EDEXCEL HIGHERS
ENGINEERING THERMODYNAMICS H2
NQF LEVEL 4

TUTORIAL No. 1 – PRE-REQUISITE STUDIES

FLUID PROPERTIES

INTRODUCTION

Before you study the four outcomes that make up the module, you should be competent in finding the properties of liquids, gases and vapours. If you are already competent in this, you should skip this tutorial.

This tutorial is designed to teach you the basic concepts of thermodynamics and the properties of fluids. On completion of this tutorial you should be able to the following.

- Use the correct thermodynamic symbols.
- Determine the properties of a gas.
- Determine the properties of vapours.
- Determine the properties of liquids.

We will start by examining the symbols used.

1. SYMBOLS

The symbols used throughout in these tutorials are S.I. (International System). These are not always the ones used by the examiner. These are the main S.I. Symbols. The S.I. also recommends the use of / to mean divide rather than the use of negative indices favoured by examiners. For example ms^{-1} becomes m/s.

Quantity	**units**	**Derived Unit**	**S.I. symbol**
Length	m		various
Mass	kg		m
Time	s		t
Volume	m^3		V or Q
Specific Volume	m^3/kg		*v*
Volume Flow Rate	m^3/s		
Density	kg/m^3		ρ
Force	kg m/s^2	N	F
Weight	kg m/s^2	N	W
Pressure Head	m		h
Altitude	m		z
Area	m^2		A
Speed of Sound	m/s		a
Specific Heat Cap.	N m/kg K	Joule/kg K	c
Energy	N m	Joule	
Enthalpy	N m	Joule	H
Internal Energy	N m	Joule	U
Specific Enthalpy	N m/kg	J/kg	h
Specific Int. Energy	N m/kg	J/kg	u
Mass flow rate	kg/s		
Polytropic Index			n
Adiabatic Index			γ
Pressure	N/m^2	Pascal	p
Heat Transfer	N m	Joule	Q
Work	N m	Joule	W
Heat Transfer Rate	N m/s	Watt	Φ
Work Rate (power)	N m/s	Watt	P
Char. Gas Const	N m/kg K	J/kg K	R
Universal Gas Constant	J/kmol K		R_o
Entropy	J/K		S
Specific Entropy	J/kg K		s
Absolute Temperature	K		T
Celsius Temperature	^oC		θ
Velocity	m/s^2		v or u
Dynamic Viscosity	N s/m^2	Pa s	η or μ
Kinematic Viscosity	m^2/s		ν

Now we will examine the basic concepts required to do this tutorial successfully.

2. BASIC CONCEPTS

Throughout these tutorials you will use properties which are either EXTENSIVE or INTENSIVE.

An extensive property is one which is divisible. For example Volume when divided by a number becomes smaller. Other examples are mass and energy.

An intensive property is a property of a mass which remains the same value when the mass is divided into smaller parts. For example the temperature and density of a substance is unchanged if it is divided into smaller masses.

Throughout the tutorials you will use TOTAL and SPECIFIC quantities which relate only to extensive properties. A total quantity is always denoted by a higher case letter such as V for volume (m^3) and H for enthalpy (J). A specific quantity represents the quantity per kg and is obtained by dividing the property by the mass. Such properties are always designated by lower case letters such as v for specific volume (m^3/kg) and h for specific enthalpy (J/kg).

Specific volume is mainly used for gas and vapours. The inverse of specific volume is density (ρ) (kg/m^3) and this is mainly used for liquids and solids but also for gases. Note $\rho=1/v$.

Because the same letters are used to designate more than one property, often alternative letters are used. For example v for specific volume may occur in the same work as v for velocity so often u or c is used for velocity. h is used for height, head and specific enthalpy so z is often used for height instead.

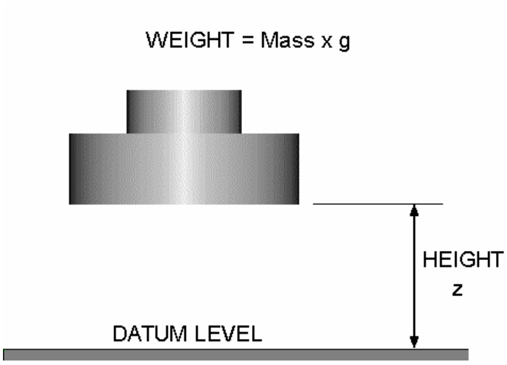
The unit of Force and Weight is the kg m/s^2. This comes from Newton's Second Law of Motion (Force = mass x acceleration). The derived name for the unit is the Newton. In the case of Weight, the acceleration is that of gravity and in order to convert mass in kg into weight in Newtons, you must use $W = mg$ where g is normally 9.81 m/s^2.

Now we will examine forms of energy a fluid may have.

3. ENERGY FORMS

A fluid may possess several forms of energy. All fluids possess energy due to their temperature and this is called INTERNAL ENERGY. All possess GRAVITATIONAL or POTENTIAL ENERGY due to their elevation relative to some datum level. If the fluid is moving it will possess KINETIC ENERGY due to its velocity. If it has pressure then it will possess FLOW ENERGY. Often pressure and temperature are the main two governing factors and we add internal energy to flow energy in order to produce a single entity called ENTHALPY. Let us look at each in more detail.

3.1. GRAVITATIONAL or POTENTIAL ENERGY



In order to raise a mass m kg a height z metres, a lifting force is required which must be at least equal to the weight mg.

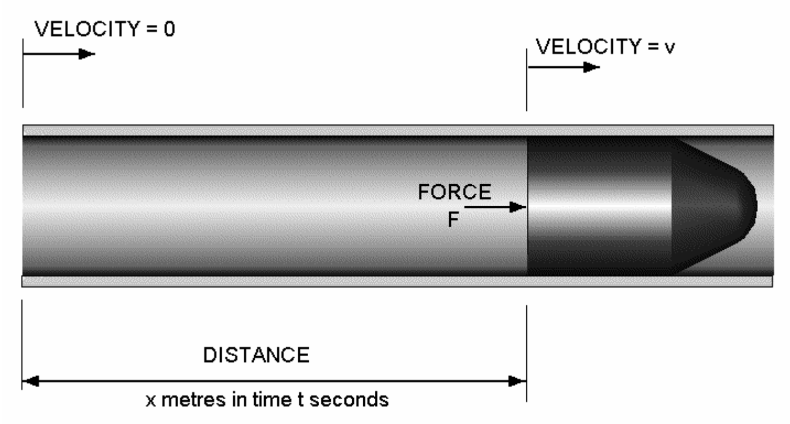
The work done raising the mass is as always, force x distance moved so

$$\text{Work} = mgz$$

Since energy has been used to do this work and energy cannot be destroyed, it follows that the energy must be stored in the mass and we call this gravitational energy or potential energy P.E. There are many examples showing how this energy may be got back, e.g. a hydro-electric power station.

$$\textbf{P.E.} = \mathbf{mgz}$$

3.2 KINETIC ENERGY



When a mass m kg is accelerated from rest to a velocity of v m/s, a force is needed to accelerate it. This is given by Newton's 2nd Law of Motion F= ma.

After time t seconds the mass travels x metres and reaches a velocity v m/s. The laws relating these quantities are

$a = v/t$ and $x = vt/2$

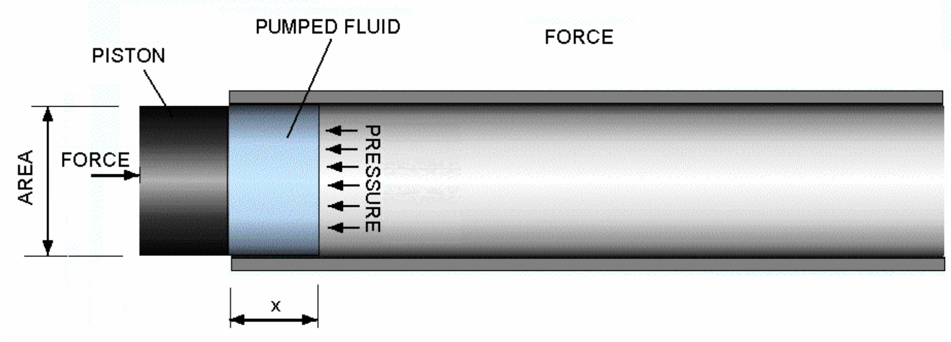
The work done is $W = Fx = max = mv^2/2$

Energy has been used to do this work and this must be stored in the mass and carried along with it. This is KINETIC ENERGY.

$$\textbf{K.E.} = \mathbf{mv^2/2}$$

3.3 FLOW ENERGY

When fluid is pumped along a pipe, energy is used to do the pumping. This energy is carried along in the fluid and may be recovered (as for example with an air tool or a hydraulic motor). Consider a piston pushing fluid into a cylinder.



The fluid pressure is p N/m^2. The force needed on the piston is

$$F = pA$$

The piston moves a distance x metres. The work done is

$$W = Fx = pAx$$

Since Ax =V and is the volume pumped into the cylinder the work done is

$$W = pV$$

Since energy has been used doing this work, it must now be stored in the fluid and carried along with it as FLOW ENERGY.

$$\textbf{F.E.} = \mathbf{pV}$$

3.4 INTERNAL ENERGY

This is covered in more detail later. The molecules of a fluid possess kinetic energy and potential energy relative to some internal datum. Usually this is regarded simply as the energy due to the temperature and very often the change in internal energy in a fluid which undergoes a change in temperature is given by

$$\Delta U = mc\Delta T$$

The symbol for internal energy is U kJ or u kJ/kg. Note that a change in temperature is the same in degrees Celsius or Kelvin. The law which states internal energy is a function of temperature only is known as **JOULE'S LAW.**

3.5 ENTHALPY

When a fluid has pressure and temperature, it must possess both flow and internal energy. It is often convenient to add them together and the result is ENTHALPY. The symbol is H kJ or h kJ/kg.

$$H = \text{F.E.} + U$$

Next you need to study the properties of fluids and the laws relating them.

4 GAS LAWS

In this section you will do the following.

- Derive basic gas laws.
- Examine the characteristic gas law.
- Examine the universal gas law.
- Define the mol.
- Solve gas law problems.

4.1 THEORY

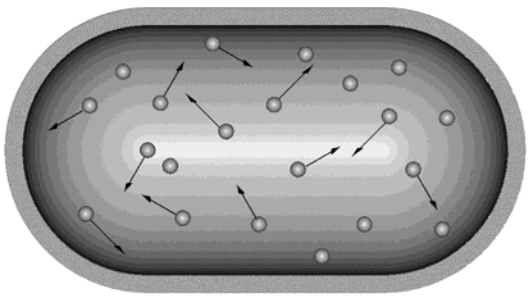
A gas is made of molecules which move around with random motion. In a perfect gas, the molecules may collide but they have no tendency at all to stick together or repel each other. In other words, a perfect gas in completely inviscid. In reality there is a slight force of attraction between gas molecules but this is so small that gas laws formulated for an ideal gas work quite well for real gas.

Each molecule in the gas has an instantaneous velocity and hence has kinetic energy. The sum of this energy is the internal energy U. The velocity of the molecules depends upon the temperature. When the temperature changes, so does the internal energy. The internal energy is for all intents and purposes zero at - 273^{o} C. This is the absolute zero of temperature. Remember that to convert from Celsius to absolute, add on 273. For example

40^{o}C is 40 + 273= 313 Kelvins.

4.2 PRESSURE

If a gas is compressed it obtains pressure. This is best explained by considering a gas inside a vessel as shown.



The molecules bombard the inside of the container. Each produces a momentum force when it bounces. The force per unit area is the pressure of the gas. Remember that pressure = Force/area

p = F/A N/m^2 or Pascals

Note that 10^3 Pa = 1 kPa 10^6 Pa = 1 MPa 10^5 Pa = 1 bar

4.3 CONSTANT VOLUME LAW

If the gas is heated the velocity of the molecules increases. If the container is rigid, then the molecules will hit the surface more often and with greater force so we expect the pressure to rise proportional to temperature.

p = c T when V is constant.

WORKED EXAMPLE No.1

A mass of gas has a pressure of 500 kPa and temperature of 150^{o}C. The pressure is changed to 900 kPa but the volume is unchanged. Determine the new temperature.

SOLUTION

Using constant volume law find p_1/T_1 =c = p_2/T_2 where

T_1 =150 + 273 = 423 K
p_1 = 500 000 p_2 =900 000
$T_2 = p_2T_1/p_1$ = 900 000 x 423/500 000 T_2 = 761.4 K

4.4 CHARLE'S LAW

If we kept the pressure constant and increased the temperature, then we would have to make the volume bigger in order to stop the pressure rising. This gives us Charle's Law:

V = c T when p is constant

WORKED EXAMPLE No.2

A mass of gas has a temperature of 150^{o}C and volume of 0.2 m^3. The temperature is changed to 50^{o}C but the pressure is unchanged. Determine the new volume.

SOLUTION

Using Charles's law we find V_1/T_1=c = V_2/T_2 where

T_1 =150 + 273 = 423 K
V_1 = 0.2
T_2 = 50 + 273 = 323 K
$V_2 = T_2V_1/T_1$ = 323 x 0.2/523
V_2 = 0.123 m^3

4.5 BOYLE'S LAW

If we keep the temperature constant and increase the volume, then the molecules will hit the surface less often so the pressure goes down. This gives Boyle's Law:

$p = c/V$ when T is constant.

