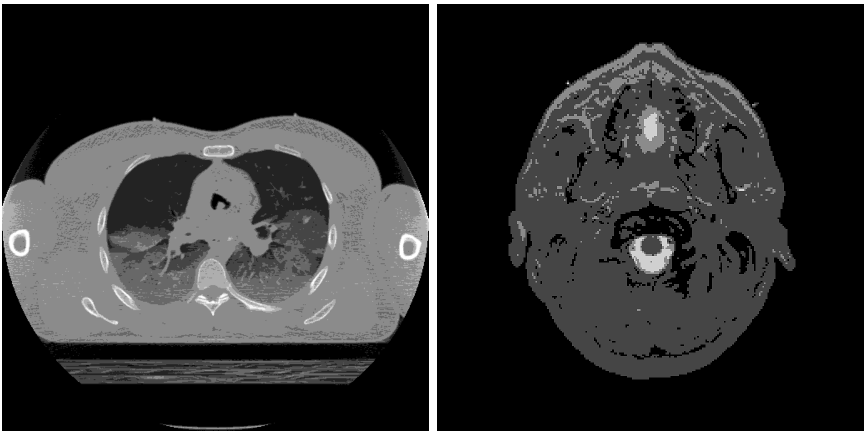

Figure 2.1: CT and MRI result. CT data [54] on the left, MRI data on the right.

Identifying muscle-bone attachments involves defining attachment areas, possibly through automatic/manual methods or additional data. Two strategies exist: 1) choosing muscle model points near or intersecting the bone premovement, or 2) users defining the area with specific points. The first approach, more straightforward, risks misalignment, especially near joints. The second, more reliable, demands mapping the entire area from provided points. Our study faced challenges in accurately reconstructing complex muscle attachments, with the best curve reconstruction algorithm achieving only 78.74%

accuracy [43]. We also explored surface reconstruction via Radial Basis Functions (RBF), showing promise for intricate boundaries, see section 3.1.

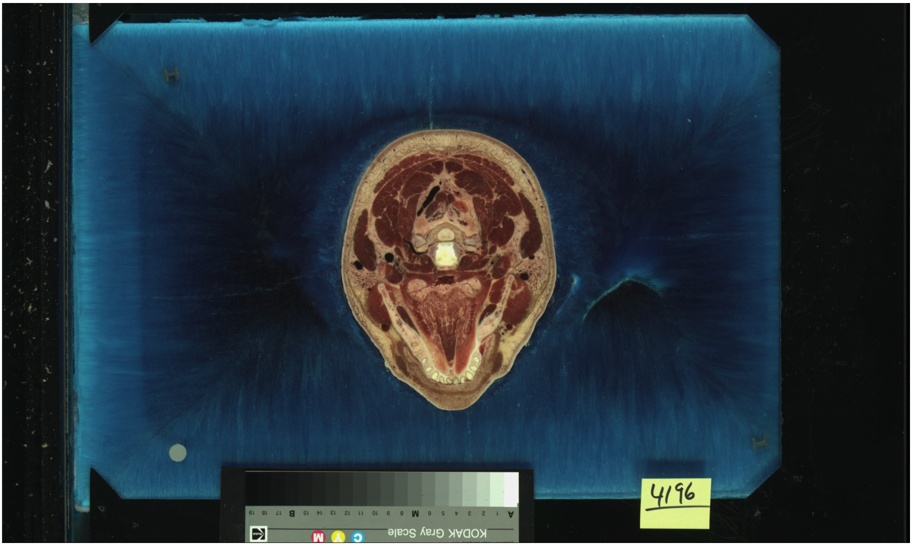

Figure 2.2: The detailed cryosection image of the Visible Human Male [70].

Non-invasive methods like CT, MRI, and PET are essential for muscle modelling, highlighting the benefits of 3D modelling and improved soft tissue visibility. Despite their advantages, these methods fail to identify muscle attachment areas accurately, necessitating personalised models or probabilistic estimates by experts [24]. In contrast, invasive techniques, such as dissection and cadaver studies, offer detailed anatomical insights crucial for muscle modelling, including muscle-tendon elements and pennate angles, albeit limited ethically to non-living subjects. These methods complement each other in developing precise musculoskeletal models despite their inherent limitations [54, 8].

Chapter 3

Estimation approaches

Estimation techniques, crucial for smooth muscle modelling even with sparse data, focus on muscles, bones, and attachment areas, assuming known model borders. Bézier curves, initially used by Delp et al. [18], faced challenges with fibre intersections, leading to Kohout and Cholt's [45] adoption of Catmull-Rom splines for smoother, intersection-free modelling.

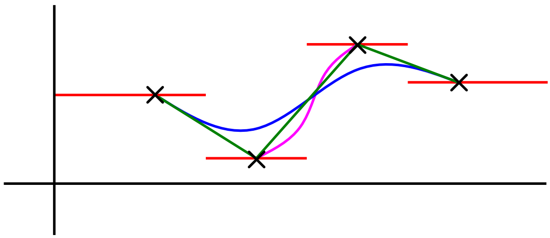

Figure 3.1: Constant (red), piecewise linear (green), Bézier (blue) and Catmull-Rom (pink) interpolation of a set of points.

Radial basis function (RBF) estimation, highlighted for its smoothness potential by Hardy [30] and applied in muscle modelling for attachment areas and overall shape in our research [43], offers a superior alternative with Gaussian RBFs enabling C^∞ smoothness. In the dissertation, the chapter discusses the applicability of these techniques across dimensions for muscle modelling, emphasising the transition from polynomial to RBF-based methods for enhanced model fidelity; however, for the self-report, only RBF, which is the most relevant technique, is further discussed.

3.1 Radial basis functions

The challenge of interpolating and approximating scattered data is prevalent in various engineering and research domains. It is exemplified by the work of Oliver et al. [59], who applied the kriging interpolation method to geographical data. Similarly, Kaymaz [40] demonstrated the utility of this approach in addressing structural reliability issues. The technique is also used in modelling, as shown by Sakata et al. [63] in their work on wing structures and in the creation of metamodels by Joseph et al. [38]. Furthermore, Radial Basis Function (RBF) methods are applicable in solving partial differential equations, particularly in engineering-related problems.

The RBF methodology, initially introduced by Hardy [31] and later refined [29], has seen continuous development and modification over time. Majdisova et al. [51] have contributed by proposing various placement methods within this framework. There has also been significant research into the behaviour of *shape parameters* (described further in the text) in RBF methods. It includes efforts by Wang et al. [76] to find optimal parameters, explorations by Afiatdoust et al. [1] into this area, and the use of different local shape parameters as investigated by Cohen et al. [17], Sarra et al. [64], and Skala et al. [69]. The mathematical formulation of the RBF approach is presented as follows:

$$f(\mathbf{x}) = \sum_{i=1}^{N} \lambda_i \varphi\left(||\mathbf{x} - \mathbf{P_i}||\right) \tag{3.1}$$

The RBF approach is a linear combination of a set of RBFs denoted as φ, with centres at points $\mathbf{P_i}$. These RBFs are adjusted to interpolate the vertices $\mathbf{P_i}$ using appropriate weights λ_i. This equation can be viewed as a linear system equation $\mathbf{Ax} = \mathbf{b}$, where the matrix $\mathbf{A}$ is composed of values of the RBF function, λ_i represents an unknown vector, and $f(\mathbf{x})$ will be populated with known values at each of the input points.

A reduced number of RBFs can be used to achieve an approximation, making the problem overdetermined and less straightforward. It can then be solved, for instance, using the ordinary least squares (OLS) approach while also considering some associated drawbacks, as mentioned in works such as Skala and Kansa [68].

$$f(\mathbf{x}) = \sum_{i=1}^{M} \lambda_i \varphi\left(||\mathbf{x} - \xi_{\mathbf{i}}||\right) \tag{3.2}$$

3.1.1 Available functions

A radial basis function is defined by its dependency on the distance from a central point. This function can be applied in spaces of any dimension, but it is commonly expressed in terms of the distance (denoted as r) rather than as a function of a spatial coordinate.

A broad range of RBF functions are generally categorised into local and global. Local RBFs have a characteristic feature where their value becomes zero beyond a certain distance, effectively limiting their influence. In contrast, global RBFs lack this restriction and maintain influence regardless of distance. This discussion will initially focus more on the characteristics and applications of local RBFs.

Local functions

The local RBF functions are limited by a sphere of influence, and beyond this sphere, it evaluates to zero. Some RBFs with those properties are listed in Table 3.1.

ID	Function	ID	Function
1	$(1-r)_+$	2	$(1-r)_+^3(3r+1)$
3	$(1-r)_+^5(8r^2+5r+1)$	4	$(1-r)_+^2$
5	$(1-r)_+^4(4r+1)$	6	$(1-r)_+^6(35r^2+18r+3)$
7	$(1-r)_+^8(32r^3+25r^2+8r+3)$	8	$(1-r)_+^3$
9	$(1-r)_+^3(5r+1)$	10	$(1-r)_+^7(16r^2+7r+1)$

Table 3.1: Typical examples of local RBF functions - compactly-supported RBF. The "+" sign means that every nonpositive value is set to zero instead. [67].

Their main advantage is computational. Due to their small influence, they produce a better conditioned linear equation system (LES). Suppose the order of vertices reflects the spatial position of the original vertices. In that case, the LES becomes diagonally dominant, which is particularly useful for most solvers (Gaussian elimination, Jacobi method, LU decomposition method and more) to obtain more accurate results.

