Approximate active fault detection and control

Jan Škach   Ivo Punčochář   Miroslav Šimandl

Department of Cybernetics
Faculty of Applied Sciences
University of West Bohemia
Pilsen, Czech Republic

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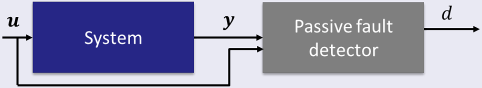
Outline

Introduction

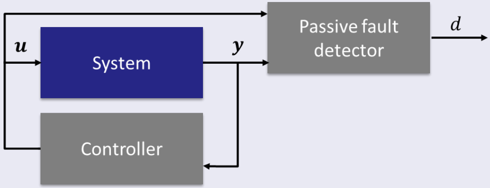
Passive fault detector and controller



- **Passive fault detector** uses the input and output data [**u**, **y**] to generate a decision *d* about faults in the system, no input signal improving the quality of detection is generated.
- **Controller** works separately. It uses the output data **y** to generate an input signal **u** that controls the system.

Introduction

Passive fault detector and controller



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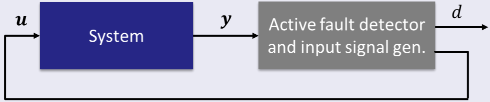
Introduction

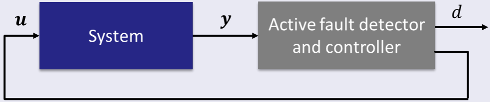
Active fault detector and controller



- **Active fault detector** uses the output data **y** to generate a decision d and an input signal **u** that **probes** the system to ensure improved quality of detection.
- **Active fault detector and controller** uses the output data **y** to generate a decision d and an input signal **u** which **probes and controls** the system.

Introduction

Active fault detector and controller



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Introduction

Goal of the paper

- To design an **active fault detector and controller** (AFDC) for nonlinear systems over an infinite time horizon with a discounted criterion.
- To demonstrate the proposed AFDC in a numerical example.

Problem formulation

System description

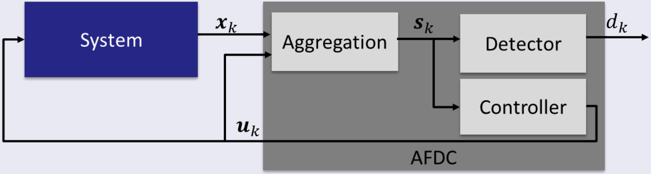
The multiple-model approach is considered (one model fault-free, other faulty, $\mu_k \in \mathcal{M} = \{1,2,\ldots,N\}$ is unknown model index).
A system with the perfect state information described by time-invariant model

$$\mathbf{s}_{k+1} = \phi(\mathbf{s}_k, \mathbf{u}_k, \mathbf{x}_{k+1}), \qquad (1)$$

$\mathbf{s}_k = [\mathbf{x}_k,\ \mathbf{b}_k]^T \in \mathcal{S}$ is a hyper-state (perfect state information),
$\mathbf{x}_k \in \mathbb{R}^{n_x}$ is a common state (x_{k+1} defined by $p(x_{k+1}|\mathbf{s}_k, \mathbf{u}_k)$),
$\mathbf{b}_k = [b_{k,1},\ldots,b_{k,i},\ldots,b_{k,N-1}]^T \in \mathcal{B}$ is a belief state of system models, $b_{k,i} = P(\mu_k = i|\mathbf{x}_0^k, \mathbf{u}_0^{k-1})$, ϕ is a nonlinear vector function,
$\mathbf{u}_k \in \mathcal{U} = \{\bar{\mathbf{u}}^1, \ldots, \bar{\mathbf{u}}^M\} \subset \mathbb{R}^{n_u}$ is an admissible control,
$P_{i,j} = P(\mu_{k+1} = j|\mu_k = i)$, $\mathbf{x}_0$, and $P(\mu_0)$ are known.