WORKED EXAMPLE No.3

A mass of gas has a pressure of 800 kPa and volume of 0.3 m^3. The pressure is changed to 100 kPa but the temperature is unchanged. Determine the new volume.

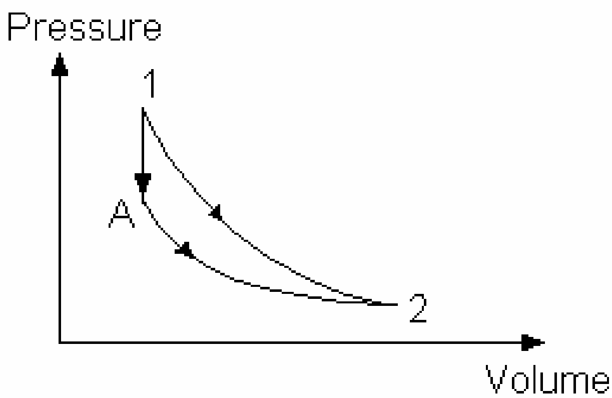
SOLUTION

Using Boyle's law we find $p_1V_1 = c = p_2 V_2$ where

$p_1 = 800 \times 10^3$ V1 = 0.3 $p_2 = 100 \times 10^3$
$V_2 = p_1V_1/p_2 = 800 \times 10^3 \times 0.3/100 \times 10^3$
$V_2 = 2.4 m^3$.

4.6 GENERAL GAS LAW

Consider a gas which undergoes a change in pV and T from point (1) to point (2) as shown. It could have gone from (1) to (A) and then from (A)to (2) as shown.



Process (1) to (A) is constant volume
$p_A/T_A = p_1/T_1$

Process (A) to (2) is constant temperature
$T_2 = T_A$

Hence $p_A/T_2 = p_1/T_1$ and $p_A = p_1T_2/T_1$ ....(1)

For the process (A) to (2) Boyle's Law applies so $p_AV_A = p_2V_2$

Since $V_A = V_1$ then we can write $p_AV_1 = p_2V_2$

So $p_A = p_2V_2/V_1$............(2)

Equating (1) and (2) we get $\mathbf{p_1V_1/T_1 = p_2V_2/T_2}$ **= constant**

This is the GENERAL GAS LAW to be used to calculate one unknown when a gas changes from one condition to another.

WORKED EXAMPLE No.4

A mass of gas has a pressure of 1.2 MPa, volume of 0.03 m^3 and temperature of 100^oC.
The pressure is changed to 400 kPa and the volume is changed to 0.06 m^3. Determine the new temperature.

SOLUTION

Using the general gas law we find $p_1V_1/T_1 = p_2V_2/T_2$ where
$p_1 = 1.2 \times 10^6$
$V_1 = 0.03$
$p_2 = 400 \times 10^3$

$T_1 = 100 + 273 = 373$ K
$T_2 = p_2V_2 T_1/p_1V_1 = 400 \times 10^3 \times 0.06 \times 373/(1.2 \times 10^6 \times 0.03)$

$T_2 = 248.7$ K

4. 7 CHARACTERISTIC GAS LAW

The general gas law tells us that when a gas changes from one pressure, volume and temperature to another, then

$$pV/T = \text{constant}$$

Thinking of the gas in the rigid vessel again, if the number of molecules was doubled, keeping the volume and temperature the same, then there would be twice as many impacts with the surface and hence twice the pressure. To keep the pressure the same, the volume would have to be doubled or the temperature halved. It follows that the constant must contain the mass of the gas in order to reflect the number of molecules. The gas law can then be written as $pV/T = mR$
where m is the mass in kg and R is the remaining constant which must be unique for each gas and is called the CHARACTERISTIC GAS CONSTANT. If we examine the units of R they are J/kg K.

The equation is usually written as **pV = mRT**

Since m/V is the density ρ, it follows that **ρ=p/RT**

Since V/m is the specific volume ,*v*, then ***v*=RT/p**

WORKED EXAMPLE No.5

A mass of gas has a pressure of 1.2 MPa, volume of 0.03 m^3 and temperature of 100°C. Given the characteristic gas constant is 300 J/kg K find the mass.

SOLUTION

From the characteristic gas law we have pV = mRT where

$p = 1.2 \times 10^6 \text{ N/m}^2$ $V = 0.03 \text{ m}^3$ $T = 100 + 273 = 373 \text{ K}$

$m = pV/RT = 1.2 \times 10^6 \times 0.03/300 \times 373 = 0.322 \text{ kg}$

SELF ASSESSMENT EXERCISE No.1

All pressures are absolute.

1. Calculate the density of air at 1.013 bar and 15 °C if R = 287 J/kg K. (1.226 kg/m^3)

2. Air in a vessel has a pressure of 25 bar, volume 0.2 m^3 and temperature 20°C. It is connected to another empty vessel so that the volume increases to 0.5 m^3 but the temperature stays the same. Taking R = 287 J/kg K. Calculate

   i. the final pressure. (10 bar)
   ii. the final density. (11.892 kg/m^3)

3. 1 dm^3 of air at 20°C is heated at constant pressure of 300 kPa until the volume is doubled. Calculate

   i. the final temperature. (586 K)
   ii. the mass. (3.56 g)

4. Air is heated from 20°C and 400 kPa in a fixed volume of 1 m^3. The final pressure is 900 kPa. Calculate

   i. the final temperature.(659 K)
   ii. the mass. (4.747 kg)

5. 1.2 dm^3 of gas is compressed from 1 bar and 20°C to 7 bar and 90°C. Taking R = 287 J/kg K calculate

   i. the new volume. (212 cm^3)
   ii. the mass. (1.427 g)

4.8 THE UNIVERSAL GAS LAW

The Characteristic Gas Law states $pV = mRT$
where R is the characteristic constant for the gas.
This law can be made universal for any gas because $R = R_o/M_m$.
where M_m is the mean molecular mass of the gas (numerically equal to the relative molecular mass).

The formula becomes $pV = mR_oT/M_m$.

R_o is a universal constant with value 8314.3 J/kmol K. It is worth noting that in the exam, this value along with other useful data may be found in the back of your fluids tables.

The kmol is defined as the number of kg of substance numerically equal to the mean molecular mass. Typical values are

GAS	Symbol	M_m.
Hydrogen	H_2	2
Oxygen	O_2	32
Carbon Dioxide	CO_2	44
Methane	CH_4	16
Nitrogen	N_2	28
Dry Air		28.96

Hence 1 kmol of hydrogen (H_2) is 2 kg
1 kmol of oxygen (O_2) is 32 kg
1 kmol of Nitrogen is 28 kg and so on.

For example if you had 3 kmol of nitrogen (N_2) you would have
$$3 \times 28 = 84 \text{ kg}$$

It follows that the M_m must have units of kg/kmol

In order to calculate the characteristic gas constant we use
$$R = R_o/M_m$$

For example the characteristic gas constant for air is

$$R = 8314.3/28.96 = 287$$

Examine the units

$$R = R_o/M_m = (\text{J/kmol K})/(\text{kg/kmol}) = \text{J/kg K}$$

WORKED EXAMPLE No.6

A vessel contains 0.2 m^3 of methane at 60°C and 500 kPa pressure. Calculate the mass of Methane.

SOLUTION

$pV = mR_oT/M_m$. $M_m = 16$

$500\,000 \times 0.2 = m \times 8314.3 \times (273 + 60)/16$ $m = 0.578$ kg

SELF ASSESSMENT EXERCISE No. 2

1. A gas compressor draws in 0.5 m^3/min of Nitrogen at 10°C and 100 kPa pressure. Calculate the mass flow rate.
(0.595 kg/min)

2. A vessel contains 0.5 m^3 of Oxygen at 40°C and 10 bar pressure.
Calculate the mass.
(6.148 kg)

Next we will examine the meaning of specific heat capacities.

5. SPECIFIC HEAT CAPACITIES

In this section you will do the following.

- Learn how to calculate the change in internal energy of gases and liquids.
- Learn how to calculate the change in enthalpy of gases and liquids.
- Define the specific heats of fluids.
- Relate specific heat capacities to the characteristic gas constant.

5.1 SPECIFIC HEAT CAPACITIES

The specific heat capacity of a fluid is defined in two principal ways as follows:

1. Constant Volume

The specific heat which relates change in specific internal energy 'u' and change in temperature 'T' is defined as :

$$c_V = du/dT$$

If the value of the specific heat capacity c_V is constant over a temperature range ΔT then we may go from the differential form to the finite form

$$c_V = \Delta u/\Delta T \quad \text{J/kg}$$

Hence $\Delta u = c_V \Delta T$ J/kg

For a mass m kg the change is **$\Delta U = mc_V \Delta T$** Joules

This law indicates that the internal energy of a gas is dependant only on its temperature. This was first stated by Joule and is called JOULE'S LAW.

2. ***Constant Pressure***.

The specific heat which relates change in specific enthalpy 'h' and change in temperature 'T' is defined as :

$$c_p = dh/dT$$

If the value of the specific heat capacity c_p is constant over a temperature range ΔT then we may go from the differential form to the finite form

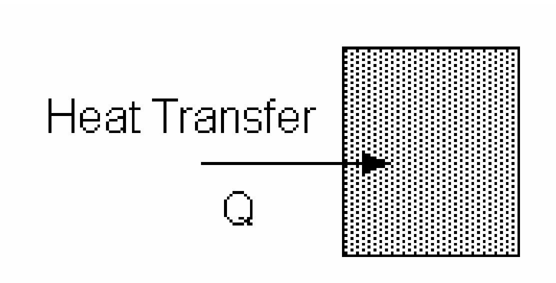
$$c_p = \Delta h/\Delta T \quad \text{J/kg}$$

Hence $\Delta h = c_p \Delta T$ J/kg

For a mass m kg the change is **$\Delta H = mc_p \Delta T$** Joules

The reasons why the two specific heats are given the symbols c_V and c_p will be explained next. They are called the PRINCIPAL SPECIFIC HEATS.

5.2 CONSTANT VOLUME HEATING

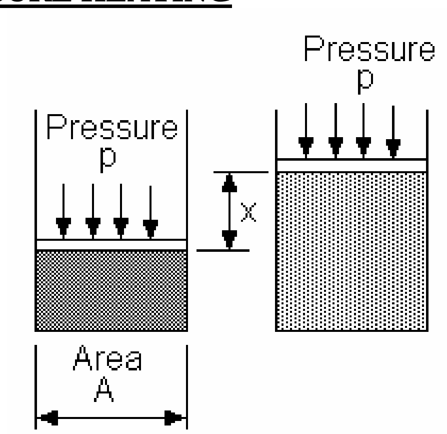


When a fluid is heated at constant volume, the heat transfer 'Q' must be the same as the increase in internal energy of the fluid ΔU since no other energy is involved. It follows that :

$$Q = \Delta U = mc_V\Delta T \text{ Joules}$$

The change in internal energy is the same as the heat transfer at constant volume so the symbol c_V should be remembered as applying to constant volume processes as well as internal energy.

5.3 CONSTANT PRESSURE HEATING



When a fluid is heated at constant pressure, the volume must increase against a surrounding pressure equal and opposite to the fluid pressure p.

The force exerted on the surroundings must be $F = pA$ Newtons
The work done is force x distance moved hence :
Work Done = $F x = pAx = p\,\Delta V$ where ΔV is the volume change.

The heat transfer Q must be equal to the increase in internal energy plus the work done against the external pressure. The work done has the same formula as flow energy p V
Enthalpy was defined as $\Delta H = \Delta U + p\Delta V$

The heat transfer at constant pressure is also: $Q = \Delta U + p\Delta V$
Since specific heats are used to calculate heat transfers, then in this case the heat transfer is by definition : $Q = mc_p\Delta T$

It follows that $\Delta H = Q = mc_p\Delta T$

For the same temperature change ΔT it follows that the heat transfer at constant pressure must be larger than that at constant volume. The specific heat capacity c_p is remembered as linked to constant pressure.

5.4 LINK BETWEEN c_v, c_p AND R.

From the above work it is apparent that $\Delta H = mc_p\Delta T = \Delta U + p\Delta V$
We have already defined $\Delta U = mc_v\Delta T$
Furthermore for a gas only, $p\Delta V = mR\Delta T$
Hence $mc_p\Delta T = mc_v\Delta T + mR\Delta T$
Hence $\mathbf{c_p = c_v + R}$

5.5 LIQUIDS

Since the volume of a liquid does not change much when heated or cooled, very little work is done against the surrounding pressure so it follows that c_v and c_p are for all intents and purposes the same and usually the heat transfer to a liquid is given as :

$$Q = mc\,\Delta T$$

Where c is the specific heat capacity.

5.6 VAPOURS

Vapour is defined as a gaseous substance close to the temperature at which it will condense back into a liquid. In this state it cannot be considered as a perfect gas and great care should be taken applying specific heats to them. We should use tables and charts to determine the properties of vapours and this is covered in the next section.

WORKED EXAMPLE No.7

Calculate the change in enthalpy and internal energy when 3 kg of gas is heated from 20°C to 200°C. The specific heat at constant pressure is 1.2 kJ/kg K and at constant volume is 0.8 kJ/kg K. Also determine the change in flow energy.

SOLUTION

i. Change in enthalpy.

$\Delta H = mc_p\Delta T = 3 \times 1.2 \times 180 = 648$ kJ

ii. Change in internal energy.

