APPLIED THERMODYNAMICS

TUTORIAL 1

REVISION OF ISENTROPIC EFFICIENCY

ADVANCED STEAM CYCLES

INTRODUCTION

This tutorial is designed for students wishing to extend their knowledge of thermodynamics to a more advanced level with practical applications.

- Before you start this tutorial you should be familiar with the following.

- The basic principles of thermodynamics equivalent to level 2.

- Basic steam cycles, mainly the Rankine and Carnot cycles.

- Fluid property tables and charts mainly a set of standard thermodynamic tables and a h - s chart for steam which you must have in your possession.

- The use of entropy.

On completion of the tutorial you should be able to

- understand isentropic efficiency for turbines and compressors.
- describe the use of process steam.
- describe the use of back pressure turbines.
- describe the use of pass out turbines.
- solve steam cycles involving pass out and back pressure turbines.
- describe the use of feed heating and superheating in steam cycles.
- solve problems involving feed heating and re-heating.

You may be very familiar with all these studies in which case you should proceed directly to section 2. For those who wish to revise the basics, section 1 should be completed. This covers

- entropy.
- isentropic processes.
- property diagrams.
- isentropic efficiency.

1. REVISION OF ENTROPY

1.1 DEFINITION

Entropy is a property which measures the usefulness of energy. It is defined most simply as
$dS = dQ/dT$ where

S is entropy
T is temperature
Q is heat transfer

The units of entropy is hence J/k. The units of specific entropy are J/kg K.

1.2. ISENTROPIC PROCESS

ISENTROPIC means constant entropy. Usually (but not always) this means a process with no heat transfer. This follows since if dQ is zero so must be dS.

A process with no heat transfer is called ADIABATIC. An adiabatic process with no friction is hence also ISENTROPIC.

1.3. PROPERTY DIAGRAMS

The two most commonly used property diagrams are

      i. Enthalpy - Entropy (h - s) diagrams and

      ii. Temperature - Entropy (T - s) diagrams.

h-s diagrams are commonly used for steam work. The diagram will hence show the saturation curve. You should familiarise yourself with the h-s diagram for steam and ensure that you can use it to find values of h and s for any pressure, temperature or dryness fraction.
T-s diagrams are commonly used for gas.

1.4. ISENTROPIC EFFICIENCY

Real expansion and compression processes have a degree of friction and this will

- generate heat which is in effect a heat transfer.

- increase the entropy.

- make the final enthalpy bigger than it would otherwise be.

- make the final temperature bigger than it would otherwise be if it is a gas or superheated vapour.

An adiabatic process with friction has no external heat transfer (ΦWatts or Q Joules) but the internal heat generated causes an increase in entropy. Consider the expansion and compression processes on fig.1 and 2.
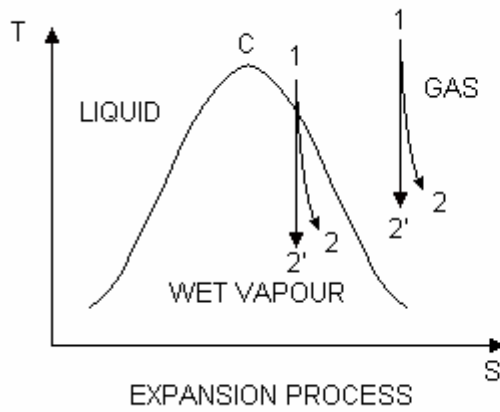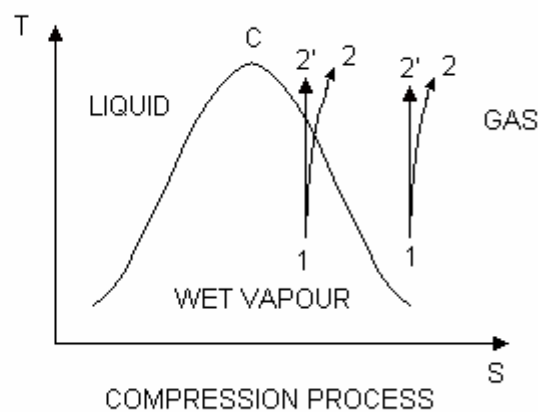

fig.1

fig.2

The ideal change in enthalpy is $\quad\quad\quad\quad h_{2'} - h_1$

The actual change is $\quad\quad\quad\quad\quad\quad h_2 - h_1$

The isentropic efficiency is defined as

$$\eta_{is} = \frac{\Delta h\,(\text{actual})}{\Delta h\,(\text{ideal})} = \frac{h_2 - h_1}{h_{2'} - h_1}\quad\quad\text{for an expansion.}$$

$$\eta_{is} = \frac{\Delta h\,(\text{ideal})}{\Delta h\,(\text{actual})} = \frac{h_{2'} - h_1}{h_2 - h_1}\quad\quad\text{for a compression.}$$

In the case of a perfect gas $h = c_p T$ $\quad\quad\quad$ hence

$$\eta_{is} = \frac{T_2 - T_1}{T_{2'} - T_1}\quad\quad\text{for an expansion}$$

$$\eta_{is} = \frac{T_{2'} - T_1}{T_2 - T_1}\quad\quad\text{for a compression}$$

Note that for an expansion this produces a negative number on the top and bottom lines that cancels out.

WORKED EXAMPLE No.1

A steam turbine takes steam at 70 bar and $500^{o}C$ and expands it to 0.1 bar with an isentropic efficiency 0.9. The process is adiabatic.

The power output of the turbine is 35 MW. Determine the enthalpy at exit and calculate the flow rate of steam in kg/s.

Note you need the tables and h-s chart for steam.

SOLUTION

h_1= 3410 kJ/kg (tables) s_1= 6.796 kJ/kg K   for an ideal expansion s_1=s_2'

Assuming that the steam becomes wet during the expansion, then
$$s_2' = s_f + x' s_{fg}. \text{ at 0.1 bar}$$

$$6.796 = 0.649 + x' \, 7.500 \text{ (tables)} \qquad x' = 0.8196$$

Note if x' is larger than 1 then the steam is still superheated and the solution does not involve x.

Now find h_2'.
$h_2' = h_f + x' h_{fg}.$ at 0.1 bar

h_2'= 192 + (0.8196)(2392) = 2152.2 kJ/kg.

Ideal change in enthalpy $\Delta h'$ =2152.2 - 3410 = -1257.5 kJ/kg

actual change in enthalpy $\Delta h = 0.9(-1257.5) = -1131.7$ kJ/kg

actual change in enthalpy $\Delta h = (h_2 - h_1) = -1131.7$
h_2 -3410 = -1131.7
h_2 =2278.3 kJ/kg

From the steady flow energy equation (with which you should already be familiar) we have
$$\Phi + P = \Delta H/s$$
Since there is no heat transfer then this becomes
$$P = \Delta H/s = m (h_2 - h_1)$$
$$P = m(-1131.7) = -35\,000 \text{ kW}$$
hence   m = 30.926 kg/s

(Note the sign convention used here is negative for energy leaving the system)

WORKED EXAMPLE No.2

A gas turbine expands gas from 1 MPa pressure and 600°C to 100 kPa pressure. The isentropic efficiency 0.92. The mass flow rate is 12 kg/s. Calculate the exit temperature and the power output.

Take $c_v = 718$ J/kg K and $c_p = 1005$ J/kg K

SOLUTION

The process is adiabatic so the ideal temperature $T_{2'}$ is given by

$$T_{2'} = T_1 (r_p)^{1-1/\gamma}$$

r_p is the pressure ratio
$r_p = p_2/p_1 = 0.1$

$\gamma = c_p /c_v = 1.005/0.718 = 1.4$

$$T_{2'} = 873(0.1)^{1-1/1.4} = 451.9 \text{ K}$$

Now we use the isentropic efficiency to find the actual final temperature.

$$\eta_{is} = (T_2 - T_1)/(T_{2'} - T_1)$$

$$0.92 = (T_2 - 873)/(451.9 - 873)$$

$$T_2 = 485.6 \text{ K}$$

Now we use the SFEE to find the power output.

$$\Phi + P = m \, c_p (T_2 - T_1)$$

The process is adiabatic $\Phi = 0$.

$$P = 12(1.005)(485.6 - 873) = -4672 \text{ kW (out of system)}$$

2. BACK-PRESSURE AND PASS-OUT TURBINES

It is assumed that the student is already familiar with steam cycles as this is necessary for this tutorial.

If an industry needs sufficient quantities of process steam (e.g. for sugar refining), then it becomes economical to use the steam generated to produce power as well. This is done with a steam turbine and generator and the process steam is obtained in two ways as follows.

- By exhausting the steam at the required pressure (typically 2 bar) to the process instead of to the condenser.

A turbine designed to do this is called a **BACK-PRESSURE TURBINE.**

- By bleeding steam from an intermediate stage in the expansion process.

A turbine designed to do this is called a **PASS-OUT TURBINE.**

The steam cycle is standard except for these modifications.

2.1. BACK-PRESSURE TURBINES

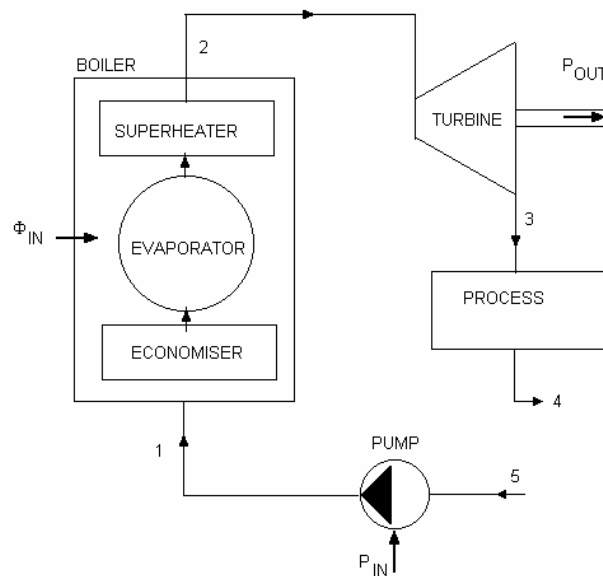The diagram shows the basic circuit. The cycle could use reheat as well but this is not normal.

Figure 3

WORKED EXAMPLE No.3

For a steam circuit as shown previously, the boiler produces superheated steam at 50 bar and 400°C. This is expanded to 3 bar with an isentropic efficiency of 0.9. The exhaust steam is used for a process.

The returning feed water is at 1 bar and 40°C. This is pumped to the boiler. The water leaving the pump is at 40°C and 50 bar. The net power output of the cycle is 60 MW. Calculate the mass flow rate of steam.

SOLUTION

Referring to the cycle sketch previous for location points in the cycle we can find:

$$h_2 = 3196 \text{ kJ/kg} \qquad s_2 = 6.646 \text{ kJ/kg K}$$

For an ideal expansion

$$s_1 = s_2 = 6.646 = s_f + x's_{fg} \text{ at 3 bar}$$
$$6.646 = 1.672 + x'(5.321)$$
$$x' = 0.935$$
$$h_4 = h_f + x'h_{fg} \text{ at 3 bar}$$
$$h_4 = 561 + 0.935(2164)$$
$$h_4 = 2583.9 \text{ kJ/kg}$$

ideal change in enthalpy $\qquad = 2583.9 - 3196 = -612 \text{ kJ/kg}$
actual change in enthalpy $\qquad = 0.9(-612) = -550.9 \text{ kJ/kg}$

The power output of the turbine is found from the steady flow energy equation so :
$$P = m(-550.9) \text{ kW}$$
$$P = -550.9 \text{ m kW (output)}$$

Next we examine the enthalpy change at the pump.
$$h_1 = 168 \text{ kJ/kg at 1 bar and 40}^o\text{C}$$
$$h_2 = 172 \text{ kJ/kg at 50 bar and 40}^o\text{C.}$$

Actual change in enthalpy $= 172 - 169 = 3 \text{ kJ/kg}$

The power input to the pump is found from the steady flow energy equation so :
$$P = -m(3) \text{ kW}$$
$$P = -3 \text{ m kW(input)}$$
Net Power output of the cycle $= 60$ MW hence
$$60\ 000 = 550.9 \text{ m} - 3 \text{ m}$$
$$m = 109.51 \text{ kg/s}$$

2.2. <u>PASS-OUT TURBINES</u>

The circuit of a simple pass-out turbine plant is shown below. Steam is extracted between stages of the turbine for process use. The steam removed must be replaced by make up water at point 6.
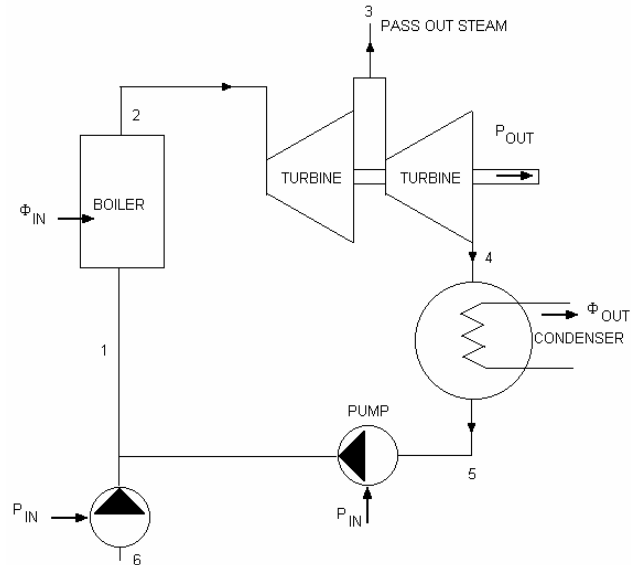

Figure 4

In order to solve problems you need to study the energy balance at the feed pumps more closely so that the enthalpy at inlet to the boiler can be determined. Consider the pumps on their own, as below.

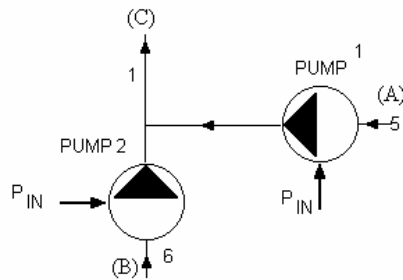

Figure 5

The balance of power is as follows.

$$P_1 + P_2 = \text{increase in enthalpy per second.}$$
$$= m_C h_C - m_A h_A - m_B h_B$$

From this the value of h_C or the mass m_C may be determined. This is best shown with a worked example.

WORKED EXAMPLE No.4

The circuit below shows the information normally available for a feed pump circuit. Determine the enthalpy at entry to the boiler.
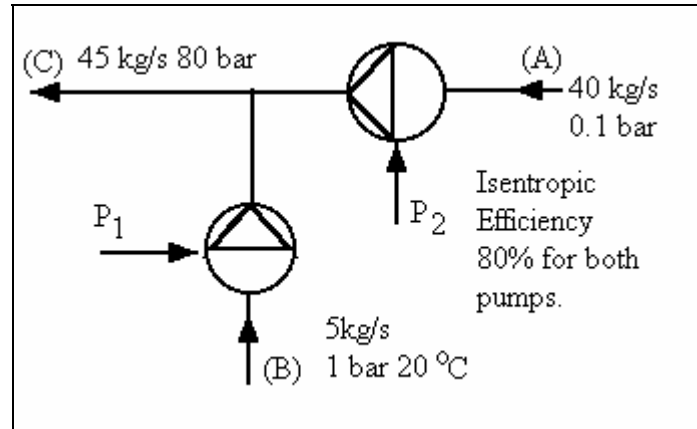
Figure 6

SOLUTION

$$P_1(\text{ideal}) = (5)(0.001)(80-1)(100) = 39.5 \text{ kW}$$
$$P_1(\text{actual}) = 39.5/0.8 = 49.375 \text{ kW}$$

$$P_2(\text{ideal}) = (40)(0.001)(80 - 0.1)(100) = 319.6 \text{ kW}$$
$$P_2(\text{actual}) = 319.6/0.8 = 399.5 \text{ kW}$$

$$\text{Total power input} = 49.375 + 399.5 = 448.9 \text{ kW}$$

$h_A = h_f = 192$ kJ/kg at 0.1 bar

$h_B = 84$ kJ/kg (from water tables or approximately h_f at 20°C) hence

$$448.9 = 45\, h_C - 40\, h_A - 5h_B$$

$$448.9 = 45\, h_C - 40(192) - 5(84)$$

$$h_C = 190 \text{ kJ/kg}$$

WORKED EXAMPLE No.5

The following worked example will show you to solve these problems.

A passout turbine plant works as shown in fig. 4. The boiler produces steam at 60 bar and 500ºC which is expanded through two stages of turbines. The first stage expands to 3 bar where 4 kg/s of steam is removed. The second stage expands to 0.09 bar. The isentropic efficiency is 0.9 for the overall expansion. Assume that the expansion is a straight line on the h - s chart.

The condenser produces saturated water. The make up water is supplied at 1bar and 20ºC. The isentropic efficiency of the pumps is 0.8. The net power output of the cycle is 40 MW. Calculate:

1. the flow rate of steam from the boiler.
2. the heat input to the boiler.
3. the thermal efficiency of the cycle.

SOLUTION

TURBINE EXPANSION $\quad$ h_3 = 3421 kJ/kg from tables.
$\quad\quad\quad\quad\quad\quad\quad\quad\quad$ h_5' = 2165 kJ/kg using isentropic expansion and entropy.
$\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $0.9 = (3421 - h_5) / (3421 - 2165)$ hence h_5 = 2291 kJ/kg

Sketching the process on the h - s chart as a straight line enables h_4 to be picked off at 3 bar. h_4 = 2770 kJ/kg.

POWER OUTPUT $\quad$ $P_{out} = m(h_3 - h_4) + (m - 4)(h_4 - h_5)$
$\quad\quad\quad\quad\quad\quad\quad$ $P_{out} = m(3421 - 2770) + (m - 4)(2770 - 2291)$
$\quad\quad\quad\quad\quad\quad\quad$ $P_{out} = 651 m + 479 m - 1916$

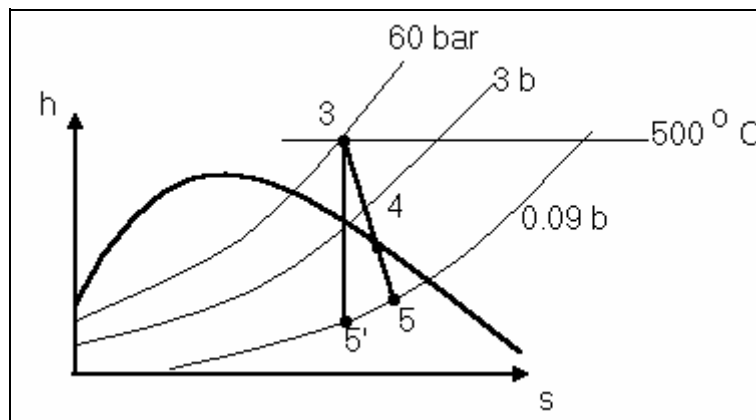Figure 7

POWER INPUT

The power input is to the two feed pumps.
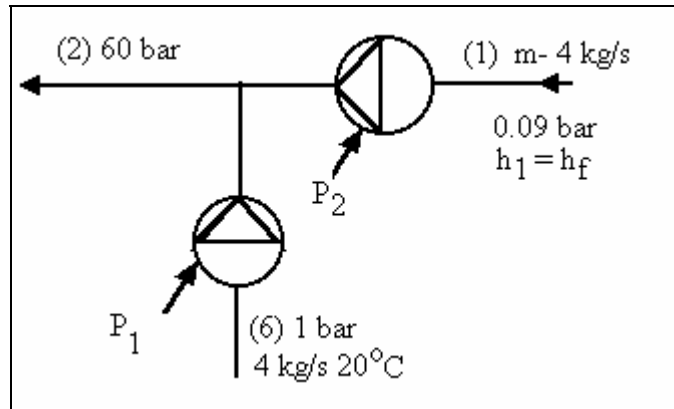
Figure 8

h_6 = 84 kJ/kg (water at 1 bar and 20°C)
h_1 = h_f at 0.09 bar = 183 kJ/kg.

P_1 (ideal) = change in flow energy = 4 x 0.001 x (60 - 1) x 100 kW = 23.6 kW
P_1 (actual) = 23.6 /0.8 = 29.5 kW
P_2(actual) = (m-4) x 0.001 x (60 - 0.09) x 100/0.8 = 7.49m - 29.96 kW

NET POWER

40 000 kW = P_{out} - P_1 - P_2
40 000 = 651 m + 479 m - 1916 - 29.5 - 7.49 m + 29.96
40000 = 1122.5 m - 1916   hence m = 37.34 kg/s

ENERGY BALANCE ON PUMPS

P_1 = 29.5 kW
P_2= 249.4 kW (using the value of m just found)
m h_2 = (m-4) h_1 + P_1 + P_2
37.3 h_2 = 33.34 x 183 + 29.5 + 249.7
hence h_2 = 171 kJ/kg

HEAT INPUT

Heat input = m(h_3 - h_2) = 121355 kW

EFFICIENCY

Efficiency = η = 40/121.3 = 33 %

SELF ASSESSMENT EXERCISE No.3

1. A steam turbine plant is used to supply process steam and power. The plant comprises an economiser, boiler, superheater, turbine, condenser and feed pump. The process steam is extracted between intermediate stages in the turbine at 2 bar pressure. The steam temperature and pressure at outlet from the superheater are 500°C and 70 bar, and at outlet from the turbine the pressure is 0.1 bar. The overall isentropic efficiency of the turbine is 0.87 and that of the feed pump is 0.8.

   Assume that the expansion is represented by a straight line on the h-s chart. The make-up water is at 15°C and 1 bar and it is pumped into the feed line with an isentropic efficiency 0.8 to replace the lost process steam.

   If due allowance is made for the feed pump-work, the net mechanical power delivered by the plant is 30 MW when the process steam load is 5 kg/s. Calculate the rate of steam flow leaving the superheater and the rate of heat transfer to the boiler including the economiser and superheater. Sketch clear T- s and h-s and flow diagrams for the plant. (29.46 kg/s 95.1 MW)

2. The demand for energy from an industrial plant is a steady load of 60 MW of process heat at 117°C and a variable demand of up to 30 MW of power to drive electrical generators. The steam is raised in boilers at 70 bar pressure and superheated to 500°C. The steam is expanded in a turbine and then condensed at 0.05 bar. The process heat is provided by the steam bled from the turbine at an appropriate pressure, and the steam condensed in the process heat exchanger is returned to the feed water line.

   Calculate the amount of steam that has to be raised in the boiler. Assume an overall isentropic efficiency of 0.88 in the turbine. The expansion is represented by a straight line on the h-s diagram. Neglect the feed pump work.

   (Answer 36 kg/s).

3. <u>ADVANCED STEAM CYCLES</u>

In this section you will extend your knowledge of steam cycles in order to show that the overall efficiency of the cycle may be optimised by the use of regenerative feed heating and steam re-heating.

Regenerative feed heating is a way of raising the temperature of the feed water before it reaches the boiler. It does this by using internal heat transfer within the power cycle. Steam is bled from the turbines at several points and used to heat the feed water in special heaters.

In this way the temperature of the feed water is raised along with the pressure in stages so that the feed water is nearly always saturated. The heat transfers in the heaters and in the boiler are conducted approximately isothermally.

Studies of the Carnot cycle should have taught you that an isothermal heat transfer is reversible and achieves maximum efficiency.

The ultimate way of conducting feed heating is to pass the feed water through a heat exchanger inside the turbine casing. In this way the temperature of the steam on one side of heat exchanger tubes is equal to the temperature of the water on the other side of the tubes. Although the temperature is changing as water and steam flow through heat exchanger, at any one point, the heat transfer is isothermal. If no superheating nor undercooling is used then the heat transfers in the boiler and condenser are also isothermal and efficiencies equal to those of the Carnot cycle are theoretically possible.
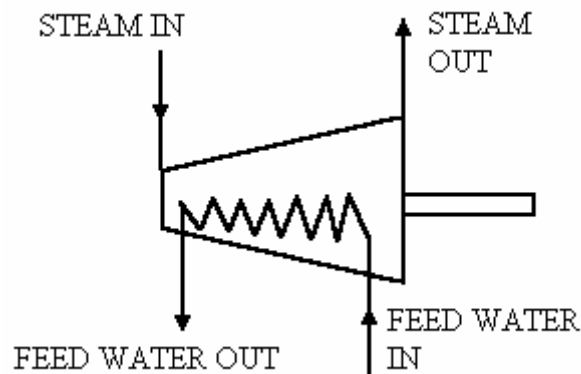

Figure 9

There are several reasons why this arrangement is impractical. Most of them are the same reasons why a Carnot cycle is impractical.

   i.   The steam would be excessively wet in the turbine.
   ii. Placing a heat exchanger inside the turbine casing is mechanically impossible.
   iii. The power output would be small even though the cycle efficiency would be high.

Steam reheating is another way of improving the thermodynamic efficiency by attempting to keep the steam temperature more constant during the heat transfer process inside the boiler.
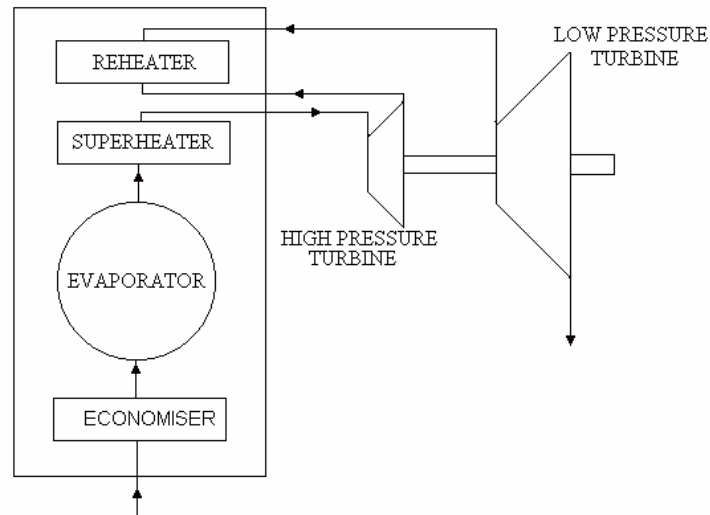

Figure 10

Superheated steam is first passed through a high pressure turbine. The exhaust steam is then returned to the boiler to be reheated almost back to its original temperature. The steam is then expanded in a low pressure turbine. In theory, many stages of turbines and reheating could be done thus making the heat transfer in the boiler more isothermal and hence more reversible and efficient.

If a steam cycle used many stages of regenerative feed heating and many stages of reheating, the result would be an efficiency similar to that of the Carnot cycle. Although practicalities prevent this happening, it is quite normal for an industrial steam power plant to use several stages of regenerative feed heating and one or two stages of reheating. This produces a significant improvement in the cycle efficiency.

There are other features in advanced steam cycles which further improve the efficiency and are necessary for practical operation. For example air extraction at the condenser, steam recovery from turbine glands, de-superheaters, de-aerators and so on. These can be found in details in textbooks devoted to practical steam power plant.

4. FEED HEATING

4.1. PRACTICAL DESIGNS

Practical feed heaters may be heat exchangers with indirect contact. The steam is condensed through giving up its energy and the hot water resulting may be inserted into the feed system at the appropriate pressure. The type which you should learn is the open or direct contact mixing type. The bled steam is mixed directly with the feed water at the appropriate pressure and condenses and mixes with the feed water. Compare a basic Rankine cycle with a similar cycle using one such feed heater.
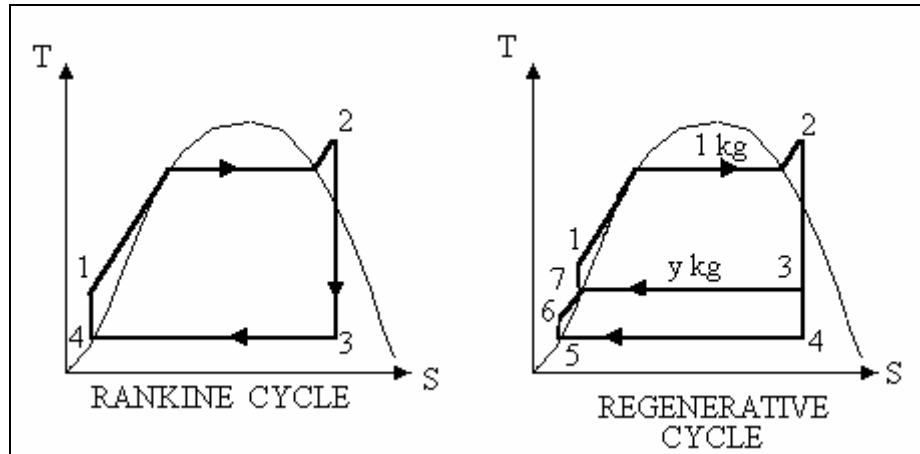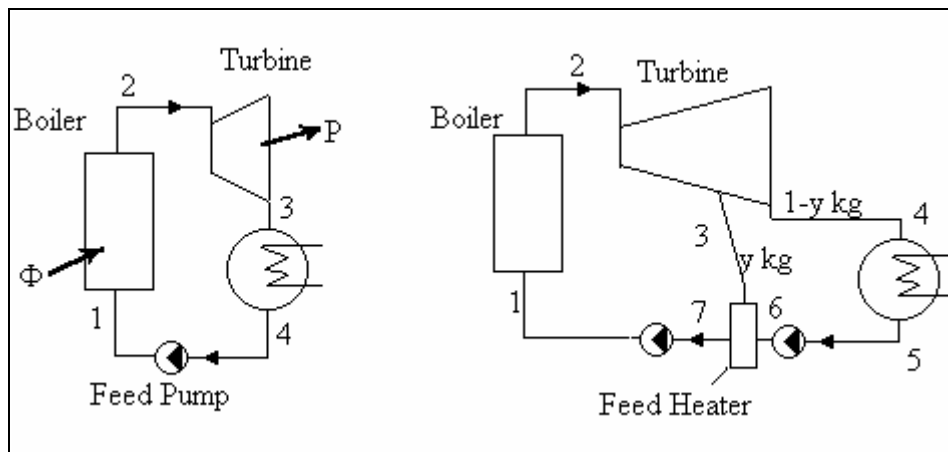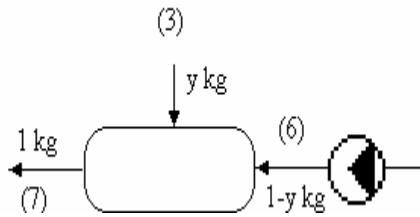

Figure 11

Figure 12

4. 2. ENERGY BALANCE FOR MIXING FEED HEATER

Consider a simple mixing type feed heater. The bled steam at (3) is mixed directly with incoming feed water (6) resulting in hotter feed water (7).

Mass of bled steam = y kg
Mass of feed water entering= 1 - y kg
Doing an energy balance we find

$$y\, h_3 + (1-y)h_6 = h_7$$

Figure 13

WORKED EXAMPLE No.6

A feed heater is supplied with condensate at 0.1 bar. The bled steam is taken from the turbine at 30 bar and 0.95 dry. Calculate the flow rate of bled steam needed to just produce saturated water at outlet.

SOLUTION
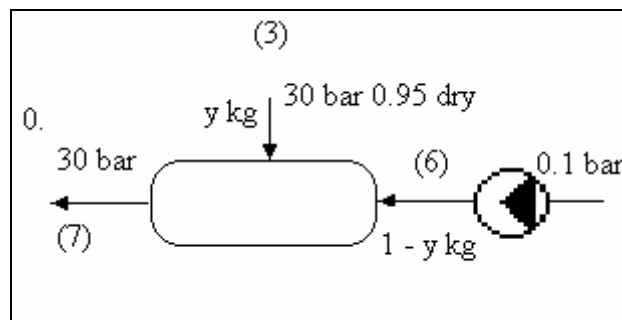
Figure 14

Assumptions

1. Energy input from pump is negligible.
2. No energy is lost.
3. The heater pressure is the same as the bled pressure.

In this case $h_6 = h_f$ at 0.1 bar = 192 kJ/kg
$h_7 = h_f$ at 30 bar = 1008 kJ/kg
$h_3 = h_f + x_{hfg}$ at 30 bar
$h_3 = 11008 + 0.95(1795) = 2713.3$ kJ/kg

ENERGY BALANCE
$y(2713.3) = (1-y)(192) + 1008$

hence $y = 0.414$ kg

Note that it is usual to calculate these problems initially on the basis of 1 kg coming from the boiler and returning to it.

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4. 3. CYCLE WITH ONE FEED HEATER
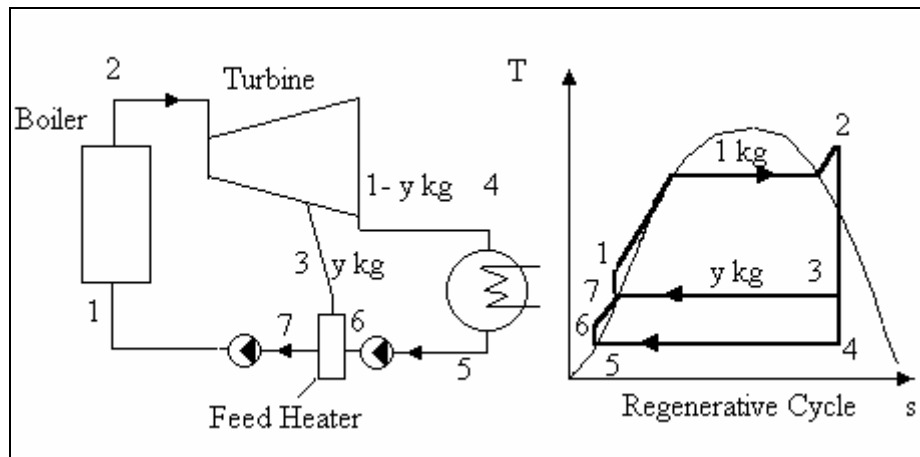
Figure 15

If only one feed heater is used, the steam is bled from the turbine at the point in the expansion where it just becomes dry saturated and the saturation temperature is estimated as follows.

$$t_S(\text{bleed}) = \{t_S(\text{high pressure}) + t_S(\text{low pressure})\}/2$$

For example a cycle operating between 40 bar and 0.035 bar.

$$t_S \ (40 \text{ bar }) = 250.3 \ ^{\circ}C$$

$$t_S \ (0.035 \text{ bar }) = 26.7 \ ^{\circ}C$$

$$t_S \ (\text{bleed }) = (250.3 + 26.7)/2 = 138.5 ^{\circ}C$$

The pressure corresponding to this is 3.5 bar so this is the bleed pressure.

WORKED EXAMPLE No.7

A Rankine cycle works between 40 bar, 400°C at the boiler exit and 0.035 bar at the condenser. Calculate the efficiency with no feed heating. Assume isentropic expansion. Ignore the energy term at the feed pump.

SOLUTION

Figure 16

$h_2 = 3214$ kJ/kg $s_2 = 6.769$ kJ/kg K

$s_2 = s_3 = 0.391 + 8.13$ x

$x = 0.785$

$h_3 = h_f + x\ h_{fg} = 112 + 0.785(2438) = 2024.6$ kJ/kg

$h_4 = h_f$ at 0.035 bar = 112 kJ/kg

$\Phi = h_2 - h_1 = 3102$ kJ/kg into boiler.

$P = h_2 - h_3 = 1189.4$ kJ/kg (out of turbine)

$\eta = P/\Phi = 38.3$ %

WORKED EXAMPLE No.8
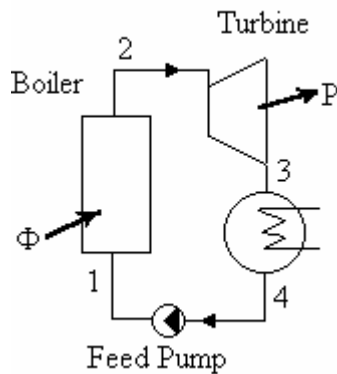
Repeat the last example but this time there is one feed heater.

SOLUTION
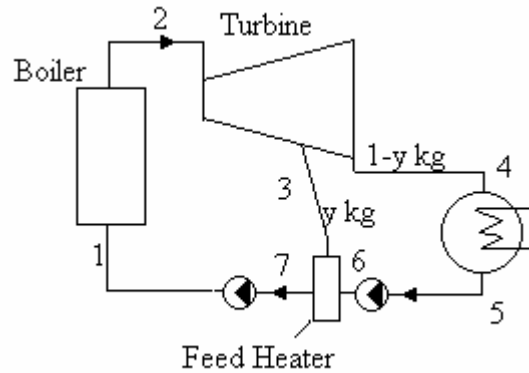
Figure 17

The bleed pressure was calculated in an earlier example and was 3.5 bar.

$s_2 = s_3 = 6.769$ kJ/kg K $= 1.727 + 5.214x_3$
$x_3 = 0.967$ (not quite dry).
$h_3 = h_f + x_{hfg} = 584 + 0.967(2148)$
$h_3 = 2661$ kJ/kg
$h_7 = h_f$ at 3.5 bar $= 584$ kJ/kg

Neglecting pump power
$h_6 = h_5 = h_f = 112$ kJ/kg
$h_1 = h_7 = 584$ kJ/kg

Conducting an energy balance we have
$yh_3 + (1-y) h_6 = h_7$ hence y =0.185 kg

$\Phi = h_2 - h_1 = 2630$ kJ/kg into boiler.

Rather than work out the power from the turbine data, we may do it by calculating the heat transfer rate from the condenser as follows.

$\Phi_{out} = (1-y)(h4 - h5) = 0.815(2024.6 - 112) = 1558.8$ kJ/kg
$P = \Phi_{in} - \Phi_{out} = 1072$ kJ/kg (out of turbine)
$\eta = P/ \Phi_{in} = 40.8$ %
Note that the use of the feed heater produced an improvement of 2.5 % in the thermodynamic efficiency.

4. 4. CYCLE WITH TWO FEED HEATERS

When two (or more) feed heaters are used, the efficiency is further increased. the principles are the same as those already explained. The mass of bled steam for each heater must be determined in turn starting with the high pressure heater. It is usual to assume isentropic expansion that enables you to pick off the enthalpy of the bled steam from the h-s chart at the pressures stated.

WORKED EXAMPLE No.9

A steam power plant works as follows. The boiler produces steam at 100 bar and 600°C. This is expanded isentropically to 0.04 bar and condensed. Steam is bled at 40 bar for the h.p. heater and 4 bar for the l.p. heater. Solve the thermodynamic efficiency.

