Positioning Using GNSS Signals and Digital Maps

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Introduction

- A global navigation satellite system (GNSS) provides positioning, navigation and timing services, so important for safety critical applications.
- In adverse conditions (urban areas, tunnels, etc.) GNSS needs to be complemented by information form external sources (intertial sensors, digital maps, etc.) to obtain desired performance.
- The purpose of the paper is to employ information from digital maps and respect different local Earth-Centered Earth-Fixed (ECEF) coordinate systems.

Modeling real transportation network

- A digital map is a model of a real transportation network with properties determined by application requirements.
  - Topological map represents connectivity in the network by nodes and links.
  - Geometrical map geometrically represents connection between nodes.
  - Topographical map includes supplementary information about close surroundings of the path.

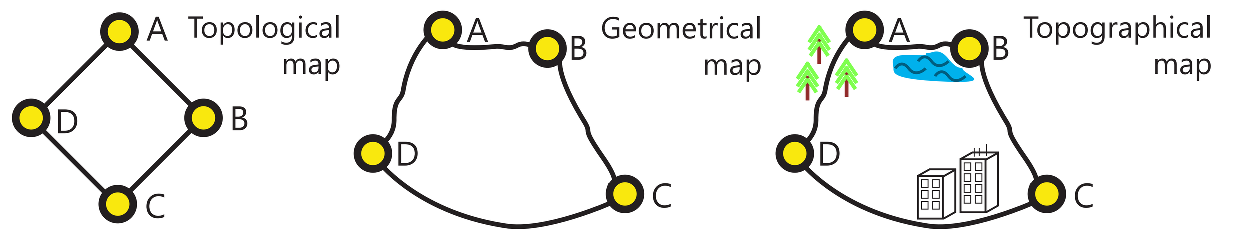

Figure: Different digital maps

Map-matching algorithms

- Current map-matching algorithms find a correct segment of the path and determine position on the segment. They can be divided into two groups.
  - Loosely coupled algorithms use initial position estimate and compute partial position estimate without map. The final position estimate is determined as the minimum distance projection to the selected segment, which represents additional constraints.
  - Tightly coupled algorithms find final position estimate in preselected segments. No initial position estimate is needed, instead preselected segments represent additional constraints.

Goal

Introduce a loosely coupled and a tightly coupled map-matching algorithm which improve a quality of the position estimate.

Model of pseudoranges

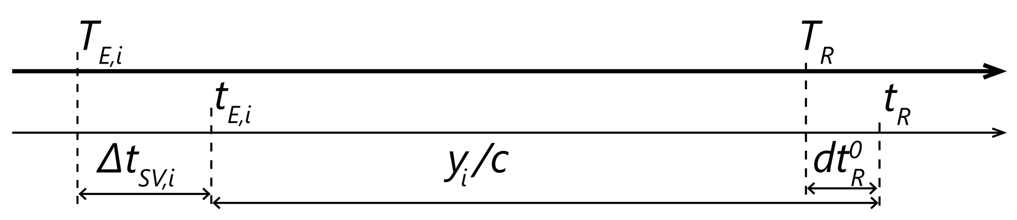

Figure: Time diagram of the measurements in GNSS

- A correct approach to a measurement model must respect different local ECEF satellite coordinate systems. A rotation to a common Earth-Centered Inertial (ECI) is necessary.
  - Relation between coordinates $\mathbf{r}_i$ in the ECEF and ECI is $\mathbf{r}_i(T_1;T_2) = \mathbf{R}_3(T_1 - T_2)\,\mathbf{r}_i(T_1;T_1)$ with time moments T_1 and T_2 and z-axis rotation matrix $\mathbf{R}_3$.
- The fundamental positioning measurement model using GNSS is a model of pseudoranges represented as follows

$$\mathbf{y} = \mathbf{g}\left(\mathbf{x}^0(T_R;T_R)\right) + \mathbf{v},$$
$$g_i\left(\mathbf{x}^0(T_R;T_R)\right) = \left\|\mathbf{r}^0(T_R;T_R) - \mathbf{r}_i(T_{E,i};T_R)\right\|_2 + b^0 - c\cdot\Delta t_{SV,i}.$$