Global functions

The scope of the global RBFs is not limited, so their influence can be infinite. It is particularly advantageous in situations where, for example, an RBF centre point change should influence the whole space. Table 3.2 lists some of the most notable ones.

The advantage of the global function over the local one is that the single change of a single parameter (one centre point, for example) would influence the whole space. The disadvantage of some is the presence of a shape parameter α. The parameters define the variance (indirectly) of the function. The higher the

Name	Function
Gaussian RBF	$e^{-\alpha r}$
Thin-plate spline	$r^2 \log r$
An RBF proposed in [10]	$r^2 (r^\alpha - 1)$

Table 3.2: Some of the most notable global RBFs.

value, the higher the variance and the further the influence. However, there is also an inverse proportion between the shape parameter and the computational stability of the LES. To take it to the extreme, the tiny shape parameter would lead to nonzero values only for a vertex with itself, producing a very much solvable LES. On the other hand, a nearly infinite shape parameter would lead to a situation where no matter how far apart two vertices would be, the value would stay the same, leading to the limit to a singular and constant matrix.

3.1.2 Polynomial extension

Consider the scenario where we must approximate a nonzero constant function using the RBF approach. Without a polynomial extension, accurately approximating such a function would be challenging, as the centre points would need to be distributed uniformly throughout the entire function domain. Therefore, a polynomial extension is introduced to handle functions that exhibit constant- and polynomial-like behaviour.

The extension involves incorporating a polynomial approximation into the RBF equation, expanding the matrix by the same number of rows and columns as the degree of the polynomial used. In the case of a function with a degree of $2\frac{1}{2}D$, this is achieved as follows:

$$
\begin{bmatrix}
\varphi_{11} & \dots & \varphi_{1N} & 1 & x_1 & y_1 \\
\vdots & \ddots & \vdots & 1 & \vdots & \vdots \\
\varphi_{N1} & \dots & \varphi_{NN} & 1 & x_N & y_N \\
1 & 1 & 1 & 0 & 0 & 0 \\
x_1 & \dots & x_N & 0 & 0 & 0 \\
y_1 & \dots & y_N & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\lambda_1 \\ \vdots \\ \lambda_N \\ a_0 \\ a_1 \\ a_2
\end{bmatrix}
=
\begin{bmatrix}
f_1 \\ \vdots \\ f_N \\ 0 \\ 0 \\ 0
\end{bmatrix}
\tag{3.3}
$$

Here, φ_{ij} represents the value of the Radial Basis Function (RBF) based on the distance between the vertex i and the vertex j, while x_i and y_i denote the coordinates of the centre points. The coefficients include a_0 as the constant coefficient and a_1, a_2 as the linear coefficients of the linear expression $a_0 +$

$a_1x + a_2y$. Variables f_i represent the function values of the given vertices, and λ_i means the unknown weight of the RBF to be determined.

This matrix can be expressed more succinctly as follows:

$$\begin{bmatrix} \mathbf{A} & \mathbf{P} \\ \mathbf{P}^{\mathbf{T}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \lambda \\ \mathbf{a} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix} \tag{3.4}$$

In this formulation, **A** represents an RBF submatrix, **P** stands for a polynomial submatrix, **a** is a subvector containing polynomial coefficients and **f** is a subvector containing the values of the interpolated or approximated function.

Usually, filling this equation yields satisfactory results. However, the underlying theoretical issue arises from using different units within the matrix. φ_{ij} represents some form of distance that has passed through the RBF function (with units somewhat unknown in the case of the Gaussian RBF, akin to "exponential metres" (e^m) when all constants are excluded). On the other hand, x_i and y_i are typically measured in units related to the function domain, commonly in metres (m). This amalgamation of distinct units raises theoretical challenges rarely addressed in the literature [48].

3.1.3 Centre point distribution

The fundamental issue at hand revolves around determining the optimal placement of the centre points ξ_i. There are several commonly employed options, each with its own set of advantages and drawbacks.

Iterative greedy search

In many mathematical scenarios, iterative methods often compete with direct methods. The search for centre points is no exception. The most straightforward iterative approach involves initially searching for a single centre point, which reduces the discrepancy between the approximated function and the approximated one. Subsequently, more centres are sought within the space defined by the difference between these two functions to minimise this difference as much as possible. This iterative process continues until the overall error reaches an acceptable level or the available number of centre points is exhausted.

The primary advantage of this method is that the approximant preserves the essential features of the original function. Furthermore, the shape parameter can be adjusted at each step of the greedy search to achieve a better fit. However, the disadvantage lies in its computational complexity, as each "guess" entails solving a potentially significant linear equation and the summation of

Radial Basis Functions (RBFs). Some RBFs can be computationally expensive to evaluate.

Grid distribution

Grid centre point distribution entails arranging all centre points in a grid format, which can take various forms, such as regular, cartesian, rectilinear, or curvilinear grids. Although the grid setup is straightforward, it presents significant challenges. The most prominent challenge is the potential for poor conditioning of the RBF matrix [11] if the shape parameter is not carefully selected. Another drawback is that it does not capture the intrinsic features of the interpolated or approximated function.

Halton distribution

The Halton distribution [28] represents a quasi-random point distribution. Its original version is one-dimensional and operates within the interval $(0,1)$ (scalable by a constant). The sequence's elements are defined as:

$$\text{Halton}_k\left(p\right) = \sum_{i=0}^{\lfloor \log_p k \rfloor} \frac{1}{p^{i+1}} \left(\left\lfloor \frac{k}{p^i} \right\rfloor \bmod p \right) \tag{3.5}$$

Here, p is an arbitrary prime number, and k represents the index of the sequence element. One can achieve this in the case of multidimensional sequences by selecting multiple prime numbers, resulting in a vector of Halton sequences with different prime values for each dimension. For example, the Halton sequence $[2,3]$ begins with $\left[\left[\frac{1}{2},\frac{1}{3}\right],\left[\frac{1}{4},\frac{2}{3}\right],\left[\frac{3}{4},\frac{1}{9}\right]\right]$. Essentially, it partitions the $(0,1)$ interval into p subintervals of equal size and outputs the boundary points. Subsequently, each subspace is subdivided, yielding additional boundary points. A breadth-first approach is employed for traversal, resulting in a more evenly spread distribution than a depth-first (recursive) approach.

The same sequence can be generated by expressing k in base p, inverting it, and placing it after the decimal point. For example, when $p = 2$:

$$0.1_2 = \frac{1}{2}, 0.01_2 = \frac{1}{4}, 0.11_2 = \frac{3}{4}, 0.001_2 = \frac{1}{8}, 0.101_2, 0.011_2, 0.111_2 ... \tag{3.6}$$

The Halton sequence offers a significant advantage by exhibiting fewer regularities, which helps avoid ill-conditioned matrices and ensures excellent interval coverage. However, a notable drawback is that the endpoints (i.e., 0 and 1) must be explicitly included in most cases, as they are not part of the sequence by default.

Distribution concerning the original function

An entirely justifiable approach is to select the centre points based on specific features of the original function. The likely candidates are the minima and maxima since RBFs often exhibit extrema at the centre points. Additionally, inflexion points can be included to cover a broader range of features.

We have previously addressed this issue in a different context in [12], which resulted in the "sophisticated placement of radial basis functions significantly improving the quality of the RBF approximation" [12]. The primary takeaway from this research was to minimise the use of grid or equidistant centre point distributions whenever possible.

3.1.4 RBFs for muscle modelling

Our in-depth studies of RBF [10, 12, 11, 66, 73, 67] have revealed its potential for approximation purposes. To my knowledge, RBF approximation methods have not been widely applied in muscle modelling. We have contributed to the use of the RBF approach in muscle modelling in the article titled "Nonplanar Surface Shape Reconstruction from a Point Cloud in the Context of Muscle Attachment Estimation" [43], where RBF is used to reconstruct surfaces from sets of attachment points.

Chapter 4

Related work

Human movement and muscle modelling methods vary, focusing here on one-dimensional models and inverse kinematics, which requires knowing the desired motion outcome. Dereshgi et al. [19] and Lee et al. [47] provide critical reviews, though the dissertation delves into fewer methods and notes the overestimated understanding of muscle properties. It introduces the Hill-type muscle model, using lines or curves to mimic muscle fibres.

4.1 Hill-Type Model

Hill-type muscle models, conceptualised by Hill in 1938 through frog muscle studies [34], represent muscles using a series-parallel arrangement of elements to describe contraction dynamics. Over time, enhancements like the parallel element [78] and viscous damping [37] have been integrated, evolving into the modern Hill-type models. Despite its foundational role, the Hill-type model's simplifications limit its accuracy for internal muscle force calculations, particularly in complex, multidimensional muscle structures like the pelvic floor [52]. Consequently, newer musculoskeletal modelling techniques have moved beyond the pure Hill-type model, adopting its principles within more sophisticated frameworks that either refine parameter estimation or simulate muscle behaviour with reduced complexity [55], [72].

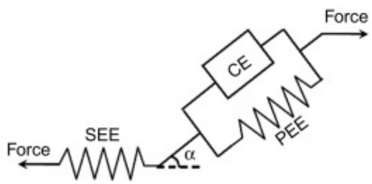

Figure 4.1: Hill-type model of a muscle fiber [3]. PEE = parallel element, SEE = serial element, CE = contractile element, α - pennate angle.

4.2 Via-points

A via-points approach works with a predefined set of points defining the muscle fibre model. There are many options for defining these points. The most common ones are points directly fixed to a bone, so whenever the bone moves, the point moves accordingly. The second option is that the point is present only if a condition is met (for example, if the joint flexion angle is more significant than x), so the natural shape of the fibre model is partially restored. The third option is a point that may move depending on some state (for example, depending on some angle, typically between two bones), following a predefined curve (see Fig. 4.2).