SOLUTION
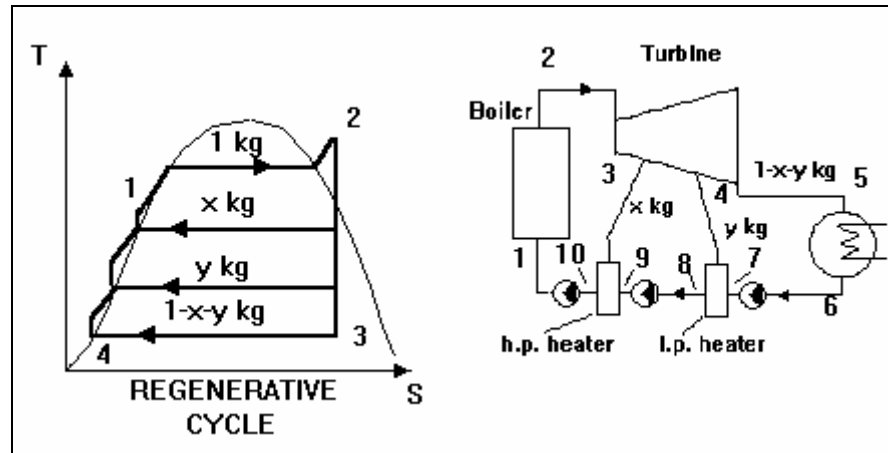
Figure 18
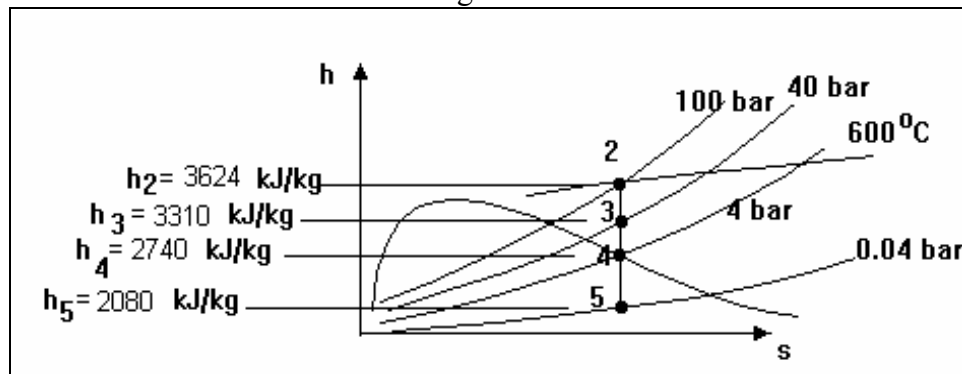
Figure 19

Ignoring the energy input from the pump we find:

$$h_1 = h_{10} = h_f \ 40 \ bar = 1087 \ kJ/kg$$
$$h_9 = h_8 = h_f \ 4 \ bar = 605 \ kJ/kg$$
$$h_7 = h_6 = h_f \ 0.04 \ bar = 121 \ kJ/kg$$

H.P. HEATER

$$xh_3 + (1-x)h_9 = h_{10}$$
$$3310x + 605(1-x) = 1087$$
hence $\quad\quad\quad\quad x = 0.178 \ kg$

<u>L.P. HEATER</u>

$$(1-x)h_8 = yh_4 + (1-x-y)h_7$$
$$0.822(605) = 2740y + (0.822-y)(121)$$
$$y = 0.152 \text{ kg}$$

<u>BOILER</u>

heat input $\Phi_{in} = h_2 - h_1 = 3624 - 1087 = 2537 \text{ kJ/kg}$

<u>CONDENSER</u>

heat output $\Phi_{out} = (1-x-y))(h_5 - h_6)$
$$\Phi_{out} = 0.67(2080-121) = 1312.5 \text{ kJ/kg}$$

<u>POWER OUTPUT</u>

$$P = \Phi_{in} - \Phi_{out} = 1224.5 \text{ kJ/kg}$$

$$\eta = P/ \Phi_{in} = 48.3 \text{ %}$$

<u>**SELF ASSESSMENT EXERCISE No. 5**</u>

1. Explain how it is theoretically possible to arrange a regenerative steam cycle which has a cycle efficiency equal to that of a Carnot cycle.

In a regenerative steam cycle steam is supplied from the boiler plant at a pressure of 60 bar and a temperature of 500°C. Steam is extracted for feed heating purposes at pressures of 30 bar and 3.0 bar and the steam turbine exhausts into a condenser operating at 0.035 bar.

Calculate the appropriate quantities of steam to be bled if the feed heaters are of the open type, and find the cycle efficiency; base all calculations on unit mass leaving the boiler.

Assume isentropic expansion in the turbine and neglect the feed pump work.

(Answers 0.169 kg/s, 0.145 kg/s and 45 %)

2. The sketch shows an idealised regenerative steam cycle in which heat transfer to the feed water in the turbine from the steam is reversible and the feed pump is adiabatic and reversible. The feed water enters the pump as a saturated liquid at 0.03 bar, and enters the boiler as a saturated liquid at 100 bar, and leaves as saturated steam.

Draw a T-s diagram for the cycle and determine, not necessarily in this order, the dryness fraction in state 2, the cycle efficiency and the work per unit mass.
(Answers 0.269 kg/s, 50% and 658.5 kJ/kg).

Outline the practical difficulties that are involved in realising this cycle and explain how regenerative cycles are arranged in practice.

Figure 20

Note point (6) is the point in the steam expansion where the feed water enters and presumably the temperatures are equal. There is further expansion from (6) to (2).

5. REHEAT CYCLES

We shall only examine cycles with one stage of reheating and two turbine stages, high pressure and low pressure. You should refer to text books on practical steam turbine layouts to see how low, medium and high pressure turbines are configured and laid out in order to produced axial force balance on the rotors. The diagram below shows a basic circuit with one stage of reheating.
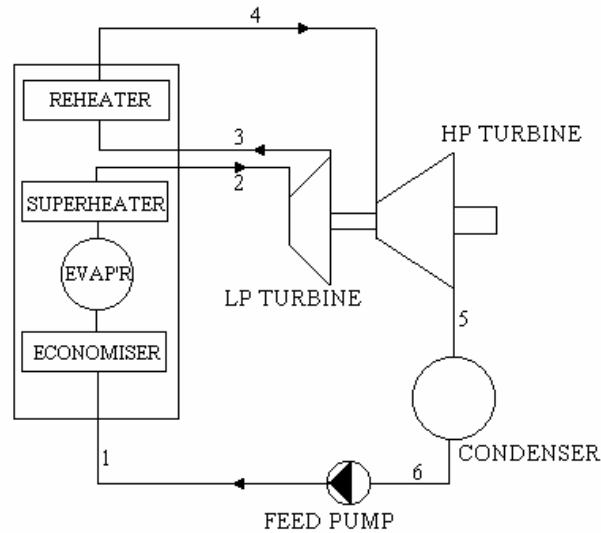

Figure 21

You should be proficient at sketching the cycle on a T - s diagram and a h - s diagram. They are shown below for the cycle shown above.
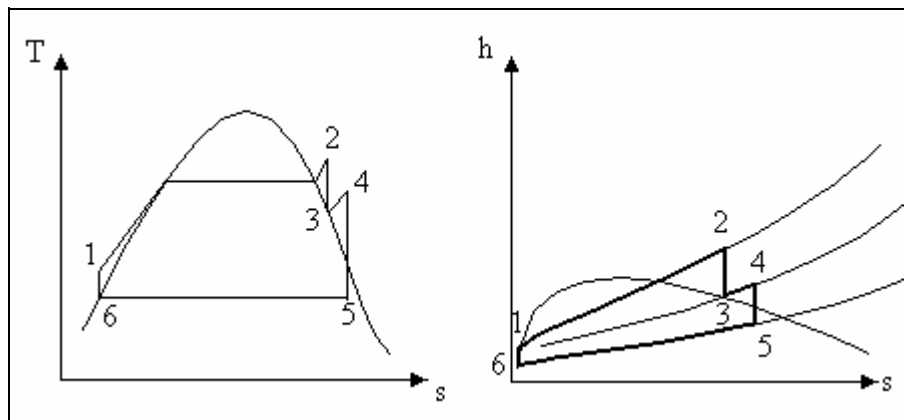
Figure 22

The calculations for this cycle are not difficult. You need only take into account the extra heat transfer in the reheater.

WORKED EXAMPLE No.10

A reheat cycle works as follows. The boiler produces 30 kg/s at 100 bar and 400°C. This is expanded isentropically to 50 bar in the h.p. turbine and returned for reheating in the boiler. The steam is reheated to 400°C. This is then expanded in the l.p. turbine to the condenser which operates at 0.2 bar. The condensate is returned to the boiler as feed.

Calculate the net power output and the cycle efficiency.

SOLUTION

$h_6 = h_f$ at 0.2 bar = 251 kJ/kg
h_2 = 3097 kJ/kg at 100 bar and 400°C.

From the h-s chart we find

h_3 = 2930 kJ/kg h_4 = 3196 kJ/kg h_5 = 2189 kJ/kg

If we ignore the feed pump power then

Φin at boiler = $30(h_2 - h_1) + 30(h_4 - h_3)$ = 93 360 kW or 93.360 MW

Φ_{out} at condenser = $30(h_5 - h_6)$ = 58.14kW

$P_{(net)} = \Phi_{in} - \Phi_{out}$ =35220 kW or 35.22 MW

$\eta = P_{(net)}/\Phi_{in}$ = 37.7 %

1. Repeat worked example No.10 but this time do not ignore the feed pump term and assume an isentropic efficiency of 90% for each turbine and 80% for the pump.
   (Answers 32.1 MW ,35%)

2. A water-cooled nuclear reactor supplies dry saturated steam at a pressure of 50 bar to a two-cylinder steam turbine. In the first cylinder the steam expands with an isentropic efficiency of 0.85 to a pressure of 10 bar, the power generated in this cylinder being 100 MW. The steam then passes at a constant pressure of 10 bar through a water separator from which all the water is returned to the reactor by mixing it with the feed water. The remaining dry saturated steam then flows at constant pressure through a reheater in which its temperature is raised to 250°C before it expands in the second cylinder with an isentropic efficiency of 0.85 to a pressure of 0.1 bar, at which it is condensed before being returned to the reactor.

   Calculate the cycle efficiency and draw up an energy balance for the plant. Neglect the feed pump work. (Answer 30.3%)

3. Steam is raised in a power cycle at the supercritical pressure of 350 bar and at a temperature of 600°C. It is then expanded in a turbine to 15 bar with an overall isentropic efficiency of 0.90. At that pressure some steam is bled to an open regenerative feed heater, and the remainder of the steam is, after reheating to 600°C, expanded in a second turbine to the condenser pressure of 0.04 bar, again with an isentropic efficiency of 0.90. The feed pumps each have an overall isentropic efficiency of 0.90.

   Calculate the amount of steam to be bled into the feed heater, making the usual idealising assumptions. Also calculate the cycle efficiency. Use the h-s chart wherever possible and do not neglect feed pump work.

   (Answers 0.279 kg/s and 47%)

APPLIED THERMODYNAMICS

TUTORIAL 2

GAS COMPRESSORS

In order to complete this tutorial you should be familiar with gas laws and polytropic gas processes.

You will study the principles of reciprocating compressors in detail and some principles of rotary compressors. On completion you should be able to the following.

- Describe the working principles of reciprocating compressors.

- Define and calculate swept volume.

- Define and calculate volumetric efficiency.

- Define and calculate isothermal efficiency.

- Define and calculate indicated power.

- Dtate the benefits of cooling.

- Calculate the heat rejected through cooling.

- Define and calculate the interstage pressures for multiple compressors.

- Define polytropic efficiency.

Let's start by considering the general use of compressed air.

1. COMPRESSED AIR

1.1 TYPES

Air is an expansive substance and dangerous when used at high pressures. For this reason, most applications are confined to things requiring low pressures (10 bar or lower) but there are industrial uses for high pressure air up to 100 bar.

The common source of the air is the compressor. There are many types of compressors with different working principles and working conditions. These are examples.

- Reciprocating.
- Sliding vane compressors.
- Lobe compressors.
- Helical screws.
- Centrifugal.
- Axial turbine compressors.

The function of all of them is to draw in air from the atmosphere and produce air at pressures substantially higher. Usually a storage vessel or receiver is used with the compressor. The same principles are applied to the compression of other gasses. This tutorial is mainly about reciprocating compressors. The other types are covered briefly. The reciprocating compressor is probably the most versatile of all the types and is only out performed by rotary types when large volumes at low pressures are required. For high pressures, the reciprocating compressor is almost universal.

1.2 ATMOSPHERIC VAPOUR

Air and vapour mixtures are covered in detail in a later tutorial. We should note, however, the effects it has on the performance of an air compressor. Atmospheric air contains *WATER VAPOUR* mixed with the other gases. When the air is cooled to the dew point, the vapour condenses into water and we see rain or fog. The ratio of the mass of water vapour in the air to the mass of the air is called the *ABSOLUTE HUMIDITY*. The quantity of water that can be absorbed into the air at a given pressure depends upon the temperature. The hotter the air, the more water it can evaporate. When the air contains the maximum possible amount of vapour it is at its dew point and rain or fog will appear. The air is then said to have 100% humidity. When the air contains no water vapour at all (dry air), it has 0% humidity. This refers to *RELATIVE HUMIDITY*. For example if the air has 40% humidity it means that it contains 40% of the maximum that it could contain. There are various ways to determine the humidity of air and instruments for doing this are called *HYGROMETERS*.

The importance of humidity to air compressors is as follows. When air is sucked into the compressor, it brings with it water vapour. When the air is compressed the pressure and the temperature of the air goes up and the result is that the compressed air will have a relative humidity of about 100% and it will be warm. When the air leaves the compressor it will cool down and the water vapour will condense. Water will then clog the compressor, the receiver and the pipes.

Water causes damage to air tools, ruins paint sprays and corrodes pipes and equipment. For this reason the water must be removed and the best way is to use a well designed compressor installation and distribution network.

1.3 TYPICAL COMPRESSOR LAYOUT

The diagram below shows the layout of a two stage reciprocating compressor typically for supplying a workshop.
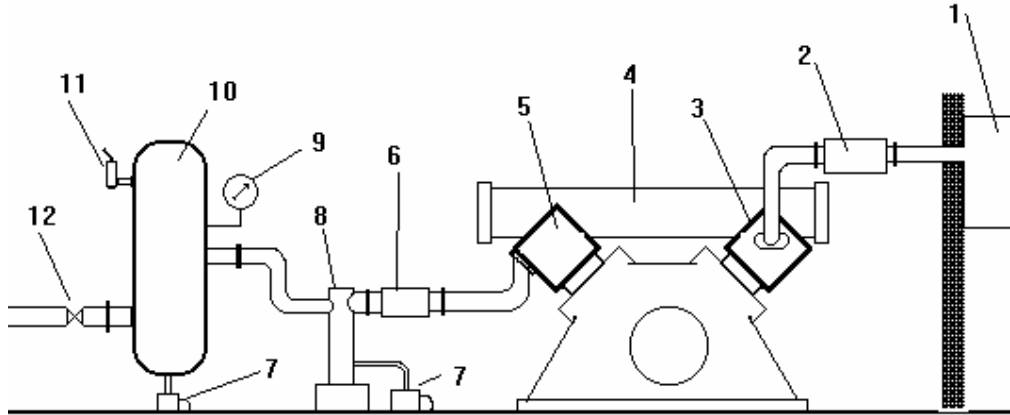

Figure 1

1. Induction box and silencer on outside of building with course screen.
2. Induction filter.
3. Low pressure stage.
4. Intercooler.
5. High pressure stage.
6. Silencer.
7. Drain trap.
8. After cooler
9. Pressure gauge.
10. Air receiver.
11. Safety pressure relief valve.
12. Stop valve

1.4 FREE AIR DELIVERY

When a gas such as air flows in a pipe, the mass of the air depends upon the pressure and temperature. It would be meaningless to talk about the volume of the air unless the pressure and temperature are considered. For this reason the volume of air is usually stated as FREE AIR DELIVERY or FAD.

FAD refers to the volume the air would have if let out of the pipe and returned to atmospheric pressure at the same temperature.

The FAD is the volume of air drawn into a compressor from the atmosphere. After compression and cooling the air is returned to the original temperature but it is at a higher pressure. Suppose atmospheric conditions are p_aT_a and V_a (the FAD) and the compressed conditions are p, V and T.

Applying the gas law we have

$$\frac{pV}{T} = \frac{p_a V_a}{T_a}$$

$$V_a = \frac{pVT_a}{Tp_a} = F.A.D.$$

2. CYCLE FOR RECIPROCATING COMPRESSOR

2.1 THEORETICAL CYCLE

The diagram shows the basic design of a reciprocating compressor. The piston reciprocates drawing in gas, compressing it and expelling it when the pressure inside the cylinder reaches the same level as the pressure in the delivery pipe.
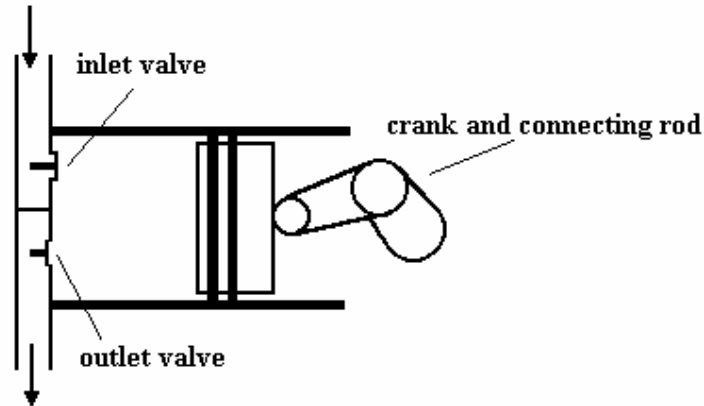
Figure 2

If the piston expels all the air and there is no restriction at the valves, the pressure - volume cycle is as shown below.
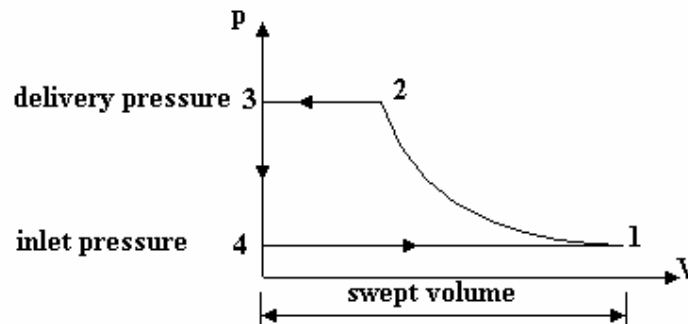
Figure 3

Gas is induced from 4 to 1 at the inlet pressure. It is then trapped inside the cylinder and compressed according the law $pV^n = C$. At point 2 the pressure reaches the same level as that in the delivery pipe and the outlet valve pops open. Air is then expelled at the delivery pressure. The delivery pressure might rise very slightly during expulsion if the gas is being compacted into a fixed storage volume. This is how pressure builds up from switch on.

2.2 VOLUMETRIC EFFICIENCY

In reality, the piston cannot expel all the gas and a clearance volume is needed between the piston and the cylinder head. This means that a small volume of compressed gas is trapped in the cylinder at point 3. When the piston moves away from the cylinder head, the compressed gas expands by the law $pV^n = C$ until the pressure falls to the level of the inlet pressure. At point 4 the inlet valve opens and gas is drawn in. The volume drawn in from 4 to 1 is smaller than the swept volume because of this expansion.
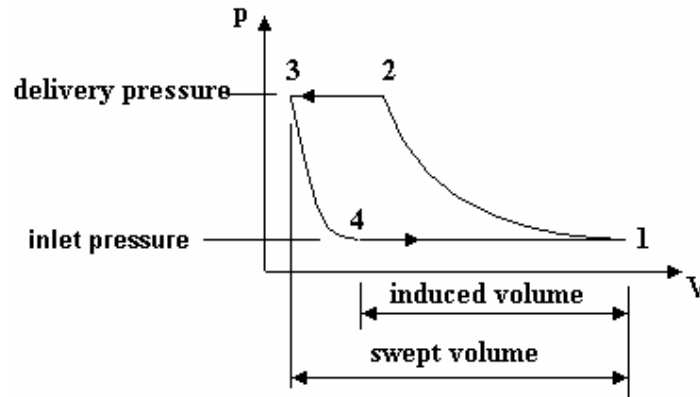

Figure 4

The volumetric efficiency is defined as

$$\eta_{vol} = \frac{\text{Induced Volume}}{\text{Swept Volume}}$$

This efficiency is made worse if leaks occur past the valves or piston.

The clearance ratio is defined as c = Clearance volume/Swept volume.

Ideally the process 2 to 3 and 4 to 1 are isothermal. That is to say, there is no temperature change during induction and expulsion.

WORKED EXAMPLE No.2

Show that if the clearance ratio of an ideal single stage reciprocating compressor is c that the volumetric efficiency is given by

$$\eta_{vol} = 1 - c\left[\left(\frac{p_H}{p_L}\right)^{(1/n)} - 1\right]$$

where p_L is the inlet pressure and p_H the outlet pressure.

SOLUTION

Swept volume = $V_1 - V_3$ Induced volume = $V_1 - V_4$

Clearance volume = V_3

$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3} \qquad c = \frac{V_3}{V_1 - V_3}$$

$$V_1 - V_3 = \frac{V_3}{c} \qquad \frac{V_1}{V_3} = \frac{(1+c)}{c}$$

$$\eta_{vol} = \frac{c(V_1 - V_3)}{V_3} = c\left\{\left(\frac{V_1}{V_3}\right) - \left(\frac{V_4}{V_3}\right)\right\}$$

$$\frac{V_4}{V_3} = \left(\frac{p_3}{p_4}\right)^{1/n} = \left(\frac{p_H}{p_L}\right)^{1/n}$$

$$\eta_{vol} = c\left[\left\{\left(\frac{1+c}{c}\right)\right\} - \left(\frac{p_H}{p_L}\right)^{1/n}\right]$$

$$\eta_{vol} = 1 + c - c\left(\frac{p_H}{p_L}\right)^{1/n}$$

$$\eta_{vol} = 1 - c\left\{\left(\frac{p_H}{p_L}\right)^{1/n} - 1\right\}$$

In real compressors the warm cylinder causes a slight temperature rise over the induction from 4 to 1. The gas is restricted by the valves and p_1 is slightly less than p_4. The valves also tend to move so the real cycle looks more like this.
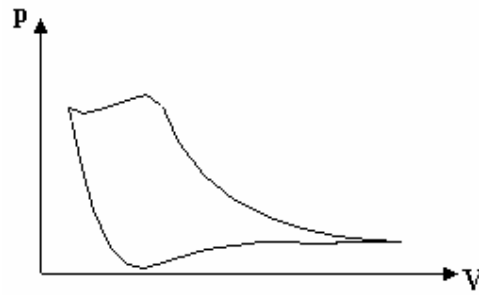
Figure 5

WORKED EXAMPLE No.3

A single stage reciprocating compressor produces a FAD of 2 dm³/s at 420 rev/min. The inlet conditions are 1 bar and 10°C. The polytropic index is 1.2 for the compression and expansion. The outlet pressure is 8 bar. The clearance volume is 10 cm³.

Due to the restriction of the inlet valve and the warming effect of the cylinder walls, the pressure at the start of compression is 0.97 bar and the temperature is 17°C.

Determine the volumetric efficiency.

SOLUTION

Because the induction stroke is neither at constant pressure nor constant temperature, we must solve the swept volume by using the expulsion stroke, which is assumed to be at constant pressure and temperature. The numbers of the cycle points are as before.

$$T_2 = 290\left(\frac{8}{0.97}\right)^{\frac{0.2}{1.2}} = 412.2K$$

$$F.A.D. \text{ per stroke} = \frac{2x60}{420} = 0.2857 \text{ dm}^3 \text{per stroke}$$

$$\text{The compressed volume expelled} = \frac{0.2857xp_aT_H}{T_ap_H}$$

$$\text{Expulsion volume} = V_2 - V_3 = \frac{0.2857x1x412.2}{283x8} = 0.052 \text{dm}^3$$

$$p_3V_3^n = p_4V_4^n$$

$$8x0.01^{1.2} = 1xV_4^{1.2} \quad \text{hence } V_4 = 0.0566 \text{ dm}^3$$

$$V_2 = 0.01 + 0.052 = 0.062 \text{dm}^3$$

$$p_1V_1^n = p_2V_2^n$$

$$0.96V_1^{1.2} = 8x0.062^{1.2}$$

$$V_1 = 0.363 \text{dm}^3$$

$$\text{Induced volume} = V_1 - V_4 = 0.306 \text{ dm}^3$$

$$\text{Swept volume} = V_1 - V_3 = 0.353 \text{ dm}^3$$

$$\eta_{vol} = \frac{0.306}{0.353} = 86.7\%$$

2.3 INDICATED POWER

The indicated work per cycle is the area enclosed by the p - V diagram. The easiest way to find this is by integrating with respect to the pressure axis.
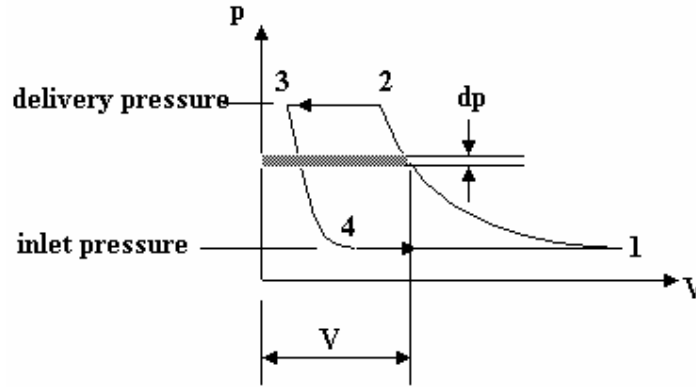
Figure 6

The processes 1 to 2 and 3 to 4 are polytropic $pV^n = C$.

$$V = C^{1/n} p^{-1/n} \qquad C = V p^{1/n}$$

Consider the expression

$$\int V dp = C^{1/n} \int p^{-1/n} dp = \left[\frac{C^{1/n} p^{1-1/n}}{(1 - 1/n)} \right] = \left[\frac{nV p^{1/n} p^{1-1/n}}{(n-1)} \right] = \left[\frac{npV}{(n-1)} \right]$$

Between the limits of p_2 and p_1 this becomes $\dfrac{n[p_2 V_2 - p_1 V_1]}{(n-1)}$

Between the limits p_4 and p_3 this becomes $\dfrac{n[p_4 V_4 - p_3 V_3]}{(n-1)}$

The indicated work (input) is then $\quad W = \dfrac{n[p_2 V_2 - p_1 V_1]}{(n-1)} - \dfrac{n[p_3 V_3 - p_4 V_4]}{(n-1)}$

$$W = \left(\frac{n}{n-1} \right) p_1 V_1 \left\{ \left(\frac{p_2 V_2}{p_1 V_1} \right) - 1 \right\} - \left(\frac{n}{n-1} \right) p_4 V_4 \left\{ \left(\frac{p_3 V_3}{p_4 V_4} \right) - 1 \right\}$$

$$W = \left(\frac{n}{n-1} \right) p_1 V_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\} - \left(\frac{n}{n-1} \right) p_4 V_4 \left\{ \left(\frac{p_3}{p_4} \right)^{\frac{n-1}{n}} - 1 \right\}$$

$$W = \left(\frac{n}{n-1} \right) p_1 V_1 \left\{ r^{\frac{n-1}{n}} - 1 \right\} - \left(\frac{n}{n-1} \right) p_4 V_4 \left\{ r^{\frac{n-1}{n}} - 1 \right\}$$

Where r is the pressure ratio. Since $p_1 = p_4$ this reduces to

$$W = \left(\frac{n}{n-1} \right) p_1 \left\{ r^{\frac{n-1}{n}} - 1 \right\} [V_1 - V_4]. \quad \text{Where } V_1 - V_4 \text{ is the swept volume.}$$

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If the clearance volume is neglected this becomes

$$W = \left(\frac{n}{n-1}\right) p_1 V_1 \left\{ r^{\frac{n-1}{n}} - 1 \right\}$$

Since pV = mRT

$$W = mRT_1 \left(\frac{n}{n-1}\right) \left\{ r^{\frac{n-1}{n}} - 1 \right\}$$

Note that if the process was isothermal and n= 1 then the integration would yield $W = mRT_1 l_n (p_2 / p_1)$

m is the mass compressed each cycle and W is the indicated work per cycle. The indicated power is found by multiplying W by the strokes per second.

$$\text{I.P.} = W \times N \quad \text{where} \quad N \text{ is the shaft speed in Rev/s}$$

If the clearance volume is neglected, the mass compressed is the mass expelled. In this case the actual mass flow rate delivered may be used for m and W becomes the indicated power.

2.4 ISOTHERMAL EFFICIENCY

The minimum indicated power is obtained when the index n is a minimum. The ideal compression is hence isothermal with n=1. The isothermal efficiency is defined as

$$\eta_{(iso)} = \text{Isothermal work/Actual work} = \frac{(n-1)T_1 \ln(p_2 / p_1)}{n(T_2 - T_1)}$$

Note that in the ideal case, T_1 and T_2 are the inlet and outlet temperatures.

Show how the volumetric efficiency of an ideal single stage reciprocating air compressor may be represented by the equation

$$\eta_{vol} = 1 - c\left[\left(\frac{p_H}{pL}\right)^{(1/n)} - 1\right]$$

Where c is the clearance ratio, p_H the delivery pressure and p_L the induction pressure.

A reciprocating air compressor following the ideal cycle has a free air delivery of 60 dm^3/s. The clearance ratio is 0.05. The inlet is at atmospheric pressure of 1 bar. The delivery pressure is 7 bar and the compression is polytropic with an index of 1.3.

Calculate the following.

   i. The ideal volumetric efficiency. (82.7%)

   ii. The ideal indicated power. (14.7 kW)

3. MULTIPLE COMPRESSOR STAGES

3.1 THE EFFECT OF INTERCOOLING

The advantage of compressing the fluid in stages is that intercoolers may be used and the overall compression is nearer to being isothermal. Consider the p - V diagram for a two stage compressor.
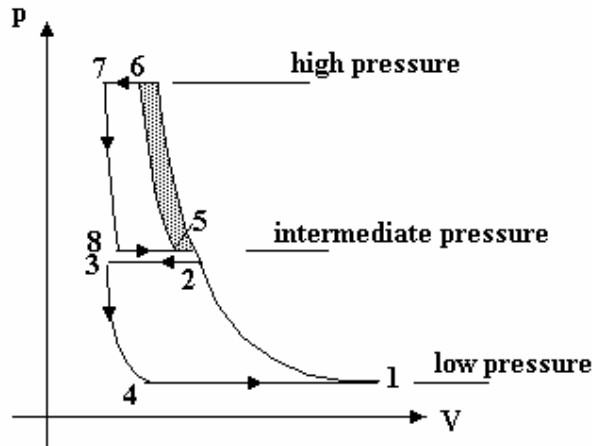
Figure 7

The cycle 1 to 4 is a normal cycle conducted between p_L and p_M. The air is expelled during process 3 to 4 at p_M and constant temperature. The air is then cooled at the intermediate pressure and this causes a contraction in the volume so that the volume entering the high pressure stage is V_5 and not V_2. The high pressure cycle is then a normal cycle conducted between p_M and p_H.

The shaded area of the diagram represents the work saved by using the intercooler. The optimal saving is obtained by choosing the correct intermediate pressure. This may be found as follows.

3.2 OPTIMAL INTERSTAGE PRESSURE

$W = W_1 + W_2$ where W_1 is the work done in the low pressure stage and W_2 is the work done in the high pressure stage.

$$W = \frac{mRn(T_2 - T_1)}{(n-1)} + \frac{mRn(T_6 - T_5)}{(n-1)}$$

Since $T_2 = T_1 \left(\dfrac{p_2}{p_1}\right)^{(1-1/n)}$ and $T_6 = T_5 \left(\dfrac{p_6}{p_5}\right)^{(1-1/n)}$

then assuming the same value of n for each stage

$$W = mR\left[\left\{\frac{nT_1}{(n-1)}\right\}\left\{\frac{p_2}{p_1}\right\}^{1-(1/n)} - 1\right] + mR\left[\left\{\frac{nT_6}{(n-1)}\right\}\left\{\frac{p_6}{p_5}\right\}^{1-(1/n)} - 1\right]$$

Since $p_2 = p_5 = p_m$ and $p_6 = p_H$ and $p_1 = p_L$

$$W = mR\left[\left\{\frac{nT_1}{(n-1)}\right\}\left\{\frac{p_M}{p_L}\right\}^{1-(1/n)} - 1\right] + mR\left[\left\{\frac{nT_6}{(n-1)}\right\}\left\{\frac{p_H}{p_M}\right\}^{1-(1/n)} - 1\right]$$

For a minimum value of W we differentiate with respect to p_M and equate to zero.

$$\frac{dW}{dp_M} = mRT_1 p_L^{(1-n)/n} p_M^{-1/n} - mRT_5 p_H^{(n-1)/n} p_M^{(1-2n)/n}$$

If the intercooler returns the air to the original inlet temperature so that $T_1 = T_5$, then equating to zero reveals that for minimum work

$$p_M = (p_L p_H)^{½}$$

It can further be shown that when this is the case, the work done by both stages are equal.

When K stages are used, the same process reveals that the minimum work is done when the pressure ratio for each stage is $(p_L/p_H)^{1/K}$

WORKED EXAMPLE No.5

A single acting reciprocating compressor runs at 360 rev/min and takes in air at 1 bar and 15°C and compresses it in 3 stages to 64 bar. The free air delivery is 0.0566 m^{3}/s. There is an intercooler between each stage, which returns the air to 15°C. Each stage has one piston with a stoke of 100 mm. Calculate the following.

The ideal interstage pressure.
The ideal indicated power per stage.
The heat rejected from each cylinder.
The heat rejected from each intercooler.
The isothermal efficiency.
The swept volume of each stage.
The bore of each cylinder.

Ignore leakage and the effect of the clearance volume. The index of compression is 1.3 for all stages.

SOLUTION

Pressure ratio for each stage = $(64/1)^{1/3} = 4$

Hence the pressure after stage 1 is 1 x 4 = 4 bar.

The pressure after the second stage is 4 x 4 = 16 bar

The final pressure is 16 x 4 = 64 bar.

T_1 = 288 K.   m = $p_1 V/RT_1$ = 1 x 10^{5} x 0.0566/(287 x 288) = 0.06847 kg/s

$T_2 = 288(4)^{0.3/1.3} = 396.5$ K

The indicated power for each stage is the same so it will be calculated for the 1st. stage.

$$I.P. = \frac{mRnT_1}{n-1} \times \left\{ \left(\frac{p_2}{p_1} \right)^{1-\frac{1}{n}} - 1 \right\} \quad \text{since m is the mass compressed.}$$

$$I.P. = \frac{0.06847 \times 287 \times 1.3 \times 288}{1.3 - 1} \left\{ 4^{\frac{0.3}{1.3}} - 1 \right\} = 9246 \text{ Watts}$$

CYLINDER COOLING

Consider the energy balance over the first stage.
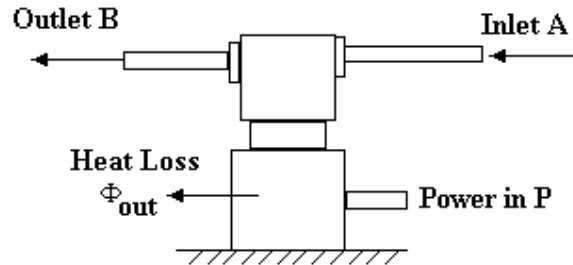
Figure 8

Balancing the energy we have

$H_A + P_{(in)} = H_B + \Phi_{(out)}$

$\Phi_{(out)} = P_{(in)} - mC_p(T_B - T_A)$

$\Phi_{(out)} = 9.246 - 0.06847 \times 1.005 (396.5 - 288)$

$\Phi_{(out)} = 1.78$ kW (rejected from each cylinder)

INTERCOOLER

Now consider the Intercooler. No work is done and the temperature is cooled from T_2 to T_5.

$\Phi_{(out)} = mC_p(T_2 - T_5) = 0.0687 \times 1.005 (396.5 - 288) = 7.49$ kW

ISOTHERMAL EFFICIENCY

The ideal isothermal power $= mRT_1 \ln(p_1/p_2)$ per stage.

P(isothermal) = 0.06847 x 287 x 288 ln4 = 7.846 kW

$\eta_{(iso)} = 7.846/9.246 = 84.9$ %

SWEPT VOLUMES

Consider the first stage.

The F.A.D. is 0.0566 m^3/s. In the ideal case where the air is drawn in at constant temperature and pressure from the atmosphere, the FAD is given by

FAD = Swept Volume x Speed   and the speed is 6 rev/s

Hence S.V. (1st. Stage) = 0.0566/6 = 0.00943 m^3

S.V. = Bore Area x Stroke

0.00943 = π D^2/4   x 0.1   D$_1$ = 0.347 m.

Now consider the second stage. The air is returned to atmospheric pressure at inlet with a pressure of 4 bar. The volume drawn is hence 1/4 of the original FAD.

The swept volume of the second stage is hence 0.00943/4 = 0.00236 m^3.

0.00236 = π D^2/4   x 0.1 hence D$_2$ = 0.173 m

By the same reasoning the swept volume of the third stage is

SV(3rd stage) = 0.00943/16 = 0.000589 m^3.

0.000589 = π D^2/4   x 0.1   D$_3$ = 0.0866 m

1. A single acting 2 stage compressor draws in 8.5 m^3/min of free air and compresses it to 40 bar. The compressor runs at 300 rev/min. The atmospheric conditions are 1.013 bar and 15^0C. There is an intercooler between stages which cools the air back to 15^oC. The polytropic index for all compressions is 1.3. The volumetric efficiency is 90% for the low pressure stage and 85% for the high pressure stage. Ignore the effect of the clearance volume. Calculate the following.

    The intermediate pressure for minimum indicated work. (6.365 bar)
    The theoretical indicated power for each stage. (32.85 kW)
    The heat rejected in each cylinder. (6.31 kW)
    The heat rejected by the intercooler. (26.53 kW)
    The swept volumes of both stages. (31.4 dm^3 and 5.3 dm^3)

    What advantage is there in using an after-cooler?

    State the effect on your answers of not ignoring the clearance volume and leakages.

2. A single acting 2 stage compressor draws in free air and compresses it to 8.5 bar. The compressor runs at 600 rev/min. The atmospheric conditions are 1.013 bar and 15^oC.

    The interstage pressure is 3 bar and the intercooler cools the air back to 30^oC. The polytropic index for all compressions is 1.28.

    Due to the effect of warming from the cylinder walls and the pressure loss in the inlet valve, the pressure and temperature at the start of the low pressure compression stroke is 0.96 bar and 25^oC. The high pressure cycle may be taken as ideal.

    The clearance volume for each stages is 4% of the swept volume of that stage. The low pressure cylinder is 300 mm diameter and the stroke for both stages is 160 mm.

    Calculate the following.

    The free air delivery. (5.858m^3/min)
    The volumetric efficiency of the low pressure stage. (86.3 %)
    The diameter of the high pressure cylinder. (171 mm)
    The indicated power for each stage. (14.6 kW and 13.4 kW)

3. A 2 stage reciprocating air compressor has an intercooler between stages. The induction and expulsion for both stages are at constant pressure and temperature. All the compressions and expansions are polytropic.