  - Measured pseudoranges $\mathbf{y}$ from n visible satellites, an unknown state of the receiver $\mathbf{x}^0 = \left[\mathbf{r}^0, b^0\right]^T$, a position of the receiver $\mathbf{r}^0$, coordinates of visible satellites $\mathbf{r}_i$, a clock offset of the receiver $b^0 = c\cdot dt_R^0$, the speed of light c, a local GNSS time of signal arrival at the receiver t_R and the same time instant in the global GNSS time T_R, a local GNSS time of signal departure from i-th satellite $t_{E,i}$ and the same instant in global GNSS time $T_{E,i}$, and finally a measurement noise $\mathbf{v}$ with Gaussian distribution, zero mean and known covariance $\Sigma = \sigma^2\mathbf{I}$.

Constraint represented by a digital map

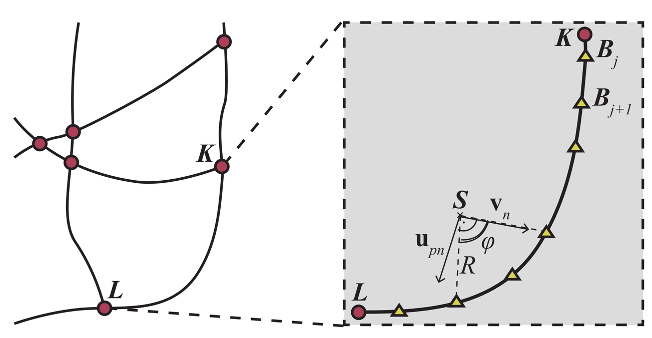

Figure: Segment representation of transportation network

- A suitable segment of a geometric map is selected by a map-matching algorithm.
- The segment is represented by general parametric curve $\mathbf{r} = \mathbf{h}(p)$ with known vector function $\mathbf{h}$ and parameter p, it holds that $\mathbf{r}^0(T_R;T_R) = \mathbf{h}(p^0)$.
- Two types of map segments were chosen.
  - A line segment is defined as

$$\mathbf{r} = \mathbf{B}_j + \mathbf{d}_j p$$

  with starting point $\mathbf{B}_j$ and ending point $\mathbf{B}_{j+1}$, nonzero vector $\mathbf{d} = \mathbf{B}_{j+1} - \mathbf{B}_j$, and $p \in \langle 0;1\rangle$.
  - A circular segment represented as

$$\mathbf{r} = \mathbf{S} + R\left(\mathbf{u}_{pn}\sin(p) + \mathbf{v}_n\cos(p)\right),$$

  where $\mathbf{v}_n$ and $\mathbf{u}_{pn}$ are orthogonal normalized vectors pointing from center $\mathbf{S}$, $R > 0$ is the radius, parameter $p \in \langle 0;\phi\rangle$, and $\phi \neq 0$.

GNSS positioning as an optimization problem

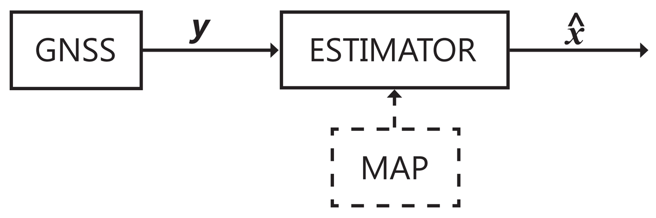

Figure: A general scheme for GNSS map-aided positioning

- The main objective is to design a state estimator which minimizes the least square criterion V written as follows

$$\hat{\mathbf{x}} = \arg\min V(\mathbf{x}),$$
$$V(\mathbf{x}) = (\mathbf{y} - \mathbf{g}(\mathbf{x}))^T\,\Sigma^{-1}\,(\mathbf{y} - \mathbf{g}(\mathbf{x})).$$

- Minimization problem is solved by Gauss-Newton's method.