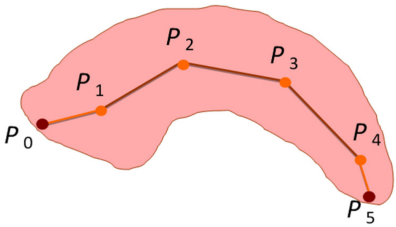

Figure 4.2: Via-points control curve inside a muscle[27]

There are, however, some catches. The first obvious one lies in the definition of via-points. There must be an approach to define these points because user-defined points will be time-consuming, costly, and subjective for the physician. The second main problem is the intersections of muscle models with the closest adjacent bone model when performing an inverse kinematic movement. Similarly, self-intersections should be handled correctly.

Furthermore, the muscle fibre model will sometimes not be smooth. Modenese stated that the "straight line representation of the muscles surrounding the hip joint was limiting the accuracy of hip contact force predictions" [55]. The better approach might not go through a set of points but "wrap around" a predefined set of geometric objects [25].

4.3 Wrapping Obstacles

The wrapping obstacles approach has been developed to address some problems of the via-points approach.

The improvement lies in the smoothness of the curve. Unlike the via-points method, a curve that wraps around a sphere can be formed smoothly. Figure 4.3 illustrates wrapping around a cylinder.

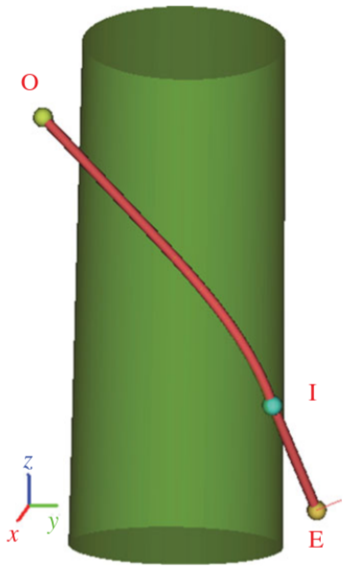

Figure 4.3: Wrapping obstacles approach. A line of action is wrapped around a single cylinder [44].

The main challenge is that infinitely many curves can wrap around a volumetric object. From these curves, one can select the shortest, the least average curvature, the one with maximum curvature, and so on. Selection depends primarily on the specific application, the required precision, the computing power, and other factors. This issue becomes even more complex with extreme bone arrangements.

The wrapping obstacles approach has been used in muscle modelling by researchers such as Lloyd et al. [49] and Kohout et al. [44]. Lloyd et al. wrapped around an arbitrary geometrical model, dividing the curve into segments with knots connected by elastic forces to prevent them from penetrating obstacles. The latter approach, which is significantly older, is limited to spheres, single cylinders, and sphere-capped cylinders. The issue is that some of the previously described problems persist:

- The geometric objects must be specified in the same fashion as the via-points.
- In the case of incorrect curve selections, the intersection problem remains.

4.4 Finite Element Method

The Finite Element Method (FEM) offers a means to achieve precise physical modelling. It has proven effective in tackling various challenges, such as solving problems related to heat transfer, fluid dynamics, and more. An example of its application is seen in the work of George-Ghiocel et al., who used the stress-and-strain approximation approach in Storz coupling within fire hose coupling.

It is essential to begin with the mathematical formulation to grasp the fundamentals, as these problems are often defined by partial differential equations (PDEs).

The finite element method has also been applicable in muscle modelling. This section provides an overview of the relevant approaches in chronological order.

Although multiple FE methods have been published for soft tissue modelling, one of the first papers on musculoskeletal modelling using the FE method is likely from Martins et al. [53], published in 1998. They incorporated the Hill-type model into their FE model, although a coauthor later demonstrated that the Hill-type model may be insufficient in some cases [52]. Their model divided the brachialis muscle into 4050 tetrahedra and assumed constant material properties without considering muscle anisotropy. External forces were applied to an arbitrary part of the muscle ("right end").

Delp and Blemker [18], in 2005, employed a template that was projected onto the target mesh. The projected template was deformed using the finite element method. They used magnetic resonance imaging (MRI) resolution to create the template. They identified the tendon region from MR images to determine boundary conditions, although the complete process was not described in detail.

Boubacker et al. [6] conducted significant work to publish a survey on this topic, providing valuable insights into the problem. However, it should be noted that the study was published in 2006, so some of the information therein may be outdated.

Oberhofer et al. [58], in 2009, used cubic Hermite interpolation functions in their FE approach to ensure C^1 smoothness of the model. They also used the via-point approach (see Section 4.2) for boundary conditions to prevent muscles from penetrating the bones. These via-points were integrated into the FE method, and the objective function included a distance term between landmarks and targets and a Sobolev smoothing constraint.

Kaze et al. [20], in 2017, utilised FE to partition the model using tetrahedra. They derived boundary conditions from anatomical muscle attachment areas and used a mass-spring-like system to simulate tendons (for further details, see Section 4.5). Their primary focus was on estimating the maximal strain.

Wei et al. [77], in 2019, used FE to model a human hand, employing the Nolan hyperelastic soft tissue model [57] to accurately represent human skin. Their study mainly focused on analysing the pressure generated by different hand grips.

Currently, several methods use the pure finite element method for various applications. For example, Fougeron et al. [23] applied FE for the analysis of the load of the upper knee socket. In contrast, using FE, Vila Pouca et al. [74] studied muscle fatigue in the pelvic floor. Sun et al. [71] modelled spine movement using FE.

Despite the capability of the described FE methods to produce high-quality results, they present challenges in setup due to the numerous parameters required [62]. Additionally, FE methods are often computationally demanding. For example, Fougeron et al. [23] reported that their approach ran for 40 minutes on a 2-CPU machine, far from real-time simulation. Consequently, this work only briefly introduces FE methods, which deserve more comprehensive coverage. The primary goal of this research is to explore a faster method, ideally capable of real-time simulation, while maintaining comparable results.

Many other methods for muscle modelling, such as position-based dynamics (PBD), mass-spring system (MSS), and as-rigid-as-possible (ARAP), are currently available and promising alternatives that require fewer computational resources while delivering comparable outcomes.

4.5 Mass-Spring System (MSS)

The Mass-Spring System (MSS) functions as its name implies, simulating complex motion by connecting masses (individual points of mass) with virtual springs. Motion propagation occurs through force transfer across a network of these springs. A relatively straightforward equation describes the spring's behaviour:

$$\mathbf{F}_{ij} = -k\mathbf{d}_{ij} \tag{4.1}$$

Here, $\mathbf{F}_{ij}$ represents the force generated by the spring between the i-th and j-th particles, where the variable k denotes the spring constant (the force required to restore the spring to its original shape per unit of spring extension). The displacement of the spring is represented by the variable $\mathbf{d}_{ij}$.

Although the fundamental concept is not overly complicated, various implementations of this approach exist. A notable implementation comes from Georgii and Westermann [26], who effectively applied this technique to GPU hardware.

Another approach, as presented by Aubel and Thalmann [5], utilises a 1D mass-spring system model for each muscle fibre independently, although with some problems, particularly regarding gaps between muscles. They also introduced angular springs to maintain the angle between the two spring segments. A significant drawback of their work is their somewhat lenient approach to collision detection: "Attempting to handle all muscle-muscle and muscle-bone collisions is unreasonable. In our framework, preventing muscle-bone collisions is easy and fast using repulsive force fields" [5].

A noteworthy contribution in this field is Janak's work [35], which combines the mass-spring system with muscle modelling. Unlike Aubel and Thalmann, Janak's approach operates on muscle fibre models in continuous space. He decomposes the triangular mesh muscle model into a muscle fibre model, uniformly sampling these fibres to obtain nodes (masses), which are then connected by edges (springs). Janak's results show some promise, but a notable challenge lies in his collision detection approach, which approximates the surface mesh with spheres, leading to an "imprecise collision response and partial surface intersection between objects" [35].

Uniform sampling and connection of mass points are depicted in Figure 4.4. It is worth mentioning that the author also considers randomisation and its implications.

4.6 ARAP - As-Rigid-As-Possible Deformation

The acronym ARAP, which stands for As-Rigid-As-Possible, represents a technique for determining minimal non-rigid transformations within a surface mesh. Its primary innovation lies in its ability to operate without requiring an internal structure, distinguishing it from methods like Kelnhofer & Kohout [41] that incorporate volume constraints but require the definition of an internal "skeleton."

ARAP has found application in the medical field, as demonstrated by Fasser et al. [22]. They used ARAP to morph a pelvic bone template into subject-specific landmarks. However, their approach overlooks crucial bone properties, such as volume preservation, which the original ARAP method does not inher-

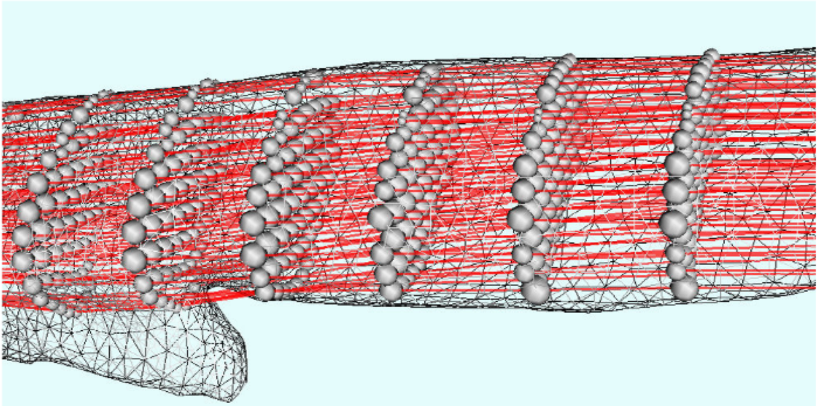

Figure 4.4: Mass-spring system simulating an unspecified human muscle in close detail. An issue arises with the lower portion, which lacks proper approximation. Image source: [36].

ently ensure.