Neglecting the effect of the clearance volume show that the intermediate pressure, which gives minimum, indicated work is
$$p_M = (p_L p_H)^{\frac{1}{2}}$$

Explain with the aid of a sketch how the delivery temperature from both cylinders varies with the intermediate pressure as it changes from p_L to p_H.

4.a. Prove that the ideal volumetric efficiency of a single stage reciprocating compressor is

$$\eta_{vol} = 1 - c(r^{1/n} - 1)$$

r is the pressure ratio, n is the polytropic index and c the clearance ratio.

Sketch curves of η_{vol} against r for typical values of n and c.

b. A two stage reciprocating air compressor works between pressure limits of 1 and 20 bar. The inlet temperature is 15°C and the polytropic index is 1.3. Intercooling between stages reduces the air temperature back to 15°C.

Find the free air delivery and mass of air that can be compressed per kW h of work input.

(10.06 m^3/kW h   12.17 kg/kW h)

Find the ratio of the cylinder diameters if the piston have the same stroke. Neglect the effect of the clearance volume.

(d/D = 0.473)

4. POLYTROPIC or SMALL STAGE EFFICIENCY

This is an alternative way of approaching isentropic efficiency. In this method, the compression is supposed to be made up of many stages, each raising the pressure a small amount. The theory applies to any type of compressor.

For an adiabatic gas compression the law of compression and the gas law may be combined.

$$dT = C\left(\frac{\gamma-1}{\gamma}\right)p^{-\frac{1}{\gamma}}dp$$

Divide by p

$$\frac{dT}{p} = C\left(\frac{\gamma-1}{\gamma}\right)p^{-\frac{1}{\gamma}}\frac{dp}{p}$$

$$\frac{dT}{p^{1-\frac{1}{\gamma}}} = C\left(\frac{\gamma-1}{\gamma}\right)\frac{dp}{p}$$

Since $\dfrac{T}{C} = p^{1-\frac{1}{\gamma}}$ differentiate and $\dfrac{dT}{T} = \left(\dfrac{\gamma-1}{\gamma}\right)\dfrac{dp}{p}$

For an isentropic compression, let the final temperature be designated T' and the change in temperature be ΔT'.
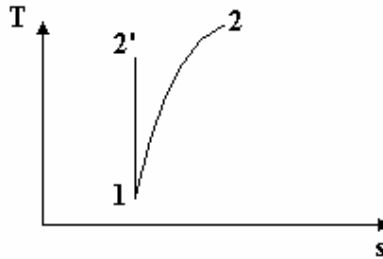
Figure 9

$$\frac{dT'}{T} = \left(\frac{\gamma-1}{\gamma}\right)\frac{dp}{p} \quad ....(1)$$

Isentropic efficiency $= \dfrac{T_{2'} - T_1}{T_2 - T_1}$

Let the change be infinitesimally small such that

$T_2 = T_1 + dT$ and $T_{2'} = T_1 + dT'$

The ratio $\dfrac{dT'}{dT}$ is designated η_∞

η_∞ is called the POLYTROPIC EFFICIENCY.

If we think of the compression as being made up of many tiny steps each with the same value of η_∞.

$$dT' = \eta_\infty dT$$

$$\frac{dT'}{T} = \eta_\infty \frac{dT}{T}$$

$$\left(\frac{\gamma-1}{\gamma}\right)\frac{dp}{p} = \eta_\infty \frac{dT}{T}$$

Integrate

$$\left(\frac{\gamma-1}{\gamma}\right)\int_{p_1}^{p_2}\frac{dp}{p} = \eta_\infty \int_{T_1}^{T_2}\frac{dT}{T}$$

$$\left(\frac{\gamma-1}{\gamma}\right)\ln\frac{p_2}{p_2} = \eta_\infty \ln\frac{T_2}{T_1}$$

$$\left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = \left(\frac{T_2}{T_1}\right)^{\eta_\infty}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma\eta_\infty}\right)}$$

T_1 is the starting temperature and T_2 is the final temperature.

The overall efficiency is $\eta_0 = \dfrac{T_{2'} - T_1}{T_2 - T_1}$

ADIABATIC PROCESS $\quad \dfrac{T_{2'}}{T_1} = r^{\frac{\gamma-1}{\gamma}} \quad\quad T_{2'} = T_1 r^{\frac{\gamma-1}{\gamma}}$

POLYTROPIC PROCESS $\quad \dfrac{T_{2'}}{T_1} = r^{\frac{n-1}{n}} = r^{\frac{\gamma-1}{\gamma\eta_\infty}} \quad\quad T_{2'} = T_1 r^{\frac{\gamma-1}{\gamma\eta_\infty}}$

SUBSTITUTE

$$\eta_O = \frac{T_1 r^{\frac{\gamma-1}{\gamma}} - T_1}{T_1 r^{\frac{\gamma-1}{\gamma\eta_\infty}} - T_1} = \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\frac{\gamma-1}{\gamma\eta_\infty}} - 1}$$

Comparing $\dfrac{T_2}{T_1} = r^{\frac{n-1}{n}}$ with $\dfrac{T_2}{T_1} = r^{\frac{\gamma-1}{\gamma\eta_\infty}} \quad \eta_\infty = 1$ as expected.

This theory may be applied to expansions as well as compressions. It may also be applied to expansions in nozzles. In steam work, it is more usual to use the REHEAT FACTOR, which is based on the same principle.

WORKED EXAMPLE 6

A compressor draws in air at 15°C and 0.3 bar. The air is compressed to 1.6 bar with a polytropic efficiency of 0.86. Determine the temperature and the isentropic efficiency. Take   γ =1.4

SOLUTION

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\eta_\infty}}$$

$$T_2 = 288\left(\frac{1.6}{0.3}\right)^{\frac{1.4-1}{1.4(0.86)}} = 288(5.33)^{0.332} = 502K$$

$$T_2' = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 288(5.33)^{0.286} = 464.5K$$

$$\eta_{is} = \frac{464.5-288}{502-288} = 0.825$$

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SELF ASSESSMENT EXERCISE No. 3

1. Show that for any compression process the overall efficiency is given by

    $$\eta_O = \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\frac{\gamma-1}{\eta\infty}} - 1}$$

    where η_∞ is the polytropic efficiency.

    Determine the index of compression for a gas with an adiabatic index of 1.4 and a polytropic efficiency of 0.9. (1.465)

    Determine the overall efficiency when the pressure compression ratio is 4/1 and 8/1. (0.879 and 0.866)

2. A compressor draws in air at 223.3 K temperature and 0.265 bar pressure. The compression ratio is 6. The polytropic efficiency is 0.86. Determine the temperature after compression. Take $\gamma = 1.4$
    (405 K)

APPLIED THERMODYNAMICS

TUTORIAL No.3

GAS TURBINE POWER CYCLES

In this tutorial you will do the following.

- **Revise gas expansions in turbines.**

- **Revise the Joule cycle.**

- **Study the Joule cycle with friction.**

- **Extend the work to cycles with heat exchangers.**

- **Solve typical exam questions.**

1. REVISION OF EXPANSION AND COMPRESSION PROCESSES.

When a gas is expanded from pressure p_1 to pressure p_2 adiabatically, the temperature ratio is given by the formula $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}}$

The same formula may be applied to a compression process. Always remember that when a gas is expanded it gets colder and when it is compressed it gets hotter. The temperature change is $T_2 - T_1$

If there is friction the isentropic efficiency (η_{is}) is expressed as

$$\eta_{is} = \Delta T \text{ (ideal)}/\Delta T\text{(actual)} \quad \text{for a compression.}$$

$$\eta_{is} = \Delta T \text{ (actual)}/\Delta T\text{(ideal)} \quad \text{for an expansion.}$$

An alternative way of expressing this is with POLYTROPIC EFFICIENCY η_∞
For a compression from (1) to (2) the temperature ratio is expressed as follows.

$T_2 = T_1 \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\eta_\infty}}$ and for an expansion from (1) to (2)

$T_2 = T_1 \left(\dfrac{p_2}{p_1}\right)^{\frac{(\gamma-1)\eta_\infty}{\gamma}}$ where η_∞ is called the polytropic efficiency.

WORKED EXAMPLE No.1

A gas turbine expands 4 kg/s of air from 12 bar and 900ºC to 1 bar adiabatically with an isentropic efficiency of 87%. Calculate the exhaust temperature and the power output. $\gamma = 1.4$ $c_p = 1005$ J/kg K

SOLUTION

$T_2 = T_1 (1/12)^{1-1/1.4} = 1173 (1/12)^{0.2958} = 562.48$ K
Ideal temperature change = 1173 - 562.48 = 610.52 K
Actual temperature change = 87% x 610.52 = 531.15 K
Exhaust temperature = 1173 - 531.15 = 641.85 K
The steady flow energy equation states
$\quad$ $\Phi + P$ = change in enthalpy/s
Since it is an adiabatic process $\Phi = 0$ so
$\quad$ $P = \Delta H/s = m\, c_p\, \Delta T = 4$ x 1005 x (531.15)
$\quad$ $P = -2.135$ x 10^6 W   (Leaving the system)

P(out) = 2.135 MW

---

1. A gas turbine expands 6 kg/s of air from 8 bar and 700ºC to 1 bar isentropically. Calculate the exhaust temperature and the power output. $\gamma = 1.4$ $c_p = 1005$ J/kg K
    (Answers 537.1 K and 2.628 MW)

2. A gas turbine expands 3 kg/s of air from 10 bar and 920ºC to 1 bar adiabatically with an isentropic efficiency of 92%. Calculate the exhaust temperature and the power output. $\gamma = 1.41$ $c_p = 1010$ J/kg K
    (Answers 657.3 K and 1.62 MW)

3. A gas turbine expands 7 kg/s of air from 9 bar and 850ºC to 1 bar adiabatically with an isentropic efficiency of 87%. Calculate the exhaust temperature and the power output.
    $\gamma = 1.4$ $c_p = 1005$ J/kg K
    (Answers 667.5 K and 3.204 MW)

2. THE BASIC GAS TURBINE CYCLE

The ideal and basic cycle is called the JOULE cycle and is also known as the constant pressure cycle because the heating and cooling processes are conducted at constant pressure. A simple layout is shown on fig. 1.
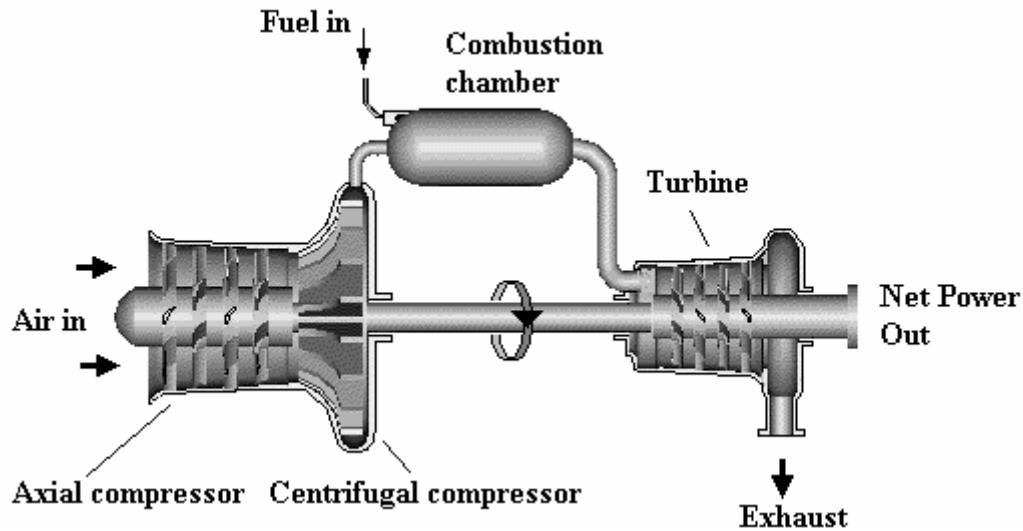

Figure 1 Illustrative diagram.

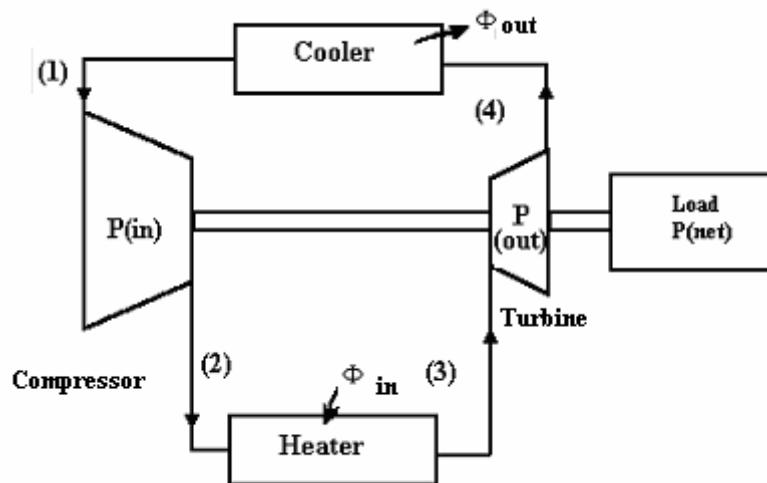The cycle in block diagram form is shown on fig. 2.

Fig.2 Block diagram

There are 4 ideal processes in the cycle.

1 - 2   Reversible adiabatic (isentropic) compression requiring power input.
             $P_{in} = \Delta H/s = m\, c_p\, (T_2-T_1)$

2 - 3   Constant pressure heating requiring heat input.
             $\Phi_{in} = \Delta H/s = m\, c_p\, (T_3-T_2)$

3 - 4   Reversible adiabatic (isentropic) expansion producing power output.
             $P_{out} = \Delta H/s = m\, c_p\, (T_3-T_4)$

4 - 1   Constant pressure cooling back to the original state requiring heat removal.
             $\Phi_{out} = \Delta H/s = m\, c_p\, (T_4-T_1)$

The pressure - volume, pressure - enthalpy and temperature-entropy diagrams are shown on figs. 3a, 3b and 3c respectively.
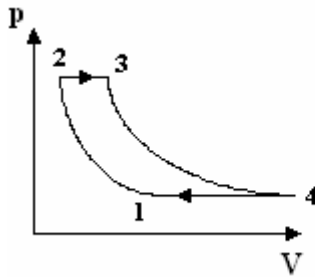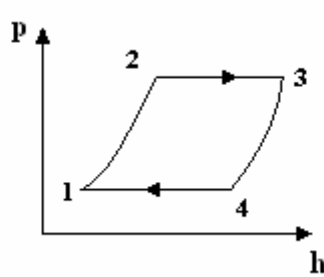
Fig 3a p-V diagram   Fig 3b p-h diagram   Fig 3c T-s diagram
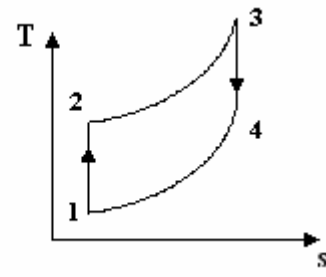
2.1 EFFICIENCY

The efficiency is found by applying the first law of thermodynamics.

$$\Phi_{nett} = P_{nett}$$
$$\Phi_{in} - \Phi_{out} = P_{out} - P_{in}$$
$$\eta_{th} = \frac{P_{nett}}{\Phi_{in}} = 1 - \frac{\Phi_{out}}{\Phi_{in}} = 1 - \frac{mc_p(T_4-T_1)}{mc_p(T_3-T_2)} = 1 - \frac{(T_4-T_1)}{(T_3-T_2)}$$

It assumed that the mass and the specific heats are the same for the heater and cooler.

It is easy to show that the temperature ratio for the turbine and compressor are the same.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}} = r_p^{\,1-\frac{1}{\gamma}} \qquad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{1-\frac{1}{\gamma}} = r_p^{\,1-\frac{1}{\gamma}} \qquad \frac{T_3}{T_4} = \frac{T_2}{T_1}$$

r_p is the pressure compression ratio for the turbine and compressor.

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{\left(\dfrac{T_3 T_1}{T_2} - T_1\right)}{\left(\dfrac{T_2 T_4}{T_1} - T_2\right)} = 1 - \frac{T_1\left(\dfrac{T_3}{T_2} - 1\right)}{T_2\left(\dfrac{T_4}{T_1} - 1\right)}$$

$$\frac{T_3}{T_2} = \frac{T_4}{T_1} \qquad \frac{T_3}{T_2} - 1 = \frac{T_4}{T_1} - 1$$

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} = 1 - \frac{1}{r_p^{1-\frac{1}{\gamma}}} = 1 - r_p^{-0.286} \quad \text{since } \gamma = 1.4$$

This shows that the efficiency depends only on the pressure ratio which in turn affects the hottest temperature in the cycle.

A gas turbine uses the Joule cycle. The pressure ratio is 6/1. The inlet temperature to the compressor is 10°C. The flow rate of air is 0.2 kg/s. The temperature at inlet to the turbine is 950°C. Calculate the following.

i. The cycle efficiency.

ii. The heat transfer into the heater.

iii. The net power output.

$\gamma = 1.4$ $c_p = 1.005$ kJ/kg K

SOLUTION

$$\eta_{th} = 1 - r_p^{-0.286} = 1 - 6^{-0.286} = 0.4 \text{ or } 40\%$$

$$T_2 = T_1 r_p^{0.286} = 283 \times 6^{0.286} = 472.4K$$

$$\Phi_{in} = mc_p(T_3 - T_2) = 0.2 \times 1.005 \times (1223 - 472.4) = 150.8 \text{ kW}$$

$$\eta_{th} = \frac{P_{nett}}{\Phi_{in}}$$

$$P_{nett} = 0.4 \times 150.8 = 60.3 \text{ kW}$$

SELF ASSESSMENT EXERCISE No.2

A gas turbine uses the Joule cycle. The inlet pressure and temperature to the compressor are respectively 1 bar and -10°C. After constant pressure heating, the pressure and temperature are 7 bar and 700°C respectively. The flow rate of air is 0.4 kg/s. Calculate the following.

1. The cycle efficiency.

2. The heat transfer into the heater.

3. the nett power output.

$\gamma = 1.4$ $c_p = 1.005$ kJ/kg K

(Answers 42.7 % , 206.7 kW and 88.26 kW)

3. THE EFFECT OF FRICTION ON THE JOULE CYCLE

3.1 TURBINE

The isentropic efficiency for a gas turbine is given by:

η_i = (Actual change in enthalpy)/(Ideal change in enthalpy)

η_i = (Actual change in temperature)/(Ideal change in temperature)

3.2 COMPRESSOR

For a compressor the isentropic efficiency is inverted and becomes as follows.

η_i = (Ideal change in enthalpy)/(Actual change in enthalpy)

h_i = (Ideal change in temperature)/(Actual change in temperature)

Remember that friction always produces a smaller change in temperature than for the ideal case. This is shown on the T-s diagrams (fig.4a and 4b).
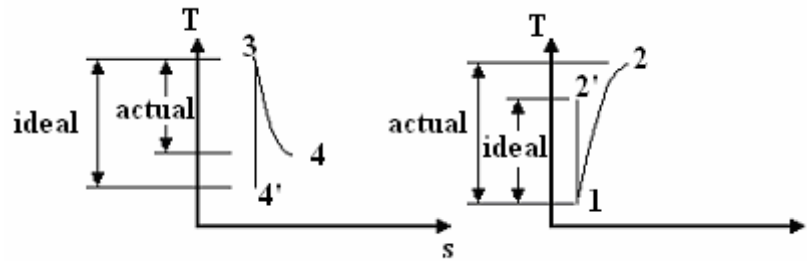
Fig.4a Turbine expansion.   Fig.4b Compression process.

$\eta_i = ((T_3 - T_4)/(T_3 - T_{4'})$ $\eta_i = (T_{2'} - T_1)/(T_2 - T_1)$

The power output from the turbine is hence $P(out) = m\, c_p\, (T_3 - T_{4'})\ \eta_i$

The power input to the compressor is hence $P(in) = m\, c_p\, (T_{2'} - T_1)/\eta_i$

3.3 THE CYCLE WITH FRICTION

It can be seen that the effect of friction on the gas turbine cycle is reduced power output and increased power input with an overall reduction in nett power and thermal efficiency. Figs. 5a and 5b show the effect of friction on T-s and p-h diagrams for the Joule cycle.
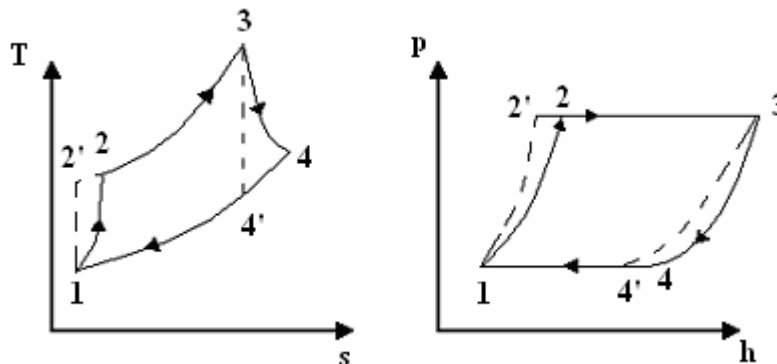
Fig.5a Temperature - Entropy   Fig.5b. Pressure - Enthalpy

Note the energy balance which exists is

$P(\text{in}) + \Phi(\text{in}) = P(\text{out}) + \Phi(\text{out})$ $P(\text{nett}) = P(\text{out}) - P(\text{in}) = \Phi(\text{nett}) = \Phi(\text{in}) - \Phi(\text{out})$

WORKED EXAMPLE No.3

A Joule Cycle uses a pressure ratio of 8. Calculate the air standard efficiency. The isentropic efficiency of the turbine and compressor are both 90%. The low pressure in the cycle is 120 kPa. The coldest and hottest temperatures in the cycle are 20°C and 1200°C respectively. Calculate the cycle efficiency with friction and deduce the change. Calculate the nett power output. $\gamma = 1.4$ and $c_p = 1.005$ kJ/kg K. Take the mass flow as 3 kg/s.

SOLUTION

No friction $\eta_{th} = 1 - r_p^{1/\gamma -1} = 0.448$ or 48.8 %

With friction $T_{2'} = 293 \times 8^{0.286} = 531$ K

$\quad\quad \eta_i = 0.9 = (531\text{-}293)/(T_2\text{-}293)$ $T_2 = 531$ K
$\quad\quad T_{4'} = 1473/8^{0.286} = 812.7$ K
$\quad\quad \eta_i = 0.9 = (1473\text{-}T_4)/(1473\text{-}812.7)$ $T_4 = 878.7$
$\quad\quad \eta_{th} = 1 - \Phi(\text{out})/\Phi(\text{in}) = 1 - (T_4\text{-}T_1)/(T_3\text{-}T_2)$
$\quad\quad \mathbf{\eta_{th} = 0.36\ or\ 36\%}$

The change in efficiency is a reduction of 8.8%

$\quad\quad \Phi(\text{in}) = m\ c_p\ (T_3\text{-}T_2) = 3 \times 1.005 \times (1473\text{-}557) = 2760$ kW

Nett Power Output = P(nett) = η_{th} x Φ(in) = 0.36 x 2760 = 994 kW

4. VARIANTS OF THE BASIC CYCLE

In this section we will examine how practical gas turbine engine sets vary from the basic Joule cycle.

4.1 GAS CONSTANTS

The first point is that in reality, although air is used in the compressor, the gas going through the turbine contains products of combustion so the adiabatic index and specific heat capacity is different in the turbine and compressor.

4.2 FREE TURBINES

Most designs used for gas turbine sets use two turbines, one to drive the compressor and a free turbine. The free turbine drives the load and it is not connected directly to the compressor. It may also run at a different speed to the compressor.

Fig.6a. shows such a layout with turbines in parallel configuration. Fig.6b shows the layout with series configuration.
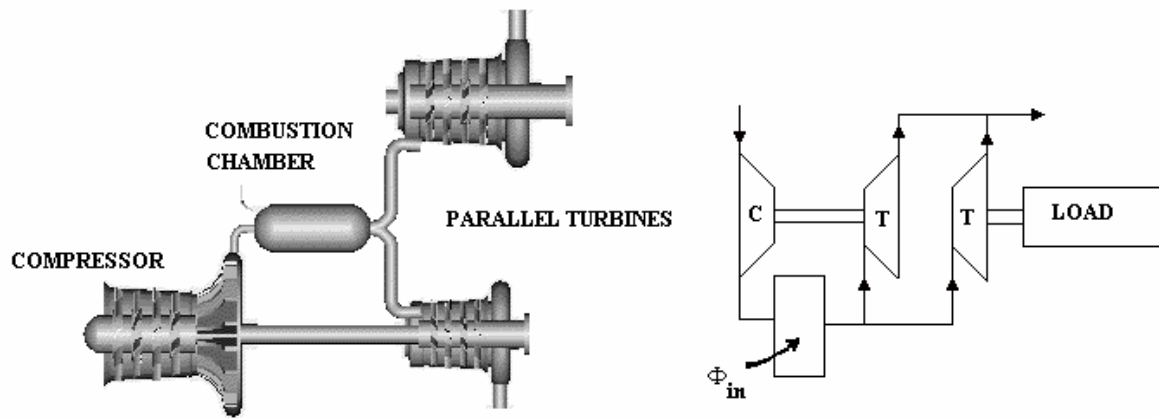
Fig. 6a Parallel turbines
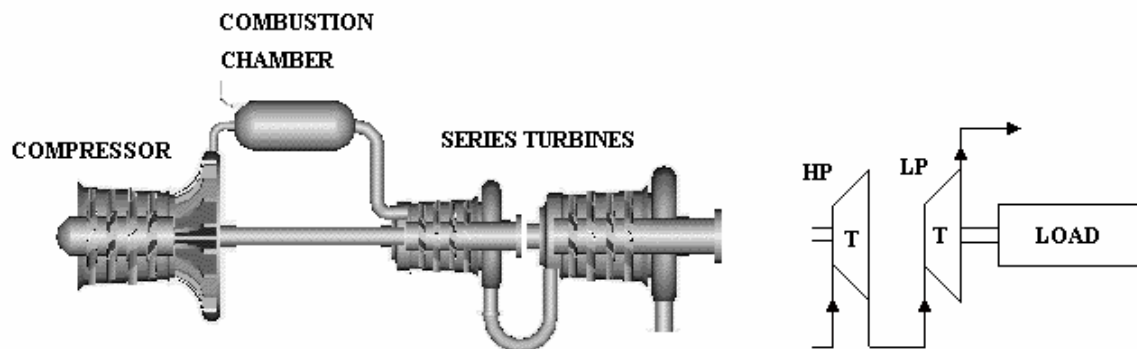
Fig.6b. Series turbines

4.3 INTERCOOLING

This is not part of the syllabus for the power cycles but we will come across it later when we study compressors in detail. Basically, if the air is compressed in stages and cooled between each stage, then the work of compression is reduced and the efficiency increased. The layout is shown on fig. 7a.

4.4 REHEATING

The reverse theory of intercooling applies. If several stages of expansion are used and the gas reheated between stages, the power output and efficiency is increased. The layout is shown on fig. 7b.
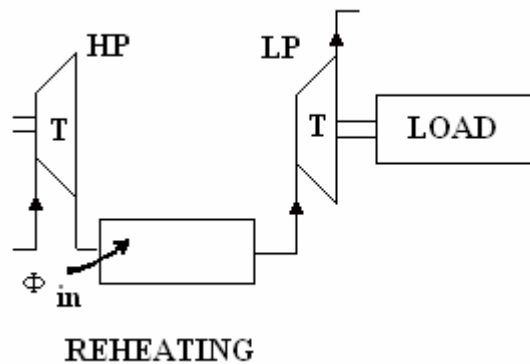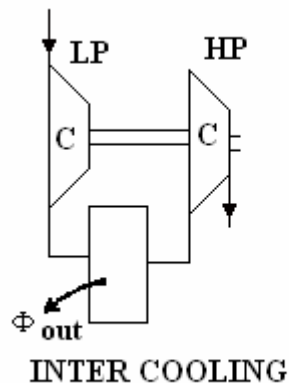
Fig.7a. Intercooler   Fig.7b. Reheater

WORKED EXAMPLE No.4

A gas turbine draws in air from atmosphere at 1 bar and 10ºC and compresses it to 5 bar with an isentropic efficiency of 80%. The air is heated to 1200 K at constant pressure and then expanded through two stages in series back to 1 bar. The high pressure turbine is connected to the compressor and produces just enough power to drive it. The low pressure stage is connected to an external load and produces 80 kW of power. The isentropic efficiency is 85% for both stages.

Calculate the mass flow of air, the inter-stage pressure of the turbines and the thermal efficiency of the cycle.

For the compressor $\gamma = 1.4$ and for the turbines $\gamma = 1.333$.
The gas constant R is 0.287 kJ/kg K for both.

Neglect the increase in mass due to the addition of fuel for burning.

SOLUTION

$$\frac{c_p}{c_v} = \gamma \text{ and } R = c_p - c_v \text{ hence } c_p = \frac{R}{1 - \frac{1}{\gamma}}$$

Hence c_p= 1.005 kJ/kg K for the compressor and 1.149 kJ/kg K for the turbines.
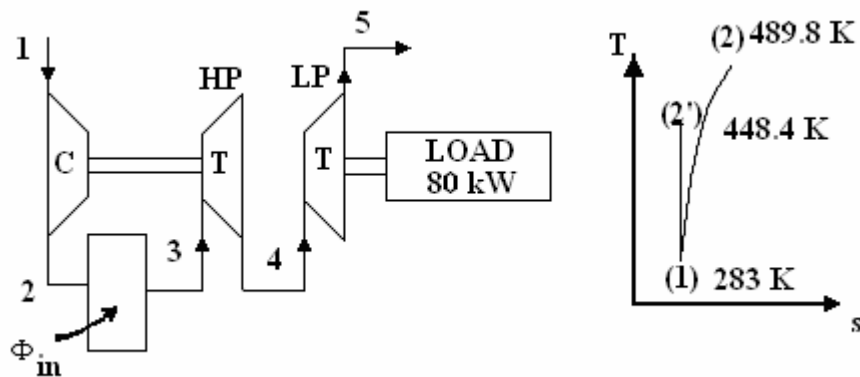
COMPRESSOR

$$T_{2'} = T_1 r_p^{1-\frac{1}{\gamma}} = 283 \text{ x } 5^{0.286} = 448.4 \text{ K}$$

Power input to compressor $= m c_p (T_2\text{-}T_1)$
Power output of h.p. turbine $= m c_p (T_3\text{-}T_4)$
Since these are equal it follows that
$1.005(489.8\text{-}283)=1.149(1200\text{-}T_4)$
$T_4 =1019.1 \text{ K}$

HIGH PRESSURE TURBINE

$$\eta_i = 0.85 = \frac{T_3 - T_4}{T_3 - T_{4'}} \text{ hence } T_{4'} = 987.2 \text{ K}$$

$$\frac{T_{4'}}{T_3} = \left(\frac{p_4}{p_3}\right)^{1-\frac{1}{\gamma}} = \left(\frac{p_4}{5}\right)^{0.25} \text{ hence } p_4 = 2.29 \text{ bar}$$

LOW PRESSURE TURBINE

$$\frac{T_{5'}}{T_1} = \left(\frac{1}{2.29}\right)^{1-\frac{1}{\gamma}} = 1.746^{0.25} \text{ hence } T_{5'} = 828.5 \text{ K}$$

$$\eta_i = 0.85 = \frac{T_4 - T_5}{T_4 - T_{5'}} \text{ hence } T_5 = 854.5 \text{ K}$$

NETT POWER
The nett power is 80 kW hence
80 = m c$_p$(T$_4$-T$_5$) = m x 1.149(1019.1 - 854.5)   **m = 0.423 kg/s**

HEAT INPUT
Φ(in) = m c$_p$ (T$_3$-T$_2$) = 0.423 x 1.149 (1200 - 489.8) = 345.2 kW

THERMAL EFFICIENCY
η_{th} = P(nett)/Φ(in) = 80/345.2 = 0.232 or 23.2%

A gas turbine draws in air from atmosphere at 1 bar and 15°C and compresses it to 4.5 bar with an isentropic efficiency of 82%. The air is heated to 1100 K at constant pressure and then expanded through two stages in series back to 1 bar. The high pressure turbine is connected to the compressor and produces just enough power to drive it. The low pressure stage is connected to an external load and produces 100 kW of power. The isentropic efficiency is 85% for both stages.

For the compressor $\gamma = 1.4$ and for the turbines $\gamma = 1.3$. The gas constant R is 0.287 kJ/kg K for both.

Neglect the increase in mass due to the addition of fuel for burning.

Calculate the mass flow of air, the inter-stage pressure of the turbines and the thermal efficiency of the cycle.

(Answers 0.642 kg/s and 20.1 %)

4.5. EXHAUST HEAT EXCHANGERS

Because the gas leaving the turbine is hotter than the gas leaving the compressor, it is possible to heat up the air before it enters the combustion chamber by use of an exhaust gas heat exchanger. This results in less fuel being burned in order to produce the same temperature prior to the turbine and so makes the cycle more efficient. The layout of such a plant is shown on fig.8.
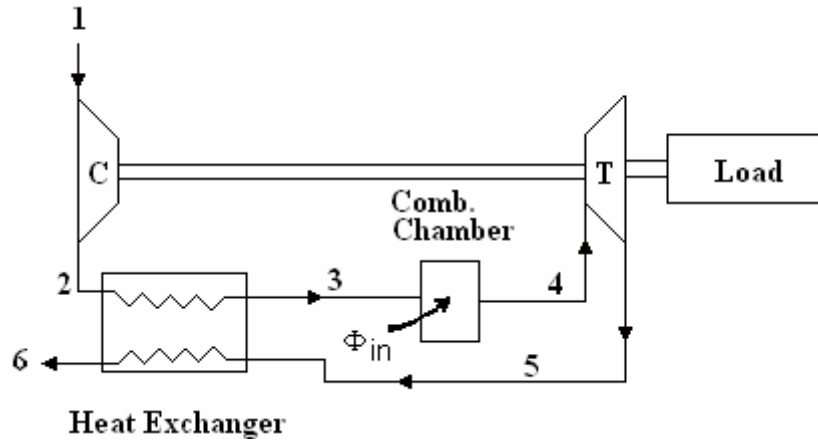
Fig.8 Plant layout

In order to solve problems associated with this cycle, it is necessary to determine the temperature prior to the combustion chamber (T_3).

A perfect heat exchanger would heat up the air so that T_3 is the same as T_5. It would also cool down the exhaust gas so that T_6 becomes T_2. In reality this is not possible so the concept of **THERMAL RATIO** is used. This is defined as the ratio of the enthalpy given to the air to the maximum possible enthalpy lost by the exhaust gas. The enthalpy lost by the exhaust gas is

$$\Delta H = m_g c_{pg}(T_5\text{-}T_6)$$

This would be a maximum if the gas is cooled down such that $T_6 = T_2$. Of course in reality this does not occur and the maximum is not achieved and the gas turbine does not perform as well as predicted by this idealisation.

$$\Delta H(\text{maximum}) = \Delta H = m_g c_{pg}(T_5\text{-}T_6)$$

The enthalpy gained by the air is

$$\Delta H(\text{air}) = m_a c_{pa}(T_3\text{-}T_2)$$

Hence the thermal ratio is

$$\text{T.R.} = m_a c_{pa}(T_3\text{-}T_2)/\ m_g c_{pg}(T_5\text{-}T_2)$$

The suffix 'a' refers to the air and g to the exhaust gas. Since the mass of fuel added in the combustion chamber is small compared to the air flow we often neglect the difference in mass and the equation becomes

$$T.R. = \frac{c_{pa}\left(T_3 - T_2\right)}{c_{pg}\left(T_5 - T_2\right)}$$

WORKED EXAMPLE No.5

A gas turbine uses a pressure ratio of 7.5/1. The inlet temperature and pressure are respectively 10°C and 105 kPa. The temperature after heating in the combustion chamber is 1300 °C. The specific heat capacity c_p for air is 1.005 kJ/kg K and for the exhaust gas is 1.15 kJ/kg K. The adiabatic index is 1.4 for air and 1.33 for the gas. Assume isentropic compression and expansion. The mass flow rate is 1kg/s.

Calculate the air standard efficiency if no heat exchanger is used and compare it to the thermal efficiency when an exhaust heat exchanger with a thermal ratio of 0.88 is used.

SOLUTION

Referring to the numbers used on fig.8 the solution is as follows.

Air standard efficiency = $1 - r_p^{(1-1/\gamma)}$ = $1-7.5^{0.286}$ = **0.438 0r 43.8%**

Solution with heat exchanger

$T_2 = T_1 r_p^{(1-1/\gamma)}$ = $283 \, (7.5)^{0.286}$ =503.6 K

$T_5 = T_4 / r_p^{(1-1/\gamma)}$ = $1573/(7.5)^{0.25}$ = 950.5 K

Use the thermal ratio to find T_3.
$$0.88 = \frac{1.005(T_3 - T_2)}{1.15(T_5 - T_2)} = \frac{1.005(T_3 - 503.6)}{1.15(950.5 - 503.6)}$$
$$T_3 = 953.6 \text{ K}$$

In order find the thermal efficiency, it is best to solve the energy transfers.

P(in)= $mc_{pa}(T_2-T_1)$ = 1 x 1.005 (503.6-283) = 221.7 kW

P(out) = $mc_{pg}(T_4-T_5)$ = 1 x 1.15 (1573-953.6) = 712.3 kW

P(nett) = P(out) - P(in) = 397.3 kW

Φ(in)combustion chamber) = $mc_{pg}(T_4-T_3)$
Φ(in)= 1.15(1573-953.6) = 712.3 kW

η_{th} = P(nett)/Φ(in) = 494.2/712.3 = **0.693 or 69.3%**

SELF ASSESSMENT EXERCISE No. 5

1. A gas turbine uses a pressure ratio of 7/1. The inlet temperature and pressure are respectively 10°C and 100 kPa. The temperature after heating in the combustion chamber is 1000 °C. The specific heat capacity c_p is 1.005 kJ/kg K and the adiabatic index is 1.4 for air and gas. Assume isentropic compression and expansion. The mass flow rate is 0.7 kg/s.

    Calculate the net power output and the thermal efficiency when an exhaust heat exchanger with a thermal ratio of 0.8 is used.
    (Answers 234 kW and 57%)

2. A gas turbine uses a pressure ratio of 6.5/1. The inlet temperature and pressure are respectively 15°C and 1 bar. The temperature after heating in the combustion chamber is 1200 °C. The specific heat capacity c_p for air is 1.005 kJ/kg K and for the exhaust gas is 1.15 kJ/kg K. The adiabatic index is 1.4 for air and 1.333 for the gas. The isentropic efficiency is 85% for both the compression and expansion process. The mass flow rate is 1kg/s.

    Calculate the thermal efficiency when an exhaust heat exchanger with a thermal ratio of 0.75 is used.
    (Answer 48.3%)

WORKED EXAMPLE No.6

A gas turbine has a free turbine in parallel with the turbine which drives the compressor. An exhaust heat exchanger is used with a thermal ratio of 0.8. The isentropic efficiency of the compressor is 80% and for both turbines is 0.85.

