Infinite Time Horizon Active Fault Diagnosis based on Approximate Dynamic Programming

Ivo Punčochář Jan Škach Miroslav Šimandl

Identification and Decision Making Research Group (IDM)
Department of Cybernetics, Faculty of Applied Sciences
University of West Bohemia
Pilsen, Czech Republic
janskach@kky.zcu.cz, idm-group@control.zcu.cz

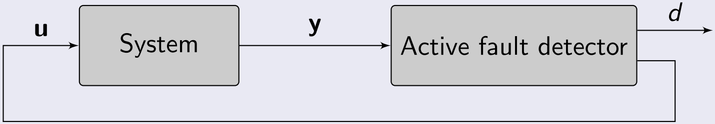
54th Conference on Decision and Control, Osaka, Japan

Outline

Introduction

Fault detection

- Detection of abrupt changes is important to prevent economical losses and environmental disasters.
- **Passive fault detectors** generate a decision d about faults in the system, no input signal improving quality of detection is generated.
- **Active fault detectors** use the output data $\mathbf{y}$ to generate the decision d and an input signal $\mathbf{u}$ that **probes** the system to improve quality of detection.



Introduction

Classification of active fault detection approaches

- **Deterministic** approaches assume disturbances to be bounded signals.
- **Stochastic** approaches assume disturbances to be random processes with known distributions.
- Stochastic active fault detection based on a general detection design criterion is considered.

Goal of the paper

- Previous papers [1]-[3] dealt with optimal active fault detector design on infinite time horizon with directly observable state.
- A goal is to formulate and solve the problem when the state is observed only through noisy measurements.

Problem formulation

Imperfect state information model of the system

The following multiple model is considered

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}_{\mu_k}\mathbf{x}_k + \mathbf{B}_{\mu_k}\mathbf{u}_k + \mathbf{G}_{\mu_k}\mathbf{w}_k, \\ \mathbf{y}_k &= \mathbf{C}_{\mu_k}\mathbf{x}_k + \mathbf{H}_{\mu_k}\mathbf{v}_k, \end{aligned} \quad k = 0, 1, \dots,$$

- $\mathbf{x}_k \in \mathbb{R}^{n_x}$ is a unknown common state, $\mu_k \in \mathcal{M} = \{1, 2, \dots, N_\mu\}$ is an unknown index of model ($\mu_k = 1$: fault-free model, $\mu_k > 1$: faulty models), $\mathbf{x}_k^a = [x_k^T, \mu_k]^T \in \mathbb{R}^{n_x} \times \mathcal{M}$ is an unknown hybrid-state,
- $P_{ji} = P(\mu_{k+1} = j | \mu_k = i)$ are known transition probabilities,
- $\mathbf{y}_k \in \mathbb{R}^{n_y}$ is a measured output, $\mathbf{u}_k \in \mathcal{U} \subset \mathbb{R}^{n_u}$ is an input,
- $\mathbf{w}_k \in \mathbb{R}^{n_w}$, $\mathbf{v}_k \in \mathbb{R}^{n_v}$ are mutually independent white noises with known Gaussian distributions, initial values $\mathbf{x}_0$ and μ_0 are known.
- $\mathbf{A}_{\mu_k}$, $\mathbf{B}_{\mu_k}$, $\mathbf{C}_{\mu_k}$, $\mathbf{G}_{\mu_k}$, and $\mathbf{H}_{\mu_k}$ are known matrices.

Problem formulation

Perfect state information model

- A sufficient statistic for the hybrid-state $\mathbf{x}^a_k$ is needed.

State estimation for linear stochastic Markovian switching system

- The conditional probability of μ_k and $\mathbf{x}_k$ can be expressed as

$$P(\mu_k|\mathbf{I}_0^k) = \sum_{\mu_0^{k-1}} P(\mu_0^k|\mathbf{I}_0^k), \qquad \mathbf{I}_0^k = [\mathbf{y}_0^k, \mathbf{u}_0^{k-1}],$$

$$P(\mathbf{x}_k|\mathbf{I}_0^k) = \sum_{\mu_0^k} P(\mu_0^k|\mathbf{I}_0^k) P(\mathbf{x}_k|\mathbf{I}_0^k, \mu_0^k),$$