$\Delta H = mc_v\Delta T = 3 \times 0.8 \times 180 = 432$ kJ

iii. Change in flow energy

$\Delta FE = \Delta H - \Delta = 216$ kJ

WORKED EXAMPLE No.8

A vertical cylinder contains 2 dm^3 of air at 50^oC. One end of the cylinder is closed and the other end has a frictionless piston which may move under the action of weights placed on it. The weight of the piston and load is 300 N. The cylinder has a cross sectional area of 0.015 m^2. The outside is at atmospheric conditions.

Determine

i. the gas pressure.

ii. the gas mass.

iii. the distance moved by the piston when the gas is heated to 150^oC.

For air take c_p= 1005 J/kg K and c_v= 718 J/kg K. Atmospheric pressure = 100 kPa

SOLUTION

The pressure of the gas is constant and always just sufficient to support the piston so
p = Weight/Area + atmospheric pressure

p = 300/0.015 + 100 kPa = 20 kPa + 100 kPa = 120 kPa

$T_1 = 50 + 273 = 323$ K

$V_1 = 0.002\ m^3$

$R = c_p - c_v = 1005 - 718 = 213$ J/kg K

m = pV/RT = 120 000 x 0.002/(213 x 323) = 0.00348 kg

$T_2 = 150 + 273 = 423$ K

$V_2 = p_1V_1T_2/p_2T_1$ but $p_1 = p_2$

$V_2 = V_1T_2/T_1$ = 0.02 x 423/323 = 0.0262 m^3

Distance moved = Volume change/Area = (0.0262 - 0.02)/0.015 = 0.412 m

WORKED EXAMPLE No.9

Convert the principal specific heats and characteristic gas constant for dry air into molar form.

SOLUTION

The normal values for dry air are found in the back of your fluids tables and are:
c_p= 1.005 kJ/kg K c_v= 0.718 kJ/kg K   R = 0.287 kJ/kg K

In order to convert these into molar form we must multiply them by the molar mass. The molar mass for dry air is a mean value for a gas mixture and is found on the back page of your fluids tables and is 28.96 kg/kmol.

In molar form
c_p= 1.005 [kJ/kg K] x 28.96 [kg/kmol] = 29.1 kJ/kmol K

c_v= 0.718 [kJ/kg K] x 28.96 [kg/kmol] = 20.8 kJ/kmol K

R = 0.287 [kJ/kg K] x 28.96 [kg/kmol] = 8.31 kJ/kmol K

Note that the last value is the universal gas constant R_o.

SELF ASSESSMENT EXERCISE No. 3

For air take c_p= 1005 J/kg K and c_v= 718 J/kg K unless otherwise stated.

1. 0.2 kg of air is heated at constant volume from 40°C to 120°C. Calculate the heat transfer and change in internal energy. (11.49 kJ for both)

2. 0.5 kg of air is cooled from 200°C to 80°C at a constant pressure of 5 bar. Calculate the change in internal energy, the change in enthalpy, the heat transfer and change in flow energy. (-43 kJ), (-60.3 kJ), (-17.3 kJ)

3. 32 kg/s of water is heated from 15°C to 80°C.
Calculate the heat transfer given c = 4186J/kg K.
(8.7 MW)

4. Air is heated from 20°C to 50°C at constant pressure. Using your fluid tables (pages 16 and 17) determine the average value of c_p and calculate the heat transfer per kg of air. (30.15 kJ)

5. The diagram shows a cylinder fitted with a frictionless piston. The air inside is heated to 200°C at constant pressure causing the piston to rise. Atmospheric pressure outside is 100 kPa. Determine :

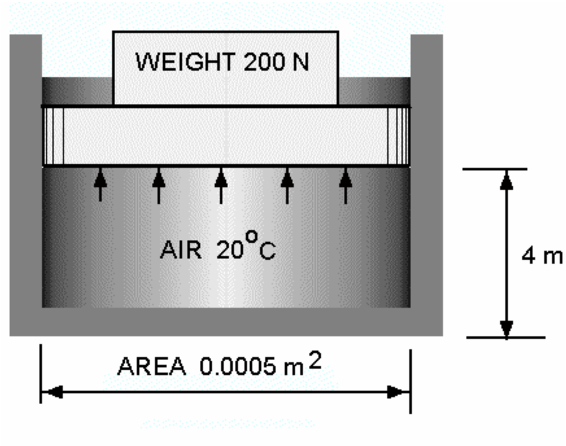
i. the mass of air. (11.9 g)

ii. the change in internal energy. (1.537 kJ)

iii. the change in enthalpy. (2.1553 kJ)

iv. the pressure throughout. (500 kPa)

v. the change in volume. (1.22 dm^3)



6. Define the meaning of a mole as a means of measuring the amount of a substance.

Calculate the volume occupied at a temperature of 25°C and a pressure of 3 bar, by 60 kg of (i) Oxygen gas, O_2, (ii)atomic oxygen gas,O, and (iii) Helium gas, H_e. The respective molar masses ,M, and the molar heats at constant volume, C_V, of the three gases, and the molar (universal) gas constant, R_M, are as follows:

	M (kg/kmol)	c_V(kJ/kmol K)	R_M(kJ/kmol K)
O_2	32	20.786	8.3144
O	16	12.4716	8.3144
H_e	4	12.4716	8.3144

Go on to calculate the values of the specific heats C_p and C_V. Using these values, calculate the specific gas constant R for all three gases. Show not only numerical work, but also the manipulation of units in arriving at your results.

You should now be able to determine the properties of gases. Next we will examine the properties of liquids and vapours.

6. PROPERTIES OF LIQUIDS AND VAPOURS

In this section you will do the following.

- Learn about the properties and definitions concerning vapours.
- Learn how to find the properties of vapours and liquids from your tables and charts.

You should ensure that you have a copy of 'Thermodynamic and Transport Properties of Fluids' by Mayhew and Rogers.

6.1 GENERAL THEORY

When a liquid changes into a vapour by the process of evaporation, it undergoes a change of state or phase. The reverse process is called liquefaction or condensing. The following work should lead you to an understanding of this process and by the end of it you should be able to find the same quantities and do the same type of problems as you have already done for gas.

When a liquid is heated, the temperature rises directly proportional to the heat transferred, 'Q'. Q is given by $Q = mc\Delta T$
The specific heat c is reasonably constant but changes significantly if the pressure or temperature change is very large.

When discussing heat transfer and energy of a fluid, we may wish to consider the internal energy U or the enthalpy H. In the following, the energy of the fluid may be construed as either. In tables this is tabulated as specific internal energy or enthalpy u and h.

When the liquid receives enough heat to bring it to the boiling point , the energy it contains is called SENSIBLE ENERGY. In tables this is denoted as u_f or h_f.

A liquid starts to evaporate because it becomes saturated with heat and can absorb no more without changing state (into a vapour and hence gas). For this reason the boiling point is more correctly described as the SATURATION TEMPERATURE and the liquid in this state is called SATURATED LIQUID. The saturation temperature is denoted as $\mathbf{t_s}$ in tables.

If a boiling liquid is supplied with more heat, it will evaporate and vapour is driven off. The vapour, still at the saturation temperature is called DRY SATURATED VAPOUR.

A vapour is a gas near to the temperature at which it will condense. In order to convert liquid into vapour, extra heat must be transferred into it. The amount of enthalpy and internal energy required to evaporate 1 kg is denoted $\mathbf{h_{fg}}$ and $\mathbf{u_{fg}}$ in tables and this is called the LATENT ENTHALPY and LATENT INTERNAL ENERGY respectively.

The energy contained in 1 kg of dry saturated vapour must be the sum of the sensible and latent energy and this is denoted h_g and u_g. It follows that :

$$\mathbf{h_g = h_f + h_{fg}}$$
$$\mathbf{u_g = u_f + u_{fg}}$$

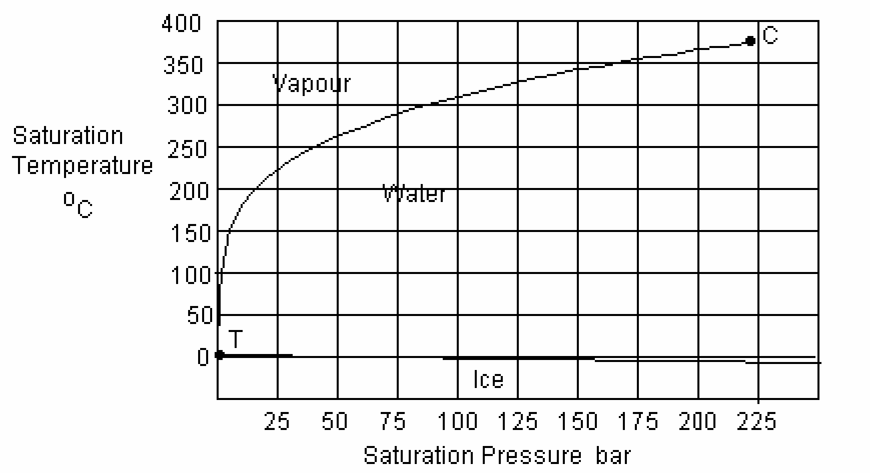
TABLE FOR THE ENTHALPY OF HIGH PRESSURE WATER

Temp ^oC	Pressure in bar										
	0	25	50	75	100	125	150	175	200	221	250
0	0	2.5	5	7.5	10	12.6	15	17.5	20	22	25
20	84	86	87	91	93	96	98	100	103	105	107
40	168	170	172	174	176	179	181	183	185	187	190
60	251	253	255	257	259	262	264	266	268	270	272
80	335	337	339	341	343	345	347	349	351	352	355
100	419	421	423	425	427	428	430	432	434	436	439
120	504	505	507	509	511	512	514	516	518	519	521
140	589	591	592	594	595	597	599	600	602	603	605
160	675	677	678	680	681	683	684	686	687	688	690
180	763	764	765	767	767	769	790	772	773	774	776
200	852	853	854	855	856	857	858	859	861	862	863

All enthalpy values are given in kJ/kg

The temperature at which evaporation occurs 't_s' depends upon the pressure at which it takes place. For example we all know that water boils at 100^{o}C at atmospheric pressure (1.013 bar). At pressure below this, the boiling point is less. At higher pressures the boiling point is higher. If we look up the values of t_s and p for water in the tables and plot them we get the graph below. It should also be noted that if the temperature of a liquid is kept constant, it may be made to boil by changing the pressure. The pressure at which it boils is called the SATURATION PRESSURE and is denoted as $\mathbf{p_s}$ in the tables.

The graph below also shows the freezing point of water plotted against pressure (pressure has little effect on it).

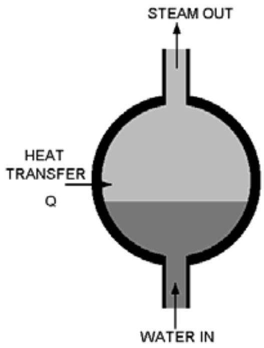


The two graphs cross at 0.01^{o}C and 0.006112 bar. This point is called the TRIPLE POINT. The graph shows the three phases of ice, water and steam. At the triple point, all three can occur together. Below the triple point, ice can change into steam without a liquid stage (and vice versa). All substances have a triple point.

If you did the exercise of plotting the graph of t_s against p for water/steam you would find that the tables stop at 221.2 bar and 374.15^{o}C. Above this pressure and temperature, the phenomenon of evaporation does not occur and no latent energy stage exists. This point is called the CRITICAL POINT and every substance has one.

If vapour is heated, it becomes hotter than the boiling point and the more it is heated, the more it becomes a gas. Such vapour is referred to as SUPERHEATED VAPOUR, except when it is a substance at pressures and temperatures above the critical point when it is called SUPERCRITICAL VAPOUR.

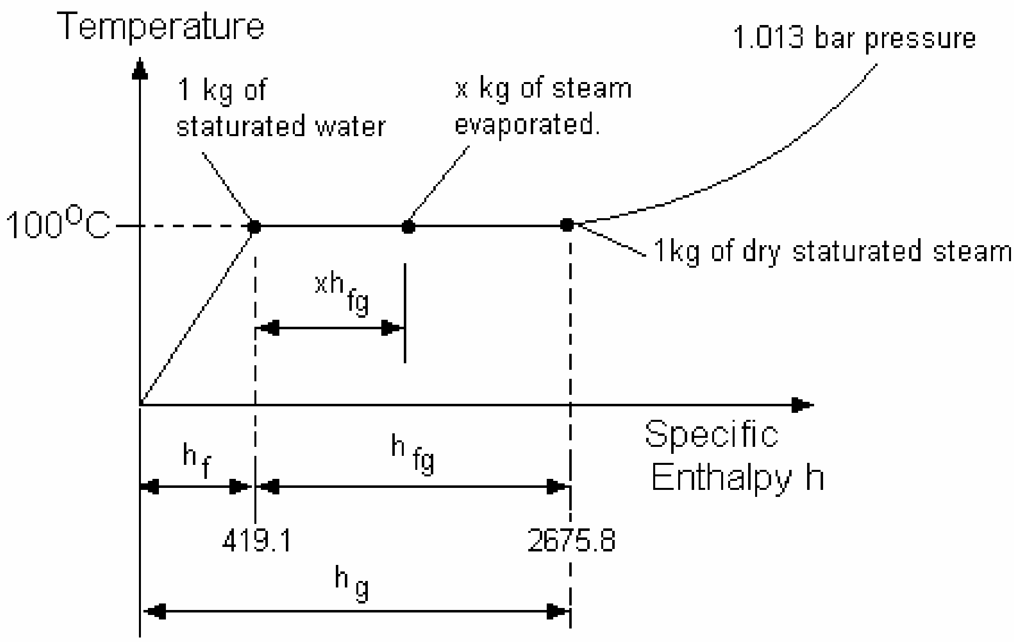
6.2 CONTINUOUS EVAPORATION



A simple boiler or evaporator as shown is needed to continuously produce vapour from liquid. The liquid is pumped in at the same rate at which the vapour is driven off. The heat transfer rate needed to do this must supply the internal energy to the process and the flow energy. In other words, the heat transfer is equal to the increase in the enthalpy from liquid to vapour. This is why enthalpy is such an important property.