Wang et al. [75] also explored ARAP but abandoned it due to its inability to produce satisfactory results. They noted issues like non-smooth shapes with spikes resulting from ARAP, among others. Furthermore, their proposed approach did not address the volume preservation requirement.

The essence of As-Rigid-As-Possible deformation is to minimise the model deformation as much as possible. Any non-rigid transformation is penalised through a cost function. This problem is mathematically formulated as solving a system of linear equations, where the matrix is a discrete Laplace operator of the mesh (after applying boundary conditions, particularly fixed points), and the right-side vector contains second differences of each vertex concerning its local neighbourhood.

4.6.1 Laplace Operator

The Laplace operator, Δ, has already been discussed in the context of FE methods (see Section 4.4), and its primary motivation behind using it in ARAP is its ability to capture significant local shape changes. When applied to a triangular mesh, the continuous space definition is adapted to discrete space, typically called the Laplace-Beltrami operator.

4.6.2 Volume Preservation

The original ARAP method, while fast and precise, does not inherently address the preservation of the initial model's volume. Volume preservation is crucial in muscle modelling as muscles do not significantly change their volume while contracting.

To achieve proper volume preservation, a straightforward approach is to scale the entire model by a scalar value to restore the original volume. This scaling factor can be calculated using Equation 4.2:

$$\sqrt[3]{\frac{V_0}{V}} \tag{4.2}$$

Here, V_0 represents the initial volume, and V represents the current volume. The cube root is applied to account for the three-dimensional nature of the scaling.

However, this method may not work when the muscle is attached to multiple bones at specific fixed coordinates, as these points cannot be moved, leading to volume discrepancies. Alternative methods have been explored, such as the one proposed by Aubel and Thalman [4], which describes the scale factor as the square root of muscle elongation. However, practical experiments have shown that this approach may not yield accurate results in all cases, especially compared to more straightforward methods such as Position Based Dynamics (PBD) [9].

Seylan et al. [65] have employed ARAP for shape deformation with volume preservation. They addressed volume preservation by adding more edges to the mesh, improving the results. However, they acknowledge that some volume loss still occurs. However, Dvorak et al. [21] demonstrated promising results with ARAP and volume tracking for surfaces that vary over time, significantly exceeding previous experiments. While their approach is practical for noiseless data and general meshes, it may require further adaptation for muscle modelling, particularly in addressing specific challenges such as muscle anisotropy, collision detection and response, and execution speed.

4.7 Position-Based Dynamics

Position-based dynamics, often abbreviated as PBD [56], is a fast and widely used approach, primarily in the animation industry, for modelling elastic deformations of objects, including cloth. It has also gained popularity in the realm of physical simulations. The original PBD algorithm is designed for general objects and does not inherently account for object anisotropy. It takes a manifold surface mesh as input and produces its deformed variant.

An extended version of PBD, known as xPBD, incorporates the concept of elastic potential energy and eliminates the need to specify the time step and iteration count. For a more detailed understanding, the readers can refer to Macklin et al. work [50].

Romeo et al. [62] made the initial significant contribution by applying the PBD algorithm to muscle modelling problems. They recognised the limitations of the traditional finite element method (FEM) and finite volume method (FVM), which, although providing excellent results, lacked qualities such as fast simulation convergence, ease of setup, intuitive controls, and artistic control [62]. Their fundamental idea involved creating an internal structure above the surface mesh to account for muscle anisotropy, aligning with the general direction of muscle fibres. Through an intelligent edge-creation process, they could construct a volumetric model better suited for the PBD algorithm. It is important to note that they used XPBD (eXtended PBD), which incorporates the concept of elastic potential energy.

In 2019, Angles et al. [2] developed a PBD-based approach for muscle modelling. Their method virtually decomposes the muscle into "rods" that approximate muscle fibres. These rods are allowed to change their diameter to preserve volume. The essential contribution of their work was to achieve real-time simulation, a capability that Romeo's approach lacked due to its substantial processing time of approximately 40 seconds per frame [2].

The journey of utilising PBD for muscle modelling continued with the author's master's thesis [15], which expanded the basic PBD approach to include anisotropy considerations. This work coincided with Romeo's article [62], published in the same year. Following the thesis, the article "Fast and Realistic Approach to Virtual Muscle Deformation" [9] extensively tested and integrated the approach into an existing framework. Subsequently, the article "Muscle Deformation using Position-Based Dynamics" [42] further assessed the approach and compared its results with a current FEM approach. A notable advantage of the proposed approach is that it does not require an interior representation, and anisotropy is computed exclusively on the mesh surface using muscle fibres defined on the mesh surface, indicating the fibre direction. During the author's PhD studies, the implementation was integrated into OpenSim, a well-established platform for modelling various physical phenomena. Additionally, a publication [32] extended the work, improving collision detection and response methods, based on the bachelor's thesis of a colleague [33].

The output of the PBD algorithm is a deformed triangular mesh suitable for visualisation. However, the malformed muscle must be transformed into a set of fibres to calculate properties like muscle force. This process is called muscle decomposition, and detailed information can be found in Kohout and Kukacka [46] or Kohout and Cholt [45] articles.

The pseudocode for the PBD algorithm is presented in Listing 1. In this pseudocode, x_i represents the position of each vertex, v_i represents its velocity, Δt denotes the discretisation step (smaller values lead to better accuracy), w_i is the inverse of the weight associated with each vertex, and p_i is a "working" position for each vertex. The variable C_i represents a constraint; more details will be provided in the following text.

Algorithm 1 PBD algorithm [15].

for all vertices i **do**
    initialise $\mathbf{x}_i = \mathbf{x}_i^0$, $\mathbf{v}_i = \mathbf{v}_i^0$, $w_i = \frac{1}{m_i}$.
end for
loop
    for all verticies i **do**
        $\mathbf{v}_i \leftarrow \mathbf{v}_i + \Delta t w_i \mathbf{f}_{ext}(\mathbf{x}_i)$
    end for
    dampVelocities($\mathbf{v}_1, \ldots, \mathbf{v}_N$)
    for all verticies i **do**
        $\mathbf{p}_i \leftarrow \mathbf{x}_i + \Delta t \mathbf{v}_i$
    end for
    for all verticies **do**
        generateCollisionConstraints($\mathbf{x}_i \rightarrow \mathbf{p}_i$)
    end for
    loop solverIterations **times**
        projectConstraints($C_1, \ldots, C_{M+M_{coll}}, \mathbf{p}_1, \ldots, \mathbf{p}_N$)
    end loop
    for all verticies i **do**
        $\mathbf{v}_i \leftarrow \frac{\mathbf{p}_i - \mathbf{x}_i}{\Delta t}$
        $\mathbf{x}_i \leftarrow \mathbf{p}_i$
    end for
    velocityUpdate($\mathbf{v}_1, \ldots, \mathbf{v}_N$)
end loop

Chapter 5

Overview of the contribution

5.1 A New Strategy for Scattered Data Approximation Using Radial Basis Functions Respecting Points of Inflection

In the first contribution [12], the author introduced using radial basis functions (RBFs) for 2D function approximation, focusing on selecting optimal centres from four categories: border vertices, local extrema, inflexion points, and pseudo-randomly chosen positions to minimise approximation errors. The method struggled with sharp edges. Later works adjusted RBF placement, particularly for musculoskeletal data, by prioritising high Mean Squared Error (MSE) locations over inflexion points, acknowledging the unique challenges of higher-dimensional data and musculoskeletal shapes. This refinement was essential for precise musculoskeletal model approximations, with boundary RBFs remaining key to the technique's success.

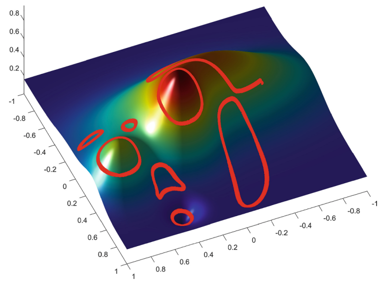

Figure 5.1: An example of $2\frac{1}{2}$D function with the curves of inflection points. The red curve represents the location of inflexion points

Publication [12]:

M. Cervenka, M. Smolik, and V. Skala. "A New Strategy for Scattered Data Approximation Using Radial Basis Functions Representing Points of Inflection". In: *Computational Science and Its Application, ICSSA 2019 proceedings, Part I, LNCS 11619* (2019). UT WoS: 000661318700024, EID: 2-s2.0-85069157052, OBD: 43926678, pp. 322–226. ISSN: 0302-9743. DOI: `https://doi.org/10.1007/978-3-030-24289-3_24`

5.2 Novel RBF Approximation Method Based on Geometrical Properties for Signal Processing with a New RBF Function: Experimental Comparison

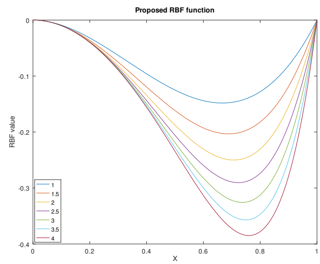

Figure 5.2: Proposed RBF function with various shape parameters.