Problem formulation

Active fault detector and controller



Two actions: **decision** $d_k \in \mathcal{M}$ and **control** $\mathbf{u}_k \in \mathcal{U}$,

$$\begin{bmatrix} d_k \\ \mathbf{u}_k \end{bmatrix} = \begin{bmatrix} \sigma(\mathbf{s}_k) \\ \gamma(\mathbf{s}_k) \end{bmatrix} = \bar{\rho}(\mathbf{s}_k), \tag{2}$$

$\bar{\rho} : \mathcal{S} \mapsto \mathcal{M} \times \mathcal{U}$ is an unknown policy, $\sigma : \mathcal{S} \mapsto \mathcal{M}$ is a detector, $\gamma : \mathcal{S} \mapsto \mathcal{U}$ is a controller.

Problem formulation

Optimality criterion

Optimality criterion is given by

$$\bar{J}(\bar{\boldsymbol{\rho}}, \mathbf{s}_0) = \lim_{F \to \infty} \mathrm{E}\left\{ \sum_{k=0}^{F} \lambda^k \bar{L}(d_k, \mathbf{s}_k, \mathbf{u}_k) | \mathbf{s}_0 \right\}, \tag{3}$$

where $\bar{L}(d_k, \mathbf{s}_k, \mathbf{u}_k) = \alpha \bar{L}^{\mathrm{d}}(d_k, \mathbf{s}_k) + (1-\alpha)\bar{L}^{\mathrm{c}}(\mathbf{s}_k, \mathbf{u}_k)$ is a cost function (CF), $\alpha \in [0; 1]$ is a weighting factor,
$\bar{L}^{\mathrm{d}}(d_k, \mathbf{s}_k) = \mathrm{E}\{L^{\mathrm{d}}(\mu_k, d_k) | d_k, \mathbf{x}_0^k, \mathbf{u}_0^{k-1}\}$, is a detection CF ($L^{\mathrm{d}} : \mathcal{M} \times \mathcal{M} \mapsto \mathbb{R}^+$ is the original detection CF),
$\bar{L}^{\mathrm{c}}(\mathbf{s}_k, \mathbf{u}_k) = L^{\mathrm{c}}([\mathbf{s}_{k,1}, \ldots, \mathbf{s}_{k,n_x}]^{\mathrm{T}}, \mathbf{u}_k)$ is a control CF ($L^{\mathrm{c}} : \mathbb{R}^{n_x} \times \mathcal{U} \mapsto \mathbb{R}^+$ is the original control CF).
Assume L^{d}, L^{c}, and L are bounded making the criterion (3) well defined for any policy $\bar{\boldsymbol{\rho}}$.

Optimal active fault detector and controller

Design

The goal is to find Bellman function V^* that solves the following Bellman functional equation

$$V^*(\mathbf{s}_k) = \min_{d_k \in \mathcal{M}, \mathbf{u}_k \in \mathcal{U}} \mathrm{E}\left\{ \bar{L}(d_k, \mathbf{s}_k, \mathbf{u}_k) + \lambda V^*(\mathbf{s}_{k+1}) | d_k, \mathbf{s}_k, \mathbf{u}_k \right\}. \quad (4)$$

Optimal detector σ^* and optimal controller γ^* are given as

$$d_k^* = \sigma^*(\mathbf{s}_k) = \arg\min_{d_k \in \mathcal{M}} \alpha \bar{L}^{\mathrm{d}}(\mathbf{s}_k, d_k), \quad (5)$$

$$\mathbf{u}_k^* = \gamma^*(\mathbf{s}_k) = \arg\min_{\mathbf{u}_k \in \mathcal{U}} \mathrm{E}\left\{ (1-\alpha)\bar{L}^{\mathrm{c}}(\mathbf{s}_k, \mathbf{u}_k) + \lambda V^*(\mathbf{s}_{k+1}) | \mathbf{s}_k, \mathbf{u}_k \right\}. \quad (6)$$

The Bellman function V^* is computed offline by solving (4), then the AFDC is implemented online by means of (5) and (6).

Approximate active fault detector and controller

Analytical solution to the Bellman equation is impossible to find in this case. Numerical methods are employed.