- where $\sum_{\mu_0^k}$ is a sum over all $(N_\mu)^{k+1}$ possible model sequences μ_0^k,

$$P(\mu_0^k|\mathbf{I}_0^k) = \frac{p(\mathbf{y}_k|\mathbf{I}_0^{k-1}, \mathbf{u}_{k-1}, \mu_0^k) P(\mu_0^k|\mathbf{I}_0^{k-1})}{p(\mathbf{y}_k|\mathbf{I}_0^{k-1}, \mathbf{u}_{k-1})}.$$

Problem formulation

State estimation for linear stochastic Markovian switching system

- For a given model sequence μ_0^k the optimal state estimate is obtained by the Kalman filter, that is to estimate:
  1. the output mean value and the covariance matrix of the Gaussian distribution $p(\mathbf{y}_k|\mathbf{I}_0^{k-1}, \mathbf{u}_{k-1}, \mu_0^k)$,
  2. the filtering state mean value and the covariance matrix of the Gaussian distribution $p(\mathbf{x}_k|\mathbf{I}_0^k, \mu_0^k)$,
  3. the predictive state mean value and the covariance matrix of the Gaussian distribution $p(\mathbf{x}_{k+1}|\mathbf{I}_0^k, \mathbf{u}_k, \mu_0^k)$.
- The number of possible model sequences grows exponentially with k.
- Only h-step history of the model sequences is considered whenever $k \geq h$, i.e. μ_{k-h}^k.

Active fault detector design

Perfect state information model using hyper-state - reformulation

The earlier system model is reformulated such that

$$\boldsymbol{\xi}_{k+1} = \phi(\boldsymbol{\xi}_k, \mathbf{u}_k, \mathbf{y}_{k+1}),$$

- $\boldsymbol{\xi}_k \in \mathcal{G} \subset \mathbb{R}^{n_\xi}$ is a hyper-state that consists of the filtering estimates of $\mathbf{x}^a_k$, that is:
  1. common state mean values $\hat{\mathbf{x}}_{k|k}(\mu^k_{k+h-1})$,
  2. common state covariance matrices $\boldsymbol{\Sigma}^{\mathbf{x}}_{k|k}(\mu^k_{k+h-1})$,
  3. probabilities of model sequences $P(\mu^k_{k-h+1}|\mathbf{I}^k_0)$,
- $\phi : \mathcal{G} \times \mathcal{U} \times \mathbb{R}^{n_y} \mapsto \mathcal{G}$ is a hyper-state time-invariant function that includes the state estimation algorithm,
- $\mathbf{y}_{k+1}$ is a random variable with Gaussian sum pdf $p(\mathbf{y}_{k+1}|\boldsymbol{\xi}_k, \mathbf{u}_k)$ that can be computed.

Active fault detector design

Active fault detector structure and design

Active fault detector has a form of a time-invariant mapping

$$\begin{bmatrix} d_k \\ \mathbf{u}_k \end{bmatrix} = \boldsymbol{\rho}(\boldsymbol{\xi}_k) = \begin{bmatrix} \sigma(\boldsymbol{\xi}_k) \\ \boldsymbol{\gamma}(\boldsymbol{\xi}_k) \end{bmatrix},$$

- $d_k \in \mathcal{M}$ is the decision, $\mathbf{u}_k$ is the auxiliary input signal, and $\boldsymbol{\rho} : \mathcal{G} \mapsto \mathcal{M} \times \mathcal{U}$ is an unknown function, $\sigma : \mathcal{G} \mapsto \mathcal{M}$, $\boldsymbol{\gamma} : \mathcal{G} \mapsto \mathcal{U}$,

$\boldsymbol{\rho}$ is designed such that it minimizes a general detection design criterion

$$J^{\mathrm{OAFD}}(\boldsymbol{\rho}) = \lim_{F\to\infty} \mathbb{E}\left\{ \sum_{k=0}^{F} \lambda^k \bar{L}^{\mathrm{d}}(\boldsymbol{\xi}_k, d_k) \right\},$$

- $\lambda \in (0,1)$ is a discount factor, $\bar{L}^{\mathrm{d}} : \mathcal{G} \times \mathcal{M} \mapsto \mathbb{R}^+$ is a general detection cost function penalizing wrong decisions about faults.