The primary objective of the second contribution, as described in [66], was to investigate the behaviour of a chosen subset of RBFs exhaustively. The study involved testing the performance of CSRBF, Gaussian RBF, and TPS RBF, together with a newly proposed radial basis function. A set of signals designed to expose potential weaknesses in each type of RBF was used for the testing. The method used in the previous article [12] was a similar centre placement strategy.

The experiment results revealed that accurate approximations could be achieved, with a mean square error consistently below 1% for all global RBFs. In particular, the proposed RBF, denoted $\varphi(r) = r^2(r^\alpha - 1)$, outperformed all other RBFs in these cases. However, more extensive testing is required to ensure its relevance to real-world data, especially when dealing with higher-dimensional datasets. The Gaussian RBF showed suboptimal performance in the testing scenarios, although it was stable in terms of the conditionality of the equation system and increased predictability.

The exploration of individual RBFs in this study laid the foundation for a more efficient selection of RBFs for muscle modelling. The novel and the Gaussian RBFs were tested as potential candidates to approximate a muscle model, emerging as the best options among those investigated in this study.

Publication [66]:

V. Skala and M. Cervenka. "Novel RBF Approximation Method Based on Geometrical Properties for Signal Processing with a New RBF Function: Experimental Comparison". In: *Informatics 2019, IEEE proceedings* (2019). UT WoS: 000610452900074, EID: 2-s2.0-85087090327, OBD: 43929007, pp. 357–362. DOI: `https://doi.org/10.1109/Informatics47936.2019.9119276`

5.3 Modified Radial Basis Functions Approximation Respecting Data Local Features

The subsequent paper [73] builds on the foundations established in the initial paper [12]. This specific contribution exploits diverse features of the input function as optimal locations for RBF centres. These features contain edges, stationary curvature points, pseudorandom positions, and centres at the function's borders. Including the latter two is necessary according to Section 5.1. The centres situated at edges (identified, e.g. through the Canny edge detector on the height map image of the function) and stationary curvature points yield an even more enhanced interpolation result than inflexion points and local extrema.

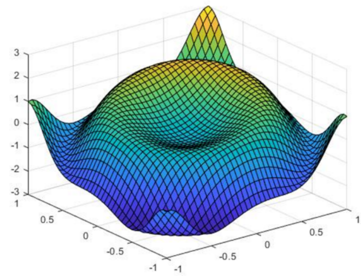

Figure 5.3: An RBF approximation of $2\frac{1}{2}$D function

The exploration of curvature concepts was further developed and ultimately integrated into the most recent publication, as elucidated in Section 5.13.

Furthermore, the discussion in Section 5.1 emphasised the importance of including pseudorandom positions and centres at the border. This deliberate integration stems from the overarching belief that centres located at edges and stationary curvature points possess the potential to enhance results beyond what can be achieved with inflexion points and local extrema alone.

The research presented in [73] not only builds on the concepts introduced in [12] but also advances the understanding of optimal RBF centre placement by considering a broader spectrum of features in the input function. The subsequent developments, as outlined in Section 5.13, underscore the curvature-centric approach's continued evolution and practical applicability, showing its relevance in subsequent research.

Publication [73]:

J. Vasta, V. Skala, M. Smolik, and M. Cervenka. "Modified Radial Basis Functions Approximation Respecting Data Local Features". In: *Informatics 2019, IEEE proceedings* (2019). UT WoS: 000610452900015, EID: 2-s2.0-8508762067, OBD: 43928987, pp. 445–449. DOI: https://doi.org/10.1109/Informatics47936.2019.9119330

5.4 Fast and Realistic Appr. to Virtual Muscle Deformation

The fourth article [9] provided an overview of the contemporary landscape at its creation. It delved into a muscle modelling approach rooted in position-based dynamics, serving as a fundamental algorithm. Although this approach was operated directly on a triangular mesh representing the muscle surface, it is essential to note that it is still undergoing development, as indicated in sections 5.8 and 5.10. However, the author has since redirected their focus towards muscle modelling methods using Radial Basis Function (RBF) approximation techniques, as detailed throughout the text.

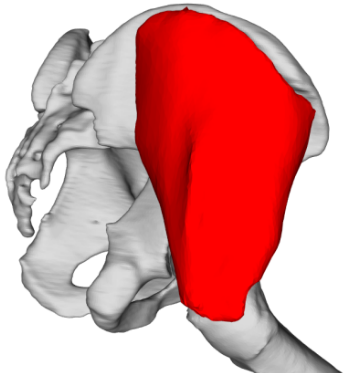

Figure 5.4: The gluteus medius muscle under the maximum femur flexion.

Within the paper, Section 6, titled "Discussion," not only expounded upon the state-of-the-art but also scrutinised various challenges and impediments associated with employing techniques such as Position-Based Dynamics, Finite Element Method, Mass-spring systems, wrapping obstacles, and more.

The identified issues encompassed the lack of smoothness in the model, the difficulties in collision detection and response (hip joint issue discussed in the article, subsequently addressed in further research, as evidenced in Section 5.10), and the excessive amount of data utilised for the model compared to its actual requirements. Faced with these challenges, the notion of adopting a different geometrical description for a muscle emerged at the time of writing. However, this concept has not yet been formally presented in the paper.

Publication [9]:

M. Cervenka and J. Kohout. "Fast and Realistic Approach to Virtual Muscle Deformation". In: *in Proceedings of the 14th International Joint Conference on Biomedical Engineering Systems and Technologies - Volume 5: HEALTHINF* (2020). UT WoS: 000571479400020, EID: 2-s2.0-85083710925, OBD: 43929104, pp. 217–227. DOI: `https://doi.org/10.5220/0009129302170227`

5.5 Behavioral Study of Various Radial Basis Functions for Approximation and Interpolation Purposes

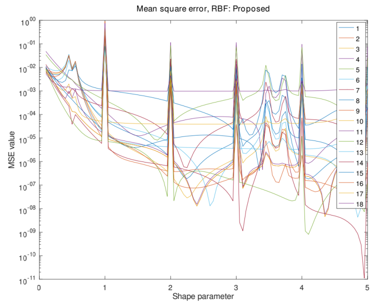

Figure 5.5: Mean square error of the proposed RBF depending on the shape parameter.

The study [10] further explores various RBFs, where the placement of the centre was mainly adopted from the previous paper [66] described in Section 5.2. The new RBF of that paper has also been tested. These more in-depth tests of the narrowed subset of RBFs showed some weaknesses of the author's RBF, mainly that there is a pattern (whole-numbered shape parameter), while the RBF approximation is ill-conditioned. The Gaussian RBF does not provide that shortcoming in the case of 1D signals; however, it also has its issues with shape parameter selection considering conditionality (see Section 5.7).

Specifically, the results presented in the article show that selecting a whole number as the shape parameter α for the proposed RBF can lead to peaks or singularities in the mean square error and conditionality plots. These singularities indicate instability or significant variations in the approximation error and conditionality, which can affect the reliability and performance of the RBF approximation. Therefore, considering and potentially avoiding whole-numbered shape parameters might be advisable in RBF approximation tasks to ensure more stable and consistent results.

Publication [10]:

M. Cervenka and V. Skala. "Behavioral Study of Various Radial Basis Functions for Approximation and Interpolation Purposes". In: *IEEE 18th World Symposium on Applied Machine Intelligence and Informatics, SAMI 2020* (2020). UT WoS: 000589772600026, EID: 2-s2.0-85087093548, OBD: 43929006, pp. 135–140. DOI: `https://doi.org/10.1109/SAMI48414.2020.9108712`

5.6 Finding Points of Importance for RBF Approximation of Large Scattered Data

This article [67] focuses mainly on describing the experimental results demonstrating the previously proposed approaches' effectiveness. The results showed high approximation precision in tests involving various functions, using only a fraction of the available data points. For instance, the proposed approximation method exhibits a compression ratio between 5% and 10%, maintaining precision and a high compression ratio. It shows that muscle modelling can reach data reduction and simplification.

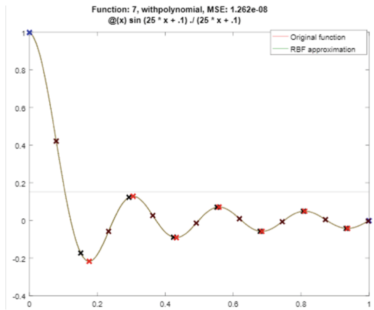

Figure 5.6: A various techniques of RBF centre placement.

The study's conclusion highlights the simplicity and efficiency of the RBF-based approximation method, which achieves relatively low error rates and high data compression. However, there are still many other function points of importance, which may even enhance the approximation's precision by including them. However, evaluating neighbouring points is crucial for scattered data to identify the points of importance accurately. The study highlights the potential for future research, mainly in analysing behaviour at interval borders in 3D, which is recognised as a critical aspect for further development. Studying behaviour on borders in 3D is beneficial for muscle modelling.

The article also contributes to handling large and complex data sets. The proposed method stands out for its ability to simplify the approximation process while maintaining high accuracy and efficiency. It mainly benefits engineering and scientific computations involving large scattered data sets.

