A New Bipartizing Matching Conjecture

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Definition. Given a dominating cycle (DC) C in a cubic graph G, we call a matching M in G a *bipartizing matching with respect to C (BM)*, if
(i) $M \cap E(C) = \emptyset$,
(ii) M covers every vertex of $V(G) - V(C)$,
(iii) $G - M$ is homeomorphic to a cubic bipartite (hamiltonian) graph.
If it is obvious with respect to which cycle C the BM M is referred to, we just speak of a BM.

Observation 1. The existence of BMs with respect to any dominating cycle is guaranteed as a side product of the cycle-plus-triangles theorem.

Observation 2. If G has two edge-disjoint BMs with respect to some dominating cycle C, then G has
(a) a 5-CDC (5-cycle double cover) S with $C \in S$,
(b) a NZ5F (nowhere-zero 5-flow) with flow values $1, 2, 3, 4$, and C being cyclically oriented.

The Bipartizing Matching Conjecture (BMC) was the basis of Observation 2.

Bipartizing Matching Conjecture (BMC). *Given a DC in a cyclically 4-edge-connected cubic graph G, then there are two disjoint BMs with respect to C.*

This conjecture was finally disproved by A. Hoffmann-Ostenhof who observed that it suffices to solve the BMC for just one DC to prove the 5CDC and the NZ5FC.

We present an even weaker BMC. We say that a cycle C is *stable* if there is no other cycle C' with $V(C) \subset V(C')$.

Conjecture. *Let C be a stable DC in a cyclically 4-edge-connected cubic graph G. Then there exist two disjoint BMs with respect to C.*

We note that the validity of this conjecture and the validity of the DCC imply the validity of the CDCC, but not the validity of the NZ5FC.