Numerical solution to the Bellman equation

- The hyper-state space is quantized by a uniform grid with grid points $\bar{\mathbf{s}} \in \mathbb{R}^{n_s}$. Hyper-states are projected to the grid using an aggregation function.
- The Bellman function is approximated by a piecewise constant function $\bar{V}$ found by a numerical method of dynamic programming (e.g. value iteration method).
- Due to nonlinearity of the system, the expectation in the Bellman equation is approximated by the Unscented transform.

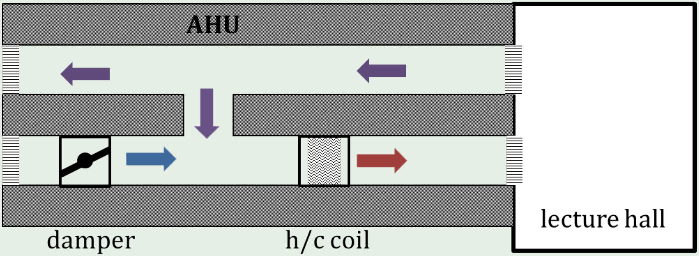
Numerical example

Air handling unit

- An example of an air handling unit (AHU) is considered.
- The AHU mixes the ambient air and indoor air together with ratio proportional to a damper position. The mixed air is then heated or cooled by a coil before entering a lecture hall.
- Two nonlinear discretized state-space models are considered: a fault-free model and a faulty model with a stuck damper in fully opened position.
- Discretization by the forward Euler method with sampling period $T_{\mathrm{s}} = 300s$.

Numerical example

Air handling unit



- A goal is to detect a stuck damper in the AHU and to control the indoor air temperature (AT) in a lecture hall.
- $\mathbf{x}_k = [\mathbf{x}_{k,1}, \mathbf{x}_{k,2}]^{\mathsf{T}}$, $\mathbf{x}_{k,1}$ is the indoor AT, $\mathbf{x}_{k,2}$ is the ambient AT, $\mathbf{u}_k = [\mathbf{u}_{k,1}, \mathbf{u}_{k,2}]^{\mathsf{T}}$, $\mathbf{u}_{k,1} \in \mathcal{U}^{\mathsf{L}} = \{-1, 0, 1\}$ is the coil control, $\mathbf{u}_{k,2} \in \mathcal{U}^{\mathsf{N}} = \{0, 0.1, \ldots, 0.9\}$ is the damper position.

Numerical example

Detection

The original detection cost function aims at correct system model detection

$$L^{\mathrm{d}}(\mu_k, d_k) = \begin{cases} 0 & \text{if } d_k = \mu_k, \\ 1 & \text{otherwise.} \end{cases}$$

Control

An objective of the original control cost function is to control the indoor air temperature $\mathbf{x}_{k,1}$ to a reference temperature $x^{\mathrm{ref}} = 23\,^{\circ}\mathrm{C}$

$$L^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k) = \sum_{i=1}^{n_u} |\mathbf{p}_i^{\mathrm{hc}} \mathbf{u}_{k,i}| + \mathbf{q}_1 \left(1 - e^{-\mathbf{q}_2 (\mathbf{x}_{k,1} - x^{\mathrm{ref}})^2}\right),$$

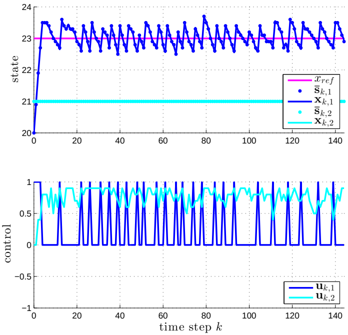
where $\mathbf{p}^{\mathrm{hc}} = \left[\mathbf{p}_1^{\mathrm{hc}}, \mathbf{p}_2^{\mathrm{hc}}\right]^{\mathsf{T}} = [1, 0]^{\mathsf{T}}$ and $\mathbf{q} = [\mathbf{q}_1, \mathbf{q}_2]^{\mathsf{T}} = [60, 1]^{\mathsf{T}}$ are parameters. Note that control actions $\mathbf{u}_k$ are penalized as well.