Publication [67]:

V. Skala, S. Karim, and M. Cervenka. "Finding Points of Importance for Radial Basis Function Approximation of Large Scattered Data". In: *Computational Science - ICCS 2020, Part VI, LNCS 12142* (2020). OBD: 43932925, UT WoS: 000841676000019, EID: 2-s2.0-85087274721, pp. 239–250. DOI: `https://doi.org/10.1007/978-3-030-50433-5_19`

5.7 Conditionality Analysis of the Radial Basis Function Matrix

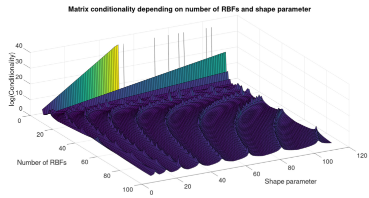

Figure 5.7: Conditionality analysis of Gaussian RBF depending on the number of RBFs and shape parameter.

The conditionality analysis [11] paper is a more in-depth study of a Gaussian RBF. The goal was to take the knowledge from the previous paper and take the approach to the higher (2D) dimensions to ensure there are no problems with the approach. Also, the uniform centre point distribution was tested thoroughly.

The research aimed to figure out the most suitable shape parameter, and the testing scenario was simplified for that purpose. The testing scenario involves a $\langle 0,1\rangle \times \langle 0,1\rangle$ domain, where variable number of RBF centres N was put uniformly with a variable (global) shape parameter β.

During the research, two significant outcomes emerged. The first outcome is that if the uniform distribution is used, some shape parameters lead to an ill-conditional linear equation system. We were also able to find those experimentally and analytically. The second one is that the uniform distribution is unsuitable for the RBF approximation because the resulting linear equation system is ill-conditioned, in contrast to using some pseudorandom, e.g., Halton distribution.

Also, previously less discussed TPS RBF was rigorously tested. In the case of this RBF, a shape parameter also exists, which leads to ill-conditionality; however, there is only one for each number of RBFs tested.

This research proved the facts from the previous ones, that the pseudorandom distribution on points (where no other viable placement option exists) is better than the uniform one, considering the conditionality of the RBF equation system to solve. Due to that fact, further research did not consider the uniform distribution for placement at all.

Publication [11]:

M. Cervenka and V. Skala. "Conditionality Analysis of the Radial Basis Function Matrix". In: *ICCSA 2020 proceedings, part II, LNCS* (2020). UT WoS: 000719685200003, EID: 2-s2.0-85093112881, OBD: 43932697, pp. 30–43. DOI: `https://doi.org/10.1007/978-3-030-58802-1_3`

5.8 Muscle Deformation Using Position Based Dynamics

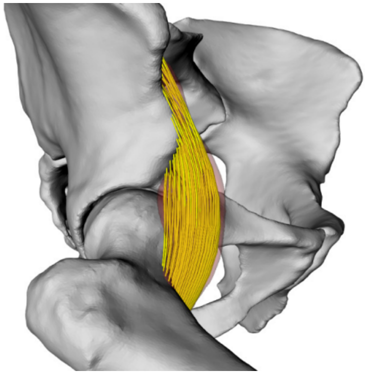

Figure 5.8: The deformation and decomposition of the iliacus muscle.

In this article [42], we build on our earlier work [9] (Section 5.4) detailing muscle deformation through a triangular mesh model of the musculoskeletal system. Our method prioritises volume preservation, achieving less than 1% error in volume maintenance across all tests. This focus greatly enhances the realism and accuracy of muscle simulations. Additionally, we explore the displacement dynamics of the adductor brevis muscle during hip flexion, examining the effects of varying iterations of the PBD solver. This analysis advances our understanding of muscle deformation, providing an evaluation of our previous method [9] and describing several challenges:

1. The approach detects muscle points moving with bones, relying on manual expert input for muscle attachment areas, which is time-consuming.
2. The collision handling method may cause surface spikes, especially with coarse voxel bone representations, and refined representations are impractical due to cubic memory growth.
3. Simulation outcomes depend heavily on parameters; while it runs in real-time, even unoptimised, accurate results require parameter calibration.

The challenges in simulating muscle deformation, including efficient collision handling and accurate muscle attachment setup, are significant. We addressed attachment point accuracy in Section 5.11 and improved collision handling in Section 5.10. The resolution of the third issue is still underway.

Publication [42]:

J. Kohout and M. Cervenka. "Muscle Deformation Using Position Based Dynamics". In: *Ye X. et al. (eds) Biomedical Engineering Systems and Technologies. BIOSTEC 2020. Communications in Computer and Information Science* 1400 (2021). EID: 2-s2.0-85107281398, OBD: 43932927.
DOI: `https://doi.org/10.1007/978-3-030-72379-8_24`

5.9 Geometry Algebra and Gauss Elimination method for solving a linear system of equations without division

The results of this paper [14] can be applied to the RBF equation system solving if Gaussian elimination (GE) is used. The research introduces an innovative approach to GE that eliminates the need for division. This method is significant in contexts where division is expensive, not optimised, or inconvenient. The GE process includes division operations, generating computational expense and numerical instabilities. The division avoidance approach reduces the computational costs and maintains numerical stability.

Proposed approach	
/ (Division)	0
* (Multiplication)	$N^3 + N^2 - 2N$
- (Subtraction)	$\frac{3}{2}(N^3 + 2N^2 + 3N)$
$\wedge$ (Bitwise AND)	$\frac{1}{2}(N^3 + 2N^2 - 3N)$
$\vee$ (Bitwise OR)	$\frac{1}{2}(N^3 + 2N^2 - 3N)$
$\oplus$ (Bitwise XOR)	$\frac{1}{2}(N^3 + 2N^2 - 3N)$
Memory	N^2

Figure 5.9: The complexity of the provided GE approach.

The proposed method involves additional multiplication and addition steps to substitute for division operations. By avoiding division, the process reduces computational expense, with only a slight increase in execution time compared to the standard approach on modern computers. Experiments were conducted using the Hilbert matrix, which is known for its numerical instability during inversion. The paper compares the proposed method with the traditional GE and another approach for reducing division operations. The results demonstrate that the new method maintains numerical stability.

The proposed method maintains accuracy and stability while being slightly slower than the GE method on a traditional PC because the division on this hardware is already optimised. The performance of the approach was evaluated using the normalised Frobenius norm and conditionality of the inverses.

The paper contributes to computational mathematics by offering an approach to a fundamental process impacting areas where division is not feasible.

Publication [14]:

M.: Cervenka. "Geometry Algebra and Gauss Elimination method for solving a linear system of equations without division". In: *Informatics 2022, IEEE proceedings* (2022). OBD: 43937872, EID: 2-s2.0-85153355665, pp. 55–59. DOI: `https://doi.org/10.1109/Informatics57926.2022.10083445`

5.10 Collision detection and resp. approaches for computer muscle modelling

In our research, addressing collision detection and response presented significant challenges (refer to the second point in Section 5.8 and also see Section 5.4), primarily due to the lack of a practical solution initially. The study highlighted in this article [32] sought to improve collision handling by substituting the original voxelisation method with a scalar distance field (SDF) approach.

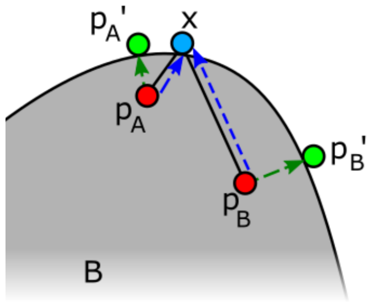

Figure 5.10: A proposed approach when a collision between two objects occurs.

Initially, we considered two alternatives: the scalar distance field (SDF) and the flexible collision library (FCL). Our findings indicated that SDF was the superior option, offering enhanced computational speed and reduced memory usage without compromising quality.

The implementation of SDF in this study markedly enhanced our collision handling technique. Its success is primarily attributed to its lower discretisation resolution than voxelisation, with trilinear interpolation in each voxel providing sufficient accuracy.

Furthermore, we successfully settled a persistent issue where muscles would get stuck in joints due to the coarse surface of the voxel grid, hindering smooth movement out of the joint. The new method also significantly reduces the likelihood of muscles entering narrow spaces between bones.

Publication [32]:

O. Havlicek, M. Cervenka, and J. Kohout. “Collision detection and response approaches for computer muscle modelling”. In: *Informatics 2022, IEEE proceedings* (2022). EID: 2-s2.0-85153333554, OBD: 43937869, pp. 120–125. DOI: `https://doi.org/10.1109/Informatics57926.2022.10083500`

5.11 Nonplanar Surface Shape Reconstruction from a Point Cloud in the Context of Muscles Attachments Estimation

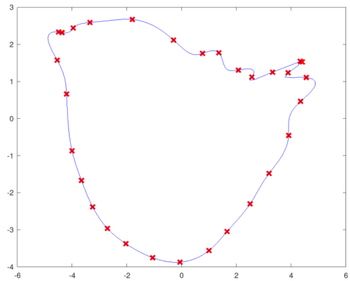

Figure 5.11: A curve reconstruction from a projected nonplanar attachment points.

The attachment estimation described in the following paper [43] plays a crucial role in muscle modelling. The article mainly tackles the issue described in Section 5.8, already described in that paper. The option of automatically searching for the attachment area according to the information about the closeness of the muscle to the bone proved insufficient because it often happens that the muscle is adjacent to some bone but not attached to it. Hence, additional data is required. Those data already exist as vertices measured at the border of the attachment area as a part of the TLEM 2.0 [8]. The main issue is finding all points inside the attachment area, which would be fixed afterwards to the adjacent bone.

The article tested 15 different curve reconstruction algorithms to reconstruct the whole attachment boundary on the bone, and the subsequent surface bounded by it can be restored by the radial basis function (RBF) approach, which has already been researched before. In this research, we also tested the RBF approach for curve reconstruction, which worked to some extent but was unstable in terms of the parameters selected, which must be chosen carefully with the prior and deep knowledge of the RBF approximation technique.