The heat transfer rate to the combustion chamber is 1.48 MW. The gas leaves the combustion chamber at 1100°C. The air is drawn into the compressor at 1 bar and 25°C. The pressure after compression is 7.2 bar.

The adiabatic index is 1.4 for air and 1.333 for the gas produced by combustion. The specific heat c_p is 1.005 kJ/kg K for air and 1.15 kJ/kg K for the gas. Determine the following.

i. The mass flow rate in each turbine.
ii. The net power output.
iii. The thermodynamic efficiency of the cycle.

SOLUTION

$T_1 = 298$ K
$T_2 = 298(7.2)^{(1-1/1.4)} = 524$ K
$T_4 = 1373$ K
$T_5 = 1373(1/7.2)^{(1-1/1.333)} = 838.5$ K

COMPRESSOR
$\eta_i = 0.8 = (524-298)/(T_2-298)$ hence $T_2 = 580.5$ K

TURBINES
Treat as one expansion with gas taking parallel paths.
$\eta_i = 0.85 = (1373-T_5)/(1373-838.5)$ hence $T_5 = 918.7$ K

HEAT EXCHANGER
Thermal ratio $= 0.8 = 1.005(T_3-580.5)/1.15(918.7-580.5)$
hence $T_3 = 890.1$ K

COMBUSTION CHAMBER
$\Phi(in) = mc_p(T_4-T_3) = 1480$ kW
$1480 = m(1.15)(1373-890.1)$ hence m = 2.665 kg/s

COMPRESSOR
$P(in) = mc_p (T_2-T_1) = 2.665(1.005)(580.5-298) = 756.64$ kW

TURBINE A

P(out) = 756.64 kW = $m_A c_p (T_4 - T_5)$

756.64 = = 2.665(1.15)(1373-918.7)   hence m_A = 1.448 kg/s
Hence mass flow through the free turbine is 1.2168 kg/s

P(nett) = Power from free turbine =1.2168(1.15)(1373-918.7) = 635.7 kW

THERMODYNAMIC EFFICIENCY

η_{th} = P(nett)/Φ(in)= 635.7/1480 = 0.429 or 42.8 %

SELF ASSESSMENT EXERCISE No. 6

1. List the relative advantages of open and closed cycle gas turbine engines.

    Sketch the simple gas turbine cycle on a T-s diagram. Explain how the efficiency can be improved by the inclusion of a heat exchanger.

    In an open cycle gas turbine plant, air is compressed from 1 bar and 15°C to 4 bar. The combustion gases enter the turbine at 800°C and after expansion pass through a heat exchanger in which the compressor delivery temperature is raised by 75% of the maximum possible rise. The exhaust gases leave the exchanger at 1 bar. Neglecting transmission losses in the combustion chamber and heat exchanger, and differences in compressor and turbine mass flow rates, find the following.
    (i) The specific work output.
    (ii) The work ratio
    (iii) The cycle efficiency
    The compressor and turbine polytropic efficiencies are both 0.84.

    Compressor c_p = 1.005 kJ/kg K γ= 1.4
    Turbine c_p = 1.148 kJ/kg K γ= 1.333

    Note for a compression $T_2 = T_1\left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\eta_\infty}}$

    and for an expansion $T_2 = T_1\left(\dfrac{p_2}{p_1}\right)^{\frac{(\gamma-1)\eta_\infty}{\gamma}}$

2. A gas turbine for aircraft propulsion is mounted on a test bed. Air at 1 bar and 293K enters the compressor at low velocity and is compressed through a pressure ratio of 4 with an isentropic efficiency of 85%. The air then passes to a combustion chamber where it is heated to 1175 K. The hot gas then expands through a turbine which drives the compressor and has an isentropic efficiency of 87%. The gas is then further expanded isentropically through a nozzle leaving at the speed of sound. The exit area of the nozzle is 0.1 m^2. Determine the following.

        (i) The pressures at the turbine and nozzle outlets.
        (ii) The mass flow rate.
        (iii) The thrust on the engine mountings.

Assume the properties of air throughout.

The sonic velocity of air is given by $a = (\gamma RT)^{\frac{1}{2}}$. The temperature ratio before and after the nozzle is given by

$$T(in)/T(out) = 2/(\gamma+1)$$

3. (A). A gas turbine plant operates with a pressure ratio of 6 and a turbine inlet temperature of 927°C. The compressor inlet temperature is 27°C. The isentropic efficiency of the compressor is 84% and of the turbine 90%. Making sensible assumptions, calculate the following.

        (i) The thermal efficiency of the plant.
        (ii) The work ratio.

Treat the gas as air throughout.

(B). If a heat exchanger is incorporated in the plant, calculate the maximum possible efficiency which could be achieved assuming no other conditions are changed.

Explain why the actual efficiency is less than that predicted.

APPLIED THERMODYNAMICS

TUTORIAL 4

PISTON ENGINES

In this tutorial you will do a comprehensive study of piston engine cycles and all matters associated with these engines required for the examination.

On completion of this tutorial you should be able to do the following.

- Calculate the fuel power of an engine.

- Calculate the brake power of an engine.

- Calculate the indicated power of an engine.

- Calculate the various efficiencies of an engine.

- Calculate the Mean Effective Pressure of an engine.

- Describe the standard thermodynamic cycles for spark and compression ignition engines.

- Solve standard cycles.

First we will examine engine testing methods.

1. ENGINE TESTING

1.1   FUEL POWER (F.P.)

Fuel power is the thermal power released by burning fuel inside the engine.

F.P. = mass of fuel burned per second x calorific value of the fuel.

$$\textbf{F.P.} = \textbf{m}_f \textbf{ x C.V.}$$

All engines burn fuel to produce heat that is then partially converted into mechanical power. The chemistry of combustion is not dealt with here. The things you need to learn at this stage follow.

1.1.1   AIR FUEL RATIO

This is the ratio of the mass of air used to the mass of fuel burned.

$$\textbf{Air Fuel Ratio} = \textbf{m}_a/\textbf{m}_f$$

STOICHIOMETRIC RATIO

This is the theoretical air/fuel ratio which is required to exactly burn the fuel.

TRUE RATIO

In reality, the air needed to ensure complete combustion is greater than the ideal ratio. This depends on how efficient the engine is at getting all the oxygen to meet the combustible elements.

The volume of air drawn into the engine is theoretically equal to the capacity of the engine (the swept volumes of the cylinders). The mass contained in this volume depends upon the pressure and temperature of the air. The pressure in particular, depends upon the nature of any restrictions placed in the inlet flow path.

Engines with carburettors work by restricting the air flow with a butterfly valve. This reduces the pressure to less than atmospheric at inlet to the cylinder and the restriction of the inlet valve adds to the affect.

Engines with turbo charging use a compressor to deliver air to the cylinders at pressures higher than atmospheric.

The actual mass of air which enters the cylinder is less than the theoretical value for various reasons such as warming from the cylinder walls, residual gas left inside and leaks from the valves and around the piston. To deal with this we use the concept of *EFFICIENCY RATIO.*

$$\textbf{Efficiency Ratio} = \textbf{Actual mass/ Theoretical mass}$$

1.1.2   CALORIFIC VALUE

This is the heat released by burning 1 kg of fuel. There is a higher and lower value for fuels containing hydrogen. The lower value is normally used because water vapour formed during combustion passes out of the system and takes with it the latent energy. We can now define the fuel power.

FUEL POWER = Mass of fuel/s x Calorific Value

1.1.3   VOLUME FLOW RATE

A two stroke engine induces the volume of air once every revolution of the crank. A 4 stroke engine does so once every two revolutions.

Induced Volume = Capacity x speed   for a 2 stroke engine

Induced volume = Capacity x speed/2 for a 4 stroke engine.

WORKED EXAMPLE No.1

A 4 stroke carburetted engine runs at 2 500 rev/min. The engine capacity is 3 litres. The air is supplied at 0.52 bar and 15°C with an efficiency ratio of 0.4. The air fuel ratio is 12/1. The calorific value is 46 MJ/kg. Calculate the heat released by combustion.

SOLUTION

Capacity = 0.003 m^3
Volume induced = 0.003 x (2 500/60)/2 = 0.0625 m^3/s
Using the gas law pV = mRT we have
Ideal air
m = pV/RT = 0.52 x 10^5 x 0.0625/(287 x 288) = 0.03932 kg/s

Actual air
m = 0.03932 x 0.4 = 0.01573 kg/s.

Mass of fuel
        m_f = 0.01573/12 = 0.00131 kg/s

Heat released

Φ = calorific value x m_f = 46 000 kJ/kg x 0.00131 kg/s = 60.3 KW

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SELF ASSESSMENT EXERCISE No.1

A 4 stroke carburetted engine runs at 3 000 rev/min. The engine capacity is 4 litres. The air is supplied at 0.7 bar and 10ºC with an efficiency ratio of 0.5. The air fuel ratio is 13/1. The calorific value is 45 MJ/kg. Calculate the heat released by combustion.

(Answer 149 KW)

1.2 BRAKE POWER

Brake power is the output power of an engine measured by developing the power into a brake dynamometer on the output shaft. Dynamometers measure the speed and the Torque of the shaft. The Brake Power is calculated with the formula

$$B.P. = 2\pi NT \qquad \text{where N is the shaft speed in rev/s}$$
$$\text{T is the torque in N m}$$

You may need to know how to work out the torque for different types of dynamometers. In all cases the torque is   **T = net brake force x radius**

The two main types are shown below.
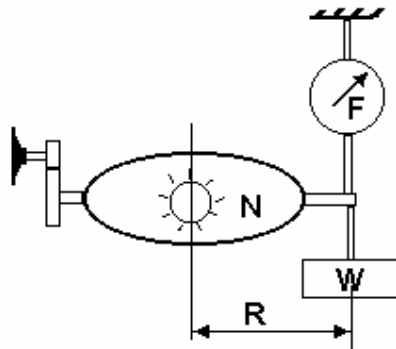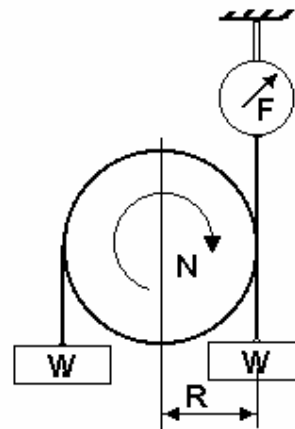
Figure 1   Figure 2

Figure 1 shows a hydraulic dynamometer which absorbs the engine power with an impeller inside a water filled casing. Basically it is a pump with a restricted flow. The power heats up the water and produces a torque on the casing. The casing is restrained by a weight pulling down and a compression spring balance pushing down also. The torque is then (F + W) x R.

Figure 2 shows a friction drum on which a belt rubs and absorbs the power by heating up the drum which is usually water cooled. The belt is restrained by a spring balance and one weight. The second equal weight acts to cancel out the other so the torque is given by T = F R.

Another form of dynamometer is basically an electric generator that absorbs the load and turns it into electric power that is dissipated in a bank of resistor as heat.

1.3 INDICATED POWER

This is the power developed by the pressure of the gas acting on the pistons. It is found by recording and plotting the pressure against volume inside the piston. Such diagrams are called indicator diagrams and they are taken with engine indicators. The diagram shows a typical indicator diagram for an internal combustion engine. Modern systems use electronic pressure and volume transducers and the data is gathered and stored digitally in a computer and then displayed and processed.
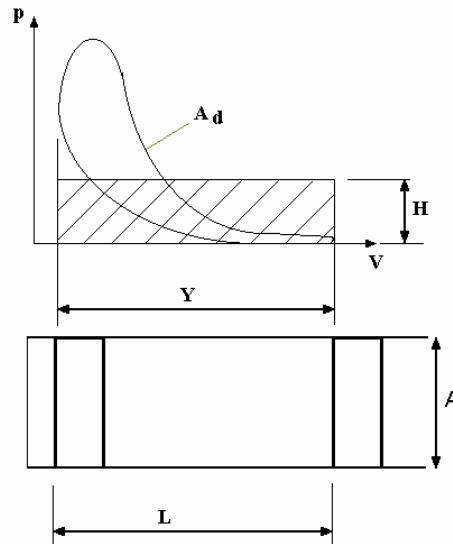
Figure 3

The average force on the piston throughout one cycle is F where   F = MEP x Area of piston = pA

The Mean Effective Pressure p is the mean pressure during the cycle.
The work done during one cycle is W = Force x distance moved = F L = pAL
L is the stroke and this is twice the radius of the crank shaft.
The number of cycles per second is N.
The Indicated Power is then   **I.P. = pLAN per cylinder.**
Note for a 4 stroke engine N = 1/2 the shaft speed.

MEAN EFFECTIVE PRESSURE

The mean effective pressure is found from the indicator diagram as follows.
The area enclosed by the indicator diagram represents the work done per cycle per cylinder. Let this area be A_d mm^2. The average height of the graph is H mm. The base length of the diagram is Y mm. The hatched area is equal to A_d and so
$$A_d = YH$$
$$H = A_d/Y$$
In order to convert H into pressure units, the pressure scale (or spring rate) of the indicator measuring system must be known. Let this be S_p kPa/mm. The MEP is then found from
$$\textbf{MEP} = \textbf{S}_\textbf{p} \ \textbf{H}$$

This is also known as the *Indicated Mean Effective Pressure* because it is used to calculate the Indicated Power. There is also a *Brake Mean Effective Pressure (BMEP)* which is the mean pressure which would produce the brake power.

$$\textbf{BP = (BMEP) LAN}$$

The BMEP may be defined from this as   **BMEP = BP/LAN**

6

1.4 EFFICIENCIES

1.4.1 BRAKE THERMAL EFFICIENCY

This tells us how much of the fuel power is converted into brake power.

$$\eta_{BTh} = \text{B.P./F.P.}$$

1.4.2 INDICATED THERMAL EFFICIENCY

This tells us how much of the fuel power is converted into brake power.

$$\eta_{ITh} = \text{I.P./F.P.}$$

1.4.3 MECHANICAL EFFICIENCY

This tells us how much of the indicated power is converted into brake power. The difference between them is due to frictional losses between the moving parts and the energy taken to run the auxiliary equipment such as the fuel pump, water pump, oil pump and alternator.

$$H_{mech} = \text{B.P./I.P.}$$

WORKED EXAMPLE No.2

A 4 cylinder, 4 stroke engine gave the following results on a test bed.

Shaft Speed N = 2 500 rev/min
Torque arm R = 0.4 m
Net Brake Load F = 200 N
Fuel consumption m_f = 2 g/s
Calorific value = 42 MJ/kg
Area of indicator diagram A_d = 300 mm^2
Pressure scale S_p = 80 kPa/mm
Stroke L = 100 mm
Bore D = 100 mm
Base length of diagram Y = 60 mm.

Calculate the B.P., F.P., I.P., MEP, η_{BTh}, η_{ITh}, and η_{mech},

SOLUTION

BP = 2 πNT = 2π x (2500/60)x (200 x 0.4) = 20.94 kW

FP = mass/s x C.V. = 0.002 kg/s x 42 000 kJ/kg = 84 kW

IP = pLAN

p = MEP = A_d/Y x S_p = (300/60) x 80 = 400 kPa

IP = 400 x 0.1 x (π x 0.1^2/4) x (2500/60)/2   per cylinder

IP = 6.54 kW per cylinder.

For 4 cylinders IP = 6.54 x 4 = 26.18 kW

η_{BTh}= 20.94/84 = 24.9%

η_{ITh} = 26.18/84 = 31.1 %

η_{mech}= 20.94/26.18 = 80%

SELF ASSESSMENT EXERCISE No.2

1. A 4 stroke spark ignition engine gave the following results during a test.

Number of cylinders	6
Bore of cylinders	90 mm
Stroke	80 mm
Speed	5000 rev/min
Fuel consumption rate	0.3 dm^3/min
Fuel density	750 kg/m^3
Calorific value	44 MJ/kg
Net brake load	180 N
Torque arm	0.5 m
Net indicated area	720 mm^2
Base length of indicator diagram	60 mm
Pressure scale	40 kPa/mm

    Calculate the following.

i) The Brake Power.	(47.12 kW)
ii) The Mean effective Pressure.	(480 kPa).
iii) The Indicated Power.	(61 kW).
iv) The Mechanical Efficiency.	(77.2%).
v) The Brake Thermal efficiency.	(28.6 %).

2. A two stroke spark ignition engine gave the following results during a test.

Number of cylinders	4
Bore of cylinders	100 mm
Stroke	100 mm
Speed	2000 rev/min
Fuel consumption rate	5 g/s
Calorific value	46 MJ/kg
Net brake load	500 N
Torque arm	0.5 m
Net indicated area	1 500 mm^2
Base length of indicator diagram	66 mm
Pressure scale	25 kPa/mm

    Calculate the following.

i) The Indicated thermal efficiency.	(26.3 %)
ii) The Mechanical Efficiency.	(87%).
iii) The Brake Thermal efficiency.	(22.8%).

3. A two stroke spark ignition engine gave the following results during a test.

Number of cylinders	4
Bore of cylinders	80 mm
Stroke	80 mm
Speed	2 200 rev/min
Fuel consumption rate	1.6 cm^3/s
Fuel density	750 kg/m^3
Calorific value	60 MJ/kg
Net brake load	195 N
Torque arm	0.4 m
Net indicated area	300 mm^2
Base length of indicator diagram	40.2 mm
Pressure scale	50 kPa/mm

    Calculate the following.

    i) The Indicated thermal efficiency.   (30.5 %)
    ii) The Mechanical Efficiency.   (81.7%).
    iii) The Brake Thermal efficiency. (25%).

4. A four stroke spark ignition engine gave the following results during a test.

Number of cylinders	4
Bore of cylinders	90 mm
Stroke	80 mm
Speed	5 000 rev/min
Fuel consumption rate	0.09 kg/min
Calorific value	44 MJ/kg
Net brake load	60 N
Torque arm	0.5 m
MEP	280 kPa

    Calculate the following.

    i) The Mechanical Efficiency.   (66.1%).
    ii) The Brake Thermal efficiency. (23.8%).

5. Define Indicated Mean Effective Pressure and Brake Mean Effective Pressure.

The BMEP for a 4 cylinder, 4 stroke spark ignition engine is 8.4 bar. The total capacity is 1.3 dm^3 (litres). The engine is run at 4 200 rev/min.

Calculate the Brake Power. (38.22 kW)

There are 10 kW of mechanical losses in the engine.

Calculate the Indicated Mean effective Pressure. (10.6 bar).

The Volumetric Efficiency is 85% and the Brake Thermal Efficiency of the engine is 28%. The air drawn in to the engine is at 5°C and 1.01 bar. The fuel has a calorific value of 43.5 MJ/kg.

Calculate the air/fuel ratio. (Answer 12.3/1).

Let's now have a look at the theoretical cycle for spark ignition engines.

2. SPARK IGNITION ENGINES

THE OTTO CYCLE

The Otto cycle represents the ideal cycle for a spark ignition engine. In an ideal spark ignition engine, there are four processes as follows.
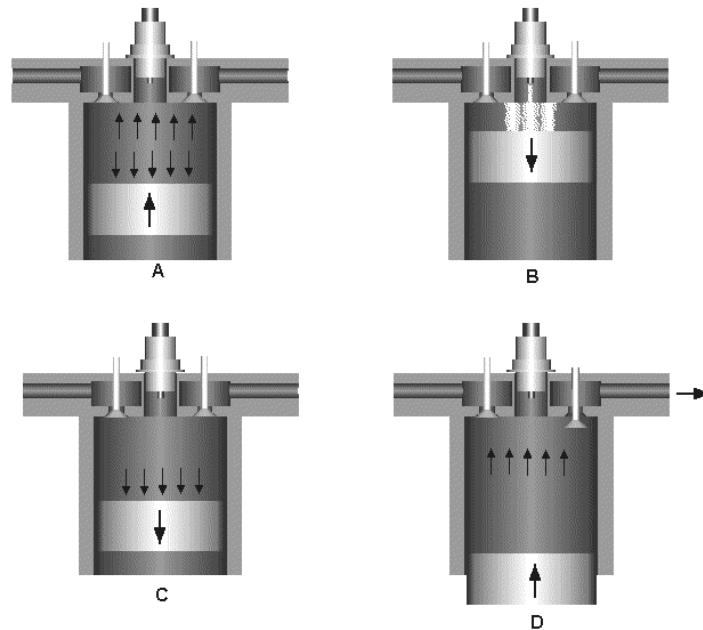
Fig.4

COMPRESSION STROKE
Air and fuel are mixed and compressed so rapidly that there is no time for heat to be lost. (Figure A) In other words the compression is adiabatic. Work must be done to compress the gas.

IGNITION
Just before the point of maximum compression, the air is hot and a spark ignites the mixture causing an explosion (Figure B). This produces a rapid rise in the pressure and temperature. The process is idealised as a constant volume process in the Otto cycle.

EXPANSION OR WORKING STROKE
The explosion is followed by an adiabatic expansion pushing the piston and giving out work. (Figure C)

EXHAUST
At the end of the working stroke, there is still some pressure in the cylinder. This is released suddenly by the opening of an exhaust valve. (Figure D) This is idealised by a constant volume drop in pressure in the Otto cycle. In 4 stroke engines a second cycle is performed to push out the products of combustion and draw in fresh air and fuel. It is only the power cycle that we are concerned with.

The four ideal processes that make up the Otto cycle are as follows.

1 to 2.   The air is compressed reversibly and adiabatically. Work is put in and no heat transfer occurs.

Fig.5

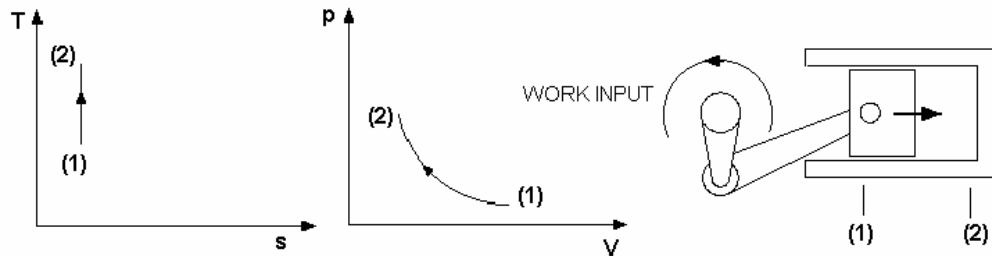2 to 3.   The air is heated at constant volume. No work is done. $Q_{in} = mc_V(T_3-T_2)$
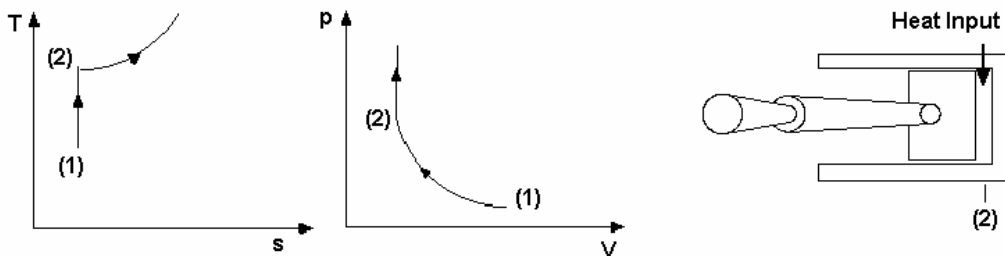
Fig.6

3 to 4.   The air expands reversibly and adiabatically with no heat transfer back to its original volume. Work output is obtained.
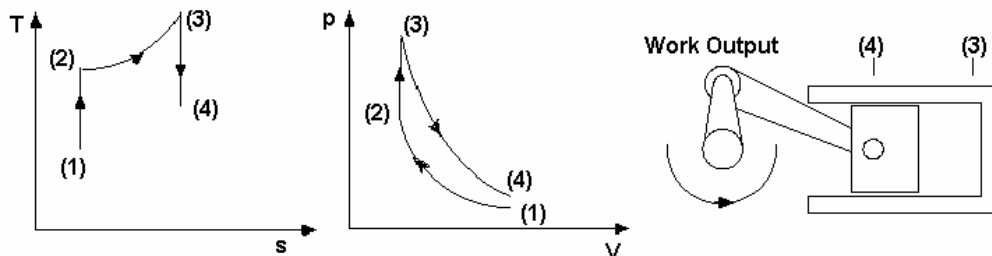
FFig.7

4 to 1.   The air is cooled at constant volume back to its original pressure and temperature. No work is done $Q_{out} = mc_V(T_4-T_1)$
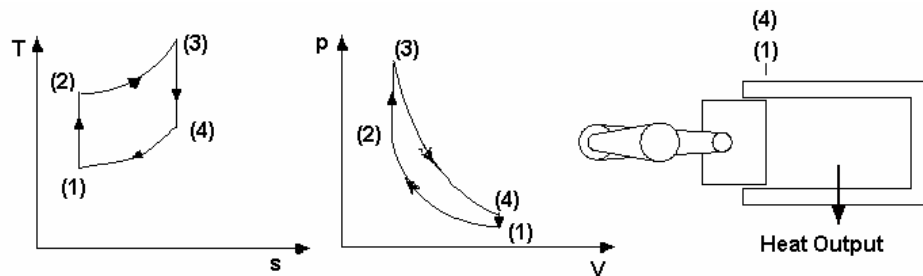
Fig.8

If the engine is successful, then W_{out} The area under the top curve) is larger than W_{in} (the area under the lower curve). The enclosed area represents the net work obtained from the cycle.
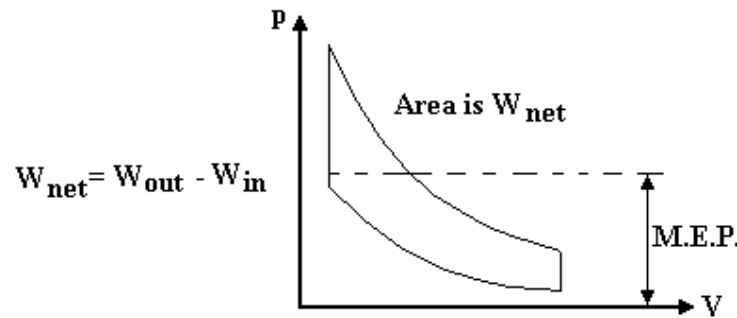
Figure 9

The Mean Effective Pressure (MEP) is the average pressure such that
$$W(net) = \text{Enclosed Area} = MEP \times A \times L$$
$$W(net) = MEP \times \text{Swept Volume}$$
This is true for all cycles and for real engines.
A corresponding net amount of heat must have been transferred into the cycle of:
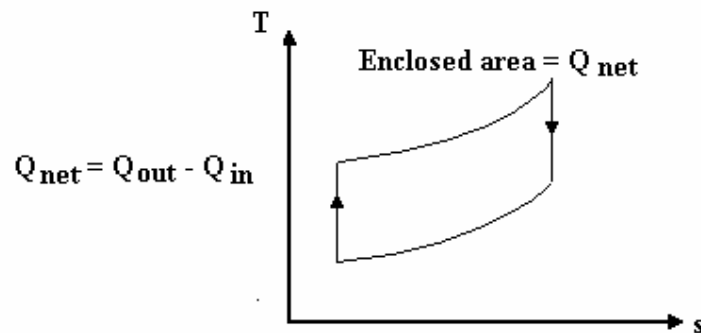
Figure 10

Applying the first law, it follows $\mathbf{Q_{net} = W_{net}}$

It also follows that since the heat transfer is equal to the area under a T - S graph, then the area enclosed by the cycle on the T - S diagram is equal to the Qnet and this is true for all cycles.

EFFICIENCY

$$\eta = \frac{W_{nett}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_4 - T_1)}{mc_v(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

For the process (1) to (2) we may use the rule $\dfrac{T_2}{T_1} = \left(\dfrac{V_1}{V_2}\right)^{\gamma-1} = r_v^{\gamma-1}$

For the process (3) to (4) we may similarly write $\quad \dfrac{T_3}{T_4} = \left(\dfrac{V_4}{V_3}\right)^{\gamma-1} = r_v^{\gamma-1}$

where r_v is the volume compression ratio $r_v = \dfrac{V_1}{V_2} = \dfrac{V_4}{V_3}$

It follows that $\dfrac{T_2}{T_1} = \dfrac{T_3}{T_4}$ and $\dfrac{T_4}{T_1} = \dfrac{T_3}{T_2}$

and that $\quad \eta = 1 - \dfrac{T_4 - T_1}{T_3 - T_2} = 1 - \dfrac{\dfrac{T_3 T_1}{T_2} - T_1}{\dfrac{T_2 T_4}{T_1} - T_2} = 1 - \dfrac{T_1\left(\dfrac{T_3}{T_2} - 1\right)}{T_2\left(\dfrac{T_4}{T_1} - 1\right)}$

$\dfrac{T_4}{T_1} = \dfrac{T_3}{T_2}$ then $\dfrac{T_4}{T_1} - 1 = \dfrac{T_3}{T_2} - 1$

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} = 1 - \frac{1}{r_v^{\gamma-1}} = 1 - r_v^{1-\gamma}$$

Since this theoretical cycle is carried out on air for which $\gamma = 1.4$ then the efficiency of an Otto Cycle is given by $\eta = 1 - r_v^{0.4}$

This shows that the thermal efficiency depends only on the compression ratio. If the compression ratio is increased, the efficiency is improved. This in turn increases the temperature ratios between the two isentropic processes and explains why the efficiency is improved.

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WORKED EXAMPLE No.3

An Otto cycle is conducted as follows. Air at 100 kPa and 20ºC is compressed reversibly and adiabatically. The air is then heated at constant volume to 1500ºC. The air then expands reversibly and adiabatically back to the original volume and is cooled at constant volume back to the original pressure and temperature. The volume compression ratio is 8. Calculate the following.

     i. The thermal efficiency.

     ii. The heat input per kg of air.

     iii. The net work output per kg of air.

     iv. The maximum cycle pressure.

     c_V = 718 kJ/kg γ= 1.4. R = 287 J/kg K

SOLUTION

Remember to use absolute temperatures throughout. Solve for a mass of 1 kg.

T_1=20 +273=293K T_3=1500+273=1773K r_v=8

$$\eta = 1 - r^{1-\gamma} = 1 - 8^{0.4} = 0.565 \quad \text{or } 56.5\%$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 293\left(8^{0.4}\right) = 673.1\,K$$

$$Q_{in} = mc_v(T_3 - T_2) = 1x718(1773 - 673.1) = 789700\,J/kg = 789.7\,kJ/kg$$

$$W_{nett} = \eta Q_{in} = 0.56x789.7 = 446.2\,kJ/kg$$

From the gas law we have

$$p_3 = \frac{p_1 V_1 T_3}{T_1 V_3} = \frac{100000 \text{ x } V_1 \text{ x } 1773}{293 \text{ x } V_3}$$

$$\frac{V_1}{V_3} = 8$$

$$p_3 = \frac{100000 \text{ x } 1773}{293} \text{ x } 8 = 4.84\,MPa$$

If you have followed the principles used here you should be able to solve any cycle.

SELF ASSESSMENT EXERCISE No.3

Take c_v = 0.718 kJ/kg K, R = 287 J/kg K and γ= 1.4 throughout.

1. In an Otto cycle air is drawn in at 20°C. The maximum cycle temperature is 1500°C. The volume compression ratio is 8/1. Calculate the following.

    i. The thermal efficiency. (56.5%)
    ii. The heat input per kg of air. (789 kJ/kg).
    iii. The net work output per kg of air. (446 kJ/kg).

2. An Otto cycle has a volume compression ratio of 9/1. The heat input is 500kJ/kg. At the start of compression the pressure and temperature are 100 kPa and 40°C respectively. Calculate the following.

    i. The thermal efficiency. (58.5%)
    ii. The maximum cycle temperature. (1450 K).
    iii. The maximum pressure. (4.17 MPa).
    iv. The net work output per kg of air. (293 kJ/kg).

3. Calculate the volume compression ratio required of an Otto cycle which will produce an efficiency of 60%. (9.88/1)

    The pressure and temperature before compression are 105 kPa and 25°C respectively. The net work output is 500 kJ/kg). Calculate the following.

    i. The heat input. (833 kJ/kg).
    ii. The maximum temperature. (1 906 K)
    iii. The maximum pressure. (6.64 MPa).

4. An Otto cycle uses a volume compression ratio of 9.5/1. The pressure and temperature before compression are 100 kPa and 40°C respectively. The mass of air used is 11.5 grams/cycle. The heat input is 600 kJ/kg. The cycle is performed 3 000 times per minute. Determine the following.

    i. The thermal efficiency. (59.4%).
    ii. The net work output. (4.1 kJ/cycle)
    iii. The net power output. (205 kW).

5. An Otto cycle with a volume compression ratio of 9 is required to produce a net work output of 450 kJ/cycle. Calculate the mass of air to be used if the maximum and minimum temperatures in the cycle are 1300°C and 20°C respectively.
    (Answer 1.235 kg).

6. The air standard cycle appropriate to the reciprocating spark ignition engine internal-combustion engine is the Otto. Using this, find the efficiency and output of a 2 litre (dm^3), 4 stroke engine with a compression ratio of 9 running at 3000 rev/min. The fuel is supplied with a gross calorific value of 46.8 MJ/kg and an air fuel ratio of 12.8.

    Calculate the answers for two cases.

    a. The engine running at full throttle with the air entering the cylinder at atmospheric conditions of 1.01 bar and 10°C with an efficiency ratio of 0.49.
    (Answers 58.5% and 65 kW)

    b. The engine running at part throttle with the air entering the cylinder at 0.48 bar and efficiency ratio 0.38.
    (Answers 58.5% and 24 kW).

7. The working of a petrol engine can be approximated to an Otto cycle with a compression ratio of 8 using air at 1 bar and 288 K with heat addition of 2 MJ/kg. Calculate the heat rejected and the work done per kg of air.
    (Answers 871 kJ/kg and 1129 kJ/kg).

Now let's move on to study engines with compression ignition.

3. COMPRESSION IGNITION ENGINES

Compression ignition is achieved by compressing the air until it is so hot that fuel sprayed into it will ignite spontaneously.

On modern engines the fuel is injected as a spray of droplets. Since it takes a finite time for the droplets to warm up to the air temperature, there is a time delay between injection and explosion. The accumulation of droplets in this time cause an initial sharp detonation (diesel knock) and a rapid rise in pressure and temperature inside the cylinder. This is a constant volume rise in pressure similar to the spark ignition and the Otto cycle. The big difference is that with fuel injection, fuel may continue to be injected after the explosion and controlled burning of the fuel in the air may take place as the piston moves away from the cylinder head. This ideally maintains the pressure constant as the volume increases. When the fuel is cut off, a natural expansion occurs and the rest of the cycle is similar to the Otto cycle. The air standard cycle for this engine is the DUAL COMBUSTION CYCLE.

The man most credited with the invention of this engine is Rudolf Diesel but many others worked on similar ideas. Diesel's first engine used coal dust blasted into the combustion chamber with compressed air. This developed into blasting in oil with compressed air. The air standard cycle for these old fashioned engines was deemed to be as described above but with no constant volume process. This cycle is called the DIESEL CYCLE. The Diesel cycle may have been a reasonable approximation of what happened in older slow running engines but it is not representative of a modern engine.

3.1   DUAL COMBUSTION CYCLE

This is the air standard cycle for a modern fast running diesel engine. First the air is compressed isentropically making it hot. Fuel injection starts before the point of maximum compression. After a short delay in which fuel accumulates in the cylinder, the fuel warms up to the air temperature and detonates causing a sudden rise in pressure. This is ideally a constant volume heating process. Further injection keeps the fuel burning as the volume increases and produces a constant pressure heating process. After cut off, the hot air expands isentropically and then at the end of the stroke, the exhaust valve opens producing a sudden drop in pressure. This is ideally a constant volume cooling process. The ideal cycle is shown in figure 11
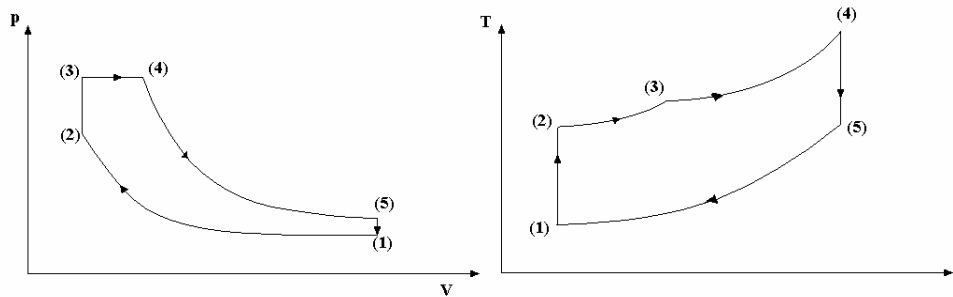
Fig. 11

The processes are as follows.

1 - 2   reversible adiabatic (isentropic) compression.
2 - 3   constant volume heating.
3 - 4   constant pressure heating.
4 – 5   reversible adiabatic (isentropic) expansion.
5 - 1   constant volume cooling.

The analysis of the cycle is as follows.

The heat is supplied in two stages hence $Q_{in} = mc_p(T_4 - T_3) + mc_v (T_3 - T_2)$
The heat rejected is $Q_{out} = mc_v (T_5 - T_1)$

The thermal efficiency may be found as follows.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_5 - T_1)}{mc_v(T_3 - T_2) + mc_p(T_4 - T_3)} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

The formula can be further developed to show that $\eta = 1 - \dfrac{k\beta^\gamma - 1}{\left[(k-1) + \gamma k(\beta - 1)\right]r_v^{\gamma-1}}$

r_v is the VOLUME COMPRESSION RATIO. $r_v = V_1/V_2$
β is the CUT OFF RATIO.
$\beta = V_4/V_3$
k is the ratio p_3/p_2.

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Most students will find this adequate to solve problems concerning the dual combustion cycle. Generally, the method of solution involves finding all the temperatures by application of the gas laws. Those requiring a detailed analysis of the cycle should study the following derivation.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

Obtain all the temperatures in terms of T_2

Isentropic compression 1 to 2

$$T_1 = T_2 \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{T_2}{r_v^{\gamma-1}}$$

Constant volume heating 2 to 3 note $V_3 = V_2$

$$T_3 = \frac{p_3 V_3 T_2}{p_2 V_2} = \frac{p_3 T_2}{p_2} = kT_2$$

Constant pressure heating 3 to 4 note $p_3 = p_4$

$$T_4 = \frac{p_4 V_4 T_3}{p_3 V_3} = \frac{V_4 T_3}{V_3} = \beta T_3 = \beta k T_2$$

Isentropic expansion 4 to 5

$$T_5 = T_4 \left(\frac{V_4}{V_5}\right)^{\gamma-1} = T_4 \left(\frac{V_4 V_2}{V_5 V_2}\right)^{\gamma-1} = T_4 \left(\frac{\beta}{r_v}\right)^{\gamma-1} = \frac{k\beta^{\gamma} T_2}{r_r^{\gamma-1}}$$

Substitute for all temperatures in the efficiency formula.

$$\eta = 1 - \frac{\dfrac{k\beta^{\gamma} T_2}{r_r^{\gamma-1}} - \dfrac{T_2}{r_v^{\gamma-1}}}{(kT_2 - T_2) + \gamma(\beta k T_2 - k T_2)} = 1 - \frac{\dfrac{k\beta^{\gamma}}{r_r^{\gamma-1}} - \dfrac{1}{r_v^{\gamma-1}}}{(k-1) + \gamma(\beta k - k)}$$

$$\eta = 1 - \frac{k\beta^{\gamma} - 1}{[(k-1) + \gamma k(\beta - 1)] r_v^{\gamma-1}}$$

Note that if β=1, the cycle becomes an Otto cycle and the efficiency formulae becomes the same as for an Otto cycle.