6.3 WET VAPOUR

Wet vapour is a mixture of dry saturated vapour and liquid droplets. It may also be thought of as a partially evaporated substance. In order to understand its properties, consider the evaporation of 1 kg of water illustrated with a temperature - enthalpy graph. Starting with water at atmospheric pressure and 0.01^{o}C, the enthalpy is arbitrarily taken as zero. Keeping the pressure constant and raising the temperature, the enthalpy of the water rises to 419.1 kJ/kg at 100^{o}C. At this point it is saturated water and the sensible enthalpy is h_f=419.1 kJ/kg. The addition of further heat will cause the water to evaporate. During evaporation, the temperature remains at 100^{o}C. When the latent enthalpy h_{fg} (2256.7 kJ/kg) has been added, the substance is dry saturated vapour and its specific enthalpy h_g is 2675.8 kJ/kg. Further addition of heat will cause the temperature to rise and the substance becomes superheated vapour.



This graph may be drawn for any pressure and the same basic shape will be obtained but of course with different values. At the critical pressure it will be found that h_{fg} is zero.

The point of interest is the enthalpy value at some point along the evaporation line. Any point on this line represents wet steam. Suppose only fraction x kg has been evaporated. The latent enthalpy added will only be xh_{fg} and not h_{fg}. The enthalpy of the water/steam mixture is then **$h = h_f + xh_{fg}$**

The fraction x is called the DRYNESS FRACTION but it is rarely given as a fraction but rather as a decimal. If no evaporation has started, then x = 0. If all the liquid is evaporated then x = 1. x cannot be larger than 1 as this would mean the vapour is superheated.

The same logic applies to internal energy and it follows that **$u = u_f + xu_{fg}$**

6.4 VOLUMES

The specific volume of saturated water is denoted v_f. The specific volume of dry saturated steam is denoted v_g. The change in volume from water to steam is v_{fg}. It follows that the specific volume of wet steam is $v = v_f + xv_{fg}$

Since the value of v_f is very small and the specific volume of dry steam is very large (in all but the extreme cases), then v_{fg} is practically the same as v_g and v_f is negligible. The specific volume of steam is then usually calculated from the formula

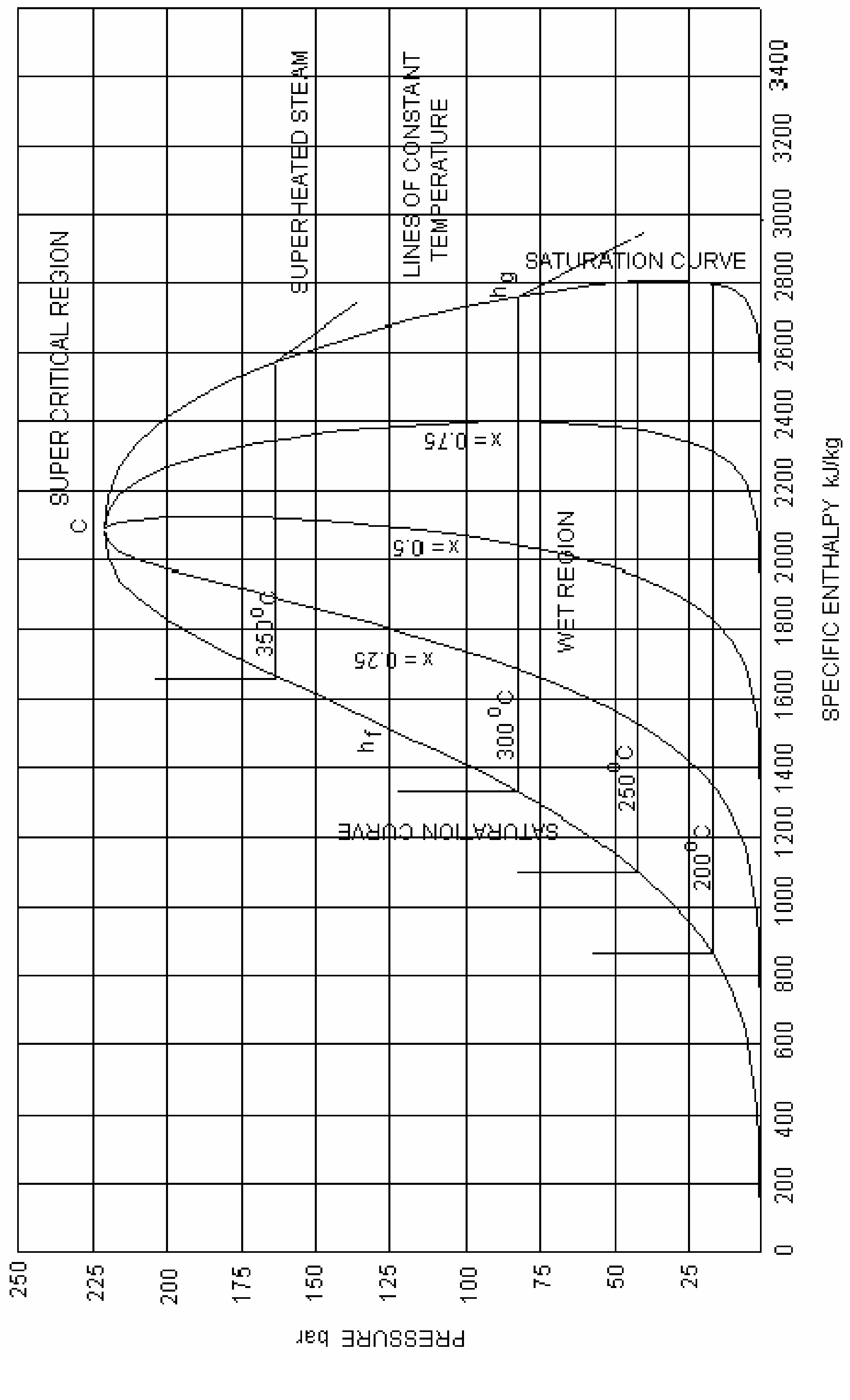
$v = xv_g$

6.5 TOTAL VALUES

All the formula above represent the values for 1 kg (specific values). When the mass is m kg, the values are simply multiplied by m . For example the volume of m kg of wet steam becomes **$V = mxv_g$**

6.6 SATURATION CURVE

If we plot a graph of h_f and h_g against either temperature or pressure, we get a property chart. The graph itself is the SATURATION CURVE. Taking the p-h graph as an example, temperatures and dryness fractions may be drawn on it and with the resulting graph, the enthalpy of water, wet, dry or superheated steam may be found. The pressure - enthalpy chart is popular for refrigerants but not for steam. A p-h chart is enclosed for arcton 12.


SUPER CRITICAL REGION
SUPERHEATED STEAM
LINES OF CONSTANT TEMPERATURE
SATURATION CURVE
C
h_g
h_f
x = 0.75
x = 0.5
x = 0.25
350 °C
300 °C
250 °C
200 °C
WET REGION
SATURATION CURVE
SPECIFIC ENTHALPY kJ/kg
PRESSURE bar
0 200 400 600 800 1000 1200 1400 1600 1800 2000 2200 2400 2600 2800 3000 3200 3400
25 50 75 100 125 150 175 200 225 250

6.7 USE OF TABLES

It is vitally important for you to be able to use the fluid tables in order to find the properties of steam. The tables are supplied in the exam but you must have a copy and become completely proficient in their use. Regarding water/steam, the tables contain a section on saturated water/steam and a section on superheated/supercritical steam.

The saturated water/steam tables are laid out as follows with an example of values. Check this out for yourself on page 4.

p	t_s	v_g	u_f	u_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
10	179.9	0.1944	762	2584	763	2015	2778			

Don't worry about the columns headed s at this stage. This is the property called entropy which is dealt with later.

The latent internal energy u_{fg} is not listed because of lack of room. However you do need to remember that it is the difference between u_f and u_g. Note that in all cases the value of h_{fg} is the difference between the values on either side of it.

The superheat tables are laid out differently. In this case the property value depends upon the pressure and temperature since the steam can exist at any pressure and temperature above the saturation values. This by necessity makes the tables very concise. More detailed tables are published. Interpolation is required to find values between those tabulated.

In the superheat tables (e.g. page 6), you must locate the temperature along the top and the pressure down the side. This results in set of values at these co-ordinates giving v, u, h and s.

WORKED EXAMPLE No.10

Find the specific enthalpy, internal energy and volume of steam at 3 bar and 200°C .

SOLUTION

On page 6 of your tables locate the column with 200°C at the top and come down the page to the row with 3 bar at the left side. At this point you have a block of 4 figures. The enthalpy value is the third figure down and is 2866 kJ/kg. The second figure down is the internal energy and is 2651 kJ/kg. The first figure is the volume and is 0.7166 m^3/kg. You don't need the fourth figure at this stage.

p/bar	t	50	100	150	200	250
3 (133.5)					0.7166.......volume 2651...........int. energy 2866...........enthalpy 7.312..........entropy	

WORKED EXAMPLE No.11

Find the enthalpy, internal energy and volume of 3 kg of steam at 11 bar and dryness 0.75.

SOLUTION

From page 4 of the steam tables determine the row corresponding to 11 bar and look up the following values.

h_f = 781 kJ/kg
h_{fg}=2000 kJ/kg
h_g=2781 kJ/kg

u_f=780 kJ/kg
u_g=2586 kJ/kg

v_g=0.1774 m^3/kg

Next deduce u_{fg} = 2586 - 780 = 1806 kJ/kg

Now find the enthalpy.
$H = m(h_f + xh_{fg}) = 3(781 + 0.75 \times 2000) = 6843$ kJ

Next find the internal energy in the same way.
$U = m(u_f + xu_{fg}) = 3(780 + 0.75 \times 1806) = 6403.5$ kJ

Finally the volume.
$V = mxv_g = 3 \times 0.75 \times 0.1774 = 0.399\ m^3$

SELF ASSESSMENT EXERCISE NO. 4

1. Using your steam tables, plot a graph of h_f and h_g against pressure horizontally and mark on the graph the following:

    i. the superheat region
    ii. the wet steam region.
    iii. the liquid region.
    iv. the critical point.

    Also label the saturation curve with dry saturated steam and saturated water.

2. Using your steam tables, plot a graph of v_g horizontally against pressure vertically. Also plot v_f

    Show on the graph:

    i. the superheated steam region.
    ii. the wet vapour region.
    iii. the liquid region.
    iv. the critical point.

    Also label the saturation curve with dry saturated steam and saturated water.

SELF ASSESSMENT EXERCISE No. 5

Use tables and charts to do the following.

1. What is the saturation temperature at 32 bars ?

2. What is the specific enthalpy and internal energy of saturated water at 16 bars?

3. What is the specific enthalpy and internal energy of dry saturated steam at 16 bars?

4. Subtract the enthalpy in 2 from that in 3 and check that it is the latent enthalpy h_{fg} at 16 bars in the tables.

5. What is the specific enthalpy and internal energy of superheated steam at 10 bar and 400°C ?

6. What is the specific volume of dry saturated steam at 20 bars ?

7. What is the volume of 1 kg of wet steam at 20 bars with dryness fraction x=0.7?

8. What is the specific enthalpy and internal energy of wet steam at 20 bars with a dryness fraction of 0.7 ?

9. What is the specific volume of superheated steam at 15 bars and 500°C.

10. What is the volume and enthalpy of 3 kg of wet steam at 5 bar with dryness fraction 0.9.

11. Using the p-h chart for arcton 12 (freon 12) determine
a. the specific enthalpy at 2 bar and 70% dry. (x = 0.7).
b. the specific enthalpy at 5 bar and 330 K
c. the specific enthalpy of the liquid at 8 bars and 300 K.

12. What is the enthalpy of 1.5 kg of superheated steam at 8 bar and 350°C ?

13. What is the internal energy of 2.2 kg of dry saturated steam at 11 bars ?

14. What is the volume of 0.5 kg of superheated steam at 15 bar and 400°C ?

Answers to Assignment 5.

Compare your answers with those below. If you find your answers are different, go back and try again referring to the appropriate section of your notes.

1. 237.4^oC.
2. 859 and 857 kJ/kg.
3. 2794 and 2596kJ/kg.
3. 1935 kJ/kg and 1739 kJ/kg.
5. 3264 and 2957 kJ/kg.
6. 0.09957 m^3/kg.
7. 0.0697 m^3/kg.
8. 2232 and 2092.1 kJ/kg.
9. 0.2351 m^3

10. 1.012 m^3, 7.61 MJ, 7.11 MJ.
11 a. 190 kJ/kg. b.286 kJ/kg. c. 130 kJ/kg
12. 4.74 MJ.
13. 5.69 MJ.
14. 0.101m^3.