Positioning with map: Minimum Distance Projection Method (MDPM)

- A constrained estimate in MDPM is found as a minimum distance point of a digital map to an unconstrained position estimate $\hat{\mathbf{r}}$. The constraint estimate has to satisfy $\hat{\mathbf{r}}^{PM}(T_R) = \mathbf{h}(\hat{p}^{PM})$,

$$\left[\frac{\partial\mathbf{h}}{\partial p}\right]^T_{p=\hat{p}^{PM}}\left(\hat{\mathbf{r}}^{PM}(T_R) - \hat{\mathbf{r}}\left(T_R;\hat{T}_R\right)\right) = 0.$$

  - The condition to a line constraint reduces to $\mathbf{d}_j^T\left(\mathbf{B}_j + \mathbf{d}_j\hat{p}^{PM} - \hat{\mathbf{r}}\left(T_R;\hat{T}_R\right)\right) = 0$.
  - The condition to a circular arc satisfies $\mathbf{u}_{pn}^T\mathbf{m}\cos\left(\hat{p}^{PM}\right) - \mathbf{v}_n^T\mathbf{m}\sin\left(\hat{p}^{PM}\right) = 0$, where $\mathbf{m} = \mathbf{S} - \hat{\mathbf{r}}\left(T_R;\hat{T}_R\right)$.

Positioning with map: Constraint Substitution Method (CSM)

- A new pseudorange measurement equation for CSM is obtained by substituting the parametric representation of the receivers position (map segment) into the pseudorange measurement equation, therefore

$$\mathbf{y} = \mathbf{g}\left(\left[\mathbf{h}(p^0), b^0\right]\right) + \mathbf{v},$$

  where the couple $\left[p^0, b^0\right]^T$ is a new state of the receiver.
- Solution found using Gauss-Newton's method with relevant Jacobian matrix.
- The position estimate of the receiver is determined from estimated parameters using suitable parametric equation of a map segment.

Illustrative example

- A position of the receiver was chosen together with passing line and circular arc segments. Ephemeris data provided positions of visible satellites.
- The quality of estimates in terms of the mean square error (MSE) at each time instant for 180 steps evaluated using 1000 Monte Carlo simulations and respecting the measurement model and variance $\sigma^2 = 2$. The quality of satellite constellations is measured by geometric dilution of precision (GDOP).

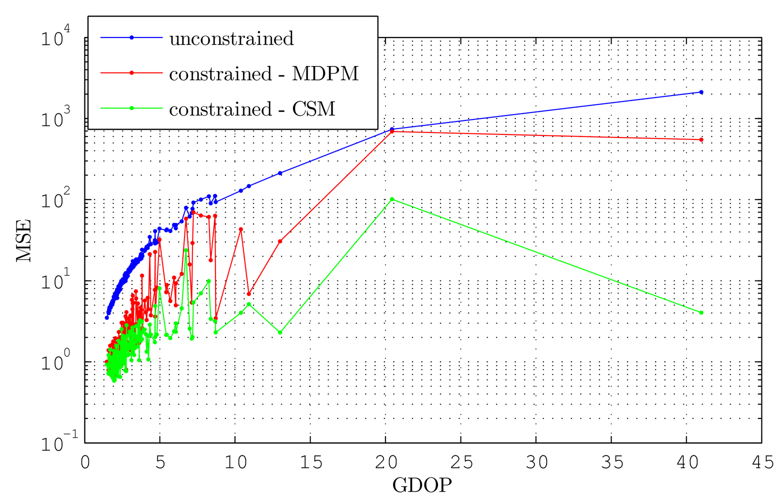

Figure: Dependence of MSE on GDOP for position estimation methods considering a line segment