Active fault detector design

Active fault detector and the Bellman function

Application of dynamic programming results in the problem of finding a function V^* that satisfies the following Bellman functional equation

$$V^*(\boldsymbol{\xi}_k) = \min_{d_k \in \mathcal{M}} \mathbb{E}\{\bar{L}^{\mathrm{d}}(\boldsymbol{\xi}_k, d_k)|\boldsymbol{\xi}_k, d_k\} + \min_{\mathbf{u}_k \in \mathcal{U}} \mathbb{E}\{\lambda V^*(\phi(\boldsymbol{\xi}_k, \mathbf{u}_k, \mathbf{y}_{k+1}))|\boldsymbol{\xi}_k, \mathbf{u}_k\},$$

- $V^* : \mathcal{G} \mapsto \mathbb{R}$ is an unknown Bellman function.

The optimal active fault detector $\boldsymbol{\rho}^*$ (in form of the optimal detector σ^* and the optimal auxiliary input signal generator γ^*) is then given as

$$d_k^* = \sigma^*(\boldsymbol{\xi}_k) = \arg\min_{d_k \in \mathcal{M}} \mathbb{E}\{\bar{L}^{\mathrm{d}}(\boldsymbol{\xi}_k, d_k)|\boldsymbol{\xi}_k, d_k\},$$

$$\mathbf{u}_k^* = \gamma^*(\boldsymbol{\xi}_k) = \arg\min_{\mathbf{u}_k \in \mathcal{U}} \mathbb{E}\{V^*(\phi(\boldsymbol{\xi}_k, \mathbf{u}_k, \mathbf{y}_{k+1}))|\boldsymbol{\xi}_k, \mathbf{u}_k\}.$$

Active fault detector design

Numerical solution by nonparametric local approximation

- The given Bellman functional equation cannot be solved analytically, therefore, a numerical solution is proposed.
- A nonparametric local approximation of the Bellman function and a value iteration algorithm are used to obtain approximate solution to the Bellman functional equation.
- A kernel average smoother with a Gaussian kernel function is used

$$V^*(\xi) \approx \tilde{V}(\xi) = \frac{\sum_{i=1}^{N_\xi} K(\xi, \xi_i^s) V_i^s}{\sum_{i=1}^{N_\xi} K(\xi, \xi_i^s)},$$

- where $\tilde{V} : \mathcal{G} \mapsto \mathbb{R}$ is an approximation to V^*, $\xi_i^s \in \Xi \subset \mathcal{G}$ is a sample hyper-state, $\Xi = \{\xi_1^s, \dots, \xi_{N_\xi}^s\}$, V_i^s is an approximate value of the Bellman function at ξ_i^s, and $K : \mathcal{G} \times \Xi \mapsto \mathbb{R}$ is the kernel function.
- The sample hyper-states are chosen through simulations of the system under various input signals.

Active fault detector design

Numerical solution by nonparametric local approximation

First, sample system trajectories based on various input signals are generated to obtain sample hyper-states $\boldsymbol{\xi}_i^s$.

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Then, the value iteration algorithm is used to train the nonparametrix approximation of the Bellman function, that is V_i^s for all $i = 1, \dots, N_\xi$.

↓

Finally, the nonparametric approximation is used to approximate a value of the Bellman function at a system trajectory point $\boldsymbol{\xi}$. Consequently, the decision d and auxiliary input signal $\mathbf{u}$ can be generated.

Numerical example

A second-order linearized model of a pendulum

The following linear continuous-time model is considered

$$\begin{bmatrix} \dot{\theta}(t) \\ \ddot{\theta}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-g}{l} & \frac{-\beta}{ml^2} \end{bmatrix} \begin{bmatrix} \theta(t) \\ \dot{\theta}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} \tau(t),$$

- $\theta(t)$ [rad] is an angle of displacement from the zero downward position, l [m] is a length of pole, m [kg] is a mass of pendulum, β [$\mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^{-1}$] is a friction coefficient, g [$\mathrm{m} \cdot \mathrm{s}^{-2}$] is the gravitational acceleration, and $\tau(t)$ [$\mathrm{N} \cdot \mathrm{m}$] is a moment of a force applied at the pendulum joint.
- Discrete-time models are obtained by discretization with a sampling period $T_s = 5 \cdot 10^{-2}$ [s] and the following parameter values.