EDEXCEL HIGHERS
ENGINEERING THERMODYNAMICS H2
NQF LEVEL 4

OUTCOME 1

TUTORIAL No. 2 – THERMODYNAMIC SYSTEMS

Thermodynamic systems

Polytropic processes: general equation pv^n=c, relationships between index `n' and heat transfer during a process; constant pressure and reversible isothermal and adiabatic processes; expressions for work flow

Thermodynamic systems and their properties: closed systems; open systems; application of first law to derive system energy equations; properties; intensive; extensive; two-property rule

Relationships: $R = c_p - c_v$. and $\gamma = c_p/c_v$

When you have completed this tutorial you should be able to do the following.

- Explain and use the First Law of Thermodynamics.
- Solve problems involving various kinds of thermodynamic systems.
- Explain and use polytropic expansion and compression processes.

1. ENERGY TRANSFER

There are two ways to transfer energy in and out of a system, by means of work and by means of heat. Both may be regarded as a quantity of energy transferred in Joules or energy transfer per second in Watts.

When you complete section one you should be able to explain and calculate the following.

- Heat transfer.
- Heat transfer rate.
- Work transfer
- Work transfer rate (Power)

1.1. HEAT TRANSFER

Heat transfer occurs because one place is hotter than another. Under normal circumstances, heat will only flow from a hot body to a cold body by virtue of the temperature difference. There are 3 mechanisms for this, *Conduction, convection and radiation.* You do not need to study the laws governing conduction, convection and radiation in this module.

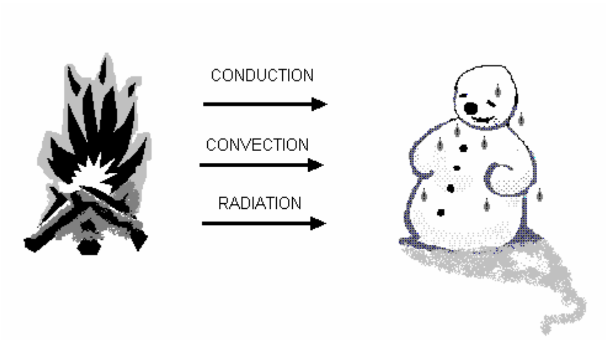


Fig.1

A quantity of energy transferred as heat is given the symbol Q and it's basic unit is the Joule. The quantity transferred in one second is the heat transfer rate and this has the symbol Φ and the unit is the Watt.

An example of this is when heat passes from the furnace of a steam boiler through the walls separating the combustion chamber from the water and steam. In this case, conduction, convection and radiation all occur together.

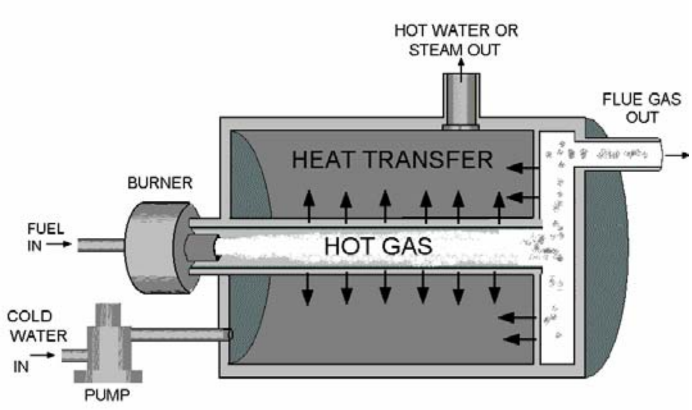


Fig.2

SELF ASSESSMENT EXERCISE No.1

1. 1 kg/s of steam flows in a pipe 40 mm bore at 200 bar pressure and 400°C.

i. Look up the specific volume of the steam and determine the mean velocity in the pipe.
(7.91 m/s)

ii. Determine the kinetic energy being transported per second.
(31.3 W)

iii. Determine the enthalpy being transported per second.
(2819 W)

1.2. WORK TRANSFER

Energy may be transported from one place to another mechanically. An example of this is when the output shaft of a car engine transfers energy to the wheels. A quantity of energy transferred as work is 'W' Joules but the work transferred in one second is the Power 'P' Watts.

An example of power transfer is the shaft of a steam turbine used to transfer energy from the steam to the generator in an electric power station.

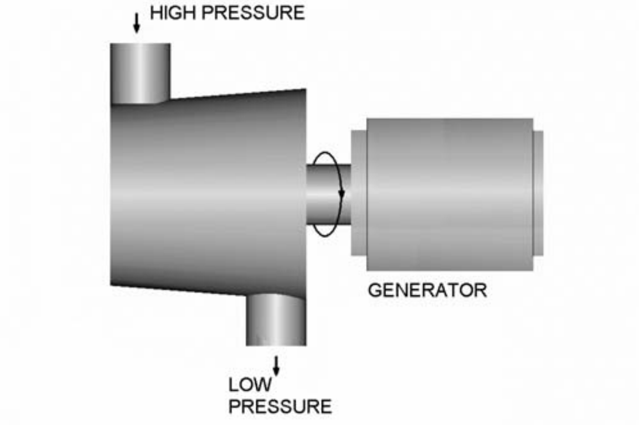


Fig.3

It is useful to remember that the power transmitted by a shaft depends upon the torque and angular velocity.

The formulae used are $P = \omega T$ or $P = 2\pi NT$

ω is the angular velocity in radian per second and N is the angular velocity in revolutions per second.

WORKED EXAMPLE No. 1

A duct has a cross section of 0.2 m x 0.4 m. Steam flows through it at a rate of 3 kg/s with a pressure of 2 bar. The steam has a dryness fraction of 0.98. Calculate all the individual forms of energy being transported.

SOLUTION

Cross sectional area = 0.2 x 0.4 = 0.08 m^2.

Volume flow rate = mxv_g at 2 bar

Volume flow rate = 3 x 0.98 x 0.8856 = 2.6 m^3/s.

velocity = c = Volume/area = 2.6/0.08 = 32.5 m/s.

Kinetic Energy being transported = $mc^2/2$ = 3 x 32.5^2 /2 = 1 584 Watts.

Enthalpy being transported = $m(h_f + x\ h_{fg})$

H= 3(505 + 0.98 x 2202) = 7988.9kW

Flow energy being transported = pressure x volume

Flow Energy = $2x10^5$ x 2.6 = 520 000 Watts

Internal energy being transported = $m(u_f + x\ u_{fg})$

U = 3(505 + 0.98 x 2025)=7468.5 kW

Check flow energy = H - U = 7988.9 - 7468.5 = 520 kW

SELF ASSESSMENT EXERCISE No.2

1. The shaft of a steam turbine produces 600 Nm torque at 50 rev/s. Calculate the work transfer rate from the steam.
   (188.5 W)

2. A car engine produces 30 kW of power at 3000 rev/min. Calculate the torque produced.
   (95.5 Nm)

2. THE FIRST LAW OF THERMODYNAMICS

When you have completed section two, you should be able to explain and use the following terms.

- The First Law of Thermodynamics.
- Closed systems.
- The Non-Flow Energy Equation.
- Open systems.
- The Steady Flow Energy Equation.

2.1 THERMODYNAMIC SYSTEMS

In order to do energy calculations, we identify our system and draw a boundary around it to separate it from the surroundings. We can then keep account of all the energy crossing the boundary. The first law simply states that

The nett energy transfer = nett energy change of the system.

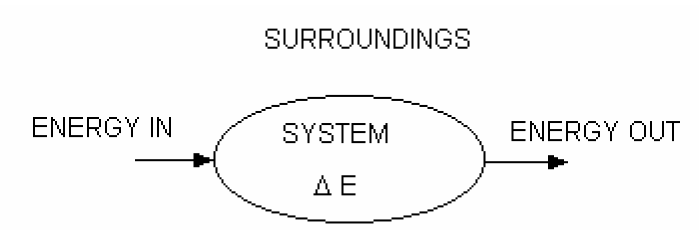


Fig. 4

Energy transfer into the system = E(in)

Energy transfer out of system = E (out)

Nett change of energy inside system = E(in) - E (out) = ΔE

This is the fundamental form of the first law.

Thermodynamic systems might contain only static fluid in which case they are called **NON-FLOW or CLOSED SYSTEMS.**

Alternatively, there may be a steady flow of fluid through the system in which case it is known as a **STEADY FLOW or OPEN SYSTEM.**

The energy equation is fundamentally different for each because most energy forms only apply to a fluid in motion. We will look at non-flow systems first.

2.2 NON-FLOW SYSTEMS

The rules governing non-flow systems are as follows.

- The volume of the boundary may change.
- No fluid crosses the boundary.
- Energy may be transferred across the boundary.

When the volume enlarges, work (-W) is transferred from the system to the surroundings. When the volume shrinks, work (+W) is transferred from the surroundings into the system. Energy may also be transferred into the system as heat (+Q) or out of the system (-Q). This is best shown with the example of a piston sliding inside a cylinder filled with a fluid such as gas.

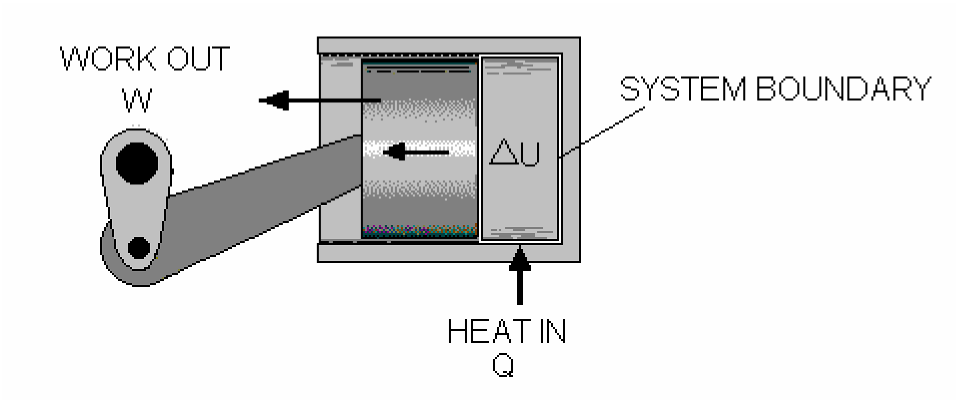


Fig.5

The only energy possessed by the fluid is internal energy (U) so the net change is Δ U. The energy equation becomes

$$\mathbf{Q + W = \Delta U}$$

This is known as the **NON-FLOW ENERGY EQUATION (N.F.E.E.)**

2.3 STEADY FLOW SYSTEMS

The laws governing this type of system are as follows.

- Fluid enters and leaves through the boundary at a steady rate.
- Energy may be transferred into or out of the system.

A good example of this system is a steam turbine. Energy may be transferred out as a rate of heat transfer Φ or as a rate of work transfer P.

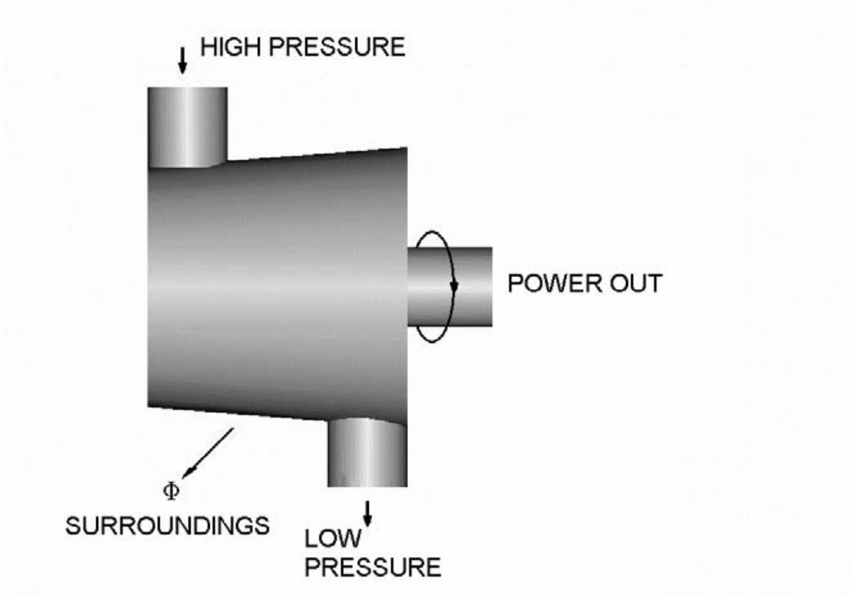


Fig.6.

The fluid entering and leaving has potential energy (PE), kinetic energy (KE) and enthalpy (H).

The first law becomes Φ + P = Nett change in energy of the fluid.

$$\mathbf{\Phi + P = \Delta(PE)/s + \Delta(KE)/s + \Delta(H)/s}$$

This is called the **STEADY FLOW ENERGY EQUATION (S.F.E.E.)**

Again, we will use the convention of positive for energy transferred into the system.

Note that the term Δ means '*change of*' and if the inlet is denoted point (1) and the outlet point (2). The change is the difference between the values at (2) and (1). For example ΔH means (H_2-H_1).

WORKED EXAMPLE No.3

A steam turbine is supplied with 30 kg/s of superheated steam at 80 bar and 400°C with negligible velocity. The turbine shaft produces 200 kNm of torque at 3000 rev/min. There is a heat loss of 1.2 MW from the casing. Determine the thermal power remaining in the exhaust steam.