Numerical example

Simulation conditions

- The ambient air temperature is constant $\mathbf{x}_{k,2} = 21\,^\circ\mathrm{C}$.
- The common state $\mathbf{x}_{k,1}$ is influenced by a state noise defined by the Laplace distribution with the location parameter $\varpi = 0$ and the scale parameter $\beta = 0.08$.
- The hyper-state $\mathbf{s}_k$ is quantized by a uniform grid defined by a Cartesian product $\mathcal{S}^{\mathrm{g}} \equiv \mathcal{S}_1^{\mathrm{g}} \times \mathcal{S}_2^{\mathrm{g}} \times \mathcal{S}_3^{\mathrm{g}} = \{5, 5.1, \ldots, 30\} \times \{21\} \times \{0, 0.01, \ldots, 1\}$.
- The weighting factor set to $\alpha = 0.99$ which compromises the detection and control objectives as shown later.
- Value iteration stopped after 30 iterations, $\lambda = 0.98$, $P(\mu_0 = 1) = 1$, and $P(\mu_{k+1} = i | \mu_k = j) = 0.02$ for $i, j \in \mathcal{M}$, $i \neq j$.

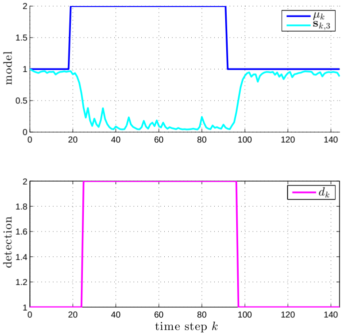
Numerical example



- Typical state trajectories and system control for the time horizon of 12 hours.

- The indoor air temperature $\mathbf{x}_{k,1}$ follows x^{ref} with oscillations caused by a discrete amount of power delivered during T_{s}.
- The damper is almost closed to the ambient air because $\mathbf{x}_{k,2} < x^{\text{ref}}$.

Numerical example

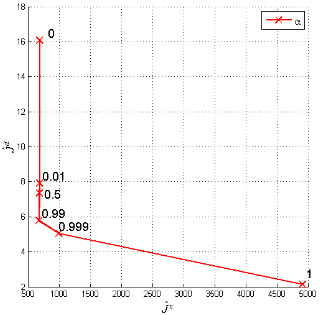


- Typical true model, probability of the fault-free model, and a decision of the AFDC for the time horizon of 12 hours.

- The actual model is correctly detected with a delay of approximately 5 steps.
- The detection and control aims are fulfilled.

Numerical example

Consider detection $J^{\mathrm{d}} = \lim_{F\to+\infty} \mathrm{E}\{\sum_{k=0}^{F} \lambda^k L^{\mathrm{d}}(\mu_k, d_k)\}$ and control $J^{\mathrm{c}} = \lim_{F\to+\infty} \mathrm{E}\{\lambda^k L^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k)\}$ parts of the criterion (3).



- Pareto front for the AHU optimization problem indicating an influence of the weighting factor α on estimates of detection and control parts of the criterion (3), $F = 42$ hours, and 10000 Monte Carlo simulations was used.

- The values of estimate $\hat{J}^{\mathrm{c}}$ remains approximately the same for $\alpha \in [0, 0.99]$, only quality of detection changes.

Conclusion

Conclusion

- The problem formulation allows a compromise between detection and control aims.
- The approximate active fault detector and controller for nonlinear stochastic systems over an infinite time horizon was designed.
- The quality of the AFDC depends on approximations employed.
- The utility of the presented approach was demonstrated in the numerical example of an air handling unit.

References

A list of references relevant to the topic.

- Bertsekas, D.P. and Tsitsiklis, J.N. (1996). *Neuro-Dynamic Programming*. Athena Scientific, Belmont, Massachusetts.
- Šimandl, M. and Punčochář, I. (2009). Active fault detection and control: Unified formulation and optimal design. *Automatica*, 45(9), 2052-2059.
- Šimandl, M., Škach, J., and Punčochář, I. (2014). Approximation Methods for Optimal Active Fault Detection. In *Proceedings of the 22nd Mediterranean Conference on Control and Automation (MED)*, 103-108. Palermo, Italy.