The RBFs are useful for muscle modelling and muscle attachment area approximation; however, parameter selection is crucial to success.

Publication [43]:

J. Kohout and M. Cervenka. "Nonplanar Surface Shape Reconstruction from a Point Cloud in the Context of Muscles Attachments Estimation". In: *Proceedings of the 17th International Joint Conference on Computer Vision, Imaging and Computer Graphics Theory and Applications* (2022). UT WoS: 000774795400024, OBD: 43936004, pp. 236–243. DOI: https://doi.org/10.5220/0010869600003124

5.12 Computerised muscle modelling and simulation for interactive applications

In this study [13], we evaluated our previously developed muscle modelling technique, addressing a future challenge and introducing a method enabling muscles to "slide" over bones using virtual edges.

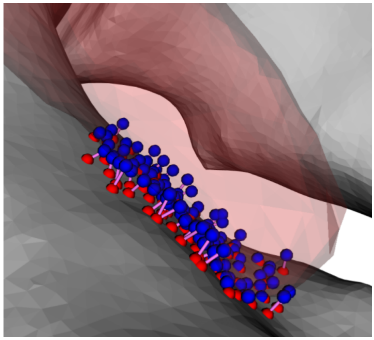

Figure 5.12: Virtual edges allow the muscle to slide.

A problem identified during our tests occurs when two bones move close to each other with a muscle sandwiched in between, akin to a muscle caught in shears. Another problem arises when bone movement is so rapid between iterations that the muscle might penetrate the bone, complicating shape restoration. This issue, while appearing unlikely, is common in long bones like the femur, where a small angular change can result in substantial displacement at the bone's opposite end. Finally, some further directions are outlined:

1. Expanding on the PBD concept to use, e.g. XPBD. This area is currently under exploration, though it has become a smaller focus of my research.
2. Integrating another algorithm with PBD to refine outcomes. The ARAP algorithm tested in this paper is a candidate for this integration. However, we decided to diverge from this research path due to unresolved issues.
3. A different geometric model could mitigate certain problems, particularly surface roughness. Each alternative geometry, however, presents its challenges.

This final examination of the triangular mesh model prompted me to shift my focus to a different geometric approach (according to the third point), employing radial basis functions rather than the triangular mesh.

Publication [13]:

M. Cervenka, O. Havlicek, J. Kohout, and L. Vasa. "Computer muscle modelling". In: *Computerised muscle modelling and simulation for interactive applications, Proceedings of the 18th International Joint Conference on Computer Vision, Imaging and Computer Graphics Theory and Applications, VISIGRAPP 2023, Volume 1: GRAPP* (2023). UT WoS: 001066254400019, OBD: 43940148, pp. 214–221. DOI: https://doi.org/10.5220/0011688000003417

5.13 A mathematical model for smooth Radial Basis Function implicit surface model for muscle modelling

This research paper [16] presents an innovative method for modelling muscle geometry. This approach is heavily theoretical and builds upon insights gleaned from our prior research. This theoretical model, but generalised, is described in Chapter 6.

The journal paper further develops knowledge acquired from earlier studies. Here's a summary of each publication and its crucial contribution to this paper:

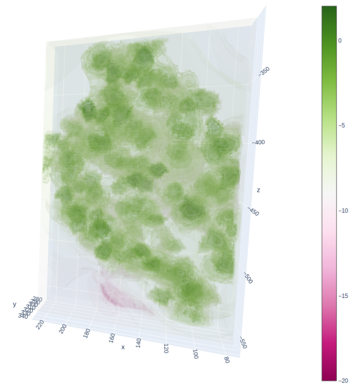

Figure 5.13: A curvature field around the gluteus maximus.

A New Strategy for Scattered Data Approximation Using Radial Basis Functions Respecting Points of Inflection [12] highlights the utilisation of Halton point distribution and the incorporation of central points at domain boundaries.

Novel RBF Approximation Method Based on Geometrical Properties for Signal Processing with a New RBF Function: Experimental Comparison [66] reveals limitations of local RBFs, suggesting three global RBF alternatives: Gaussian, TPS, and a newly developed one.

Modified Radial Basis Functions Approximation Respecting Data Local Features [73] reveals how edge detection and curvature can enhance approximation using a curvature preservation approach.

Fast and Realistic Approach to Virtual Muscle Deformation [9] is a foundational paper on muscle modelling principles that greatly influenced the journal article by outlining the requirements of the muscle model.

Behavioural Study of Various Radial Basis Functions for Approximation and Interpolation Purposes [10] concludes that while a novel RBF might be preferable for approximation, caution is needed in shape parameter selection due to potential instabilities in RBF matrix conditionality. The paper suggests Gaussian RBF is a potentially better choice.

Finding Points of Importance for Radial Basis Function Approximation of Large Scattered Data [67] recaps and tests previous research [12, 66, 73], contributing the insight that high compression ratios can maintain accuracy.

Conditionality Analysis of the Radial Basis Function Matrix [11] provides valuable insights into shape parameter selection for Gaussian RBF through analytical study.

Muscle Deformation Using Position Based Dynamics [42] examines PBD methods on a triangular mesh muscle model, highlighting the need to address muscle tissue entering the joint.

Geometry Algebra and Gauss Elimination method for solving a linear system of equations without division [14] delves into solving the RBF equation system, focusing on the methodological intricacies.

Collision detection and response approaches for computer muscle modelling [32] enhance understanding of collision handling approaches, relevant for future work with RBF geometric models.

Nonplanar Surface Shape Reconstruction from a Point Cloud in the Context of Muscles Attachments Estimation [43] discusses applying RBF to real musculoskeletal data and its unique challenges.

Computerised muscle modelling and simulation for interactive applications [13] summarises, tests, and expands the existing approach, mainly focusing on future research opportunities, which the journal paper explores. The paper is at the time of writing in the stage of submission.

Publication [16]:

Martin Cervenka, Josef Kohout, and Bogdan Lipus. "A mathematical model for smooth Radial Basis Function implicit surface model for the purpose of muscle modelling". In: *INFORMATICA* (2024, submitted)

Chapter 6

Inovative mathematical model

The derivation of the mathematical model is straightforward. It involves describing the surface as a sum of RBF implicit functions and the curvature preservation condition. At first, the innovative mathematical model starts by declaring the general notation of the RBF approximation:

$$f(\mathbf{x}) = \sum_{i=1}^{N} \lambda_i \varphi\left(||\mathbf{x} - \xi_\mathbf{i}||\right) = \sum_{i=1}^{N} \lambda_i \varphi\left(r_i\right) \tag{6.1}$$

Because the curvature calculation involves estimating the Hessian matrix, the first step is to find the gradient of the RBF approximation, which will be needed afterwards. It can be described as follows:

$$\nabla f(\mathbf{x}) = \sum_{i=1}^{N} \lambda_i \nabla\varphi\left(r_i\right) = \sum_{i=1}^{N} \lambda_i \frac{\partial\varphi}{\partial r_i}\frac{\partial r_i}{\partial x_j} = \sum_{i=1}^{N} \lambda_i \frac{r_{ij}}{r_i}\frac{\partial\varphi}{\partial r_i}, \quad r_{ij} = x_j - \xi_{ij} \tag{6.2}$$

The gradient evaluation was the first step in estimating the Hessian matrix of the approximator. Second partial derivatives declare the Hessian of a function, generally as:

$$\mathbf{H}\left(f(\mathbf{x})\right) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} & \cdots \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} & \cdots \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{6.3}$$

In our case of RBF approximation, where each second partial derivative can be evaluated from the first ones in (6.2), a single Hessian matrix element can be declared as:

$$\frac{\partial^2 f}{\partial x_j \partial x_k} = \sum_{i=1}^{N} \lambda_i \left(\frac{\partial^2 \varphi}{\partial r_i^2}\frac{r_{ij}^2}{r_i^2} + \frac{\partial\varphi}{\partial r_i}\left(\frac{\delta_{ij}}{r_i} - \frac{r_{ij}^2}{r_i^3}\right)\right) \tag{6.4}$$

The Kronecker delta function δ_{ij} indicates that it behaves differently on and off the diagonal. Luckily, for further evaluation, the derivation depends solely on the elements on the main diagonal.