SOLUTION

Shaft Power = $2\pi NT = 2\pi(3000/60) \times 200\,000 = 62.831 \times 10^6$ W = 62.831 MW

Thermal power supplied = H at 80 bar and 400°C

H = 30(3139) = 94170 kW = 94.17 MW

Total energy flow into turbine = 94.17 MW

Energy flow out of turbine = 94.17 MW = SP + Loss + Exhaust.

Thermal Power in exhaust = 94.17 -1.2 - 62.831 = **30.14 MW**

SELF ASSESSMENT EXERCISE No.3

1. A non-flow system receives 80 kJ of heat transfer and loses 20 kJ as work transfer. What is the change in the internal energy of the fluid?
(60 kJ)
2. A non-flow system receives 100 kJ of heat transfer and also 40 kJ of work is transferred to it. What is the change in the internal energy of the fluid?
(140 kJ)
3. A steady flow system receives 500 kW of heat and loses 200 kW of work. What is the net change in the energy of the fluid flowing through it?
(300 kW)
4. A steady flow system loses 2 kW of heat also loses 4 kW of work. What is the net change in the energy of the fluid flowing through it?
(-6 kW)
5. A steady flow system loses 3 kW of heat also loses 20 kW of work. The fluid flows through the system at a steady rate of 70 kg/s. The velocity at inlet is 20 m/s and at outlet it is 10 m/s. The inlet is 20 m above the outlet. Calculate the following.

   i. The change in K.E./s (-10.5 kW)
   ii. The change in P.E/s (-13.7 kW)
   iii. The change in enthalpy/s (1.23 kW)

3. MORE EXAMPLES OF THERMODYNAMIC SYSTEMS

When we examine a thermodynamic system, we must first decide whether it is a non-flow or a steady flow system. First, we will look at examples of non-flow systems.

3.1 PISTON IN A CYLINDER

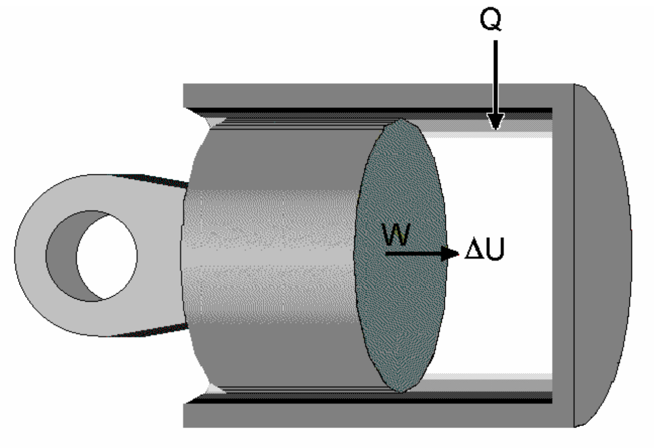


Fig. 7

There may be heat and work transfer. The N.F.E.E. is, $Q + W = \Delta U$

Sometimes there is no heat transfer (e.g. when the cylinder is insulated).

$Q = 0$ so $W = \Delta U$

If the piston does not move, the volume is fixed and no work transfer occurs. In this case

$$Q = \Delta U$$

For a GAS ONLY the change in internal energy is $\Delta U = mC_V\Delta T$.

3.2. SEALED EVAPORATOR OR CONDENSER.

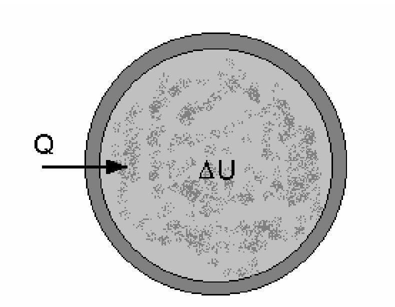


Fig. 8

Since no change in volume occurs, there is no work transfer so **$Q = \Delta U$**

WORKED EXAMPLE No.4

30 g of gas inside a cylinder fitted with a piston has a temperature of 15^oC. The piston is moved with a mean force of 200 N so that that it moves 60 mm and compresses the gas. The temperature rises to 21^oC as a result.

Calculate the heat transfer given c_V = 718 J/kg K.

SOLUTION

This is a non flow system so the law applying is $Q + W = \Delta U$
The change in internal energy is $\Delta U = mc_V \Delta T = 0.03 \times 718 \times (21 - 15)$
$\Delta U = 129.24$ J

The work is transferred into the system because the volume shrinks.

W = force x distance moved = 200 x 0.06 = 12 J

$Q = \Delta U - W = 117.24$ J

Now we will look at examples of steady flow systems.

3.3. PUMPS AND FLUID MOTORS

The diagram shows graphical symbols for hydraulic pumps and motors.

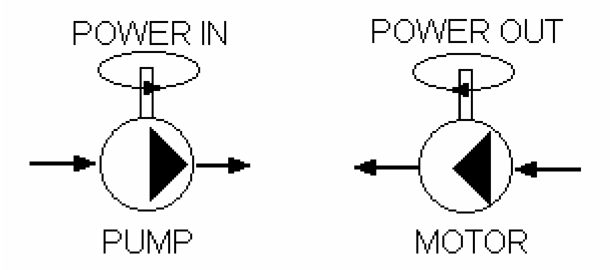


Fig.9

The S.F.E.E. states,

$$\Phi + P = \Delta KE/s + \Delta PE/s + \Delta H/s$$

In this case, especially if the fluid is a liquid, the velocity is the same at inlet and outlet and the kinetic energy is ignored. If the inlet and outlet are at the same height, the PE is also neglected. Heat transfer does not usually occur in pumps and motors so Φ is zero.

The S.F.E.E. simplifies to $\mathbf{P = \Delta H/s}$

Remember that enthalpy is the sum of internal energy and flow energy. The enthalpy of gases, vapours and liquids may be found. In the case of liquids, the change of internal energy is small and so the change in enthalpy is equal to the change in flow energy only.

The equation simplifies further to $P = \Delta FE/s$

Since FE = pV and V is constant for a liquid, this becomes $\mathbf{P = V\Delta p}$

WORKED EXAMPLE No.5

A pump delivers 20 kg/s of oil of density 780 kg/m^3 from atmospheric pressure at inlet to 800 kPa gauge pressure at outlet. The inlet and outlet pipes are the same size and at the same level. Calculate the theoretical power input.

SOLUTION

Since the pipes are the same size, the velocities are equal and the change in kinetic energy is zero. Since they are at the same level, the change in potential energy is also zero. Neglect heat transfer and internal energy.

$P = V\,\Delta p$
$V = m/\rho = 20/780 = 0.0256$ m^3/s
$\Delta p = 800 - 0 = 800$ kPa

P = 0.0256 x 800000 = 20 480 W or 20.48 kW

WORKED EXAMPLE No.6

A feed pump on a power station pumps 20 kg/s of water. At inlet the water is at 1 bar and 120°C. At outlet it is at 200 bar and 140°C. Assuming that there is no heat transfer and that PE and KE are negligible, calculate the theoretical power input.

In this case the internal energy has increased due to frictional heating.

The SFEE reduces to $P = \Delta H/s = m(h_2 - h_1)$

The h values may be found from tables.

$h_1 = 504$ kJ/kg

This is near enough the value of h_f at 120°C bar in steam tables.

$h_2 = 602$ kJ/kg

P = 20 (602 - 504) = 1969 kW or 1.969 MW

If water tables are not to hand the problem may be solved as follows.
$\Delta h = \Delta u + \Delta f.e.$

$\Delta u = c\,\Delta T$ where c = 4.18 kJ/kg K for water

$\Delta u = 4.18\,(140 - 120) = 83.6$ kJ/kg

$\Delta f.e. = V\Delta p$

The volume of water is normally around 0.001 m^3/kg.

$\Delta f.e. = 0.001 \times (200 - 1) \times 10^5 = 19\,900$ J/kg or 19.9 kJ/kg

hence $\Delta h = \Delta u + \Delta fe = 83.6 + 19.9 = 103.5$ kJ/kg

P = mΔh = 20 x 103.5 = 2070 kW or 2.07 MW

The discrepancies between the answers are slight and due to the fact the value of the specific heat and of the specific volume are not accurate at 200 bar.

3.4. GAS COMPRESSORS AND TURBINES.

Figure 10 shows the basic construction of an axial flow compressor and turbine. These have rows of aerofoil blades on the rotor and in the casing. The turbine passes high pressure hot gas or steam from left to right making the rotor rotate. The compressor draws in gas and compresses it in stages.

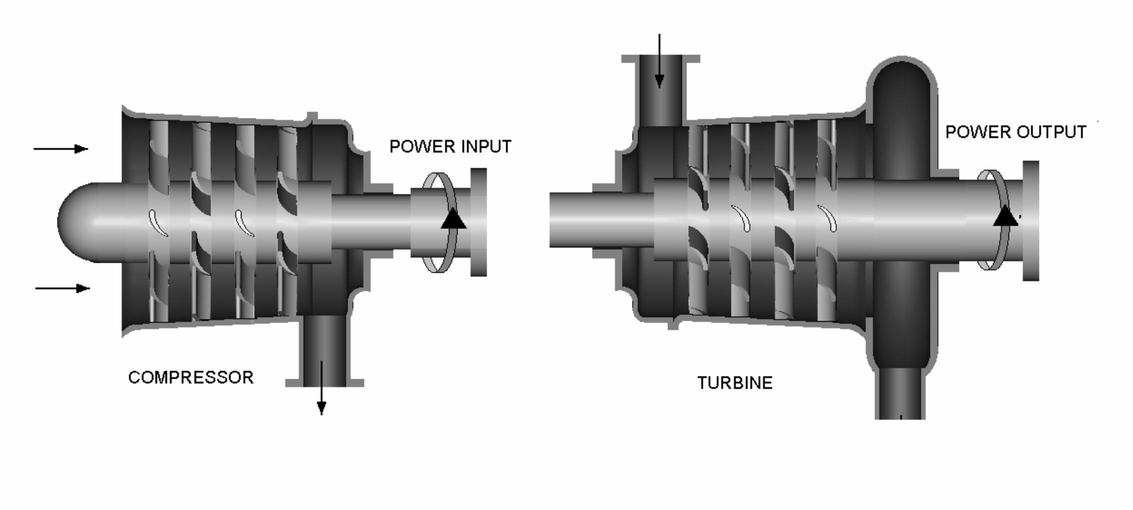


Fig. 10

Compressing a gas normally makes it hotter but expanding it makes it colder. This is because gas is compressible and unlike the cases for liquids already covered, the volumes change dramatically with pressure. This might cause a change in velocity and hence kinetic energy. Often both kinetic and potential energy are negligible. The internal energy change is not negligible. Figure 11 shows graphical symbols for turbines and compressors. Note the narrow end is always the high pressure end.

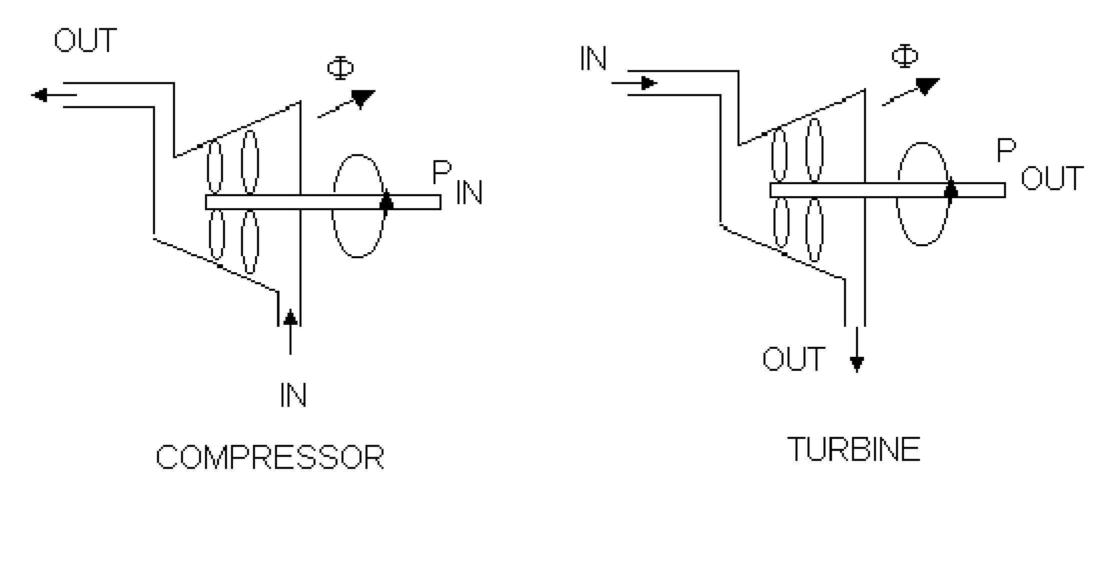


Fig.11

WORKED EXAMPLE No.7

A gas turbine uses 5 kg/s of hot air. It takes it in at 6 bar and 900^oC and exhausts it at 450^oC. The turbine loses 20 kW of heat from the casing. Calculate the theoretical power output given that c_p = 1005 J/kg K.

First identify this as a steady flow system for which the equation is

$\Phi + P = \Delta K.E./s + \Delta P.E./s + \Delta H/s$

For lack of further information we assume K.E. and PE to be negligible. The heat transfer rate is -20 kW.