WORKED EXAMPLE No. 4

In a dual combustion cycle, the compression starts from 1 bar and 20°C. The compression ratio is 18/1 and the cut off ratio is 1.15. The maximum cycle pressure is 1360 K. The total heat input is 1 kJ per cycle. Calculate the following.

i.   The thermal efficiency of the cycle.

ii.   The net work output per cycle.

Check that the efficiency does not contravene the Carnot principle.

SOLUTION

Known data.

$T_1 = 20 + 273 = 293$ K   The hottest temperature is $T_4 = 1360$K.

$\beta = 1.15$ $r_v = 18$ $\gamma = 1.4$

$T_2 = T_1 r_v^{\gamma-1} = 293 \times 18^{0.4} = 931\,K$

$T_3 = \dfrac{V_3 T_4}{V_4} = \dfrac{T_4}{\beta} = \dfrac{1360}{1.15} = 1183\,K$

$\dfrac{p_3}{p_2} = k = \dfrac{T_3}{T_2} = 1.27$

$\eta = 1 - \dfrac{k\beta^\gamma - 1}{\left[(k-1) + \gamma k(\beta-1)\right]r_v^{\gamma-1}} = 1 - \dfrac{1.27 \times 1.15^{1.4} - 1}{\left[(1.27-1) + (1.4 \times 1.27 \times (1.15-1))\right] \times 18^{0.4}}$

$\eta = 0.68$ or 68%

$W_{nett} = \eta \times Q_{in} = 0.68 \times 1 = 0.68$ kJ per cycle.

The Carnot efficiency should be higher.

$\eta = 1 - \dfrac{T_{cold}}{T_{hot}} = 1 - \dfrac{293}{1360} = 0.785$

The figure of 0.68 is lower so the Carnot principle has not been contravened.

WORKED EXAMPLE No.5

A dual combustion cycle has a compression ratio of 18/1. The maximum pressure in the cycle is 9 MPa and the maximum temperature is 2000°C. The pressure and temperature before compression is 115 kPa and 25°C respectively. Calculate the following.

> i. The cut off ratio.
> ii. The cycle efficiency.
> iii. The nett work output per kg of air.

Assume $\gamma = 1.4$ $c_p = 1.005$ kJ/kgK $c_v = 0.718$ kJ/kg K.

SOLUTION

Known data.

$T_1 = 298$ K $T_4 = 2273$ K $p_3 = p4 = 9$ MPa $p_1 = 115$ kPa

$V_1/V_2 = V_1/V_3 = 18$ $V_2 = V_3$

$T_2 = 298 \times 18^{(\gamma-1)} = 947$ K

$$T_3 = \frac{p_3 T_1 V_3}{p_1 V_1} = \frac{9 \times 10^6 \times 298}{115 \times 10^3} x \frac{V_3}{V_1} = \frac{9 \times 10^6 \times 298}{115 \times 10^3} x \frac{1}{18} = 1296K$$

Cut off ratio $= \beta = \dfrac{V_4}{V_3} = \dfrac{p_3 T_4}{p_4 T_3}$ but $p_4 = p_3$ so $\beta = \dfrac{T_4}{T_3}$

$$\beta = \frac{2273}{1296} = 1.75$$

$$T_5 = T_4 \left(\frac{V_4}{V_5}\right)^{\gamma-1} \quad \text{but} \quad \frac{V_4}{V_5} = \frac{V_4}{V_3} x \frac{V_3}{V_5} = \frac{1.75}{18} = 0.0974$$

$$T_5 = 2273 \times 0.0974^{0.4} = 895.6 \, K$$

$Q_{in} = mc_p(T4 - T3) + mc_v (T3 - T2)$ m= 1 kg

$Q_{in} = 1.005(2274 - 1296) + 0.718(1296 - 947) = 1232.5$ kJ/kg

$Q_{out} = mc_v (T_5 - T_1)$

$Q_{out} = 0.718(895.6 - 298) = 429$ kJ/g

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{429}{1232} = 0.65 \text{ or } 65\%$$

$W_{nett} = Q_{in} \quad Q_{out} = 1232 \quad 428.6 = 803.5 \, kJ/kg$

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3.2 THE DIESEL CYCLE

The Diesel Cycle proceeded the dual combustion cycle. The Diesel cycle is a reasonable approximation of what happens in slow running engines such as large marine diesels. The initial accumulation of fuel and sharp detonation does not occur and the heat input is idealised as a constant pressure process only.

Again consider this cycle as being carried out inside a cylinder fitted with a piston. The p-V and T-s cycles diagrams are shown in figure 12
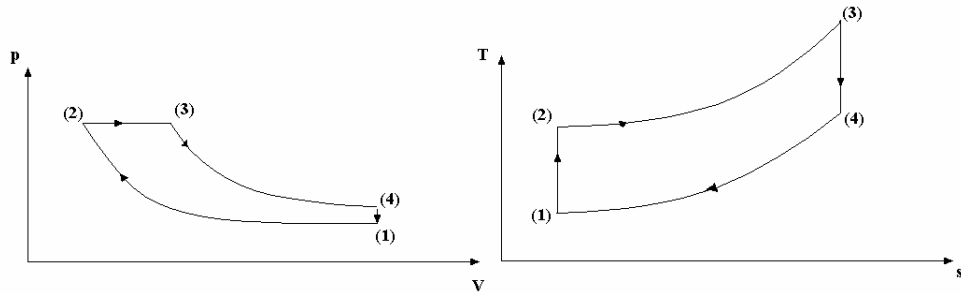
Fig.12

1 - 2   reversible adiabatic (isentropic) compression.

2 - 3   constant pressure heating.

3 - 4   reversible adiabatic (isentropic) expansion.

4 - 1   constant volume cooling.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)}$$

The cycle is the same as the dual combustion cycle without the constant volume heating process. In this case since k=1 the efficiency is given by the following formula.

$$\eta = 1 - \frac{\beta^\gamma - 1}{(\beta - 1)\gamma r_v^{\gamma-1}}$$

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WORKED EXAMPLE No.6

An engine using the Diesel Cycle has a compression ratio of 20/1 and a cut off
ratio of 2. At the start of the compression stroke the air is at 1 bar and 15°C.
Calculate the following.

i.   The air standard efficiency of the cycle.
ii.   The maximum temperature in the cycle.
iii.   The heat input.
iv.   The net work output.

SOLUTION

Initial data.

$\beta=2$ $r_v=20$ $\gamma=1.4$ $c_v = 718$ J/kg K for air $T_1=288$ K $p_1=1$ bar.

The maximum temperature is T_3 and the maximum pressure is p_3 and p_2.

$$\eta = 1 - \frac{\beta^\gamma - 1}{(\beta - 1)\gamma r_v^{\gamma-1}}$$

$$\eta = 1 - \frac{2^{1.4} - 1}{(2 - 1) \text{ x } 1.4 \text{ x } 20^{0.4}}$$

$$\eta = 1 - \frac{1.639}{1 \text{ x } 1.4 \text{ x } 3.314} = 0.647 \text{ or } 64.7\%$$

$$T_2 = T_1 r_v^{\gamma-1} = 288 \text{ x } 20^{0.4} = 954.5 \text{ K}$$

$$T_3 = \frac{V_2}{V_3} T_2 = \beta T_2 = 954.3 \text{ x } 2 = 1909 \text{ K}$$

$$Q_{in} = mc_p (T_3 - T_2)$$

$$Q_{in} = 1.005(1909 - 954.5) = 959.3 \text{ kJ}$$

$$\eta = \frac{W_{nett}}{Q_{in}}$$

$$W_{nett} = \eta Q_{in} = 0.647 \text{ x } 959.3 = 620.6 \text{ kJ}$$

SELF ASSESSMENT EXERCISE No.4

1. A Dual Combustion Cycle uses a compression ratio of 20/1. The cut off ratio is 1.6/1. The temperature and pressure before compression is 30ºC and 1 bar respectively. The maximum cycle pressure is 100 bar. Calculate the following.

          i. The maximum cycle temperature.   (2424 K).
         ii. The net work output per cycle.   (864 kJ/kg).
       iii. The thermal efficiency.   (67.5 %).

2. A Dual Combustion Cycle uses a compression ratio of 12/1. The cut off ratio is 2/1. The temperature and pressure before compression is 280 K and 1 bar respectively. The maximum temperature 2000 K. Calculate the following.

          i. The net work output per cycle.   (680 kJ/kg).
         ii. The thermal efficiency.   (57.6 %).

3. Draw a p - V and T - s diagram for the Dual Combustion Cycle.

    A reciprocating engine operates on the Dual Combustion Cycle. The pressure and temperature at the beginning of compression are 1 bar and 15ºC respectively. The compression ratio is 16. The heat input is 1800 kJ/kg and the maximum pressure is 80 bar. Calculate the following.

          i. The pressure, volume and specific volume at all points in the cycle.
         ii. The cycle efficiency.   (62.8 %).
       iii. The mean effective pressure.   (14.52 bar).

THERMODYNAMICS

TUTORIAL 5

HEAT PUMPS AND REFRIGERATION

On completion of this tutorial you should be able to do the following.

- Discuss the merits of different refrigerants.

- Use thermodynamic tables for common refrigerants.

- Define a reversed heat engine.

- Define a refrigerator and heat pump.

- Define the coefficient of performance for a refrigerator and heat pump.

- Explain the vapour compression cycle.

- Explain modifications to the basic cycle.

- Sketch cycles on a pressure - enthalpy diagram.

- Sketch cycles on a temperature - entropy diagram.

- Solve problems involving isentropic efficiency.

- Explain the cycle of a reciprocating compressor.

- Define the volumetric efficiency of a reciprocating compressor.

- Solve problems involving reciprocating compressors in refrigeration.

- Explain the ammonia vapour absorption cycle.

1. INTRODUCTION

It is possible to lower the temperature of a body by use of the thermo-electric affect (reversed thermo-couple or Peltier effect). This has yet to be developed as a serious refrigeration method so refrigerators still rely on a fluid or refrigerant which is used in a reversed heat engine cycle as follows.
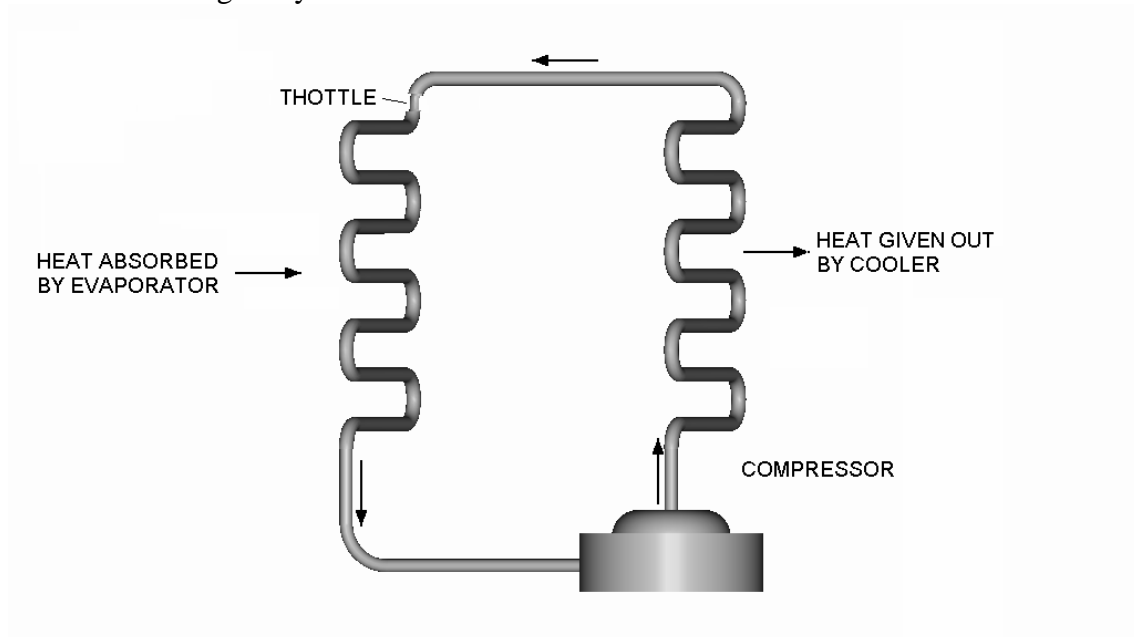

Figure 1

Heat is absorbed into a fluid (this is usually an evaporator) lowering the temperature of the surroundings. The fluid is then compressed and this raises the temperature and pressure. At the higher temperature the fluid is cooled to normal temperature (this is usually a condenser). The fluid then experiences a drop in pressure which makes it go cold (this is usually a throttle valve) and able to absorb heat at a cold temperature. The cycle is then repeated.

Various fluids or refrigerants are used in the reversed thermodynamic cycle. Refrigerants such as air, water and carbon dioxide are used but most refrigerants are those designed for vapour compression cycles. These refrigerants will evaporate at cold temperatures and so the heat absorbed is in the form of latent energy. Let's look at the properties of these and other refrigerants.

2

2. REFRIGERANTS

Refrigerants are given R numbers. Carbon dioxide, for example is R744. Some of them are dangerous if released because they are either explosive or toxic. Toxic refrigerants are placed in categories. Sulphur dioxide, for example, is classed as toxic group 1 which means that death occurs after breathing it for 5 minutes.

In the past the most popular fluids have been ammonia (R717),fluorocarbons and halo-carbons. The most popular of these is R12 or dichlorodifluoromethane (CF_2Cl_2).

The type of refrigerant used in a cycle is largely governed by the evaporation temperature required and its latent capacity. Below is a list of some of them.

Refrigerant	R number	Evaporation temp. at 1.013 bar.(^oC)	Toxic group
$C\ Cl_3\ F$	R11	24	5
$C\ Cl_2F_2$	R12	-30	6
$C\ ClF_3$	R13	-82	6
$C\ F_4$	R14	-128	6
$CH\ Cl_2F$	R21	9	4
$CH\ Cl\ F_2$	R22	-40	5
$CH\ F_3$	R23	-84	5
$C\ Cl_2\ F\ C\ Cl\ F_2$	R113	47	4
$C\ Cl_2\ F\ C\ F_3$	R114A	3	6
$C\ Cl_2\ F_2\ C\ Cl\ F_2$	R114	3	6
$C\ Cl_2\ F_2\ C\ \ F_3$	R115	-39	

All the above are Halo-Carbons and Fluro-carbons which are non-flammable and may be detected by a halide torch or electric cell sensor. Other refrigerants are shown below.

Ammonia is flammable and detected by going white in the presence of sulphur dioxide. It has a strong characteristic pungent smell. Death occurs when breathed for 30 minutes.

NH_3	R717	-33	2

Carbon Dioxide is safe and non-toxic but it can suffocate.

CO_2	R744	-78

Sulphur Dioxide is highly toxic and does not burn.

Other refrigerants are in the Hydro-Carbon groups such as Propane, Butane and Ethane. These are explosive. Because of the problems with damage to the ozone layer, new refrigerants such as R134a have been developed and are now included in the thermodynamic tables.

Now let's look at the use of thermodynamic tables for refrigerants.

3. TABLES

The section of the fluid tables devoted to refrigerants is very concise and contains only two superheat temperatures. The layout of the tables is shown below.

t	p_s	v_g	h_f	h_g	s_f	s_g	15K		30 K	
							h	s	h	s

t is the actual temperature in degrees Celsius.
p_s is the saturation pressure corresponding to the temperature.
It follows that if the refrigerant is wet or dry saturated, it must be at temperature t and pressure p_s. If the refrigerant has 15 degrees of superheat, then the actual temperature is t+15 and the properties are found under the 15 K heading. Similarly if it has 30 K of superheat, its actual temperature is t+30.

For example, R12 at 2.191 bar and 20°C must have 30 K of superheat since its saturation temperature would is -10°C. From the 30 K columns we find that h=201.97 kJ/kg and s = 0.7695 kJ/kg K.

When dealing with liquid refrigerant, take the properties as h_f and s_f at the given temperatures. The pressures are never very high so the pressure term will not cause much error.

4. VAPOUR COMPRESSION CYCLES

4.1 THE BASIC CYCLE

Refrigeration/heat pump cycles are similar to heat engine cycles but they work in reverse and are known as reversed heat engine cycles. A basic vapour cycle consists of isentropic compression, constant pressure cooling, isentropic expansion and constant pressure heating. You may recognise this as a reverse of the Rankine cycle or even the reverse of a Carnot cycle. The heating and cooling will involve evaporation and condensing. Let's consider the cycle first conducted entirely with wet vapour.
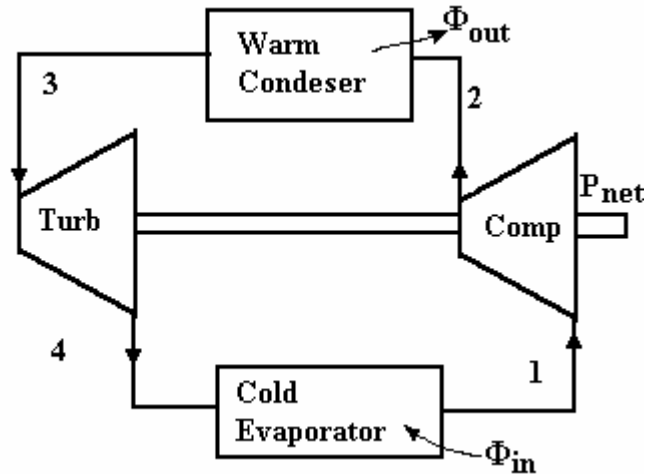

Figure 2

The basic principle is that the wet vapour is compressed and becomes dryer and warmer in the process. It is then cooled and condensed into a wetter vapour at the higher pressure. The vapour is then expanded. Because of the cooling, the expansion back to the original pressure produces a fluid which is much colder and wetter than it was before compression. The fluid is then able to absorb heat at the cold temperature becoming dryer in the process and is returned to the original state and compressed again. The net result is that heat is absorbed at a cold temperature and rejected at a higher temperature. Work is needed to drive the compressor but some of it is returned by the turbine.

The thermodynamic cycle for refrigerators is often shown on a pressure – enthalpy diagram (p – h) and professional charts are available but not used in the Engineering Council exams. Figure 3 shows the basic cycle.
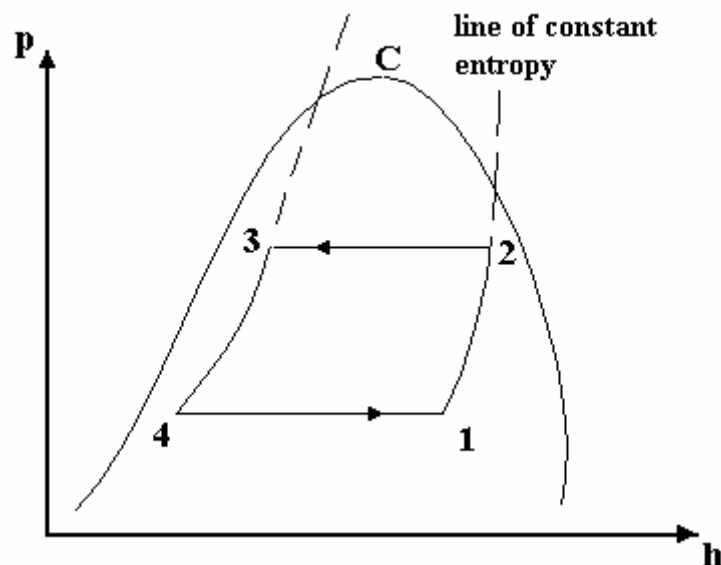

Figure 3

The four thermodynamic processes are

1 - 2 Isentropic compression.
2 - 3 Constant pressure cooling.
3 - 4 Isentropic expansion.
4 - 3 Constant pressure heating.
m_r = mass flow rate of refrigerant.

$P(in) = m_r(h_2 - h_1)$
$\Phi(out) = m_r(h_3 - h_2)$
$P(out) = m_r(h_3 - h_4)$
$\Phi(in) = m_r(h_4 - h_1)$

In practice wet vapour is difficult to compress and expand so the refrigerant is usually dry before compression and superheated after. The cooling process may produce anything from wet vapour to under-cooled liquid. The expansion of a liquid in a turbine is impractical and so a throttle is used instead.
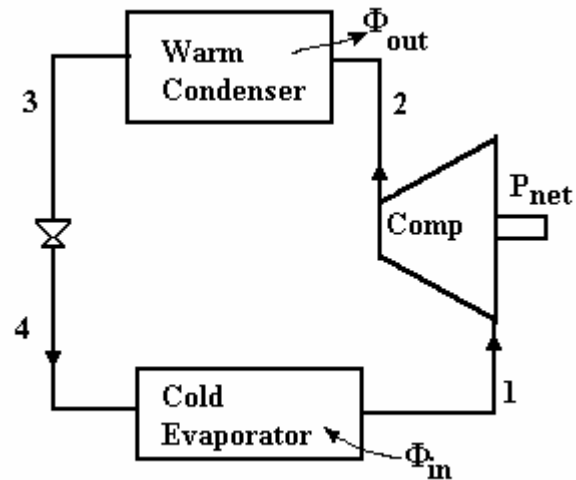

Figure 4

A throttle produces no useful work but it converts the pressure into internal energy. This makes the liquid evaporate and since the saturation temperature goes down it ends up cold. The enthalpy before and after a throttle are the same. The entropy increases over a throttle.
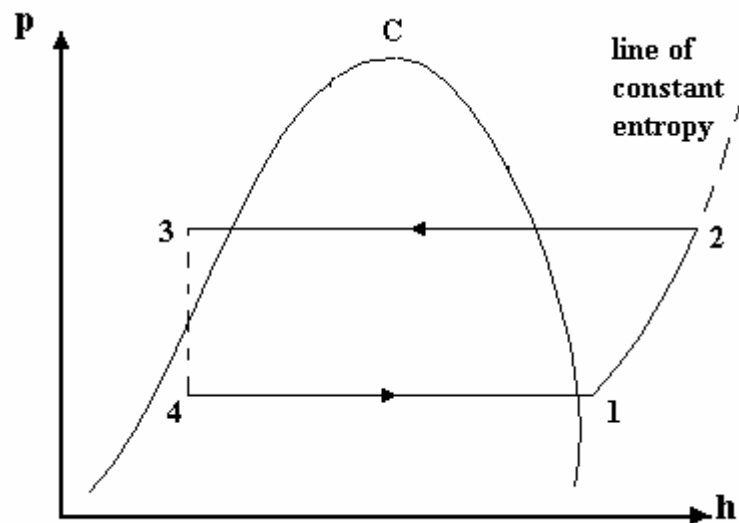

Figure 5

The cycle may also be drawn on a temperature entropy diagram as shown. The conditions shown are wet at (1), superheated at (2) and under-cooled at (3). These conditions vary.
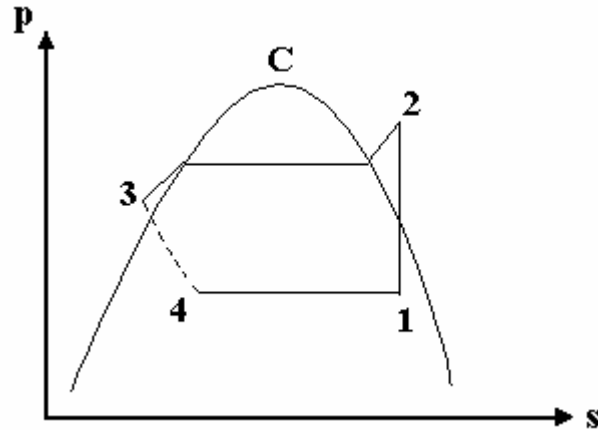
The four thermodynamic processes are

1 - 2 Isentropic compression.
$$P(in) = m_r(h_2 - h_1)$$
2 - 3 Constant pressure cooling.
$$\Phi(out) = m_r(h_3 - h_2)$$
3 - 4 Throttle ($h_3 = h_4$)
4 - 3 Constant pressure heating.
$$\Phi(in) = m_r(h_4 - h_1)$$

Figure 6

4.2 COEFFICIENT OF PERFORMANCE

The second law of thermodynamics tells us that no heat engine may be 100% efficient. In the reversed cycle, the reverse logic applies and it will be found that more energy is given out at the condenser and more absorbed in the evaporator, than is needed to drive the compressor. The ratio of heat transfer to work input is not called the efficiency, but the coefficient of performance or advantage.

There are two coefficients of performance for such a cycle, one for the refrigeration effect and one for the heat pump effect.

4.2.1 REFRIGERATOR

A refrigerator is a device for removing heat at a cold temperature so we are interested in the heat absorbed in the evaporator $\Phi(in)$. The coefficient of performance is also called the advantage and is defined as

C.O.P. = $\Phi(in)/P(in)$

The heat absorbed is called the refrigeration affect.

4.2.1 HEAT PUMP

A heat pump is a device for producing heat so we are interested in the heat given out in the cooler $\Phi(out)$. The coefficient of performance is defined as

C.O.P. = $\Phi(out)/P(in)$

It is usual to find a convenient source of low grade heat for the evaporator such as the atmosphere or a river. The heat is removed from this source and upgraded to higher temperature by the compressor. Both the work and the heat absorbed are given out at the higher temperature from the cooler.
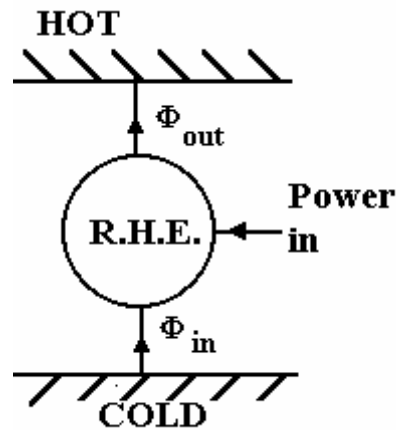
7

5. MODEL REVERSED HEAT ENGINE

Figure 7

The ideal model pumps heat from a cold source to a hot place. The 1st. Law of Thermodynamics applies so

$$\Phi(in) + P(in) = \Phi(out)$$

$$C.O.P. \text{ (refrigerator)} = \Phi(in)/P(in) = \Phi(in)/\{\Phi(out) - \Phi(in)\}$$

If the heat transfers are reversible and isothermal at temperatures $T_{(hot)}$ and $T_{(cold)}$ then
$$C.O.P. \text{ (refrigerator)} = T_{(cold)}/\{T_{(hot)} - T_{(cold)}\}$$

$$C.O.P. \text{ (heat pump)} = \Phi(out))/P(in)$$

Again for reversible isothermal heat transfers this reduces to

$$C.O.P. \text{ (heat pump)} = T_{(hot)} /\{T_{(hot)} - T_{(cold)}\}$$

This is the inverse of the Carnot efficiency expression for heat engines.

$$C.O.P. \text{ (heat pump)} = \{\Phi(in) + P(in)\}/P(in)$$

$$C.O.P. \text{ (heat pump)} = \Phi(in)/P(in) + 1$$

$$C.O.P. \text{ (heat pump)} = C.O.P. \text{ (refrigerator)} + 1$$

WORKED EXAMPLE No.1

A heat pump uses a vapour compression cycle with refrigerant 12. The compressor is driven by a heat engine with a thermal efficiency of 40%. Heat removed from the engine in the cooling system is recovered. This amounts to 40% of the energy supplied in the fuel.

The heat pump cycle uses an ideal cycle with an evaporator at 5°C and a condenser at 12.19 bar. The vapour is dry saturated at inlet to the compressor. The condenser produces liquid at 45°C.

Calculate the thermal advantage (Coefficient of Performance) for the heat pump. Compare it with a boiler running at 90% thermal efficiency.

The plant is to deliver 40 kW of heat. Determine the mass flow rate of refrigerant.
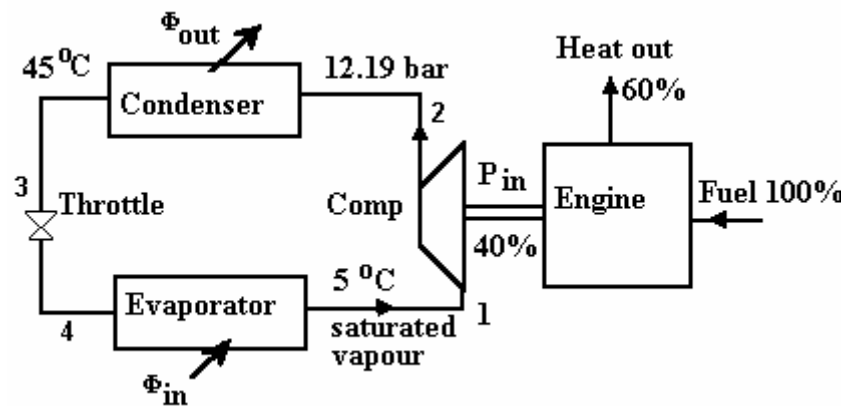
SOLUTION

Figure 8

From the R12 tables we find
$h_1 = h_g$ @ 5°C = 189.66 kJ/kg
$s_1 = s_g$ @ 5°C = 0.6943 kJ/kg K = s_2 @ 12.19 bar
If a p - h chart was available, h_2 could be found easily. We must use the tables and we can see that 0.6943 occurs between 0 K of superheat and 15 K of superheat. Using linear interpolation we may find the enthalpy as follows.

	0 K	θ	15 K	
s	0.6797	0.6943	0.7166	0.7166 - 0.6797 =0.0369
h	206.45	h_2	218.64	0.6943 - 0.6797 =0.0146
				218.64 -206.45 =12.19

$h_2 = 206.45 + (0.0146/0.0369)12.19 = 211.27$ kJ/kg
$h_3 = h_4 = h_f$ at 45°C = 79.71 kJ/kg
$h_1 = 189.66$ kJ/kg

P(in) = $h_2 - h_1$ = 21.67 kJ/kg = 40% of fuel energy

Φ(out) = $h_2 - h_3$ = 131.56 kJ/kg

Φ(in) = $h_1 - h_4$ = 109.95 kJ/kg

C.O.P (condenser). = 131.56/21.67 = 6.07

C.O.P (evaporator). = 109.95/21.67 = 5.07

Φ(out) = 40% x 10.43 = 417.2% of fuel power

Total heat from system = 242.8 % + 40% = 282.8 % of fuel energy.

Compared to a boiler which gives 90% this is 192.8 % more.

Total heat output = 40 kW = 282.8 % of fuel power.

Fuel Power = 14.14 kW

40 kW = Φ(out) + energy recovered from cooling water

40 = $m_r(h_2 - h_3)$ + 40% x 14.14 = m_r(131.56) + 5.656

m_r = 0.261 kg/s

Φ(in) = $m_r(h_1 - h_4)$ = 28.7 kW

Φ(out) = $m_r(h_2 - h_3)$ = 34.3 kW

6. ISENTROPIC EFFICIENCY

When the compression is not reversible and isentropic then the isentropic efficiency is used in the usual way for a compression process.

$$\eta_{is} = \text{ideal enthalpy change/actual enthalpy change}$$

WORKED EXAMPLE No.2

The power input to the compressor of an ammonia vapour compression plant is 8.2 kW. The mechanical efficiency is 85%. The ammonia is dry saturated at -6°C at inlet to the compressor. After compression the vapour is at 11.67 bar. The compression has an isentropic efficiency of 90%. The condenser produces saturated liquid.

Calculate the following.

        i. The flow rate.

        ii. The coefficient of performance for the refrigerator.

        iii. The coefficient of performance for the heat pump.
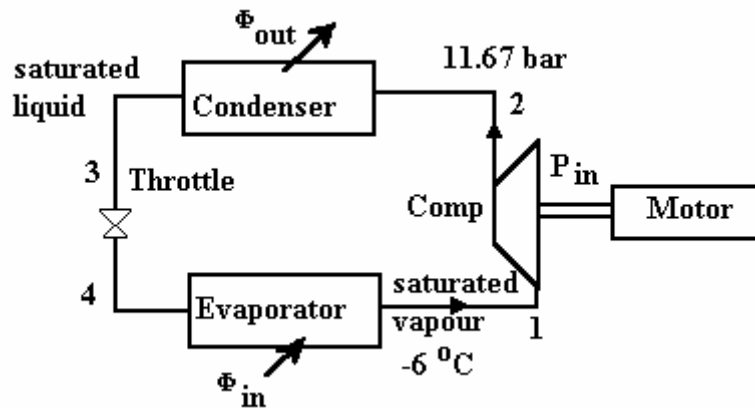
SOLUTION

Figure 9

$h_1 = h_f$ at -6°C = 1437.6 kJ/kg $h_3 = h_4 = h_f$ @ 11.67 bar = 323.1 kJ/kg

$s_1 = s_f$ at -6°C = 5.419 kJ/kg K   Ideally $s_2 = s_1 =$ 5.419 kJ/kg K

From the tables at 11.67 bar we see that the specific entropy is 5.417 kJ/kg K when there is 50 K of superheat. This is the ideal condition after compression and the corresponding enthalpy is 1610.5 kJ/kg K.

Ideal change in enthalpy = 1610.5 - 1437.6 = 172.9 kJ/kg.

Actual change = 172.9/90% = 192.1 kJ/kg.

Actual value of h_2 = 1437.6 + 192.1 = 1629.7 kJ/kg.

Power input to cycle = 8.2 kW x 85% = 6.97 kW = m_r(1629.7 - 1437.6)

m_r = 6.97/192.1 = 0.0363 kg/s

Heat input to evaporator = $m_r(h_1 - h_4)$ = 40.44 kW

Coefficient of performance (refrigerator) = 40.44/6.97 = 5.8

Heat output at condenser = $m_r(h_2 - h_3)$ = 470 kW

Coefficient of performance (heat pump) = 47.4/6.97 = 6.8

SELF ASSESSMENT EXERCISE No.1

1. A simple vapour compression refrigerator comprises an evaporator, compressor, condenser and throttle. The condition at the 4 points in the cycle are as shown.

Point	Pressure	Temperature
After evaporator	0.8071 bar	-20ºC
After compressor	5.673 bar	50ºC
After condenser	5.673 bar	15ºC
After throttle	0.8071 bar	-35ºC

    The refrigerant is R12 which flows at 0.05 kg/s. The power input to the compressor is 2 kW. Compression is reversible and adiabatic.

    Calculate the following.
        i. The theoretical power input to the compressor. (1.815 kW)
        ii. The heat transfer to the evaporator. (6.517 kW)
        iii. The coefficient of performance based answer (i.)(3.59)
        iv. The mechanical efficiency of the compressor. (90.7%)
        v. The coefficient of performance based on the true power input. (3.26)

    Is the compression process isentropic?

2. A vapour compression cycle uses R12. the vapour is saturated at -20ºC at entry to the compressor. At exit from the compressor it is at 10.84 bar and 75ºC. The condenser produces saturated liquid at 10.84 bar. The liquid is throttled, evaporated and returned to the compressor.

    Sketch the circuit and show the cycle on a p-h diagram.

    Calculate the coefficient of performance of the refrigerator. (2.0)

    Calculate the isentropic efficiency of the compressor. (71%)

7 MODIFIED CYCLES

An improvement to the basic compression cycle is the use of a flash chamber instead of a throttle valve. The condensed high pressure liquid at point 7 is sprayed into the low pressure flash chamber. The drop in pressure has the same effect as throttling and the liquid partially evaporates and drops in temperature. The dry saturated vapour is drawn into the compressor and the saturated liquid is pumped to the evaporator. The principal difference is that the evaporator now operates at a higher pressure and so the liquid at point 2 is below the saturation temperature.
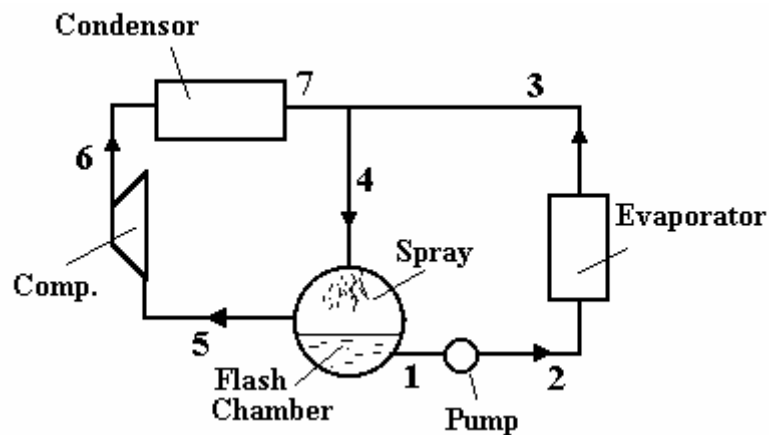

Figure 10
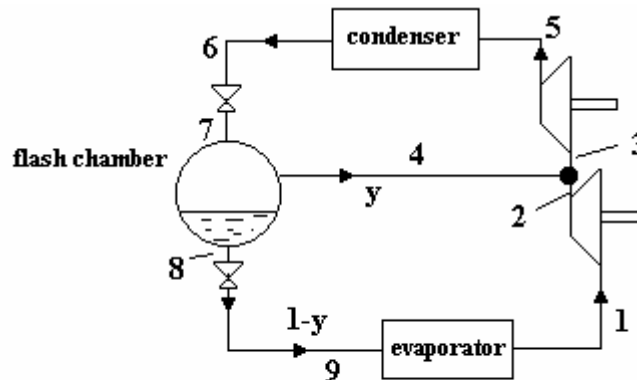
Further modifications may be made by compressing the vapour in two stages and mixing the vapour from the flash chamber at the inter-stage point. The output of the evaporator then goes to the input of the low pressure stage as shown in fig.10

These modifications require more hardware than the basic cycle so the extra cost must be justified by savings and increased capacity to refrigerate.

A refrigeration plant uses R12 in the cycle below. The evaporator temperature is -50°C and the condenser pressure is 50°C. The flash chamber is maintained at 0°C. Saturated vapour from the chamber is mixed with the compressed vapour at the inter-stage point (3). The liquid in the chamber is further throttled to -50°C in the evaporator. The vapour leaving the evaporator is dry saturated and compression is isentropic. Find the coefficient of performance for the refrigerator assuming 1 kg/s flow rate.

Figure 11

SOLUTION

At point (1) the vapour is dry saturated so $h_1 = h_g$ @ -50°C = 164.95 kJ/kg
Similarly $s_1 = s_g$ = 0.7401 kJ/kg K
    The pressure at point 2/3 must be p_s at 0°C = 3.08 bar
     $s_1 = s_2$ = 0.7401 kJ/kg K @ 3.08 bar.