6.0.1 Mean curvature

The mean curvature of a model describes the shape of the model, so it should be beneficial to preserve it. The mean curvature is defined as a mean eigenvalue of the Hessian or as the mean of the trace of the Hessian:

$$\kappa_{\mu}\left(\mathbf{H}\left(f\left(\mathbf{x}\right)\right)\right) = \kappa_{\mu f} = \overline{\lambda}_{\mathbf{H}} = \frac{\mathrm{Tr}\left(\mathbf{H}\right)}{D} = \frac{1}{D}\sum_{i=1}^{N}\lambda_i\left(\frac{\partial^2\varphi}{\partial r_i^2} + \frac{\partial\varphi}{\partial r_i}\frac{D-1}{r_i}\right) \tag{6.5}$$

The subsequent step involves determining the cost function, which is essentially the squared L2 norm between the new and original curvatures across the entire space:

$$C_f = \int\ldots\int ||\kappa_{\mu f} - \kappa_{\mu f_{\mathrm{i}}}||_2^2\, dx_1\ldots dx_d = \int_{\mathbb{R}^d} ||\kappa_{\mu f} - \kappa_{\mu f_{\mathrm{i}}}||_2^2\, d\mathbf{x} \tag{6.6}$$

One may employ the gradient descent method to discover the optimal values of ξ_i. Initially, we must compute the gradient of the curvature w.r.t. ξ_i:

$$\nabla\kappa_{\mu f} = \begin{bmatrix} \frac{\partial\kappa_{\mu f}}{\partial\xi_{i1}} & \frac{\partial\kappa_{\mu f}}{\partial\xi_{i2}} & \frac{\partial\kappa_{\mu f}}{\partial\xi_{i3}} & \ldots \end{bmatrix} \tag{6.7}$$

After the detailed following derivation, the gradient ultimately used for the gradient descent method to evaluate the new shape would take the form:

$$\begin{aligned}\frac{\partial\kappa_{\mu f}}{\partial\xi_{kj}} &= \frac{\partial}{\partial\xi_{kj}}\frac{1}{D}\sum_{i=1}^{N}\lambda_i\left(\frac{\partial^2\varphi}{\partial r_i^2} + \frac{\partial\varphi}{\partial r_i}\frac{D-1}{r_i}\right) = \frac{1}{D}\sum_{i=1}^{N}\lambda_i\left(\frac{\partial g\left(\xi_i\right)}{\partial\xi_{kj}}\right) = \\ &= \frac{\lambda_k}{D}\frac{\partial g\left(\xi_k\right)}{\partial\xi_{kj}} = \frac{\lambda_k}{D}\frac{\partial}{\partial\xi_{kj}}\left(\frac{\partial^2\varphi}{\partial r_k^2} + \frac{\partial\varphi}{\partial r_k}\frac{D-1}{r_k}\right) = \\ &= -\frac{\lambda_k r_{kj}}{D r_k^3}\left(r_k^2\frac{\partial^3\varphi}{\partial r_k^3} + r_k\left(D-1\right)\frac{\partial^2\varphi}{\partial r_k^2} - \left(D-1\right)\frac{\partial\varphi}{\partial r_k}\right)\end{aligned} \tag{6.8}$$

Now, we need to compute the gradient of the cost function C, which is expressed as follows:

$$\nabla C_f = \nabla_{\xi}\left(\int_{\mathbb{R}^d} ||\kappa_{\mu f} - \kappa_{\mu f_{\mathrm{i}}}||_2^2 d\mathbf{x}\right) = 2\int_{\mathbb{R}^d}(\kappa_{\mu f} - \kappa_{\mu f_{\mathrm{i}}})\nabla_{\xi}\kappa_{\mu f} d\mathbf{x} \tag{6.9}$$

Given the definition of the partial derivatives of κ, the complete cost function gradient can be expressed as:

$$2\int_{\mathbb{R}^d}(\kappa_{\mu f} - \kappa_{\mu f_{\mathrm{i}}})\left(-\frac{\lambda_k r_{kj}}{D r_k^3}\left(r_k^2\frac{\partial^3\varphi}{\partial r_k^3} + r_k\left(D-1\right)\frac{\partial^2\varphi}{\partial r_k^2} - \left(D-1\right)\frac{\partial\varphi}{\partial r_k}\right)\right) d\mathbf{x} \tag{6.10}$$

If we, for example, consider gaussian RBF $\varphi(r) = e^{-\alpha r^2}$ in threedimensional space $D = 3$, then the resulting gradient would take the form:

$$
\begin{gathered}
\frac{\partial\varphi}{\partial r} = -2\alpha r\varphi(r), \quad \frac{\partial^2\varphi}{\partial r^2} = \left(4\alpha^2 r^2 - 2\alpha\right)\varphi(r), \\
\frac{\partial^3\varphi}{\partial r^3} = \left(12\alpha^2 r - 8\alpha^3 r^3\right)\varphi(r), \quad E = D - 1 \\
\nabla C_{f_{kj}} = 2\int_{\mathbb{R}^d}(\kappa_{\mu f} - \kappa_{\mu f_\mathrm{i}}) \\
\left(-\frac{\lambda_k r_{kj}}{D r_k^3}\left(r_k^2\frac{\partial^3\varphi}{\partial r_k^3} + r_k E\frac{\partial^2\varphi}{\partial r_k^2} - E\frac{\partial\varphi}{\partial r_k}\right)\right)d\mathbf{x} = \\
= -\frac{2}{3}\int_{\mathbb{R}^3}(\kappa_{\mu f} - \kappa_{\mu f_\mathrm{i}})e^{-\alpha r^2}\frac{\lambda_k r_{kj}}{r^3} \\
\left(r^2\left(12\alpha^2 r - 8\alpha^3 r^3\right) + 2r\left(4\alpha^2 r^2 - 2\alpha\right) + 4\alpha r\right) = \\
= -\frac{2}{3}\int_{\mathbb{R}^3}(\kappa_{\mu f} - \kappa_{\mu f_\mathrm{i}})e^{-\alpha r^2}\lambda_k r_{kj}\left(12\alpha^2 - 8\alpha^3 r^2 + 2\left(4\alpha^2 - \frac{2\alpha}{r^2}\right) + \frac{4\alpha}{r^2}\right) = \\
= -\frac{2}{3}\int_{\mathbb{R}^3}(\kappa_{\mu f} - \kappa_{\mu f_\mathrm{i}})e^{-\alpha r^2}\lambda_k r_{kj}\alpha^2\left(20 - 8\alpha r^2\right) = \\
= \frac{8}{3}\int_{\mathbb{R}^3}(\kappa_{\mu f} - \kappa_{\mu f_\mathrm{i}})e^{-\alpha r^2}\alpha^2\lambda_k r_{kj}\left(2\alpha r^2 - 5\right)
\end{gathered} \tag{6.11}
$$

The equation follows our latest paper [16]. The gradient from the paper was described as follows:

$$
\frac{8}{d}\int_{\mathbb{R}^d}\left(\kappa_{\mu f} - \kappa_{\mu f_\mathrm{i}}\right)\alpha_k^2 g_k(\mathbf{x})\left(x_j - \xi_{kj}\right)\left(2\alpha_k||\mathbf{x} - \xi_{\mathbf{k}}||_2^2 - 2 - d\right)d\mathbf{x} \tag{6.12}
$$

Chapter 7

Conclusion & Future work

The dissertation provides a comprehensive overview of the evolution and current methodologies in muscle modelling. It traces the journey from the earliest Hill-type muscle models, which were rudimentary yet foundational, through various stages of evolution, including simplistic straight-line approximations, polyline constructs, and models incorporating lines wrapped around obstacles or bones. This progression culminates in more sophisticated, higher-dimensional frameworks such as mass-spring systems, position-based dynamics, and finite element methods. A recurring theme in this evolution is the balance between model accuracy and computational efficiency. A notable trend is observed: models demand more parameters to be accurately determined and configured as they become more complex.

A pivotal contribution of the dissertation is the detailed exploration of existing muscle modelling techniques and introducing a novel mathematical model utilising radial basis function implicit surfaces. This innovative approach marks a significant step forward, offering a memory-efficient model to generate infinitely smooth surface representations. Such a model can potentially revolutionise the field of muscle modelling by providing a more refined and scalable tool for simulating muscle morphology and dynamics.

However, the proposed model is not without limitations. It does not address certain practical aspects crucial in real-world applications, such as collision handling and preserving volume in the modelled entities. These areas are identified as avenues for future research and development. The insights and methodologies presented in the dissertation lay a solid groundwork for tackling these challenges. Building upon the foundation laid by this research, future work is anticipated to advance the field further, enhancing the realism and applicability of muscle models in various scientific and medical applications.

The dissertation, covering a broad range of topics in muscle modelling, opens up several avenues for future research and development. One of the critical areas for expansion is the proposed Radial Basis Function (RBF) mathematical model. Currently, the model primarily focuses on maintaining the initial shape of the muscle during deformation, akin to the Position-Based Dynamics (PBD) approach. However, it does not yet incorporate other crucial aspects of muscle behaviour. Notably, the interaction between muscles modelled with

RBF and bones, especially regarding collision handling, remains unaddressed. Drawing inspiration from existing literature, such as the work by Cani [7] on the collision handling between two implicit surfaces, could be beneficial. Although Cani's method is based on implicit surfaces generated by skeletons rather than RBF sets, the underlying principles could provide valuable insights for developing a similar mechanism for RBF muscles. Additionally, formulating a volume constraint for the RBF model is another crucial aspect that requires attention, ensuring that the muscle volume remains consistent during deformations.

The dissertation also delves into the complexities of the PBD approach. Despite significant advancements, challenges like muscle penetration through bones and muscle tissue getting forced into tight spaces persist. Addressing these issues involves solving intricate problems like determining the side of the bone where most muscle is located post-deformation and devising strategies for muscle tissue to escape from increasingly tight spaces. These challenges highlight the need for more sophisticated algorithms and problem-solving techniques in muscle modelling.

Furthermore, integrating the As-Rigid-As-Possible (ARAP) approach with the existing PBD framework has proven more challenging than initially anticipated. The attempt to intertwine these two methodologies resulted in a rough surface texture, as the algorithms worked at cross purposes, pushing vertices in different directions. This outcome suggests that a more intricate and harmonised cooperation between ARAP and PBD algorithms is essential to achieve the desired smoothness and accuracy in muscle modelling.

In summary, while the dissertation lays a strong foundation in muscle modelling, it also clearly outlines the need for further research in several key areas. These include developing collision handling mechanisms for RBF muscles, volume preservation techniques, resolving issues related to muscle-bone interactions in the PBD approach, and refining the integration of ARAP with PBD for smoother and more accurate muscle deformations. These challenges present exciting opportunities for future research, promising significant advancements in muscle modelling

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