The enthalpy change for a gas is $\Delta H = mC_p\Delta T$

$\Delta H = 5 \times 1005 \times (450 - 900) = -2261000$ W or -2.261 MW

$P = \Delta H - \Phi = -2261 - (-20) = -2241$ kW

The minus sign indicates that the power is leaving the turbine. Note that if this was a steam turbine, you would look up the h values in the steam tables.

3.5 STEADY FLOW EVAPORATORS AND CONDENSERS

A refrigerator is a good example of a thermodynamic system. In particular, it has a heat exchanger inside that absorbs heat at a cold temperature and evaporates the liquid into a gas. The gas is compressed and becomes hot. The gas is then cooled and condensed on the outside in another heat exchanger.

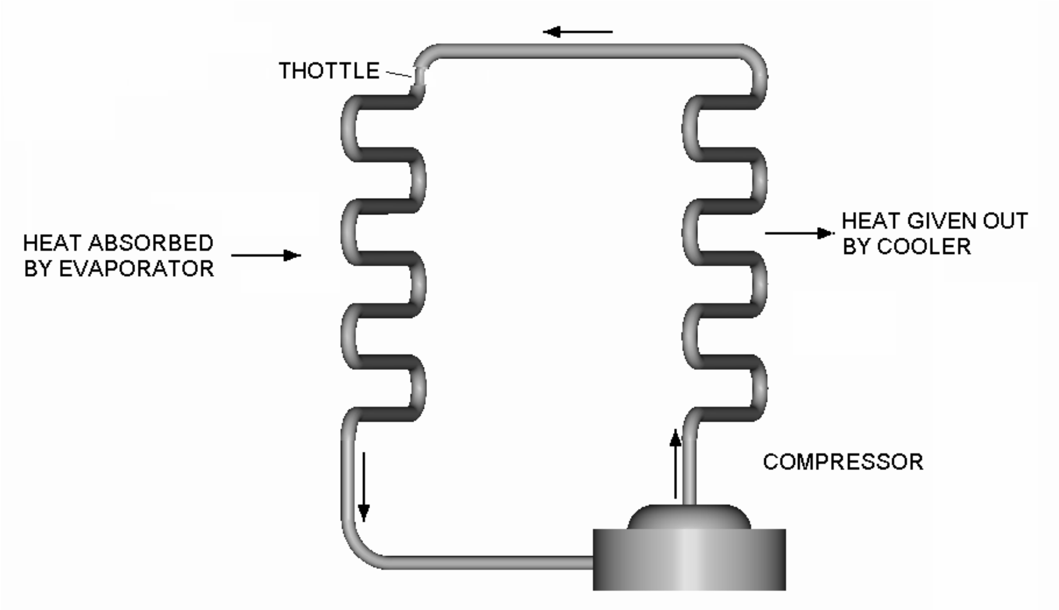


Fig. 2.12

For both the evaporator and condenser, there is no work transferred in or out. K.E. and P.E. are not normally a feature of such systems so the S.F.E.E. reduces to

$$\Phi = \Delta H/s$$

On steam power plant, boilers are used to raise steam and these are examples of large evaporators working at high pressures and temperatures. Steam condensers are also found on power stations. The energy equation is the same, whatever the application.

WORKED EXAMPLE No.8

A steam condenser takes in wet steam at 8 kg/s and dryness fraction 0.82. This is condensed into saturated water at outlet. The working pressure is 0.05 bar. Calculate the heat transfer rate.

SOLUTION

$\Phi = \Delta H/s = m(h_2 - h_1)$

$h_1 = h_f + x\,h_{fg}$ at 0.05 bar

from the steam tables we find that

$h_1 = 138 + 0.82(2423) = 2125$ kJ/kg

$h_2 = h_f$ at 0.05 bar = 138 kJ/kg

hence **$\Phi = 8(138 - 2125) = -15896$ kW**

The negative sign indicates heat transferred from the system to the surroundings.

SELF ASSESSMENT EXERCISE No.4

1. Gas is contained inside a cylinder fitted with a piston. The gas is at 20°C and has a mass of 20 g. The gas is compressed with a mean force of 80 N which moves the piston 50 mm. At the same time 5 Joules of heat transfer occurs out of the gas. Calculate the following.

    i. The work done.(4 J)

    ii. The change in internal energy. (-1 J)

    iii. The final temperature. (19.9°C)

    Take c_V as 718 J/kg K

2. A steady flow air compressor draws in air at 20°C and compresses it to 120°C at outlet. The mass flow rate is 0.7 kg/s. At the same time, 5 kW of heat is transferred into the system. Calculate the following.

    i. The change in enthalpy per second. (70.35 kW)

    ii. The work transfer rate. (65.35 kW)

    Take c_p as 1005 J/kg K.

3. A steady flow boiler is supplied with water at 15 kg/s, 100 bar pressure and 200°C. The water is heated and turned into steam. This leaves at 15 kg/s, 100 bar and 500°C. Using your steam tables, find the following.

    i. The specific enthalpy of the water entering. (856 kJ/kg)

    ii. The specific enthalpy of the steam leaving. (3373 kJ/kg)

    iii. The heat transfer rate. (37.75 kW)

4. A pump delivers 50 dm^3/min of water from an inlet pressure of 100 kPa to an outlet pressure of 3 MPa. There is no measurable rise in temperature. Ignoring K.E. and P.E, calculate the work transfer rate. (2.42 kW)

5. A water pump delivers 130 dm^3/minute (0.13 m^3/min) drawing it in at 100 kPa and delivering it at 500 kPa. Assuming that only flow energy changes occur, calculate the power supplied to the pump. (860 W)

6. A steam condenser is supplied with 2 kg/s of steam at 0.07 bar and dryness fraction 0.9. The steam is condensed into saturated water at outlet. Determine the following.

   i. The specific enthalpies at inlet and outlet. (2331 kJ/kg and 163 kJ/kg)

   ii. The heat transfer rate. (4336 kW)

7. 0.2 kg/s of gas is heated at constant pressure in a steady flow system from 10^oC to 180^oC. Calculate the heat transfer rate Φ. (37.4 kW)

   C_p = 1.1 kJ/kg K

8. 0.3 kg of gas is cooled from 120^oC to 50^oC at constant volume in a closed system. Calculate the heat transfer. (-16.8 kJ)

   C_V = 0.8 kJ/kg.

4. POLYTROPIC PROCESSES.

When you complete section four you should be able to do the following.

- Use the laws governing the expansion and compression of a fluid.
- State the names of standard processes.
- Derive and use the work laws for closed system expansions and compressions.
- Solve problems involving gas and vapour processes in closed systems.

We will start by examining expansion and compression processes.

4.1 COMPRESSION AND EXPANSION PROCESSES.

A compressible fluid (gas or vapour) may be compressed by reducing its volume or expanded by increasing its volume. This may be done inside a cylinder by moving a piston or by allowing the pressure to change as it flows through a system such as a turbine. For ease of understanding, let us consider the change as occurring inside a cylinder. The process is best explained with a pressure - volume graph.

When the volume changes, the pressure and temperature may also change. The resulting pressure depends upon the final temperature. The final temperature depends on whether the fluid is cooled or heated during the process. It is normal to show these changes on a graph of pressure plotted against volume. (p-V graphs). A typical graph for a compression and an expansion process is shown in fig.13.

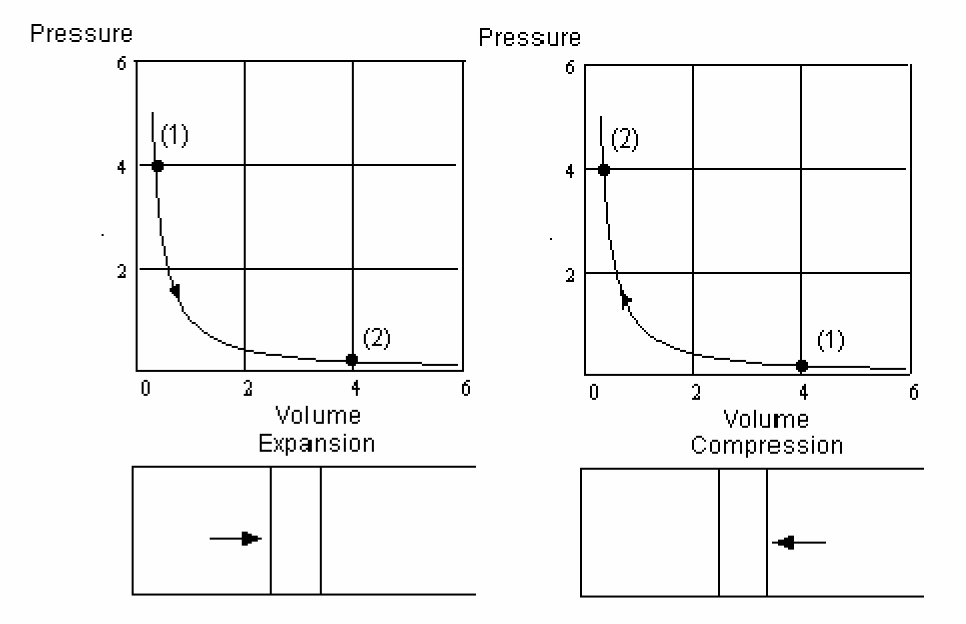


Fig. 13

It has been discovered that the resulting curves follows the mathematical law

$$pV^n = \text{constant.}$$

Depending on whether the fluid is heated or cooled, a family of such curves is obtained as shown (fig.14). Each graph has a different value of n and n is called the index of expansion or compression.

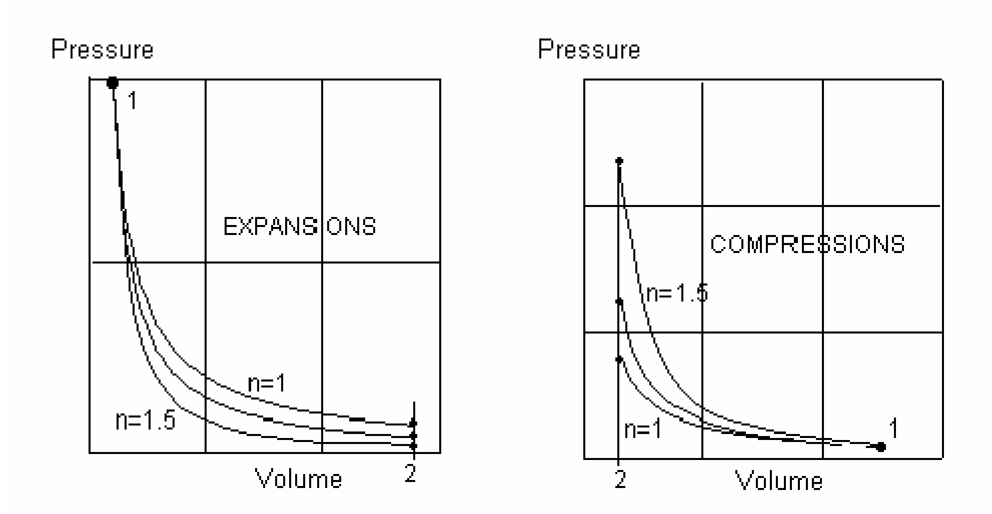


Fig.14

The most common processes are as follows.

CONSTANT VOLUME also known as ISOCHORIC

A vertical graph is a constant volume process and so it is not a compression nor expansion. Since no movement of the piston occurs no work transfer has taken place. Nevertheless, it still fits the law with n having a value of infinity.

CONSTANT PRESSURE also known as ISOBARIC

A horizontal graph represents a change in volume with no pressure change (constant pressure process). The value of n is zero in this case.

CONSTANT TEMPERATURE also known as ISOTHERMAL

All the graphs in between constant volume and constant pressure, represent processes with a value of n between infinity and zero. One of these represents the case when the temperature is maintained constant by cooling or heating by just the right amount.

When the fluid is a gas, the law coincides with Boyle's Law pV= constant so it follows that n is 1.

When the fluid is a vapour, the gas law is not accurate and the value of n is close to but not equal to 1.

ADIABATIC PROCESS

When the pressure and volume change in such a way that no heat is added nor lost from the fluid (e.g. by using an insulated cylinder), the process is called adiabatic. This is an important process and is the one that occurs when the change takes place so rapidly that there is no time for heat transfer to occur. This process represents a demarcation between those in which heat flows into the fluid and those in which heat flows out of the fluid. In order to show it is special, the symbol γ is used instead of n and the law is

$$pV^{\gamma} = C$$

It will be found that each gas has a special value for γ (e.g. 1.4 for dry air).

POLYTROPIC PROCESS

All the other curves represent changes with some degree of heat transfer either into or out of the fluid. These are generally known as polytropic processes.

HYPERBOLIC PROCESS

The process with n=1 is a hyperbola so it is called a hyperbolic process. This is also isothermal for gas but not for vapour. It is usually used in the context of a steam expansion.

WORKED EXAMPLE No.9

A gas is compressed from 1 bar and 100 cm^3 to 20 cm^3 by the law $pV^{1.3}$=constant. Calculate the final pressure.

SOLUTION.

If $pV^{1.3} = C$ then $p_1V_1^{1.3} = C = p_2V_2^{1.3}$

hence $1 \times 100^{1.3} = p_2 \times 20^{1.3}$

$1 \times (100/20)^{1.3} = p_2 = 8.1$ bar

WORKED EXAMPLE No.10

Vapour at 10 bar and 30 cm^3 is expanded to 1 bar by the law $pV^{1.2} = C$. Find the final volume.