From the tables this is seen to be superheated so the degree of superheat and the enthalpy must be estimated by interpolation as follows.

    $\Delta T/T$ = (0.7401 - 0.7311)/(0.7641 - 0.7311)
    ΔT= 4.09 K

Now the enthalpy at point 2 may be estimated as follows.
    4.09/15 = (h_2 - 197.25)/(.7641 - .7311)
    h_2 = 199.9 kJ/kg

Energy balance on Flash Chamber

Assume saturated liquid at point (6). $h_6 = h_f$ @ 50°C = 84.94 kJ/kg
Since the enthalpy is the same after throttling then h_7 = 84.94 kJ/k
    $h_4 = h_g$ at 0°C =187.53 kJ/kg

Let the flow rate be 1 kg/s at (7) and (y) at point (4). Balancing enthalpy we have

$h_7 = y(h_4) + (1-y)h_8$

$h_8 = h_f$ @ 0ºC = 36.05 kJ/kg K

$y(187.53) + (1-y)(36.05)$

$y = 0.3227$ kg/s

Energy balance at mixing point.

$(1-y)h_1 + yh_4 = h_3$

$((1-0.3227)(199.9) + (0.3227)(187.53) = h_3 = 195.9$ kJ/kg

Power of LP Turbine

$P_1 = (1-y)$ kg/s $(h_2 - h_3) = (1-0.3227)(199.9 - 164.95) = 23.67$ kW

The vapour at point 3 is clearly superheated between 0 and 15 K. Interpolation gives the degree of superheat as follows.

$\Delta T/15 = (195.908-187.53)/(197.25-187.53)$

$\Delta T = 12.93$ K

We may now interpolate to find s_3

$12.93/15 = (s_3 - 0.6966)/(0.7311 - 0.6966)$

$s_3 = 0.7263$ kJ/kg K $= s_5$ @ 12.19 bar (p_s at 50ºC)

Again we must interpolate to find the degree of superheat at point 5 which is between 0 and 15 K again.

$\Delta T/15 = (0.7263 - 0.7166)/(0.7503-0.7166)$

$\Delta T = 4.33$ K

We may now interpolate with this degree of superheat to find h_5.

$4.33/15 = (h_5 - 218.64)/(230.33 - 218.64)$

$h_5 = 222$ kJ/kg

HP Turbine Power

$P_2 = 1$ kg/s$(h_5 - h_3) = (222-195) = 26.92$ kW

Total Power input = 50.59 kW

Evaporator Heat Input

$h_9 = h_8 = h_f$ at 0ºC = 36.05 kJ/kg

$\Phi(in) = 0.6773$ kg/s $(h_1 - h_9)$

$\Phi(in) = 0.6773(164.95 - 36.05) = 87.3$ kW

Coefficient of Performance = 87.3/26.92 =1.73

SELF ASSESSMENT EXERCISE No.2

1. A refrigerator operates with ammonia. The plant circuit is shown below. The conditions at the relevant points of the cycle are as follows.

1	saturated liquid at -30ºC
3,4 and 7	saturated liquid at 10ºC
5	saturated vapour at -30ºC

    The pump and compressor have an isentropic efficiency of 80%. there are no heat losses. The specific volume of ammonia liquid is 0.0015 m3/kg.

    Determine the coefficient of performance and the mass flow rate if the refrigeration effect is 10 kW.
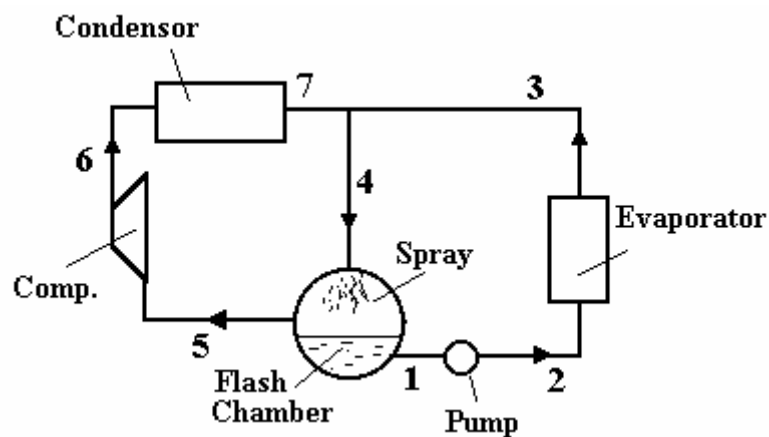
    (Ans. 3.964 and 0.0635 kg/s)

Figure 12

2. A heat pump consists of a compressor, condenser, throttle, and evaporator. The refrigerant is R12. The refrigerant is at 0°C at entry to the compressor and 80°C at exit. The condenser produces saturated liquid at 50°C. The throttle produces wet vapour at -10°C. The mass flow rate is 0.02 kg/s. The indicated power to the compressor is 1 kW.

Sketch the T - s diagram and p - h diagram for the cycle.

Calculate the coefficient of performance for the heat pump
(2.9 based on I.P.)

Calculate the rate of heat loss from the compressor.
(0.2 kW)

Calculate the coefficient of performance again for when the refrigerant is sub cooled to 45°C at exit from the condenser.
(3 based on I.P.)

Calculate the temperature at exit from the compressor if the compression is reversible and adiabatic.
(68.7°C)

3. A refrigeration cycle uses R12. The evaporator pressure is 1.826 bar and the condenser pressure is 10.84 bar. There is 5K of superheat at inlet to the compressor. The compressor has an isentropic efficiency of 90%. the condensed liquid is under-cooled by 5K and is throttled back to the evaporator.

Sketch the cycle on a T-s and p-h diagram.

Calculate the coefficient of performance. (3.04)

Explain why throttles are used rather than an expansion engine.

8. RECIPROCATING COMPRESSORS

This is covered in detail in tutorial 2. The knowledge of compressors is often required in refrigeration and heat pump studies so the basics are covered here along with example questions on compressors used in this area. The diagram shows the basic design of a reciprocating compressor. The piston reciprocates drawing in gas, compressing it and expelling it when the pressure inside the cylinder reaches the same level as the pressure in the delivery pipe.
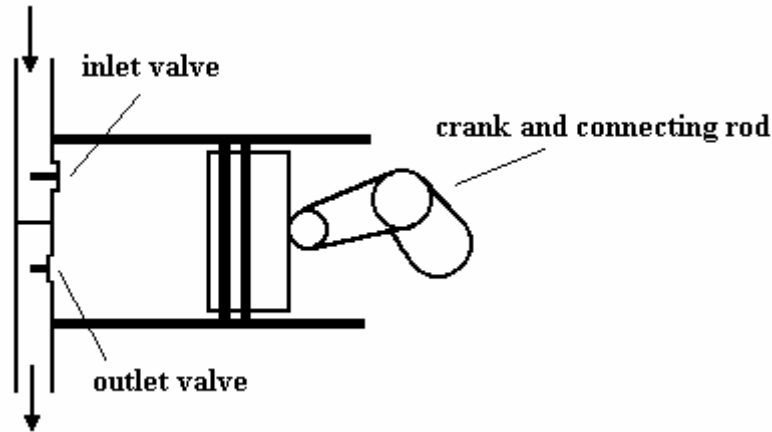

Figure 13

If the piston expels all the air and there is no restriction at the valves, the pressure - volume cycle is as shown below.
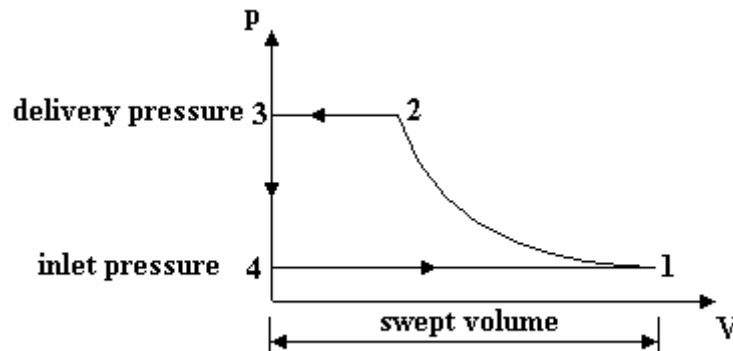

Figure 14

Gas is induced from 4 to 1 at the inlet pressure. It is then trapped inside the cylinder and compressed according the law $pV^n = C$. At point 2 the pressure reaches the same level as that in the delivery pipe and the outlet valve pops open. Air is then expelled at the delivery pressure. The delivery pressure might rise very slightly during expulsion if the gas is being compacted into a fixed storage volume. This is how pressure builds up from switch on.

In reality, the piston cannot expel all the gas and a clearance volume is needed between the piston and the cylinder head. This means that a small volume of compressed gas is trapped in the cylinder at point 3. When the piston moves away from the cylinder head, the compressed gas expands by the law $pV^n = C$ until the pressure falls to the level of the inlet pressure. At point 4 the inlet valve opens and gas is drawn in. The volume drawn in from 4 to 1 is smaller than the swept volume because of this expansion.
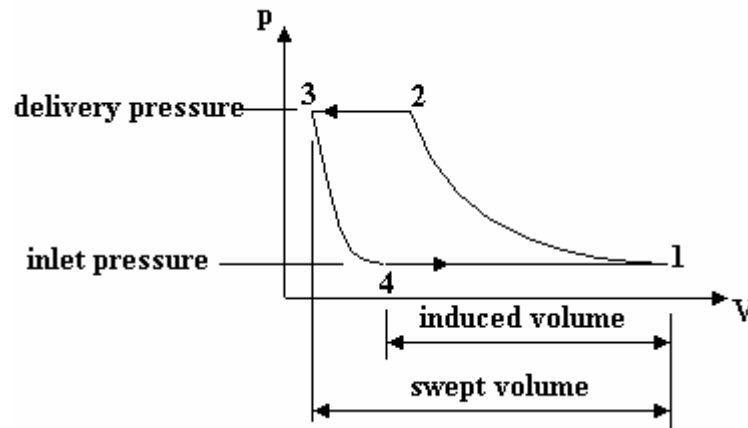

Figure 15

The volumetric efficiency is defined as

η_{vol} = Induced Volume/Swept volume.

This efficiency is made worse if leaks occur past the valves or piston.

In real compressors, the gas is restricted by the valves and the valves tend to move so the real cycle looks more like this.
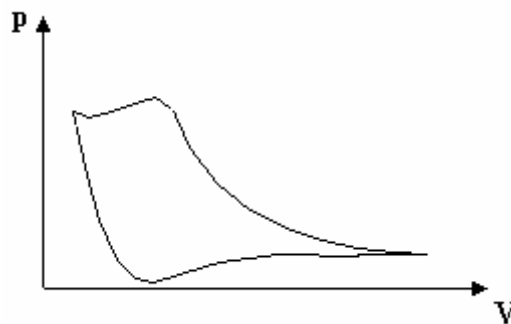

Figure 16

WORKED EXAMPLE No.4

Dry saturated Refrigerant 12 vapour at 5°C is compressed in a reciprocating compressor to 12.19 bar at a rate of 0.274 kg/s. The clearance volume is 5% of the swept volume. The expansion part of the cycle follows the law $pV^{1.2} = C$. The crank speed is 360 rev/min. Calculate the swept volume and the volumetric efficiency.

SOLUTION

Swept Volume = V   Clearance volume = 0.05 V
Consider the expansion from 3 to 4 on the p-V diagram.
The inlet pressure must be p_S at 5°C hence p_4 = 3.626 bar.

$$p_3 V_3^{1.2} = p_4 V_4^{1.2}$$

$$12.196(0.05V)^{1.2} = 3.626 (V_4^{1.2})$$
V_4 = 0.137V or 13.7% of V

V_1 = V + 0.05V = 1.05V
Induced volume = $V_1 - V_4$ = 1.05V - 0.137V = 0.913V

m_r = 0.274 kg/s

At inlet v = v_g at 5°C = 0.0475 m^3/kg.

Volume flow rate required = 0.274 x 0.0475 = 0.013 m^3/s.

Induced volume = 0.013 = 0.913V
V = 0.013/0.913 = 0.0142 m^3/s

Crank speed = 6 rev/s so the swept volume = 0.0142/6 = 0.00237 m^3.

η_{vol} = Induced Volume/Swept volume.

η_{vol} = 0.913V/V = 91.3%

1. Why is it preferable that vapour entering a compressor superheated?

    A vapour compression refrigerator uses R12. The vapour is evaporated at -10°C and condensed at 30°C. The vapour has 15 K of superheat at entry to the compressor. Compression is isentropic. The condenser produces saturated liquid.

    The compressor is a reciprocating type with double action. The bore is 250 mm and the stroke is 300 mm. The speed is 200 rev/min. The volumetric efficiency is 85%. You may treat superheated vapour as a perfect gas. Determine

    > i. the mass flow rate (0.956 kg/s)
    > ii. the coefficient of performance. (5.51)
    > iii. the refrigeration effect. (122.7 kW)

    (Note that double acting means it pumps twice for each revolution. The molecular mass for R12 is given in the tables.)

9. AMMONIA ABSORPTION CYCLE

The basic principle of the ammonia absorption cycle is similar to that of the vapour compression cycle. An evaporator is used to absorb heat at a low temperature and a condenser is used to reject the heat at a higher temperature. The difference is in the way the ammonia is passed from the evaporator to the condenser. In a compression cycle this is done with a compressor. In the absorption cycle it is done by absorbing the ammonia into water at the lower temperature. The water and ammonia is then pumped to a heater raising the pressure and temperature. The heater also separates the ammonia from the water and the ammonia vapour is driven off is at a higher pressure and temperature than it started at. The vapour is then condensed and throttled back to the evaporator.
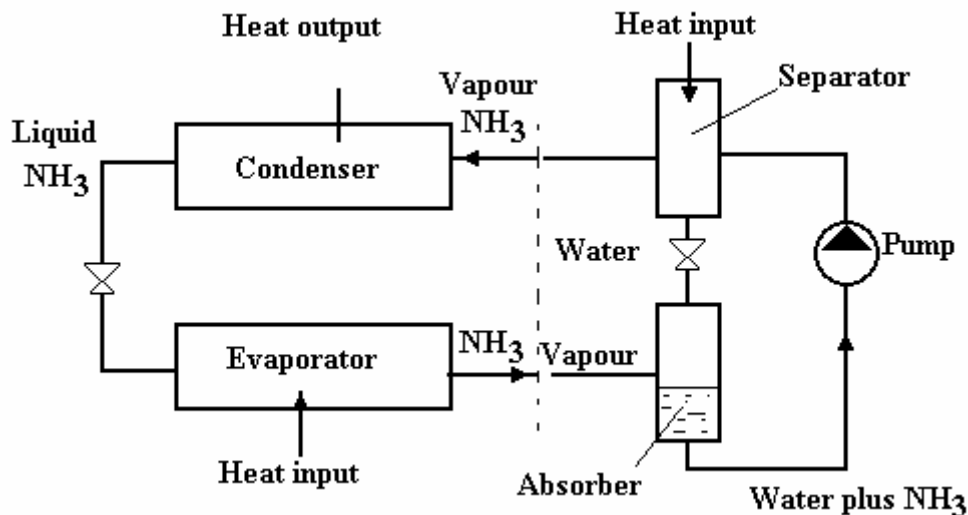

Figure 17

The advantage of this system is that a water pump replaces the vapour compressor. The pump may be done away with altogether by making use of the principle of partial pressures. When hydrogen is mixed with ammonia vapour, the total pressure of the mixture 'p' is the sum of the partial pressures such that

$$p = p(ammonia) + p(hydrogen)$$

The mixing is done in the evaporator but the total pressure stays the same. The ammonia vapour hence experiences a drop in pressure when mixing occurs and the effect is the same as throttling so a throttle valve is not needed either. Mixing causes the vapour to cool and condense so that a cold wet vapour results.

Evaporation dries it out and a mixture of ammonia and hydrogen gases leaves the evaporator. The mixture goes to the absorber where the ammonia is absorbed into the water leaving the hydrogen behind. The hydrogen goes back to the evaporator in a continuous cycle. When the ammonia and hydrogen separate out in the absorber, they both experiences a pressure rise back to p which is also the water pressure.
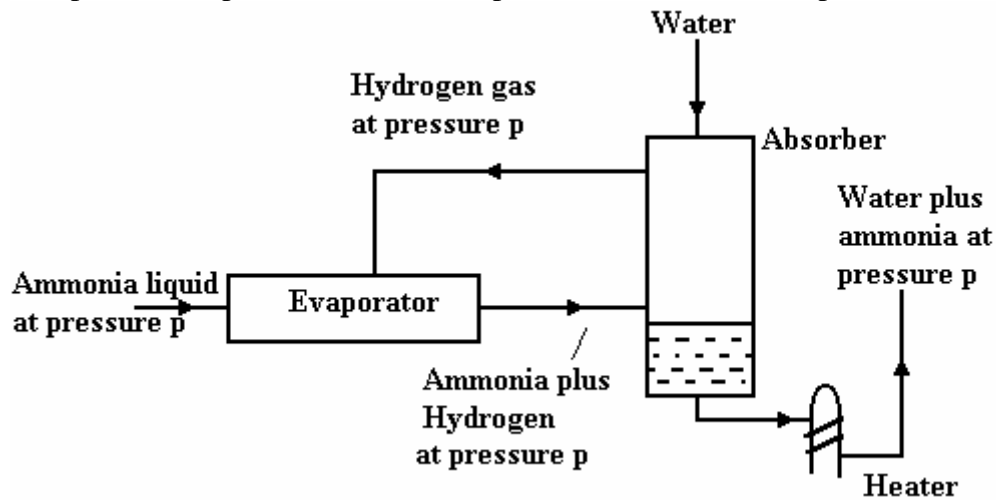

Figure 18

Since no pressure difference exists between the evaporator and condenser, circulation may be caused by a thermo-siphon which is induced by heating the water and ammonia. The p-h cycle is similar to that of a vapour compression cycle. Process 3 to 4 is due to the mixing in the evaporator. Process 1 to 2 is due to the absorption.
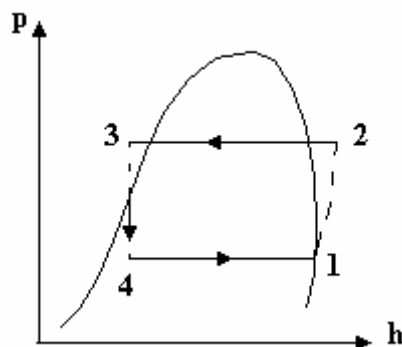
Figure 19

Heat which is put into the separator in order to make the ammonia leave the water, is carried with the water back to the absorber. This heat can be used to operate the thermo-siphon by use of a heat exchanger. A schematic of a complete plant is shown below. The heat required to operate the system may be obtained from anywhere and is commonly a gas flame (Electrolux refrigeration system). This system is popular in caravan refrigerators.
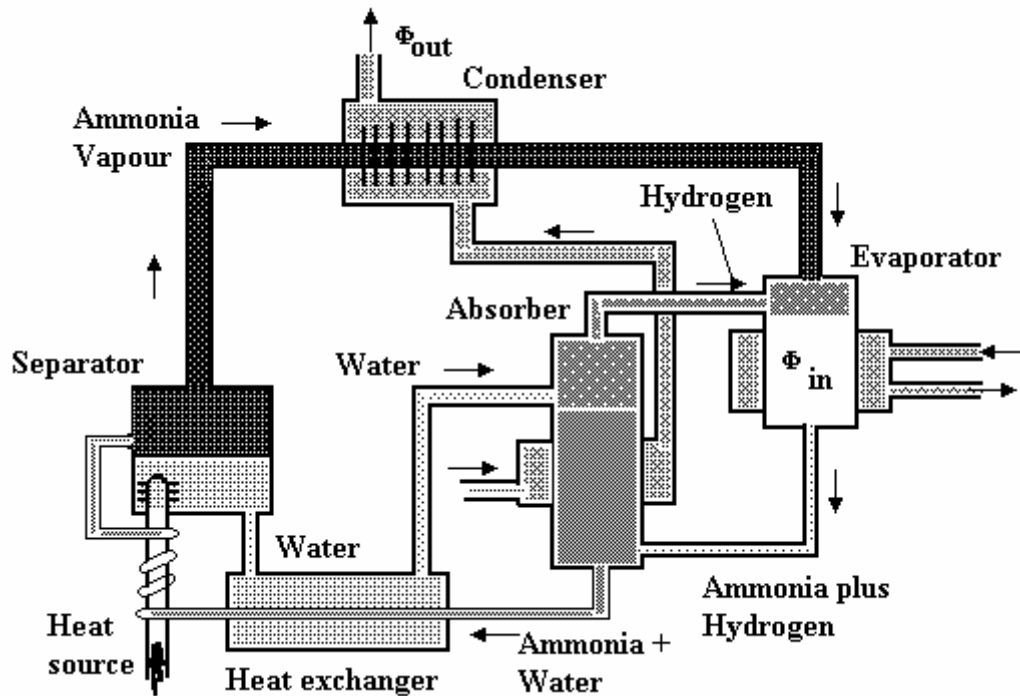
Figure 20

APPLIED THERMODYNAMICS

TUTORIAL 6

AIR-VAPOUR MIXTURES

In this tutorial you will do the following.

- ▪ **Revise the UNIVERSAL GAS LAW**

- ▪ **Learn DALTONS LAW OF PARTIAL PRESSURES**

- ▪ **Apply these laws to mixtures of water vapour and air.**

- ▪ **Solve problems involving air conditioning plant.**

- ▪ **Solve problems involving cooling towers.**

- ▪ **Solve problems involving steam condensers.**

Let's start by revising the Universal gas law and the law of partial pressures.

1.1 UNIVERSAL GAS LAW

$$pV = \frac{mR_oT}{\tilde{N}}$$

where Ro is the universal constant 8314.4 J/kmol K

$\tilde{N}$ is the relative molecular mass which is 18 for water vapour treated as a gas and 28.96 for dry air treated as a single gas.

1.2 PARTIAL PRESSURES

The pressure exerted by a gas on the surface of containment is due to the bombardment of the surface by the molecules. The relative distance between molecules is very large so if two or more gases exist in the same space, their behaviour is unaffected by the others and so each gas produces a pressure on the surface according to the gas law above. Each gas occupies the total volume V and has the same temperature T. If two gases A and B are considered, the pressure due to each is :

$$p_a = \frac{m_aR_oT}{\tilde{N}_aV}$$

$$p_b = \frac{m_bR_oT}{\tilde{N}V_b}$$

The total pressure on the surface of containment is   **p = p$_a$ + p$_b$**

This is Daltons Law of partial pressures.

Now let's see how these laws are applied to mixtures of vapour and air.

2. <u>AIR - VAPOUR MIXTURES</u>

In the following work, water vapour is treated as a gas.

Consider a mixture of dry air and vapour. If the temperature of the mixture is cooled until the vapour starts to condense, the temperature must be the saturation temperature (dew point) and the partial pressure of the vapour ps must be the value of ps in the fluids tables at the mixture temperature.

If the mixture is warmed up at constant pressure so that the temperature rises, the vapour must become superheated. It can be shown that the partial pressure of the vapour and the dry air remains the same as at the saturation temperature.

Let condition (1) be at the saturation condition and condition (2) be at the higher temperature. p is constant so it follows that :

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

The initial partial pressure of the vapour is:

$$p_{s1} = \frac{m_s R o T_1}{\tilde{N}_s V_1}$$

The final pressure of the vapour is :

$$p_{s2} = \frac{m_s R o T_2}{\tilde{N}_s V_2}$$

Since $\qquad \frac{V_1}{T_1} = \frac{V_2}{T_2}$ then $\mathbf{p_{s1} = p_{s2}}$

By the same process it can be shown that $\quad p_{a1} = p_{a2}$

If p is constant then the partial pressures are constant and the partial pressure of the vapour may easily be found by looking up the saturation pressure at the dew point if it is known.

When the air is contact with water, it will evaporate the water and the water will cool down until it is at the saturation temperature or dew point. This idea is used in wet bulb thermometers for example, which measure the dew point. When stable conditions are reached, the air becomes saturated and equal to the temperature of the water and so its temperature is the dew point (t_s) in fluids tables.

Moist air at 1 bar and 25ºC passes over water and emerges at 1 bar and 18ºC. Calculate the partial pressure of the air and vapour before cooling.

SOLUTION

When cooled 18ºC must be the saturation temperature so the partial pressure of the vapour is p_S in the fluids tables and is 0.02063 bar.

The partial pressure of the vapour was the same before cooling so the partial pressure of the air must be   **1 - 0.02063 = 0.97937 bar.**

Now let's look at the definitions and use of humidity.

3. HUMIDITY

There are two ways to express humidity SPECIFIC AND RELATIVE.

3.1. SPECIFIC HUMIDITY   ω

ω = mass of water vapour/mass of dry air

Starting with the gas law

$$m = \frac{pV\widetilde{N}}{RoT}$$

$$\omega = \frac{p_s VRoT\widetilde{N}_s}{p_a VRoT\widetilde{N}_a} = \frac{p_s \widetilde{N}_s}{p_a \widetilde{N}_a} = \frac{p_s}{p_a} \times \frac{18}{28.96} = 0.622 \frac{p_s}{p_a}$$

$$\omega = 0.622 \frac{p_s}{p - p_s}$$

This derivation is often requested in Examinations.

3.2. RELATIVE HUMIDITYϕ

ϕ = mass of vapour/maximum possible mass of vapour

The maximum possible mass of water vapour which can be held by air is when the vapour is saturated and the temperature of the mixture is the saturation temperature.

$$\text{mass} = \text{Volume/specific volume} = V/v$$

When saturated, $v = v_g$ at the mixture temperature.

$$\varphi = \frac{m_s}{m_g} = \frac{V}{v_s} \div \frac{V}{v_a} = \frac{v_a}{v_s}$$

Alternatively $v = V/m$

$$v_s = \frac{N_s p_s}{RoT} \text{ and } v_g = \frac{N_g p_g}{RoT}$$

$$\varphi = \frac{p_s}{p_g}$$

p_S = partial pressure of the actual vapour

p_g = partial pressure when saturated.

$$\omega = 0.622 \frac{p_s}{p - p_s} \text{ and } \varphi = \frac{p_s}{p_g}$$

$$\varphi = \frac{\omega(p - p_s)}{0.622 p_g}$$

WORKED EXAMPLE No.2

Moist air at 1 bar and 25°C is cooled to 18°C by passing it over water at 18°C. It emerges at 18°C and 1 bar with a relative humidity of 1.0. Assuming that there is no net water absorbed nor lost, calculate the relative and specific humidity before cooling.

SOLUTION

This is the same as the previous problem so the dew point must be 18°C and the partial pressure of the vapour is p_S at 18°C and is 0.02063 bar.

p_a = 1 - 0.02063 = 0.97937 bar

p_g at 25°C = 0.03166 bar

It follows that if no net water is gained nor lost then the specific humidity must be the same before and after and is :

$\omega = 0.622(p_S/p_a) = 0.0131$

$\phi = p_S/p_g = 0.651$

These are the humidity values which will result in no evaporation nor condensation.

If $\phi < 0.651$ then there will have been evaporation.

If $\phi > 0.651$ then condensation will have taken place on contact with the water and cooling also.

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MASS BALANCE

Consider the worked example 2 again only this time suppose the relative humidity at inlet is 0.5. This means that water is evaporated. Consider 1 kg of dry air passing through from inlet to outlet.

At outlet $\phi = 1$ and $p_{s2} = p_g$ at 18°C = 0.02063 bar.

$$p_{a2} = 1 - 0.02063 = 0.97937 \text{ bar}$$

$\omega_2 = 0.622p_s/p_g = 0.0131 = m_s/m_a$ hence for 1 kg of dry air there must be 0.0131 kg of saturated vapour.

At inlet $\phi_1 = 0.5$

This time the mass of the vapour at inlet and outlet are not the same so the specific humidity is different at inlet.

$$\phi_1 = 0.5 = \omega_1(p-p_s)/(0.622p_g) \text{ at 25°C.}$$

Remember p_s is the saturation pressure at the dew point (18°C) and p_g is the saturation pressure at the actual temperature (25°C).

$$0.5 = \omega_1(1-0.02063)/(0.622 \times 0.03166) = 49.733 \ \omega_1$$

Hence $\omega_1 = 0.01 = m_{s1}/m_a$

Since the air mass is 1 kg throughout, then the mass of vapour at inlet is 0.01 kg.

It follows that the mass of water evaporated is 0.0131 - 0.01 = 0.0031 kg

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4. ENERGY BALANCE

Consider a simple air conditioner. Moist air is drawn in and cooled so that water condenses out. The air at this point must be at the dew point.
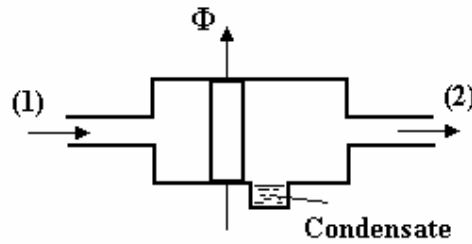

Figure 1

Applying the law of energy conservation we get :

$$M_a \, c_a \theta_1 + m_{s2} \, h_{s1} = m_w \, c_w \, \theta_2 + m_a \, c_a \, \theta_2 + m_{s2} \, h_{s2} + \text{energy removed}$$

The suffixes a, s and w refer to air, vapour and water respectively. Treating vapour as a gas, the accepted value of the specific heat capacity is 1.864 kJ/kg K.

The enthalpy of steam relative to 0^oC is then $\quad h_S = h_g + 1.864 \, \theta_S - t_S)$
t_S is the saturation temperature of the vapour.

It is probably best to use the thermodynamic tables or the h - s chart supplied in the exam whenever possible to find the enthalpy of vapour at low pressures and temperatures.

WORKED EXAMPLE No.3

Moist air enters a conditioning unit at 25^oC and 1 bar with a relative humidity of 0.7. It is passed through a cooler causing the temperature to fall to 18^oC and condensate is formed.

Calculate the mass of condensate formed per kg of dry air and the energy removed per kg.

SOLUTION

Following the same method as in the previous examples, the mass of vapour at exit is $\quad m_{s2} = 0.0131$ kg per kg of dry air.

The vapour pressure at inlet is 1.0 bar.

At inlet the relative humidity is $0.7 = \phi_1 = \omega_1(p - p_s)/(0.622 p_g)$ at 25^oC.

$\omega_1 = 0.622 \times 0.03166 \times 0.7/(1 - 0.02063) = 0.014075$

The mass of vapour at inlet is then $m_{s1} = 0.014075$ kg/kg

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The condensate formed $m_W = 0.014075 - 0.0131 = 0.000975$ kg

Conducting an energy balance:

$1 \times 1.005 \times 25 + 0.014075\ h_{s1} = 1 \times 1.005 \times 18 + 0.0131\ h_{s2} + m_W\theta_W + \Phi$

$h_{s1} = h_g + 1.864\ (\theta_S - t_S)$ at 0.02063 bar.
$h_{s1} = 2533.9 + (25-18)(1.864) = 2546.9$ kJ/kg

$h_{s2} = h_g$ at 18°C since it is saturated.
$h_{s2} = 2533.9$ kJ/kg

The balance becomes
$1 \times 1.005 \times 25 + 0.014075\ (2546.9) =$
$\qquad 1 \times 1.005 \times 18 + 0.0131\ (2533.9) + 0.000975 \times 4.168 \times 18 + \Phi$

$\Phi = 9.87$ kJ per kg of dry air.

In air conditioning it is normal to heat the air to the required temperature before it leaves the unit.
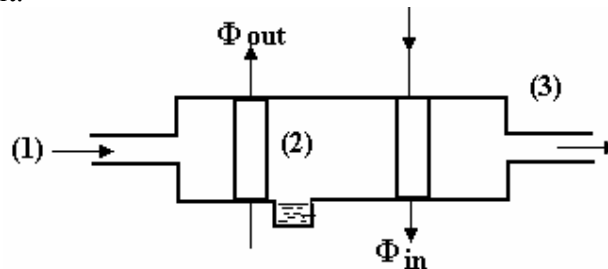
Figure 2

Suppose the air is heated to 22°C before leaving. What is the heat transfer required in the heater? The masses are unchanged so we only need an energy balance between 2 and 3.

$1 \times 1.005 \times 18 + 0.0131\ h_{s2} + \Phi = 1 \times 1.005 \times 22 + 0.0131\ h_{s3}$

$h_{s3} = h_g + 1.864\ (\theta_S - t_S)$ at 0.02063 bar. $t_S = 18$°C

$h_{s3} = 2533.9 + 1.864(22 - 18) = 2541.3$ kJ/kg

$h_{s2} = 2533.9$ kJ/kg

$1 \times 1.005 \times 18 + 0.0131 \times 2533.9 + \Phi = 1 \times 1.005 \times 22 + 0.1031 \times 2541.3$

$51.28 + \Phi = 55.27$

$\Phi = 4.11$ kJ

SELF ASSESSMENT EXERCISE No. 2

Air having a pressure, temperature and relative humidity of 1 bar, 26°C and 0.65 respectively, flows into an air conditioner at a steady rate and is dehumidified by cooling and removing water from it. The air is then heated to produce an outlet temperature and relative humidity of 24°C and 0.359 respectively. The pressure is constant throughout. Determine the heat transfers in the cooler and heater per kg of conditioned air at exit. Draw up a complete mass balance.

(36.22 kJ/kg and 16.24 kJ/kg)

5. <u>COOLING TOWERS</u>

Cooling towers fall into two types, dry and wet. Dry cooling towers are no more than very large air conditioners and the theory is the same as already outlined.

Wet cooling towers work on the principle of spraying warm water downwards so that heat and vapour is passed to the air which rises and carries away latent heat leaving the water at a lower temperature to collect in a pool at the bottom of the tower. The water is then recycled from the pool. The moist air leaves the top of the tower as a plume. The tower has a venturi shape to assist the process by causing a slight pressure reduction in the spray area followed by resurgence as the top widens. This causes condensation to form and make the plume visible. Some of the condensate rains down into the pool. We can say with certainty that the air leaves the tower with 100% humidity.

The best way to understand the problem is to do a worked example as follows.

<u>WORKED EXAMPLE No.4</u>

A cooling tower must cool 340 kg of water per minute. The water is supplied at 42°C and it is sprayed down into the column of air which enters the bottom of the tower at a rate of 540m³/min with a temperature of 18°C and relative humidity of 60%. The moist air leaves the top of the tower saturated at 27°C. The whole process occurs at a constant pressure of 1.013 bar. Determine the temperature of the cooled water in the pool and the rate at which make up water must be supplied to replace that evaporated.
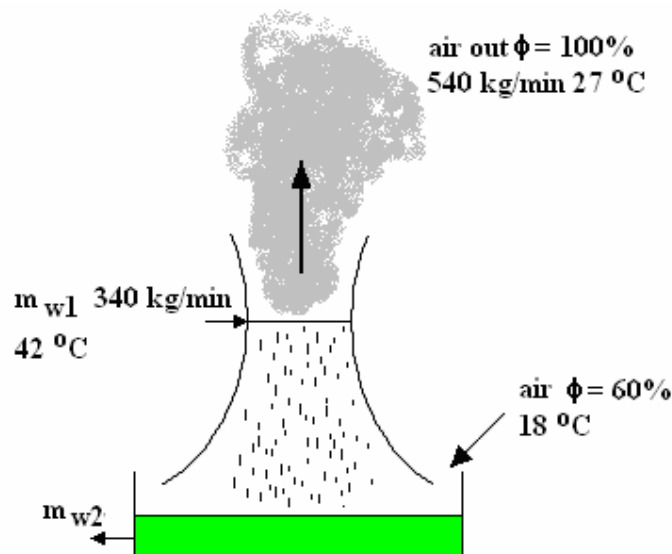

Figure 3

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SOLUTION

R = 287 J/kg K for air and 462 J/kg K for vapour.

INLET AIR

p_{g1} = 0.02063 bar at 18°C ϕ_1 = 0.6 = p_{s1}/p_{g1} p_{s1} = 0.012378 bar

hence p_{a1} = 1.013 - 0.012378 = 1 bar

m_a = pV/RT = 1 x 10^5 x 540/(287 x 291) = 646.6 kg/min

m_{s1} = pV/RT = 0.012378 x 10^5 x 540/(462 x 291) = 4.971 kg/min

OUTLET AIR

ϕ_2 = 1 p_{s2} = p_{g2} = 0.03564 bar hence p_{a2} = 0.9774 bar
ϕ_2 = 0.622 x 0.03564/0.9774 = 0.02268

m_{s2} = 0.02268 x 646.6 = 14.66 kg/min

Water evaporated = 14.66 - 4.971 = 9.693 kg/min
Make up water = 9.693 kg/min
m_{w2} = 340 – 9.693 = 330.3

ENERGY BALANCE

In this example enthalpy values from the steam tables and chart will be used.

h_{w1} = h_f @ 42°C = 175.8 kJ/kg
h_{w2} is unknown

h_{a1} = 1.005 x 27 kJ/kg

h_{s1} = h @ 0.012378 bar & 18°C = 2530 kJ/kg (from h-s chart)

h_{s2} = h_g @ 27°C = 2550.3 kJ/kg

Balancing energy we get

(340 x175.8) + {646.6 x 1.005 x (18-27)} + {4.971 x 2530) – (14.66 x 2550.3)
                                                = 330.3 h_{w2}

h_{w2} = 123.5 kJ/kg and from the tables the temperature must be 29.5 ^{o}C.

The temperature of the cooled water is 29.5 ^{o}C.

1. Derive the expression for specific humidity
    $\omega = 0.622(p_s/p_a)$

    Water flows at 5 000 kg/h and 40°C into a cooling tower and is cooled to 26°C. The unsaturated air enters the tower at 20°C with a relative humidity of 0.4. It leaves as saturated air at 30°C. The pressure is constant at 1 bar throughout.

    Calculate

           i. the mass flow of air per hour. (4 636 kg/h)

           ii. the mass of water evaporated per hour. (100.5 kg/h)

2. The cooling water for a small condenser is sent to a small cooling tower. 7 m^3/s of air enters the tower with a pressure, temperature and relative humidity of 1.013 bar, 15°C and 0.55 respectively. It leaves saturated at 32°C. The water flows out of the tower at 7.5 kg/s at 13°C. Using a mass and energy balance, determine the temperature of the water entering the tower.
    (Answer 33.9°C)

3. A fan supplies 600 dm^3/s of air with a relative humidity of 0.85, temperature 30°C and pressure 1.04 bar into an air conditioner. Moisture is removed from the air by cooling and both the air and condensate leave at the same temperature. The air is then heated to 20°C and has a relative humidity of 0.6. Determine the following.

    i. The mass of dry air and water at entrance to the conditioner.
          (0.6927 kg/s and 0.01546 kg/s)

    ii. The mass of water vapour delivered at exit.
          (0.00588 kg/s)

    iii. The mass of water extracted from the cooler.
          (0.00958 kg/s)

    iv. The temperature at exit from the cooler.
          (12°C)

    v. The heat transfer in the cooler.