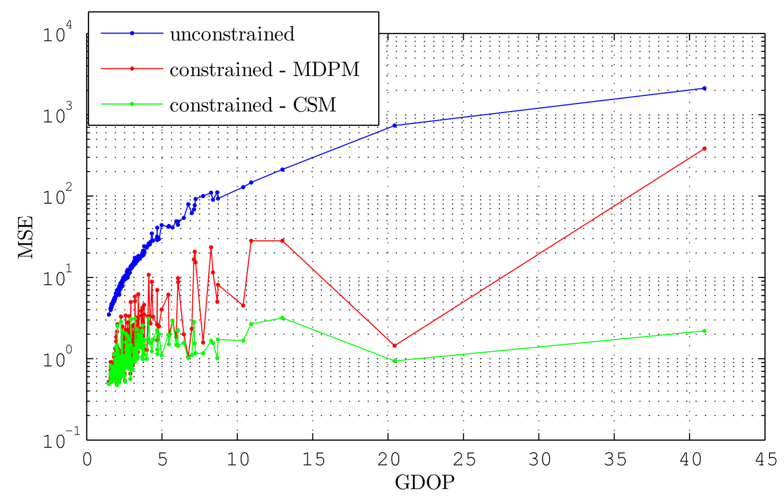

Figure: Dependence of MSE on GDOP for position estimation methods considering a circular segment

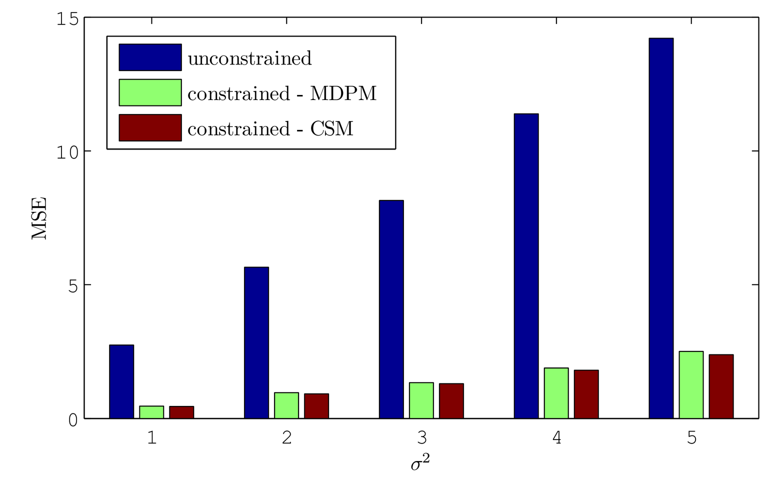

Figure: Dependance of MSE on the variance of measurement noise considering a line segment

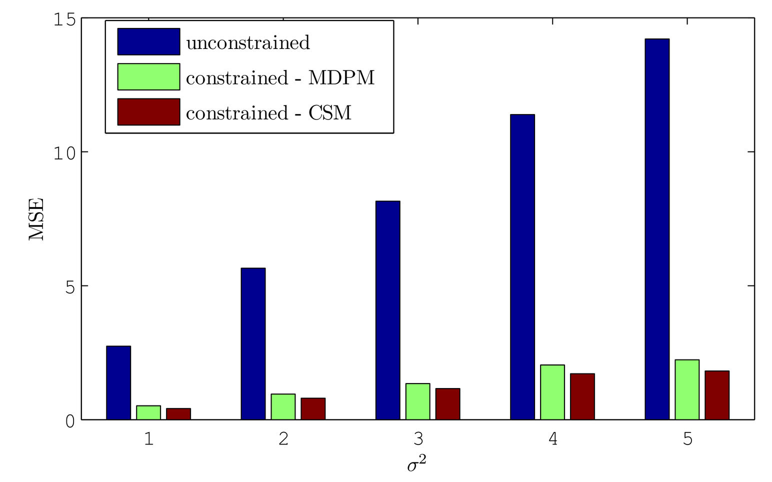

Figure: Dependance of MSE on the variance of measurement noise considering a circular segment

Conclusion

- A particular attention was paid to the differences between coordinate systems.
- Two methods for information processing using GNSS signals and digital maps were shown.
- It was shown that a precise digital map can lead to significant improvement of the estimation quality (especially in the cases of bad constellation).

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