Model	Length l	Mass m	Friction coefficient β
Fault-free $\mu_k = 1$	1	2	6
Fault 1 $\mu_k = 2$	1	2	6.2
Fault 2 $\mu_k = 3$	1.005	2	6

Numerical example

Simulation example settings

- The angle of displacement is only measured, thus, $\mathbf{C}_{\mu_k} = [1\ 0]$, $\forall \mu_k$.
- It is considered that $\mathbf{G}_{\mu_k} = 8 \cdot 10^{-4}\mathbf{I}_2$, $\mathbf{H}_{\mu_k} = 1 \cdot 10^{-3}$, $\hat{\mathbf{x}}_{0|-1}^{\mathrm{T}} = [0\ 0]$, $\mathbf{\Sigma}_{0|-1}^{\mathbf{x}} = 2 \cdot 10^{-4}\mathbf{I}_2$, $P(\mu_0 = 1) = 1$, $\lambda = 0.98$, and $h = 1$.
- The set of admissible inputs is $\mathcal{U} = \{-10, 0, 10\}$ and the model transition probabilities and the detection cost function are set to
$$P_{ji} = \begin{bmatrix} 0.98 & 0.01 & 0 \\ 0.01 & 0.98 & 0 \\ 0.01 & 0.01 & 1 \end{bmatrix}, \quad L^{\mathrm{d}}(\mu_k, d_k) = \mathbf{L}_{\mu_k, d_k} = \begin{bmatrix} 0 & 2 & 2 \\ 3 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix}.$$
- The sample hyper-states for the nonparametric approximation of the Bellman function were obtained on a time horizon of 500 steps using the following input signals:
  - constant signals with values 0 and 10, random signal (uniform distribution over $\mathcal{U}$), and sine signals with amplitude 10 and frequencies 0.1 [Hz] and 1 [Hz].

Numerical example

Simulation results

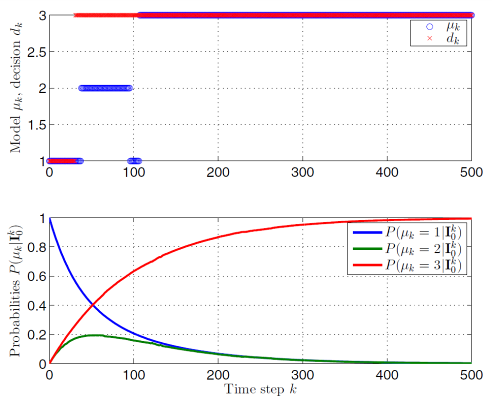
- Performance (in terms of estimates of the criterion $\hat{J}$ and variance var$\{\hat{J}\}$) of the designed active fault detector compared to the other input signals was evaluated through 1000 Monte Carlo simulations.

Input signal generator	$\hat{J}$	var$\{\hat{J}\}$
Constant (0)	36.7003	$0.2937 \cdot 10^{-3}$
Random	34.0168	$0.3364 \cdot 10^{-3}$
Sine (0.1 [Hz])	32.1226	$0.3177 \cdot 10^{-3}$
Constant (10)	31.1479	$0.2937 \cdot 10^{-3}$
Sine (1 [Hz])	28.1111	$0.2853 \cdot 10^{-3}$
AFD	26.3837	$0.2879 \cdot 10^{-3}$

Numerical example

Simulation results

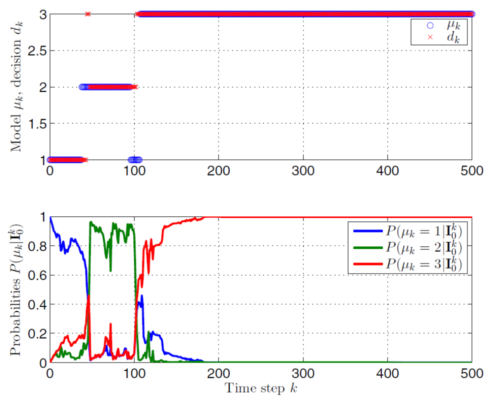
- Typical trajectories of the model μ_k, decision d_k, and conditional model probabilities $P(\mu_k|\mathbf{I}_0^k)$ for the zero input signal generator.



Numerical example

Simulation results

- Typical trajectories of the model μ_k, decision d_k, and conditional model probabilities $P(\mu_k|\mathbf{I}_0^k)$ for the designed active fault detector.



Conclusion

Contributions and conclusion

- An approximate solution to the problem of linear stochastic discrete-time active fault detection with a general detection design criterion was proposed when the system state is not directly observable.
- For the given problem, a new hyper-state structure was formulated and a suitable problem reformulation was proposed in order to find a solution using dynamic programming.
- An approximate solution using a nonparametric local approximation to the Bellman function and the value iteration algorithm was proposed.
- The simulation results indicate good performance of the active fault detector despite various approximations used.

References

A list of references relevant to the topic.

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