6. CONDENSERS

It is inevitable that air will be drawn into steam condensers operating with a vacuum. The effect of this is to reduce the saturation pressure and temperature of the steam resulting in a colder condensate that would otherwise be obtained. This in turn means more heat required to turn it back into steam in the boiler and a reduced thermal efficiency for the power plant.

The air must be removed from the condenser in order to keep the partial pressure as small as possible. This is done with an extractor pump. Some vapour will be removed with the air but this loss is tolerable because of the energy saved.
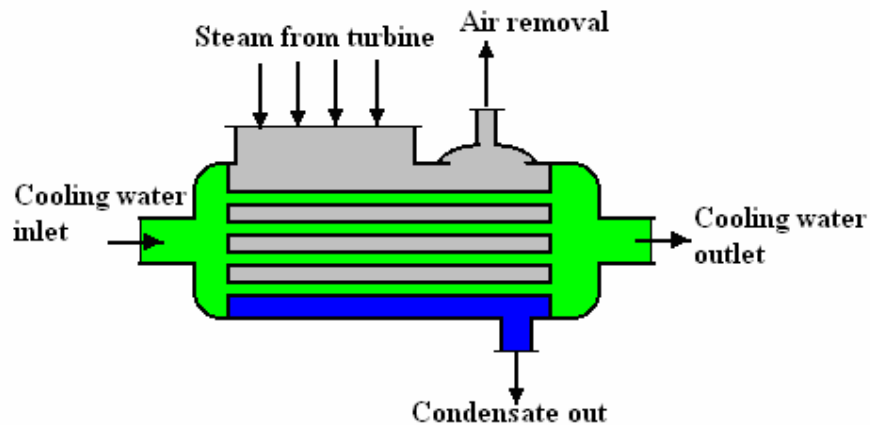
Figure 4

The solution to problems on condensers is similar to that for cooling towers and requires mass and energy balances. It is normal to neglect the partial pressure of the air at inlet as it makes little difference to the answers.

SELF ASSESSMENT EXERCISE No. 4

a. Discuss the reasons why air mixed with steam in a condenser is not desirable.

b. Wet steam with a dryness fraction of 0.9 enters a condenser at 0.035 bar pressure at a rate of 10 000 kg/h. The condensate leaves at 25°C. Air also enters with the steam at a rate of 40 kg/h. The air is extracted and cooled to 20°C. The partial pressure of the air at inlet is negligible and the process is at constant pressure. The cooling water is at 10°C at inlet and 21°C at outlet.

   i. Determine the mass of vapour extracted with the air. (50 kg/h)

   ii. Calculate the flow rate of the cooling water required. (475 484 kg/h)

THERMODYNAMICS

TUTORIAL 7

COMPRESSIBLE FLOW

On completion of this tutorial you should be able to do the following.

- Define entropy

- Derive expressions for entropy changes in fluids

- Derive Bernoulli's equation for gas

- Derive equations for compressible ISENTROPIC flow

- Solve problems involving compressible flow

Note that more work on compressible flow may be found under **FLUID MECHANICS.**

Let's start by revising entropy.

1. ENTROPY

1.1 DEFINITION

You should already be familiar with the theory of work laws in closed systems. You should know that the area under a pressure-volume diagram for a reversible expansion or compression gives the work done during the process.

In thermodynamics there are two forms of energy transfer, work (W) and heat (Q). By analogy to work, there should be a property which if plotted against temperature, then the area under the graph would give the heat transfer. This property is entropy and it is given the symbol S. Consider a p-V and T-s graph for a reversible expansion.
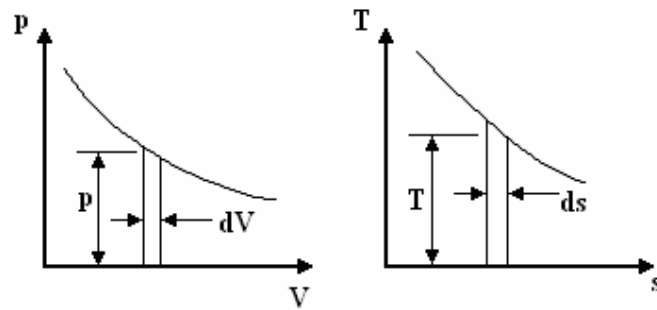
Figure 1

From the p-V graph we have $W = \int p\,dV$

From the T-S graph we have $Q = \int T\,dS$

This is the way entropy was developed for thermodynamics and from the above we get the definition

$$dS = dQ/T$$

The units of entropy are hence J/K.

Specific entropy has a symbol s and the units are J/kg K

It should be pointed out that there are other definitions of entropy but this one is the most meaningful for thermodynamics. A suitable integration will enable you to solve the entropy change for a fluid process.

2. ISENTROPIC PROCESSES

The word *Isentropic* means constant entropy and this is a very important thermodynamic process. It occurs in particular when a process is reversible and adiabatic. This means that there is no heat transfer to or from the fluid and no internal heat generation due to friction. In such a process it follows that if dQ is zero then dS must be zero. Since there is no area under the T-S graph, then the graph must be a vertical line as shown.
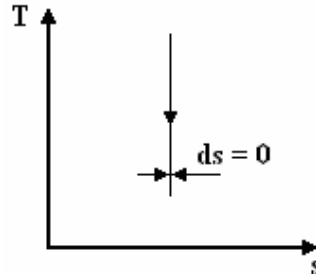
Figure 2

There are other cases where the entropy is constant. For example, if there is friction in the process generating heat but this is lost through cooling, then the nett result is zero heat transfer and constant entropy. You do not need to be concerned about this at this stage.

Entropy is used in the solution of gas and vapour problems. We should now look at practical applications of this property and study the entropy changes which occur in closed and steady flow systems for perfect gases and vapours. These derivations should be learned for the examination.

3. ENTROPY CHANGES FOR A PERFECT GAS IN A CLOSED SYSTEMS

Consider a closed system expansion of a fluid against a piston with heat and work transfer taking place.
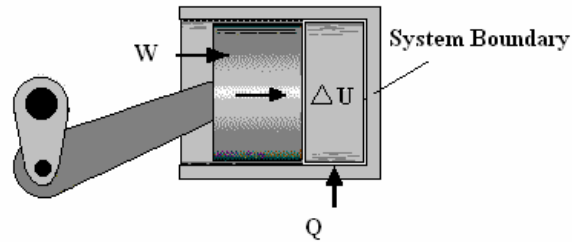
Figure 3

Applying the non-flow energy equation we have

$$Q + W = \Delta U$$

Differentiating we have

$$dQ + dW = dU$$

Since $dQ = TdS$ and $dW = -pdV$ then

$$TdS - pdV = dU$$

$$TdS = dU + pdV$$

This expression is the starting point for all derivations of entropy changes for any fluid (gas or vapour) in closed systems. It is normal to use specific properties so the equation becomes

$$Tds = du + pdv$$

but from the gas law $pv = RT$ we may substitute for p and the equation becomes

$$Tds = du + RTdv/v$$

rearranging and substituting $du = c_v\, dT$ we have

$$ds = c_v\, dT/T + Rdv/v ....(1)$$

s is specific entropy

v is specific volume.

u is specific internal energy and later on is also used for velocity.

3.1 ISOTHERMAL PROCESS
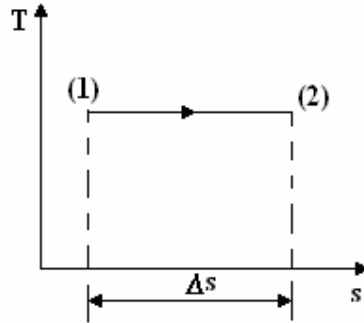
Figure 4

In this case temperature is constant. Starting with equation (1)

$$ds = c_v \, dT/T + R dv/v.$$

since $dT = 0$ then

$$s_2 - s_1 = \Delta s = R \ln(v_2/v_1)$$

A quicker alternative derivation for those familiar with the work laws is:

$$Q + W = \Delta U \text{ but } \Delta U = 0 \text{ then } Q = -W \text{ and } W = -mRT \ln\frac{V_2}{V_1}$$

$$Q = \int T ds = T\Delta S \text{ but T is constant.}$$

$$\Delta S = \frac{Q}{T} = -\frac{W}{T} = mR \ln\frac{V_2}{V_1}$$

$$\Delta S = mR \ln\frac{V_2}{V_1}$$

$$\Delta s = R \ln\frac{v_2}{v_1} \text{ and since } \frac{v_2}{v_1} = \frac{p_1}{p_2}$$

$$\Delta s = R \ln\frac{p_1}{p_2}$$

5

3.2 CONSTANT VOLUME PROCESS
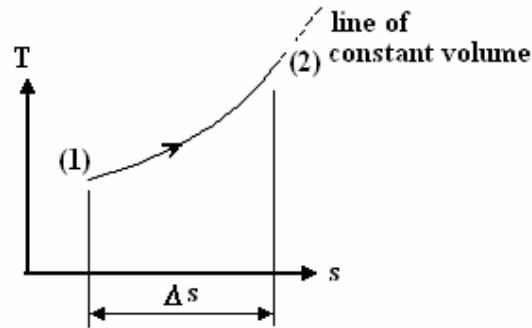
Figure 5

Starting again with equation (1) we have   ds = c_vdT/T + Rdv/v
In this case dv=0 so   ds = c_vdT/T
Integrating between limits (1) and (2) Δs= c_v ln(T$_2$/T$_1$)

3.3 CONSTANT PRESSURE PROCESS
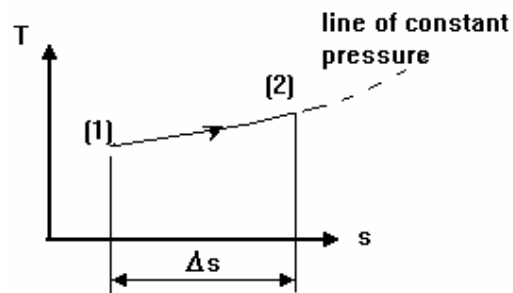
Figure 6

Starting again with equation (1) we have

$$ds = C_v \frac{dT}{T} + R\frac{dv}{v}$$ In this case we integrate and obtain

$$\Delta s = C_v \ln \frac{T_2}{T_1} R \ln \frac{v_2}{v_1}$$ For a constant pressure process, v/T = constant

$$\frac{v_2}{v_1} = \frac{T_2}{T_1}$$ so the expression becomes $\Delta s = C_v \ln \frac{T_2}{T_1} + R \ln \frac{T_2}{T_1} = (C_v + R)\ln \frac{T_2}{T_1}$

It was shown in an earlier tutorial that R = c_p - c_v hence

$$\Delta s = C_p \ln \frac{T_2}{T_1}$$

3.4 POLYTROPIC PROCESS

This is the most difficult of all the derivations here. Since all the forgoing are particular examples of the polytropic process then the resulting formula should apply to them also.

Figure 7

The polytropic expansion is from (1) to (2) on the T-s diagram with different pressures, volumes and temperatures at the two points. The derivation is done in two stages by supposing the change takes place first at constant temperature from (1) to (A) and then at constant pressure from (A) to (2). You could use a constant volume process instead of constant pressure if you wish.
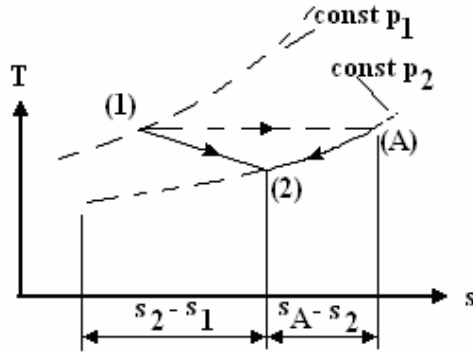
$$s_2 - s_1 = (s_A - s_1) - (s_A - s_2)$$
$$s_2 - s_1 = (s_A - s_1) + (s_2 - s_A)$$

For the constant temperature process

$$(s_A - s_1) = R \ln(p_1/p_A)$$

For the constant pressure process

$$(s_2 - s_A) = (c_p) \ln(T_2/T_A)$$

Hence

$$\Delta s = R \ln \frac{p_1}{p_A} + C_p \ln \frac{T_2}{T_A} + s_2 - s_1 \text{ Since } p_A = p_2 \text{ and } T_A = T_1$$

Then

$$\Delta s = s_2 - s_1 = R \ln \frac{p_1}{p_2} + C_p \ln \frac{T_2}{T_1} \text{ Divide through by R}$$

$$\frac{\Delta s}{R} = \ln \frac{p_1}{p_2} + \frac{C_p}{R} \ln \frac{T_2}{T_1}$$

From the relationship between c_p, c_v, R and γ we have $c_p/R = \gamma/(\gamma-1)$

Hence
$$\frac{\Delta s}{R} = \ln \frac{p_1}{p_2} + \frac{\gamma}{\gamma-1} \ln \frac{T_2}{T_1} \qquad\qquad \frac{\Delta s}{R} = \ln \frac{p_1}{p_2}\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

This formula is for a polytropic process and should work for isothermal, constant pressure, constant volume and adiabatic processes also. In other words, it must be the derivation for the entropy change of a perfect gas for any closed system process. This derivation is often requested in the exam.

7

WORKED EXAMPLE No. 1

A perfect gas is expanded from 5 bar to 1 bar by the law $pV^{1.2} = C$. The initial temperature is 200°C. Calculate the change in specific entropy.
R = 287 J/kg K   γ =1.4.

SOLUTION

$$T_2 = 473\left(\frac{1}{5}\right)^{1-\frac{1}{1.2}} = 361.7K$$

$$\frac{\Delta s}{R} = \ln\left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\Delta s}{R} = (\ln 5)\left(\frac{361.7}{472}\right)^{3.5} = 0.671$$

$$\Delta s = 0.671 \times 287 = 192.5 \text{ J/kgK}$$

SELF ASSESSMENT EXERCISE No. 1

1. Calculate the specific entropy change when a perfect gas undergoes a reversible isothermal expansion from 500 kPa to 100 kPa. R = 287 J/kg K.
   (Answer +461.9 J/kg K).

2. Calculate the total entropy change when 2 kg of perfect gas is compressed reversibly and isothermally from 9 dm3 to 1 dm3. R=300 J/kg K. (Answer -1.32 kJ/k)

3. Calculate the change in entropy when 2.5 kg of perfect gas is heated from 20°C to 100°C at constant volume. Take c_v= 780 J/kg K   (Answer 470 J/K)

4. Calculate the total entropy change when 5 kg of gas is expanded at constant pressure from 30°C to 200°C. R = 300 J/kg K c_v= 800 J/kg K   (Answer 2.45 kJ/K)

5. Derive the formula for the specific change in entropy during a polytropic process using a constant volume process from (A) to (2).

6. A perfect gas is expanded from 5 bar to 1 bar by the law $pV^{1.6} = C$. The initial temperature is 200°C. Calculate the change in specific entropy.
   R = 287 J/kg K   γ =1.4.   (Answer -144 J/kg K)

7. A perfect gas is expanded reversibly and adiabatically from 5 bar to 1 bar by the law $pV^{\gamma} = C$. The initial temperature is 200°C. Calculate the change in specific entropy using the formula for a polytropic process. R = 287 J/kg K   γ =1.4. (The answer should be zero since the process is constant entropy).

Let's go on to apply the knowledge of entropy to the flow of compressible fluids starting with isentropic flow.

4. ISENTROPIC FLOW

Isentropic means constant entropy. In this case we will consider the flow to be ADIABATIC also, that is, with no heat transfer.

Consider gas flowing in a duct which varies in size. The pressure and temperature of the gas may change.
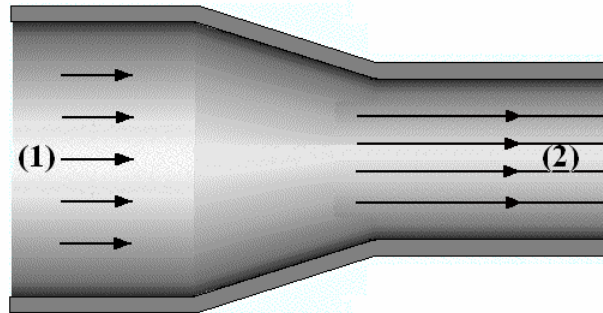
Figure 8

Applying the steady flow energy equation between (1) and (2) we have :

$$\Phi \ - \ P \ = \Delta U + \ \Delta F.E. + \ \Delta K.E. + \Delta P.E.$$

For Adiabatic Flow, $\Phi = 0$ and if no work is done then $P = 0$

$$\Delta U \ + \ \Delta F.E. \ = \ \Delta H$$
hence :
$$0 \ = \ \Delta H + \Delta K.E. + \Delta P.E.$$

In specific energy terms this becomes :

$$0 = \ \Delta h \ + \Delta k.e. + \Delta p.e.$$

rewriting we get:

$$h_1 \ + \ u_1^2/2 \ + \ g \, z_1 \ = h_2 \ + \ u_2^2/2 \ + \ g \, z_2$$

For a gas, $h = C_p T$ so we get Bernoulli's equation for gas which is :

$$C_p T_1 \ + \ u_1^2/2 \ + \ g \, z_1 \ = C_p T_2 \ + \ u_2^2/2 \ + \ g \, z_2$$

Note that T is absolute temperature in Kelvins   T = ºC + 273

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4.1 STAGNATION CONDITIONS

If a stream of gas is brought to rest, it is said to STAGNATE. This occurs on leading edges of any obstacle placed in the flow and in instruments such as a Pitot Tube. Consider such a case for horizontal flow in which P.E. may be neglected.
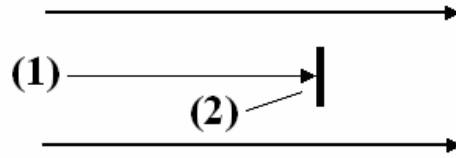
Figure 9

$u_2 = 0$ and $z_1 = z_2$ so $C_p T_1 + u_1^2/2 = C_p T_2 + 0$

$T_2 = u_1^2/2C_p + T_1$

T_2 is the stagnation temperature for this case.

$$\text{Let } T_2 - T_1 = \Delta T = u_1^2/2C_p$$

$$\Delta T = u_1^2/2C_p$$

Now $C_p - C_v = R$ and $C_p / C_v = \gamma$ γ is the adiabatic index .

hence $C_p = R / (\gamma - 1)$ and so :

$$\Delta T = u_1^2 (\gamma - 1) / (2\gamma R)$$

It can be shown elsewhere that the speed of sound   a is given by :

$$a^2 = \gamma RT$$
hence at point 1:
$$\Delta T / T_1 = u_1^2 (\gamma - 1) / (2\gamma RT_1) = u_1^2 (\gamma - 1) /2a_1^2$$

The ratio u/a is the Mach Number M_a so this may be written as :

$$\Delta T / T_1 = M_a^2 (\gamma - 1) /2$$

If M_a is less than 0.2 then M_a^2 is less than 0.04 and so $\Delta T/T_1$ is less than 0.008. It follows that for low velocities, the rise in temperature is negligible under stagnation conditions.

The equation may be written as :

$$\frac{T_2 - T_1}{T_1} = \frac{M_a^2(\gamma - 1)}{2}$$

$$\frac{T_2}{T_1} = \left\{ \frac{M_a^2(\gamma - 1)}{2} \right\} + 1$$

Since pV/T = constant and pV^γ = constant then :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}}$$

Hence :

$$\left(\frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} = \frac{M_a^2(\gamma - 1)}{2} + 1$$

$$\left(\frac{p_2}{p_1} \right) = \left[\frac{M_a^2(\gamma - 1)}{2} + 1 \right]^{\frac{\gamma}{\gamma - 1}}$$

p_2 is the stagnation pressure. If we now expand the equation using the binomial theorem we get :

$$\frac{p_2}{p_1} = 1 + \frac{\gamma Ma^2}{2} \left\{ 1 + \frac{Ma^2}{4} + \frac{Ma^4}{8} + ... \right\}$$

If M_a is less than 0.4 then : $\dfrac{p_2}{p_1} = 1 + \dfrac{\gamma Ma^2}{2}$

Now compare the equations for gas and liquids :

LIQUIDS $u = (2\Delta p/\rho)^{0.5}$

GAS $\dfrac{p_2}{p_1} = 1 + \dfrac{\gamma Ma^2}{2}$

Put $p_2 = p_1 + \Delta p$ so : $\Delta p = \dfrac{\gamma Ma^2}{2} p_1 = \dfrac{\gamma v_1^2 p_1}{2\gamma RT} = \dfrac{\rho_1 u_1^2}{2}$

where $\rho_1 = p_1 / RT$ and $M_a^2 = u_1^2 / (\gamma RT)$

hence $u = (2\Delta p / \rho_1)^{0.5}$ which is the same as for liquids.

Let's now extend the work to pitot tubes.

5. PITOT STATIC TUBE

A Pitot Static Tube is used to measure the velocity of a fluid. It is pointed into the stream and the differential pressure obtained gives the stagnation pressure.
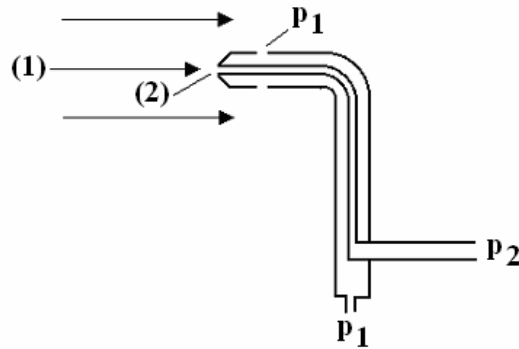
Figure 10

$$p_2 = p_1 + \Delta p$$

Using the formula in the last section, the velocity v may be found.

WORKED EXAMPLE No.2

A pitot tube is pointed into an air stream which has a pressure of 105 kPa. The differential pressure is 20 kPa and the air temperature is 20°C. Calculate the air speed.

SOLUTION

$$p_2 = p_1 + \Delta p \quad = 105 + 20 = 125 \text{ kPa}$$

$$\frac{p_2}{p_1} = \left[\left\{ \frac{Ma^2(\gamma-1)}{2} \right\} + 1 \right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{125}{105} = \left[\left\{ \frac{Ma^2(\gamma-1)}{2} \right\} + 1 \right] \text{ hence } Ma = 0.634$$

$$a = (\gamma RT)^{0.5} = (1.4 \times 287 \times 293)^{0.5} = 343 \text{ m/s}$$

$$M_a = u/a \quad \text{hence } u = 217.7 \text{ m/s}$$

Let's further extend the work now to venturi meters and nozzles.

6. VENTURI METERS AND NOZZLES

Consider the diagrams below and apply Isentropic theory between the inlet and the throat.
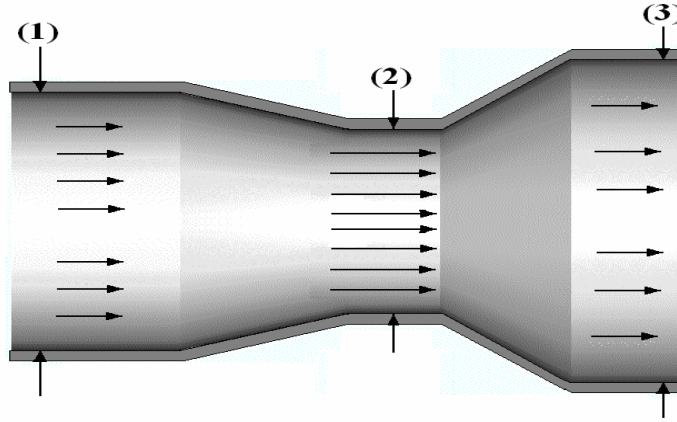
Figure 11

$$u_2^2 - u_1^2 = h_1 - h_2$$

If the Kinetic energy at inlet is ignored this gives us $\qquad u_2^2 = h_1 - h_2$

For a gas $h = C_pT$ so: $\quad u_2^2 = C_P[T_1 - T_2]$

Using $C_p = \gamma R/(\gamma-1)$ we get $\quad u_2^2 = \dfrac{2\gamma R}{\gamma - 1}[T_1 - T_2]$

$RT = pV/m = p/\rho \qquad$ so

$$u_2^2 = \frac{2\gamma}{\gamma - 1}\left[\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right]$$

$p_1 V_1^\gamma = p_2 V_2^\gamma$ so it follows that $\dfrac{p_1}{\rho_1^\gamma} = \dfrac{p_2}{\rho_2^\gamma}$

$$u_2^2 = \frac{2\gamma}{\gamma - 1}\left(\frac{p_1}{\rho_1}\right)\left[1 - \frac{p_2 \rho_1}{p_1 \rho_2}\right]$$

$$u_2^2 = \frac{2\gamma}{\gamma - 1}\left(\frac{p_1}{\rho_1}\right)\left[1 - \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}}\right]$$

The mass flow rate $m = \rho_2\, A_2\, u_2\, C_d$ where C_d is the coefficient of discharge which for a well designed nozzle or Venturi is the same as the coefficient of velocity since there is no contraction and only friction reduces the velocity.

$$\rho_2 = \rho_1\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} \qquad \text{hence} \quad m = C_d A_2 \sqrt{\left[\frac{2\gamma}{\gamma - 1}\right]\left\{[p_1 \rho_1]\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}} - \left(\frac{p_2}{p_1}\right)^{1+\frac{1}{\gamma}}\right]\right\}}$$

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If a graph of mass flow rate is plotted against pressure ratio (p_2/p_1) we get:
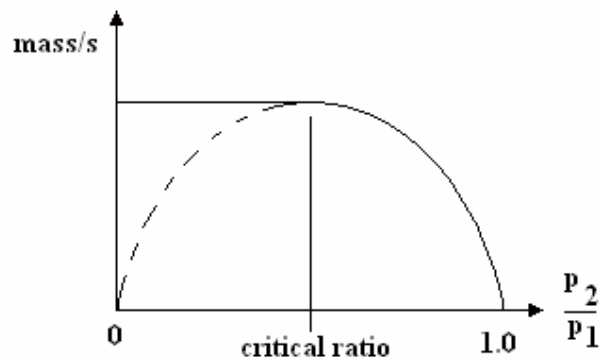
Figure 12

Apparently the mass flow rate starts from zero and reached a maximum and then declined to zero. The left half of the graph is not possible as this contravenes the 2nd law and in reality the mass flow rate stays constant over this half.

What this means is that if you started with a pressure ratio of 1, no flow would occur. If you gradually lowered the pressure p_2, the flow rate would increase up to a maximum and not beyond. The pressure ratio at which this occurs is the CRITICAL RATIO and the nozzle or Venturi is said to be choked when passing maximum flow rate. Let

$$\frac{p_2}{p_1} = r$$

For maximum flow rate, $\dfrac{dm}{dr} = 0$

The student should differentiate the mass formula above and show that at the maximum condition the critical pressure ratio is :

$$r = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}}$$

6.1 MAXIMUM VELOCITY

If the formula for the critical pressure ratio is substituted into the formula for velocity, then the velocity at the throat of a choked nozzle/Venturi is :

$$u_2^2 = \left\{\frac{\gamma p_2}{\rho_2}\right\} = \gamma R T = a^2$$

Hence the maximum velocity obtainable at the throat is the local speed of sound.

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6.2 CORRECTION FOR INLET VELOCITY

In the preceding derivations, the inlet velocity was assumed negligible. This is not always the case and especially in Venturi Meters, the inlet and throat diameters are not very different and the inlet velocity should not be neglected. The student should go through the derivation again from the beginning but this time keep v_1 in the formula and show that the mass flow rate is

$$m = \frac{C_d A_2 \sqrt{\left[p_1 \rho_1 \frac{2\gamma}{\gamma - 1} \right] \left[\left(\frac{p_2}{p_1} \right)^{\frac{2}{\gamma}} - \left(\frac{p_2}{p_1} \right)^{1+\frac{1}{\gamma}} \right]}}{\sqrt{1 - \left(\frac{A_2}{A_1} \right)^2 \left(\frac{p_2}{p_1} \right)^{\frac{2}{\gamma}}}}$$

The critical pressure ratio can be shown to be the same as before.

6.3 MORE ON ISENTROPIC FLOW

When flow is isentropic it can be shown that all the stagnation properties are constant. Consider the conservation of energy for a horizontal duct :

$$h + u^2/2 = \text{constant} \qquad h = \text{specific enthalpy}$$

If the fluid is brought to rest the total energy must stay the same so the stagnation enthalpy h_o is given by :

$h_o = h + u^2/2$ and will have the same value at any point in the duct.

since $h_o = C_p T_o$ then To (the stagnation temperature) must be the same at all points. It follows that the stagnation pressure p_o is the same at all points also. This knowledge is very useful in solving questions.

6.4 ISENTROPIC EFFICIENCY (NOZZLE EFFICIENCY)

If there is friction present but the flow remains adiabatic, then the entropy is not constant and the nozzle efficiency is defined as :

$$\eta = \text{actual enthalpy drop/ideal enthalpy drop}$$

For a gas this becomes : $(T_1 - T_2)/(T_1 - T_2')$

T_2' is the ideal temperature following expansion. Now apply the conservation of energy between the two points for isentropic and non isentropic flow :

$$C_p T_1 + u_1^2/2 = C_p T_2 + u_2^2/2 \quad \text{....... for isentropic flow}$$

$$C_p T_1 + u_1^2/2 = C_p T_{2'} + u_{2'}^2/2 \quad \text{.........for non isentropic}$$

Hence

$$\eta = (T_1 - T_2)/(T_1 - T_2') = (u_2^2 - u_1^2)/(u_{2'}^2 - u_1^2)$$

If v_1 is zero (for example Rockets) then this becomes :

$$\eta = u_2^2 / u_{2'}^2$$

SELF ASSESSMENT EXERCISE No. 3

1. A Venturi Meter must pass 300g/s of air. The inlet pressure is 2 bar and the inlet temperature is 120°C. Ignoring the inlet velocity, determine the throat area. Take C_d as 0.97. Take $\gamma = 1.4$ and R = 287 J/kg K (assume choked flow)
   (Answer 0.000758 m²)

2. Repeat problem 1 given that the inlet is 60 mm diameter and the inlet velocity must not be neglected. (Answer 0.000747 m²)

3. A nozzle must pass 0.5 kg/s of steam with inlet conditions of 10 bar and 400°C. Calculate the throat diameter that causes choking at this condition. The density of the steam at inlet is 3.263 kg/m³. Take γ for steam as 1.3 and C_d as 0.98.
   (Answer 23.2 mm)

4. A Venturi Meter has a throat area of 500 mm². Steam flows through it, and the inlet pressure is 7 bar and the throat pressure is 5 bar. The inlet temperature is 400°C. Calculate the flow rate. The density of the steam at inlet is 2.274 kg/m³.

   Take $\gamma = 1.3$. R = 462 J/kg K. Cd=0.97. (Answer 383 g/s)

5. A pitot tube is pointed into an air stream which has an ambient pressure of 100 kPa and temperature of 20°C. The pressure rise measured is 23 kPa. Calculate the air velocity. Take $\gamma = 1.4$ and R = 287 J/kg K. (Answer 189.4 m/s)

6. A fast moving stream of gas has a temperature of 25°C. A thermometer is placed into it in front of a small barrier to record the stagnation temperature. The stagnation temperature is 28°C. Calculate the velocity of the gas. Take $\gamma = 1.5$ and R = 300 J/kg K. (Answer 73.5 m/s)

Let's do some further study of nozzles of venturi shapes now.

7. CONVERGENT - DIVERGENT NOZZLES

A nozzle fitted with a divergent section is in effect a Venturi shape. The divergent section is known as a diffuser.
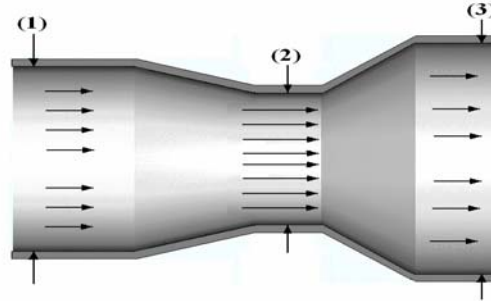

Figure 13

If p_1 is constant and p_3 is reduced in stages, at some point p_2 will reach the critical value which causes the nozzle to choke. At this point the velocity in the throat is sonic.

If p_3 is further reduced, p_2 will remain at the choked value but there will be a further pressure drop from the throat to the outlet. The pressure drop will cause the volume of the gas to expand. The increase in area will tend to slow down the velocity but the decrease in volume will tend to increase the velocity. If the nozzle is so designed, the velocity may increase and become supersonic at exit.

In rocket and jet designs, the diffuser is important to make the exit velocity supersonic and so increase the thrust of the engine.

7.1 NOZZLE AREAS

When the nozzle is choked, the velocity at the throat is the sonic velocity and the Mach number is 1. If the Mach number at exit is M_e then the ratio of the throat and exit area may be found easily as follows.

$$u_t = (\gamma R T_t)^{0.5} \qquad u_e = M_e (\gamma R T_e)^{0.5} \qquad \text{mass/s} = \rho_t A_t v_t = \rho_e A_e v_e.$$

$$\frac{A_t}{A_e} = \frac{\rho_e u_e}{\rho_t u_t} \quad \text{but earlier it was shown that} \quad \frac{\rho_e}{\rho_t} = \left(\frac{p_e}{p_t}\right)^{\frac{1}{\gamma}}$$

$$\frac{A_t}{A_e} = \left(\frac{p_e}{p_t}\right)^{\frac{1}{\gamma}} \frac{M_e (\gamma R T_e)^{0.5}}{(\gamma R T_t)^{0.5}} \qquad \text{It was also shown earlier that} \quad \frac{T_e}{T_t} = \left(\frac{p_e}{p_t}\right)^{1-\frac{1}{\gamma}}$$

$$\frac{A_t}{A_e} = \left(\frac{p_e}{p_t}\right)^{\frac{1}{\gamma}} M_e \left\{ \left(\frac{p_e}{p_t}\right)^{1-\frac{1}{\gamma}} \right\}^{0.5}$$

$$\frac{A_t}{A_e} = M_e \left(\frac{p_e}{p_t}\right)^{\frac{1+\gamma}{2\gamma}}$$

There is much more which can be said about nozzle design for gas and steam with implications to turbine designs. This should be studied in advanced text books.

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WORKED EXAMPLE No.3

Solve the exit velocity for the nozzle shown assuming isentropic flow:
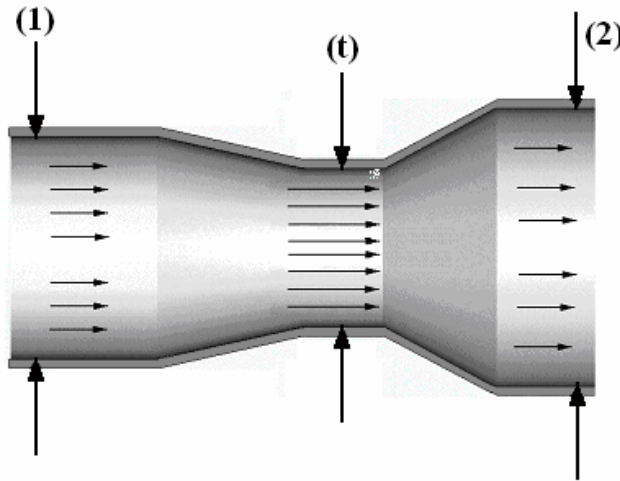
Figure 14

$T_1 = 350$ K $P_1 = 1$ MPa $p_2 = 100$ kPa

The nozzle is fully expanded (choked). Hence $M_t = 1$ (the Mach No.)
The adiabatic index $\gamma = 1.4$

SOLUTION

The critical pressure $p_t = p_1 \{2/(\gamma - 1)\}^{\gamma/(\gamma-1)} = 0.528$ MPa

$T_t/T_1 = (p_t/p_1)^{(\gamma-1)/\gamma}$ hence $T_t = 291.7$ K
$T_o/T_t = \{1 + M^2(\gamma -1)/2\}$ hence $T_o = 350$ K

It makes sense that the initial pressure and temperature are the stagnation values since the initial velocity is zero.

$T_2 = T_t (p_2/p_t)^{(\gamma-1)/\gamma} = 181.3$ K $a_2 = (\gamma RT_2)^{0.5} = 270$ m/s

$p_o/p_2 = \{1 + M_2^2(\gamma - 1)/2\}^{\gamma/(\gamma-1)}$

Hence $M_2 = 2.157$ and $u_2 = 2.157 \times 270 = 582.4$ m/s

SELF ASSESSMENT EXERCISE No. 4

1. A nozzle is used with a rocket propulsion system. The gas is expanded from complete stagnation conditions inside the combustion chamber of 20 bar and 3000K. Expansion is isentropic to 1 bar at exit. The molar mass of the gas is 33 kg/kmol. The adiabatic index is 1.2. The throat area is 0.1 m^2. Calculate the thrust and area at exit.
    (Answers 0.362 m^2 and 281.5 kN)

    Recalculate the thrust for an isentropic efficiency of 95%.
    (Answer 274.3 kN)

    Note that expansion may not be complete at the exit area. You may assume

    $$\frac{p_t}{p_o} = \left\{ \frac{2}{\gamma+1} \right\}^{\frac{\gamma}{\gamma-1}}$$

2. A perfect gas flows through a convergent-divergent nozzle at 1 kg/s. At inlet the gas pressure is 7 bar, temperature 900 K and velocity 178 m/s. At exit the velocity is 820m/s. The overall isentropic efficiency is 85%. The flow may be assumed to be adiabatic with irreversibility's only in the divergent section.

    C_p = 1.13 kJ/kg K R = 287 J/kg K.

    Calculate the cross sectional areas at the inlet, throat and exit.
    (Answers 20.8 cm^2, 10.22 cm^2 and 13.69 cm^2)

    Calculate the net force acting on the nozzle if it is stationary. The surrounding pressure is 1 bar. (-527 N) You may assume

    $$\frac{p_t}{p_o} = \left\{ \frac{2}{\gamma+1} \right\}^{\frac{\gamma}{\gamma-1}}$$

3. Dry saturated steam flows at 1 kg/s with a pressure of 14 bar. It is expanded in a convergent-divergent nozzle to 0.14 bar. Due to irreversibility's in the divergent section only, the isentropic efficiency 96%. The critical pressure ratio may be assumed to be 0.571. Calculate the following.

    The dryness fraction, specific volume and specific enthalpy at the throat.
    (Answers 0.958, 0.23 m^3/kg and 2683 kJ/kg)

    The velocity and cross sectional area at the throat and exit.
    (Answers 462.6 m/s , 497 mm^2, 1163 m/s and 73.2 cm^2.)

    The overall isentropic efficiency.
    (Answer 96.6%)

4. A jet engine is tested on a test bed. At inlet to the compressor the air is at 1 bar and 293 K and has negligible velocity. The air is compressed adiabatically to 4 bar with an isentropic efficiency of 85%. The compressed air is heated in a combustion chamber to 1175 K. It is then expanded adiabatically in a turbine with an isentropic efficiency of 87%. The turbine drives the compressor. The gas leaving the turbine is expanded further reversibly and adiabatically through a convergent nozzle. The flow is choked at exit. The exit area is 0.1 m2.

Determine the following.

The pressures at the outlets of the turbine and nozzle.
(Answers 2.38 bar and 1.129 bar)

The mass flow rate. (Answer 27.2 kg/s)
The thrust produced. (Answer 17 kN)

It may be assumed that $\dfrac{T_t}{T_o} = \dfrac{2}{\gamma+1}$ and $a = \sqrt{\gamma RT}$

5. Dry saturated steam expands through a convergent-divergent nozzle. The inlet and outlet pressures are 7 bar and 1 bar respectively at a rate of 2 kg/s. The overall isentropic efficiency is 90% with all the losses occurring in the divergent section. It may be assumed that $\gamma = 1.135$ and

$$\frac{p_t}{p_o} = \left\{\frac{2}{\gamma+1}\right\}^{\frac{\gamma}{\gamma-1}}$$

Calculate the areas at the throat and exit. (Answers 19.6 cm^2 and 38.8 cm^2).

The nozzle is horizontal and the entry is connected directly to a large vessel containing steam at 7 bar. The vessel is connected to a vertical flexible tube and is free to move in all directions. Calculate the force required to hold the receiver static if the ambient pressure is 1.013 bar.
(Answer 3.868 kN)

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THERMODYNAMICS

TUTORIAL No.8

COMBUSTION OF FUELS

- On completion of this tutorial you should be able to do the following.

  - Write down combustion equations.

  - Solve the oxygen and air requirements for the combustion of solid, liquid and gaseous fuels.

  - Determine the products of combustion.

  - Determine the air/fuel ratio from the products of combustion.

  - Solve problems involving energy released by combustion.

  - Solve problems involving dissociation of products.

  - Solve past paper questions.

Let's start by revising the basics.

1. INTRODUCTION

Combustion is the process of chemical reaction between fuel and oxygen(reactants). The process releases heat and produces products of combustion. The main elements which burn are :

CARBON

HYDROGEN

SULPHUR

The heat released by 1 kg or m^3 of fuel is called the calorific value.

The oxygen used in combustion processes normally comes from the atmosphere and this brings nitrogen in with it which normally does nothing in the process but makes up the bulk of the gases remaining after combustion.

The main elements in combustion are then :

	Symbol	Atomic Mass	Molecular Mass	Product
Carbon	C	12		CO_2
Hydrogen	H_2	1	2	H_2O
Sulphur	S	32		SO_2
Oxygen	O_2	16	32	
Nitrogen	N_2	14	28	

If the water formed during combustion leaves as vapour, it takes with it the latent heat of evaporation and thus reduces the energy available from the process. In this case the calorific value is called the lower Calorific value (LCV). If the products cool down after combustion so that the vapour condenses, the latent heat is given up and the calorific value is then the higher calorific value (HCV).

Solid and liquid fuels are normally analysed by mass to give the content of carbon, hydrogen, sulphur and any other elements present. Often there is silica, moisture and oxygen present in small quantities which have some effect on process. The silica leaves slaggy deposits on the heat transfer surfaces in boilers.

Gaseous fuels are normally analysed by volumetric content and are in the main hydrocarbon fuels.

For purposes of calculation, the content of air is considered to be :

	VOLUMETRIC	GRAVIMETRIC
Oxygen	21%	23%
Nitrogen	79%	77%

The sulphur content of the fuel is considered to be a pollutant and so undesirable. The theoretically correct quantity of air or oxygen required to just exactly burn the fuel expressed as a ratio to the fuel burned, is called the STOICHIOMETRIC RATIO. In practice it is found that not all the oxygen in the reactant reaches the fuel elements and that excess air is required in order to ensure complete combustion. This results in oxygen appearing in the products. If too little air or oxygen is supplied, the result is incomplete combustion resulting in the formation of carbon monoxide CO instead of carbon dioxide CO_2. The resulting products contain water H_2O. Industrial equipment for measuring the contents of the products usually remove the water from the sample and the products are then called the dry products.
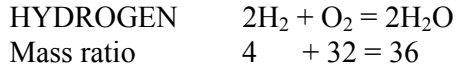
2. COMBUSTION CHEMISTRY

2.1 SOLID AND LIQUID FUELS

In the case of solid and liquid fuels, we do the combustion of each element separately. The important rule is that you must have the same number of atoms of each substance before and after the process. This may be obtained by juggling with the number of molecules.

$$\text{CARBON} \qquad C + O_2 = CO_2$$
Mass ratio $12 + 32 = 44$
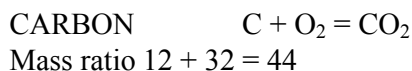
Hence 1kg of C needs 32/12kg of O_2 and makes 44/12kg of CO_2

$$\text{HYDROGEN} \qquad 2H_2 + O_2 = 2H_2O$$
Mass ratio $\qquad 4 \quad + 32 = 36$

Hence 1kg of H_2 needs 8kg of O_2 and makes 9 kg of $H_{22}O$

$$\text{SULPHUR} \qquad S + O_2 = SO_2$$
$$32 + 32 = 64$$
Hence 1 kg of S needs 1kg of O_2 and makes 2kg of SO_2.

2.2. GASEOUS FUELS

Typical hydrocarbons are :

Methane	CH_4
Ethane	C_2H_6
Propane	C_3H_8
Butane	C_4H_{10}
Pentane	C_5H_{12}
Hexane	C_6H_{14}
Heptane	C_7H_{16}
Octane	C_8H_{18}
Ethene	C_2H_4(Ethylene)
Propene	C_3H_6 (Propylene)
Ethyne	C_2H_2 (Acetylene)
Benzenol	C_6H_6 (Benzene)
Cyclohexane	C_6H_{12}

The combustion equation follows the following rule :

$$C_aH_b + (a+b/4)O_2 = (a)CO_2 + (b/2)H_2O$$
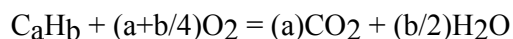
If this results in fractional numbers of molecules, then the whole equation may be multiplied up.

WORKED EXAMPLE No.1

Write out the combustion equation for C_8H_{18}

SOLUTION

$C_8H_{18} + (8+18/4)O_2 = 8CO_2 + (18/2)H_2O$
$C_8H_{18} + 12\frac{1}{2}O_2 = 8CO_2 + 9H_2O$
$2C_8H_{18} + 25O_2 = 16CO_2 + 18H_2O$
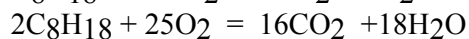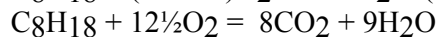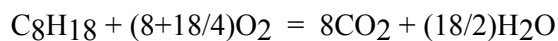
There are other gases which burn and the main one to know about is Carbon Monoxide (CO) which is partially burned carbon. The equation for the combustion of CO is :

$$2CO + O_2 = 2CO_2$$

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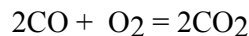
3. COMBUSTION BY MASS

The only rule to be observed in deducing the quantities of each substance is the law of conservation of mass. The proportions of the mass is that of the molecular masses. This is shown in the following example.

WORKED EXAMPLE No.2

A fuel contains by mass 88% C, 8%H_2, 1%S and 3% ash (silica). Calculate the stoichiometric air.

SOLUTION

CARBON $\qquad$ $C + O_2 = CO_2$
Mass ratio $\qquad$ $12 + 32 = 44$

Hence 0.88kg of C need (32/12)x0.88=2.347kg of oxygen.
It makes (44/12) x 0.88 = 3.227 kg of carbon dioxide.

HYDROGEN $\qquad$ $2H_2 + O_2 = 2H_2O$
Mass ratio $\qquad$ $4 + 32 = 36$

hence 0.08kg of hydrogen needs (32/4) x 0.08 = 0.64kg of oxygen.

SULPHUR $\qquad$ $S + O_2 = SO_2$
Mass ratio $\qquad$ $32 + 32 = 64$

Hence 0.01kg of sulphur needs 0.01kh of oxygen and makes 0.02kg of sulphur dioxide.

TOTAL OXYGEN needed is 2.347 + 0.64 + 0.01 = 2.997kg

TOTAL AIR needed is 2.997/23% = 13.03kg

The STOICHIOMETRIC air/fuel ratio is 13.03/1

WORKED EXAMPLE No.3

If the air supplied is 20% more than the stoichiometric value, find the analysis of the dry products by mass.

SOLUTION

If 20% excess air is supplied then the air supplied is:

$$120\% \times 13.03 = 15.637 \text{ kg}$$

Oxygen is also 20% excess so $0.2 \times 2.997 = 0.599$kg is left over.
Nitrogen in the air is $77\% \times 15.637 = 12.04$kg

List of products :

Nitrogen	12.04kg	=75.8%
Carbon dioxide	3.227kg	=20.3%
Sulphur dioxide	0.02kg	=0.1%
Oxygen	0.599kg	=3.8%
Total dry product	15.886kg	=100%

It is of interest to note that for a given fuel, the % of any product is a direct indication of the excess air and in practice the carbon dioxide and/or oxygen is used to indicate this. This is important in obtaining optimal efficiency in a combustion process.

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4.COMBUSTION BY VOLUME

First we need to revise gas mixtures and understand the meaning of VOLUMETRIC CONTENT. To do this we must understand Dalton's law of partial pressures and Avagadro's Law.

First let us define the kmol. A kmol of substance is the number of kg numerically equal to the apparent molecular mass. For example 12 kg of Carbon is a kmol, so is 32 kg of O_2 and 2 kg of H_2 and 28 kg of N_2.

The molecular mass of a substance is expressed as kg/kmol so the molecular mass of O_2, for example, is 32 kg/kmol.

Avagadro's Law states :

$1m^3$ of any gas at the same pressure and temperature contains the same number of molecules. It follows that the volume of a gas at the same p and T is directly proportional to the number of molecules. From this we find that the volume of a kmol of any gas is the same if p and T are the same.

Dalton's law states:

The total pressure of a mixture is the sum of the partial pressures. The partial pressure is the pressure each gas would exert if it alone occupied the same volume at the same temperature.

Consider two gases A and B occupying a volume V at temperature T. Using the Universal gas law for each :

$$p_A V_A = m_A R_0 T / \tilde{N}_A \qquad p_B V_B = m_B R_0 T / \tilde{N}_B$$

where $\tilde{N}$ is the relative molecular mass.

$p_A / p_B = m_A \tilde{N}_B / m_B \tilde{N}_A$ = ratio of the kmol fractions.
p_A and p_B are the partial pressures.
V_A and V_B are the partial volumes.

These are the volumes each gas would occupy if they were separated and kept at the original p and T. This concept is very useful in problems involving the combustion of gases. It also follows that the partial volumes are directly related to the partial pressures so that $V_A/V_B = p_A/p_B$
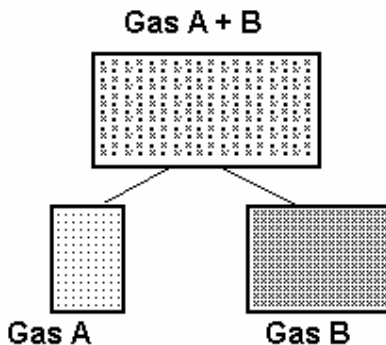

Gas A + B

Gas A   Gas B

Figure 1

When not mixed the pressure is p and the volumes are V_A and V_B. Hence :

$$pV_A/T = mR_0/\tilde{N}_A \qquad \grave{} \qquad p = m_A R_0 T/\tilde{N}_A V_A \,...........(1)$$

$$pV_B/T = mR_0/\tilde{N}_B \qquad\qquad p = m_B R_0 T/\tilde{N}_B V_B \,............(2)$$

Since (1) = (2) then :

$m_A/\tilde{N}_A V_A = m_B/\tilde{N}_B V_B$ and so $V_A/V_B = (m_A/\tilde{N}_A)(m_B/\tilde{N}_B)$ which shows that in a mixture, the partial volumes are in the same ratio as the kmol fractions which in turn are in proportion to the number of molecules of each gas.

When mixed they both have volume V, hence:

$$p_A = m_A R_0 T/\tilde{N}_A \ V.........(3)$$

$$p_B = m_B R_0 T/\tilde{N}_B \ V.........(4)$$

(3)/(1) gives $p_A/p = V_A/V$ and (4)/(2) gives $p_B/p = V_B/V$
$$\text{hence } V_A/V_B = p_A/p_B$$

Consider the combustion of Methane.

$$CH_4 + 2O_2 = CO_2 + 2H_2O$$
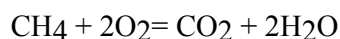
Since the volumetric content of each gas is in the same ratio as the kmol fractions then volumetric content is in the same proportion as the molecules. Hence it needs 2 volumes of oxygen to burn 1 volume of methane.

The volume of air needed is 2/21% = 9.52 volumes. Hence it burn 1 m^3 of methane we need 9.52 m^3 of air for stoichiometric combustion. If the products are at the same p and T as the original reactants, we would obtain 1 m^3 of carbon dioxide and 2 m^3 of water vapour which would probably condense and cause a reduction in volume and/or pressure.

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WORKED EXAMPLE No.4

Calculate the % CO_2 in the dry products when methane is burned with 15% excess air by volume.

SOLUTION

$$CH_4 + 2O_2 = CO_2 + 2H_2O$$

Volume ratio   1   2   1   2
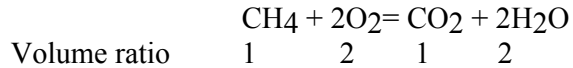
The stoichiometric air is 2/21% = 9.524 m^3
The actual air is 9.524 x 115% = 10.95 m^3

Analysis of dry products :

Nitrogen	79% x 10.95	8.65 m^3
Carbon Dioxide		1.00 m^3
Oxygen	15% x 2	0.30 m^3
Total		9.95 m^3

The % Carbon Dioxide = (1/9.95) x 100 = 10%

When the fuel is a mixture of gases, the procedure outlined must be repeated for each combustible gas and the oxygen deduced for the volume of each in 1 m^3 of total fuel.

WORKED EXAMPLE No. 5

A fuel is a mixture of 60% Methane and 30% carbon monoxide and 10% oxygen by volume. Calculate the stoichiometric oxygen needed.

SOLUTION

As before, the volume of oxygen required to burn 1 m^3 of methane is 2m^3. To burn 0.6m^3 needs 1.2m^3 of oxygen. For carbon monoxide we use the combustion equation :

$$2CO + O_2 = 2CO_2$$

Hence to burn 1 m^3 of CO need 0.5 m^3 of oxygen, so to burn 0.3 m^3 needs 0.15 m^3 of oxygen.

The total oxygen needed is 1.2 + 0.15 = 1.35 m^3 . However there is already 0.1 m^3 in the fuel so the **stoichiometric oxygen needed 1.25m^3**

SELF ASSESSMENT EXERCISE No.2

1. Find the air fuel ratio for stoichiometric combustion of Ethene by volume. (26.19/1)

2. Find the air fuel ratio for stoichiometric combustion of Butane by volume.(30.95/1). Calculate the % carbon dioxide present in the dry flue gas if 30% excess air is used. (10.6%)

3. Find the air fuel ratio for stoichiometric combustion of Propane by volume.(23.81/1). Calculate the % oxygen present in the dry flue gas if 20% excess air is used. (3.8%)

4. A gaseous fuel contains by volume :

    5% CO_2, 40% H_2 , 40% CH_4, 15% N_2

    Determine the stoichiometric air and the % content of each dry product. (4.76 m^3, 89.7%,N_2 10.3% CO_2).

5. RELATIONSHIP BETWEEN PRODUCT AND EXCESS AIR.

It follows that if we can deduce the % product then given the figure, we can work backwards to determine the air or oxygen that was used.

WORKED EXAMPLE No.6

consider the combustion of methane again.

$CH_4 + 2O_2 = CO_2 + 2H_2O$
1 vol   2 vol   1 vol   2 vols
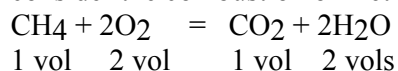
SOLUTION

Let the excess air be x (as a decimal)

The stoichiometric air is 9.52 vols.   Actual air is 9.52(1 + x)

Dry Products:

Nitrogen	0.79 x 9.52(1 + x) =	7.524x + 7.524
Oxygen		2.000x
Carbon Dioxide		1.000
Total		9.524x + 8.524

% Carbon monoxide = 100 {1/(9.524x + 8.524)}

% Oxygen   = 100{2/(9.524x + 8.524)}

For example if the % CO_2 is 10% then the excess air is found as follows :
        10% = 100 {1/(9.524x + 8.524)}
        0.1 = 1/(9.524x + 8.524)
        (9.524x + 8.524) = 10
        9.524x = 1.476
        x=0.155 or 15.5%

Similarly if the O_2 is 10% then the excess air is 81% (show this for yourself)

If the analysis of the fuel is by mass, then a different approach is needed as follows :

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WORKED EXAMPLE No.7

An analysis of the dry exhaust gas from an engine burning Benzole shows 15% Carbon Dioxide present by volume. The Benzole contains 90% C and 10% H_2 by mass. Assuming complete combustion, determine the air/fuel ratio used.

SOLUTION

1 kg of fuel contains 0.9kg of C and 0.1kg of H_2. Converting these into kmol we have 0.9/12 kmol of C and 0.1/2 kmol of H_2. For 1 kmol of dry exhaust gas we have :

$$0.15 \text{ kmol of } CO_2$$
$$Y \text{ kmol of excess } O_2$$

$$1 - 0.15 - Y = 0.85 - Y \text{ kmol of } N_2$$

1 kmol of CO_2 is 44 kg
1 kmol of N_2 is 28 kg
1 kmol of O is 32 kg
0.15 kmol of CO_2 is 0.15 x 44kg
This contains (12/44) carbon so the carbon present is 0.15 x 12 kg
The carbon in the fuel is 0.9 kmol per kmol of fuel. Hence the number of kmols of DEG must be 0.9/(.15 x 12) = 0.5

There are 0.5 kmol of DEG for each kmol of fuel burned.

The Nitrogen present in the DEG is 0.85 - Y kmol per kmol of DEG. This has a mass of
28(0.85 - Y) per kmol of DEG
The oxygen supplied to the process must be :
(23.3/76.7) x 28 x (0.85 - Y) = 7.24 - 8.5Y kg per kmol of DEG.

(using precise proportions of air for accuracy).

The oxygen contained within the carbon dioxide is:

(32/44) x 0.15 x 44 = 4.8 kg per kmol DEG

1 kmol of CO_2 contains 44 kg and 32/44 of this is oxygen. The oxygen in the CO_2 is hence 32 x 0.15 kg per kmol DEG.

The excess oxygen is
$$32Y \text{ kg per kmol DEG}$$

Total oxygen in the products excluding that used to make H_2O is :

$$32 \times 0.15 + 32Y$$

The oxygen used to burn hydrogen is hence :

$$7.24 - 8.5Y - 32 \times 0.15 + 32Y$$

O_2 used to burn H_2 is
$$2.44 - 40.5Y \text{ kg per kmol DEG}$$
For 0.5 kmol this is
$$1.22 - 20.25Y \text{ kg}$$

To burn hydrogen requires oxygen in a ratio of 8/1. There is 0.1 kg of H_2 in each kmol of fuel so 0.8 kg of O_2 is needed. Hence :

$$0.8 = 1.22 - 20.25Y$$
$$Y = 0.208 \text{kmol per kmol DEG}$$

The nitrogen in the DEG is 0.85 - Y = 0.642 kmol per kmol DEG
The actual Nitrogen = 0.642 x 0.5 x 28 = 11.61 kg
The air supplied must be 11.61/.767 = 15.14kg per kg of fuel. A simple calculation shows the stoichiometric mass of air is 13.73 so there is 10.3% excess air.

SELF ASSESSMENT EXERCISE No. 3

1. C_2H_6 is burned in a boiler and the dry products are found to contain 8% CO_2 by volume. Determine the excess air supplied. (59%)

2. The analysis of the dry exhaust gas from a boiler shows 10% carbon dioxide. Assuming the rest is oxygen and nitrogen determine the excess air supplied to the process and the % excess air. The fuel contains 85% C and 15% H_2

    (21.5 kg , 44.5%)

Now we will look at a complete example involving all the principles so far covered.

6. ENERGY RELEASED BY THE REACTION

The contents of the fuel and air or oxygen prior to combustion are called the reactants. The resulting material is called the products. The process releases energy but the amount of energy depends upon the temperature before and after the reaction.

Consider a mixture of reactants at condition (1) which is burned and the resulting products are at condition (2). In order to solve problems we consider that the reactants are first cooled to a referance condition (0) by removing energy Q_1. The reaction then takes place and energy is released. The products are then brought back to the same referance conditions (0) by removing energy Q_2. The energy Q_1 and Q_2 are then returned so that the final condition of the products is reached (2).
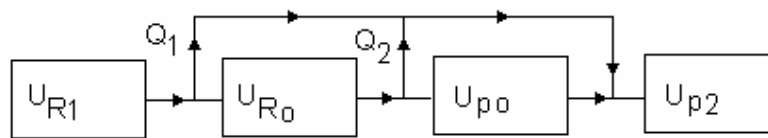

Figure 2

For constant volume combustion (closed system), we use Internal Energy. Balancing we have

$$U_{p2} - U_{R1} = (U_{Ro} - U_{R1}) + (U_{po} - U_{Ro}) + (U_{p2} - U_{po})$$

The energy released by combustion is in this case the Internal Energy of combustion and this occurs at standard conditions of 1 bar and 25^OC. This pressure is designated p^θ and the internal energy of combustion is designated ΔU^θ. When this is based on 1 kmol it is designated Δu^θ

$$U_{p2} - U_{R1} = (U_{Ro} - U_{R1}) + \Delta U_o^\theta + (U_{p2} - U_{po})$$

The standard conditions chosen for the combustion are 1 bar and 25^OC. At this temperature the internal energy of all gases is the same (-2 479 kJ/kmol). The figure is negative because the zero value of internal energy arbitrarily occurs at a higher temperature.

If the process is conducted in a steady flow system, enthalpy is used instead of internal energy. The reasoning is the same but U is replaced by H.

$$H_{p2} - H_{R1} = (H_{Ro} - H_{R1}) + \Delta H_o^\theta + (H_{p2} - H_{po})$$

Δh_o^θ may be found in the thermodynamic tables for some fuels. The figures are quoted in kJ per kmol of substance.

For the products In terms of kmol fractions $\qquad h_{po} = u_{po} + n_p R_o T_o$

For the reactants In terms of kmol fractions $\quad\quad\quad h_{Ro} = u_{Ro} + n_R R_o T_o$

where n is the kmols.

$\Delta h_o{}^\theta = (u_{po} + n_p R_o T_o) - (u_{Ro} + n_R R_o T_o)$
$\Delta h_o{}^\theta = (u_{po} - u_{Ro}) - n_R R_o T_o + n_p R_o T_o$
$\Delta h_o{}^\theta = (u_{po} - u_{Ro}) + (n_p - n_R) R_o T_o$
$\Delta u_o{}^\theta = \Delta h_o{}^\theta + (n_p - n_R) R_o T_o$

If the combustion produces equal numbers of kmols before and after, the pressure would be constant (assuming constant volume and no condensation). $n_p = n_R$ so $\Delta h_o{}^\theta$ is the same as the internal energy of reaction $\Delta v_o{}^\theta$. For example consider the combustion of ethylene.

$$C_2 H_4 + 3 O_2 = 2 CO_2 + 2 H_2 O$$
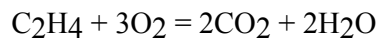
In this case there are 4 kmols before and after . When this occurs, we may use the specific heat c_p to solve the problems. $\Delta u_o{}^\theta = \Delta h_o{}^\theta = c_p \Delta T$

The specific heats are listed in the thermodynamic tables.

Note that in order to make the method of solution conform to standard data, the combustion equations should always be based on 1 kmol of fuel. The heat transfer Q_1 is found either by use of the mean specific heat or by looking up the enthalpy of the gas at the required temperatures (enthalpy of formation) and deducing the change.

In general for a constant volume we should use $\Delta u_o{}^\theta$ and c_v to solve problems.

For constant pressure with no work being done (e.g. a combustion chamber) we should use $\Delta h_o{}^\theta$ and c_p.

Since tables only list $\Delta h_o{}^\theta$ and c_p we may find

$\Delta u_o{}^\theta = \Delta h_o{}^\theta + (n_p - n_R) R_o T_o$ where is 298.1 K and R_o is 8.314 kJ/kmol K and n is the number of kmols of product.

$c_v = c_p - R = c_p - R_o/\text{molecular mass}$

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WORKED EXAMPLE No.8

1. A vessel contains 0.2 m^3 of C_2H_4O and oxygen in its stoichiometric ratio. The mixture is at 1 bar pressure and 25°C. The mixture is ignited and allowed to cool back to 25°C. Determine the following.

    i. The final pressure.

    ii. The amount of condensate formed.

    iii. The heat transfer.

    iv. The enthalpy of reaction per kmol of C_2H_4O.

The enthalpy of formation (Δh_o^θ) for the gases involved is shown below for a temperature of 298 K.

	Molecular mass. kg/kmol	Enthalpy of reaction (kJ/kmol)
CO_2(gas)	44	- 393 520
H_2O(gas)	18	- 241 830
H_2O(liquid)	18	-285 820
O_2(gas)	32	0
C_2H_4O	44	-52 630

SOLUTION

$$C_2H_4O + 2\frac{1}{2}O_2 = 2CO_2 + 2H_2O$$

mass ratio	44	80	88	36
kmol ratio	1	$2\frac{1}{2}$	2	1
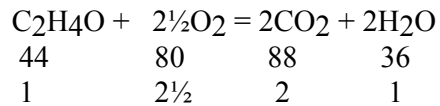
p = 1 bar   V = 0.2 m^3 T = 298 K
V = 0.2 m^3 = $V_f + V_{ox}$ = 3½ Volumes
p = 1 bar = $p_f + p_{ox}$
p_f = 1 x (1/3.5) = 0.2857 bar
p_{ox} = 1 x (2.5/3.5) = 0.7143 bar

mass of fuel = $pV\tilde{N}/R_oT$ = 0.2857 x 10^5 x 0.2 x 44/(8314.4 x 298)
mass of fuel = 0.1015 kg

mass of oxygen = $pV\tilde{N}/R_oT$ = 0.7143 x 10^5 x 0.2 x 32/(8314.4 x 298)
mass of oxygen = 0.1845 kg

total mass = 0.286 kg

Mass of CO_2 = (88/124) x 0.286 = 0.203 kg
Mass of H_2O = (36/124) x 0.286 = 0.083 kg
total mass = 0.286 kg

$p(CO_2)$ = $mR_oT/\tilde{N}V$ = 0.203 x 8314.4 x 298/(44 x 0.2)
            = 0.5716 x 10^5 = 0.5716 bar

Since condensate forms, the gas is saturated with water vapour so p(vapour) = p_g @ 28°C from the steam tables.

p(vapour) = 0.03166 bar

Total pressure = 0.5716 + 0.03166 = 0.603 bar Answer (i)

Mass of vapour = V/v_g
where v_g = 43.4 m^3/kg @ 28°C from tables.
Mass of vapour = 0.2/43.4 = 0.004608 kg
Mass of condensate formed = 0.083 - 0.0046 = 0.0784 kg
                                    Answer (ii)

Now consider the reaction. Since it starts and finishes at 25°C there is no initial cooling required (Q_1 = 0).

REACTANT

Fuel   Mass = 0.1015 kg   kmol = 0.1015/44 = 0.00231 kmol
Oxygen Mass = 0.1845 kg   kmol = 0.1845/32 = 0.00576 kmol

Enthalpy of formation for oxygen = 0
Enthalpy of formation for C_2H_4O = -52630 kJ/kmol
H_f = 0.00231(-52630) = -121.4 kJ (Minus relative to higher point of reference)

PRODUCTS

CO_2 kmol = 0.203/44 = 0.00461 kmol
H_f = 0.00461 x (-393 520) = - 1815.6 kJ

H_2O (gas)   kmol = 0.0408/18 = 0.00227 kmol
H_f = 0.00227 x (-241830) = - 548.1 kJ

H_2O (water)   kmol = 0.0784/18 = 0.00436 kmol
H_f = 0.00436 x (-285820) = - 1244.9 kJ
Total = -3608 kJ

The change in enthalpy = -3608 - (-121.4)
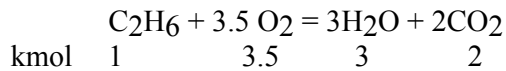
= -3486.6 kJ = ΔH_o   **Answer (iv)**

Δh_o^{θ} = -3486.6/0.00231 = **1.509 GJ/kmol**   **Answer (v)**

WORKED EXAMPLE No.9

Air and Ethane (C_2H_6) are mixed with twice the stoichiometric ratio at 600K in a vessel at 12 bar pressure. Determine the temperature and pressure after combustion assuming no energy losses. The enthalpy of combustion at 25°C is H_o = - 1 427 860 kJ/kmol

SOLUTION

$$C_2H_6 + 3.5\ O_2 = 3H_2O + 2CO_2$$

kmol   1   3.5   3   2
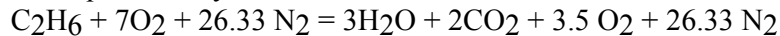
The air required = 3.5/0.21 = 16.67 kmol

Actual air = 33.33 kmol

Nitrogen = 0.79 x 33.33 = 26.33 kmol

Excess oxygen = 3.5 kmol

The equation may be rewritten as

$C_2H_6 + 7O_2 + 26.33\ N_2 = 3H_2O + 2CO_2 + 3.5\ O_2 + 26.33\ N_2$

The process may be idealised as follows
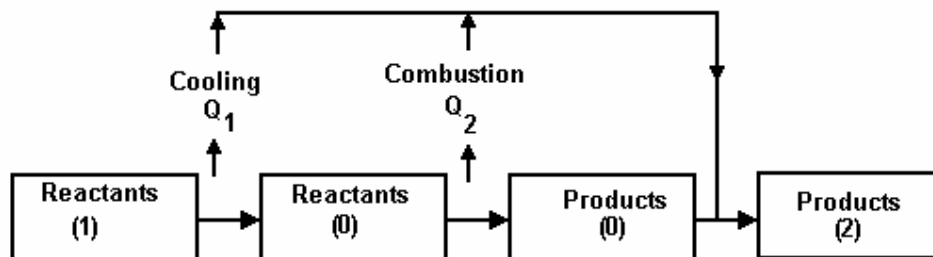
First find the enthalpy of the reactants. The mean temperature of the reactants relative to 25°C is   {(25+273) + 600}/2 = 449K near enough 450K for the tables. We look up specific heats in the thermodynamic tables at 450 K. The temperature change from 25°C to 600 K is 302 K. We proceed to work out the heat transfer based on 1 kmol of fuel, Q_1 as follows.

Table of values

C_2H_6 c_p = 2.402 kJ/kg K   mass = 1 kmol x 30 = 30 kg
                                    Q_1 = 30 x 2.402 x 302 = 21 762 kJ

O_2 c_p = 0.956 kJ/kg K   mass = 7 kmol x 32 = 224 kg
                                    Q_1 = 224 x 0.956 x 302 = 64671.5 kJ

N_2 c_p = 1.049 kJ/kg K   mass = 26.33 kmol x 28 = 737.24 kg
                                    Q_1 = 737.24 x 1.049 x 302 = 233556 kJ

Total Q_1 =- 319 989.7 kJ per kmol of fuel (negative leaving system)

Next we repeat the process for the products to find $Q_1 + Q_2$

In order to use a mean specific heat we must guess the approximate final temperature of the products. A good guess is always 2000 K so the mean of 25ºC and 2000 K is near enough 1150 K. Using this we work out the heat transfer to the products with an unknown temperature change from 25ºC to T_2 of ΔT.

H_2O $c_p = 2.392$ kJ/kg K   mass = 3 kmol x 16 = 48 kg
$$Q = 48 \text{ x } 2.392 \text{ x } \Delta T = 114.8\Delta T$$

O_2 $c_p = 1.109$ kJ/kg K   mass = 3.5 kmol x 32 = 112 kg
$$Q_1 = 112 \text{ x } 1.109 \text{ x } \Delta T = 124.2 \Delta T$$

N_2 $c_p = 1.196$ kJ/kg K   mass = 26.33 kmol x 28 = 737.24 kg
$$Q = 737.24 \text{ x } 1.196 \text{ x } \Delta T = 881.7\Delta T$$

CO_2 $c_p = 1.270$ kJ/kg K   mass = 2 kmol x 44 = 88 kg
$$Q = 88 \text{ x } 1.270 \text{ x } \Delta T = 111.76\Delta T$$

Total $Q_1 + Q_2 = 1232.5\Delta T$ kJ per kmol of fuel (positive entering system)

$Q_2 = -1\,427\,860$ kJ/kmol of fuel (from question).

Conserving energy we have $1232.5 \Delta T = 1\,427\,860 + 319\,989.7$
hence $\Delta T = 1232.5$ K and $T_2 = 1716$ K which is different from our original guess of 2000 K but more accurate.

Next we must repeat the last stage with a more accurate mean temperature.
Mean temperature = $(298 + 1716)/2 = 1007$ K. Say 1000 K.

H_2O $c_p = 2.288$ kJ/kg K $Q = 48$ x 2.288 x $\Delta T = 109.8\Delta T$

O_2 $c_p = 1.090$ kJ/kg K $Q_1 = 112$ x 1.09 x $\Delta T = 122.1\Delta T$

N_2 $c_p = 1.167$ kJ/kg K $Q = 737.24$ x 1.167 x $\Delta T = 860.4\Delta T$

CO_2 $c_p = 1.234$ kJ/kg K $Q = 88$ x 1.234 x $\Delta T = 108.6\Delta T$

Total $Q_1 + Q_2 = 1201\Delta T$ kJ per kmol of fuel (positive entering system)

Conserving energy we have $1201\Delta T = 1\,427\,860 + 319\,989.7$

hence $\Delta T = 1455$ K and $T_2 = 1753$ K which is different from our original guess of 1716K but more accurate. the true answer is between 1716 and 1753 K and may be narrowed down by making more steps but two is usually sufficient in the exam.

Finally the pressure. $p_1V_1 / N_1T_1 = R_o = p_2V_2 / N_2T_2$ and the volumes are equal.

p_1=12 bar T_1 = 600 K N_1= 4.5 kmol

T_2 = 1753 K N_2= 5 kmol

p_2 = 12 x 5 x 1753/(4.5 x 600) = 38.9 bar

SELF ASSESSMENT EXERCISE No. 4

1. The gravimetric analysis of a fuel is Carbon 78%, hydrogen 12%, oxygen 5% and ash 5%. The fuel is burned with 20% excess air. Assuming complete combustion, determine the following.

    i. The composition of the products.
    (72.6% N_2, 17.3% CO_2, 6.5%H_2O and 3.6% O_2)

    ii. The dew point of the products.47°C)

    iii. The mass of water condensed when the products are cooled to   30°C.
    (0.67 kg)

2. Carbon monoxide is burned with 25% excess oxygen in a fixed volume of 0.2 m^3. The initial and final temperature is 25°C.   The   initial pressure is 1 bar. Calculate the following.

    i. The final pressure.(0.874 bar)

    ii. The heat transfer. (574.5 kJ)

    Use your thermodynamic tables for enthalpies of reaction.

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3. Prove that the enthalpy and the internal energy of reaction are related by

$$\Delta H_0 = \Delta U_0 + R_0 T_0 (n_P - n_R)$$

where n_P and n_R are the kmols of products and reactants respectively.

Ethylene (C_2H_4) and 20% excess air at 77ºC are mixed in a vessel and burned at constant volume. Determine the temperature of the products. You should use your thermodynamic tables to find ΔU_0 or ΔH_0 and the table below. (Answer 2263 K)

U (kJ/kmol)	C_2H_4	O_2	N_2	CO_2	H_2O
T(K)					
298.15	- 2479	-2479	-2479	-2479	-2479
300	- 2415	-2440	-2440	-2427	-2432
400	-1557	-297	-355	683	126
2400		54537	50696	95833	73650
2600		60657	50696	95833	73650
2800		66864	62065	117160	92014
3000		73155	67795	127920	101420

4. An engine burns hexane (C_6H_{14}) in air. At a particular running condition the volumetric analysis of the dry products are

CO2   8.7%
CO   7.8 %
N2   83.5%

Calculate the air-fuel ratio by mass and find the stoichiometric ratio.

(Answer 12.9 and 15.17)

7. DISSOCIATION

At the high temperatures and pressures experienced in combustion, dissociation occurs. This results in some of the fuel not burning. CO is produced and in the case of hydrogen, some of it remains as hydrogen after the process even though oxygen is present. The reasons for this will not be covered here other than to say it is predicted by the 2nd law of thermodynamics and involves equilibrium in the chemical process.

When dissociation occurs, the energy released is reduced accordingly and if the amount of unburned fuel is known the previous examples may easily be modified to take account of it.

When hydrogen is burned, it can be shown that the partial pressures of the hydrogen, oxygen and water vapour present in the products are related by

$$k^{\theta} = \frac{p_{H_2O}\, p^{\theta}}{p_{H_2}\left(p_{O_2}\right)^{\frac{1}{2}}}$$

The properties tables list values of $\ln K^{\theta}$.

Similarly when dissociation occurs in the formation of carbon dioxide, the relationship between the partial pressures of CO_2, CO and O_2 is given by

$$k^{\theta} = \frac{p_{CO_2}\, p^{\theta}}{p_C\, p_{CO_2}}$$

Other similar equations for other combinations of products may be found in the tables.

SELF ASSESSMENT EXERCISE No. 5

1. Hydrogen is mixed with stoichiometric air at 25°C and burned adiabatically at constant volume. After combustion 6% of the hydrogen remains unburned. Determine the temperature and pressure of the products.

    (Answer the temperature is 2344K after two approximations)

    You need to find K^{θ} in the tables. Also find ΔH_o=241800 kJ/kmol. Deduce the partial pressures of the products as a fraction of p and then use K^{θ} to solve p.

2. A mixture of air and CO is burned adiabatically at constant volume. The air is 90% of the stoichiometric requirement. The mixture is initially at 5 bar and 400K. The only dissociation that occurs is $CO_2 \rightarrow CO + \frac{1}{2}O_2$.

    Show that the equilibrium constant at the final temperature T_2 is

    $$K_p = 1.1212a/(\{(1-a)(0.9-a)^{\frac{1}{2}}(T_2/T_1)^{\frac{1}{2}}\}$$

    where a is the amount of CO_2 kmol in the products per kmol of CO in the reactants.

    If it assumed that initially T_2 = 2900 K for which $\log K_p$ = 0.649, the solution of the above equation gives a=0.784. Check the assumed value of T_2 given that the internal energy of reaction at T_0= 298.15 K is -281750 kJ/kmol.

T (K)		U kJ/kmol			
		CO	O$_2$	N$_2$	CO$_2$
298.15		-2479	-2479	-2479	-2479
400	- 351	- 297	- 355	+ 683	
2900		+65715	+69999	+64924	+122530