SOLUTION.

$p_1V_1^{1.2} = C = p_2V_2^{1.2}$

$10 \times 30^{1.2} = 1 \times V_2^{1.2}$ $\qquad V_2 = (592.3)^{1/1.2} = 204.4 \text{ cm}^3$

WORKED EXAMPLE No.11

A gas is compressed from 200 kPa and 120 cm^3 to 30 cm^3 and the resulting pressure is 1 MPa. Calculate the index of compression n.

SOLUTION.

$200 \times 120^n = 1000 \times 30^n$

$(120/30)^n = 1000/200 = 5$

$4^n = 5$

$n\log 4 = \log 5$

$n = \log 5/\log 4 = 1.6094/1.3863 = 1.161$

Note this may be solved with natural or base 10 logs or directly on suitable calculators.

SELF ASSESSMENT EXERCISE No. 5

1. A vapour is expanded from 12 bar and 50 cm^3 to 150 cm^3 and the resulting pressure is 6 bar. Calculate the index of compression n.
(0.63)

2.a. A gas is compressed from 200 kPa and 300 cm^3 to 800 kPa by the law $pV^{1.4}=C$. Calculate the new volume. (111.4 cm^3)

2.b. The gas was at 50°C before compression. Calculate the new temperature using the gas law pV/T = C. (207°C)

3.a. A gas is expanded from 2 MPa and 50 cm^3 to 150 cm^3 by the law $pV^{1.25} = C$. Calculate the new pressure. (506 kPa)

3.b. The temperature was 500°C before expansion. Calculate the final temperature. (314°C)

4.2. COMBINING THE GAS LAW WITH THE POLYTROPIC LAW.

For gases only, the general law may be combined with the law of expansion as follows. $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$ and so $\frac{T_2}{T_1} = \frac{p_2V_2}{p_1V_1}$

Since for an expansion or compression

$$p_1V_1^n = p_2V_2^n$$

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^n$$

Substituting into the gas law we get $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$

and further since $\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = \frac{V_2}{V_1}$

substituting into the gas law gives $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}$

To summarise we have found that $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}$

In the case of an adiabatic process this is written as

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}}$$

For an isothermal process n = 1 and the temperatures are the same.

WORKED EXAMPLE No.12

A gas is compressed adiabatically with a volume compression ratio of 10. The initial temperature is 25^{o}C. Calculate the final temperature given γ = 1.4

SOLUTION

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 298(10)^{1.4-1} = 748.5K \text{ or } 475.5^oC$$

WORKED EXAMPLE No.13

A gas is compressed polytropically by the law $pV^{1.2} = C$ from 1 bar and 20°C to 12 bar. calculate the final temperature.

SOLUTION

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}} \qquad T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}} = 293(12)^{1-\frac{1}{1.2}}$$

$$T_2 = 293(12)^{0.167} = 293(1.513) = 443.3K$$

WORKED EXAMPLE No.14

A gas is expanded from 900 kPa and 1100°C to 100 kPa by the law $pV^{1.3} = C$. Calculate the final temperature.

SOLUTION

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}} \qquad T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}$$

$$T_2 = 1373\left(\frac{100}{900}\right)^{1-\frac{1}{1.3}} = 1373(0.111)^{0.2308} = 1373(0.602) = 826.9K$$

SELF ASSESSMENT EXERCISE No. 6

1. A gas is expanded from 1 MPa and 1000^oC to 100 kPa. Calculate the final temperature when the process is

   i. Isothermal (n=1) (1000^{o}C)

   ii Polytropic (n=1.2) (594^{o}C)

   iii. Adiabatic (γ =1.4) (386^{o}C)

   iv. Polytropic (n= 1.6) (264^{o}C)

2. A gas is compressed from 120 kPa and 15^{o}C to 800 kPa. Calculate the final temperature when the process is

   i. Isothermal (n=1) (15^{o}C)

   ii. Polytropic (n=1.3) (173^{o}C)

   iii Adiabatic (γ=1.4) (222^{o}C)

   iv. Polytropic (n= 1.5) (269^{o}C)

3. A gas is compressed from 200 kPa and 20^{o}C to 1.1 MPa by the law $pV^{1.3}$=C. The mass is 0.02 kg. c_p=1005 J/kg K. c_v = 718 J/kg K. Calculate the following.

   i. The final temperature. (434 K)

   ii. The change in internal energy (2.03 kJ)

   iii. The change in enthalpy (2.84 kJ)

4. A gas is expanded from 900 kPa and 1200^{o}C to 120 kPa by the law $pV^{1.4}$ = C. The mass is 0.015 kg. c_p=1100 J/kg K c_v = 750 J/kg K Calculate the following.

   i. The final temperature. (828 K)

   ii. The change in internal energy (-7.25 kJ)

   iii. The change in enthalpy (-10.72 kJ)

4.3. EXAMPLES INVOLVING VAPOUR

Problems involving vapour make use of the formulae $pV^n = C$ in the same way as those involving gas. You cannot apply gas laws, however, unless it is superheated into the gas region. You must make use of vapour tables so a good understanding of this is essential. This is best explained with worked examples.

WORKED EXAMPLE No.15

A steam turbine expands steam from 20 bar and 300°C to 1 bar by the law $pV^{1.2} = C$.

Determine for each kg flowing:

a. the initial and final volume.
b. the dryness fraction after expansion.
c. the initial and final enthalpies.
d. the change in enthalpy.

SOLUTION

The system is a steady flow system in which expansion takes place as the fluid flows. The law of expansion applies in just the same way as in a closed system.

The initial volume is found from steam tables. At 20 bar and 300°C it is superheated and from the tables we find $v = 0.1255\ m^3/kg$

Next apply the law $pV^{1.2} = C$ $\quad p_1V_1^{1.2} = p_2V_2^{1.2}$ $\quad 20 \times 0.1255^{1.2} = 1 \times V_2^{1.2}$

Hence $\quad V_2 = 1.523 m^3/kg$

Next, find the dryness fraction as follows.
Final volume = $1.523\ m^3/kg = xv_g$ at 1 bar.
From the tables we find v_g is $1.694\ m^3/kg$

hence $1.523 = 1.694x \qquad x = 0.899$

We may now find the enthalpies in the usual way.

h_1 at 20 bar and 300°C is 3025 kJ/kg
$h_2 = h_f + xh_{fg}$ at 1 bar (wet steam)
$h_2 = 417 + (0.899)(2258) = 2447$ kJ/kg

The change in enthalpy is $h_2 - h_1 = -578$ kJ/kg

SELF ASSESSMENT EXERCISE No.7

1. 3 kg/s of steam is expanded in a turbine from 10 bar and 200°C to 1.5 bar by the law $pV^{1.2}=C$. Determine the following.

   i. The initial and final volumes. (0.618 m^3 and 3 m^3)

   ii. The dryness fraction after expansion. (0.863)

   iii. The initial and final enthalpies. (2829 kJ/kg and 2388 kJ/kg)

   iv. The change in enthalpy. -1324 kW)

2. 1.5 kg/s of steam is expanded from 70 bar and 450°C to 0.05 bar by the law $pV^{1.3} = C$. Determine the following.

   i. The initial and final volumes. (0.066 m^3/kg and 17.4 m^3/kg)

   ii. The dryness fraction after expansion. (0.411)

   iii. The initial and final enthalpies. (3287 kJ/kg and 1135 kJ/kg)

   iv. The change in enthalpy. (-3228 kW)

3. A horizontal cylindrical vessel is divided into two sections each 1m^3 volume, by a non-conducting piston. One section contains steam of dryness fraction 0.3 at a pressure of 1 bar, while the other contains air at the same pressure and temperature as the steam. Heat is transferred to the steam very slowly until its pressure reaches 2 bar.

   Assume that the compression of the air is adiabatic (γ=1.4) and neglect the effect of friction between the piston and cylinder. Calculate the following.

   i. The final volume of the steam. (1.39 m^3)

   ii. The mass of the steam. (1.97 kg)

   iii. The initial internal energy of the steam. (2053 kJ)

   iv The final dryness fraction of the steam. (0.798)

   v. The final internal energy of the steam. (4172 kJ)

vi. The heat added to the steam. (2119 kJ)

4.4. CLOSED SYSTEM WORK LAWS

4.4.1. EXPANSION OF PRESSURE WITH VOLUME

We will start by studying the expansion of a fluid inside a cylinder against a piston which may do work against the surroundings.

A fluid may expand in two ways.

a) It may expand rapidly and uncontrollably doing no useful work. In such a case the pressure could not be plotted against volume during the process. This is called an **UNRESISTED EXPANSION**

b) It may expand moving the piston. The movement is resisted by external forces so the gas pressure changes in order to overcome the external force and move the piston. In this case the movement is controlled and the variation of pressure with volume may be recorded and plotted on a p-V graph. Work is done against the surroundings. This process is called a **RESISTED EXPANSION.**

Consider the arrangement shown in fig. 15. Assume that there is no pressure outside. If the string holding the weight was cut, the gas pressure would slam the piston back and the energy would be dissipated first by acceleration of the moving parts and eventually as friction. The expansion would be unresisted.

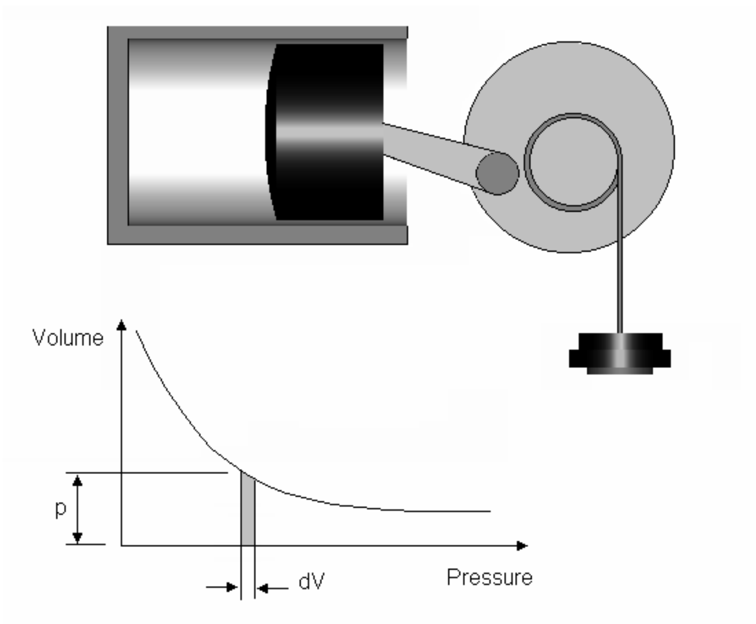


Fig. 15

If the weights were gradually reduced, the gas would push the piston and raise the remaining weights. In this way, work would be done against the surroundings (it ends up as potential energy in the weights). The process may be repeated in many small steps, with the change in volume each time being dV. The pressure although changing, is p at any time.

This process is characterised by two important factors.

1. The process may be reversed at any time by adding weights and the potential energy is transferred back from the surroundings as work is done on the system. The fluid may be returned to its original pressure, volume, temperature and energy.

2. The fluid force on one side of the piston is always exactly balanced by the external force (in this case due to the weights) on the other side of the piston.

The expansion or compression done in this manner is said to be **REVERSIBLE and CARRIED OUT IN EQUILIBRIUM.**

4.4.2. WORK AS AREA UNDER THE p - V DIAGRAM

If the expansion is carried out in equilibrium, the force of the fluid must be equal to the external force F. It follows that F = pA.

When the piston moves a small distance dx, the work done is dW

$$dW = - F\,dx =- pAdx = - pdV.$$

The minus sign is because the work is leaving the system.

For an expansion from points 1 to 2 it follows that the total work done is given by

$$W = -\int_{V_1}^{V_2} pdV$$

We must remember at this stage that our sign convention was that work leaving the system is negative.

It should be noted that some of the work is used to overcome any external pressure such as atmospheric and the useful work is reduced. Consider the system shown in fig.15 again but this time suppose there is atmospheric pressure on the outside p_a.

In this case It follows that

$F + p_aA = pA.$

$F = pA. - p_aA$

When the piston moves a small distance dx, the the useful work done is –F dx

$- F\,dx = - (pAdx - p_aAdx) = - (p - p_a)dV.$

For an expansion from points 1 to 2 it follows that the useful work done is given by

$$W = -\int_{V_1}^{V_2} (p - p_a) dV$$

4.4.3. WORK LAWS FOR CLOSED SYSTEMS

If we solve the expression $W = -\int_{V_1}^{V_2} pdV$ we obtain the work laws for a closed system.

The solution depends upon the relationship between p and V. The formulae now derived apply equally well to a compression process and an expansion process. Let us now solve these cases.

CONSTANT PRESSURE

$$W = -\int_{V_1}^{V_2} pdV$$

$$W = -p\int_{V_1}^{V_2} dV$$

$\mathbf{W = - p (V_2 - V_1)}$

CONSTANT VOLUME

If V is constant then dV = 0

W = 0.

HYPERBOLIC

This is an expansion which follows the law $pV^1 = C$ and is **ISOTHERMAL** when it is a gas. Substituting $p = CV^{-1}$ the expression becomes

$$W = -\int_{V_1}^{V_2} pdV = -C\int_{V_1}^{V_2} V^{-1}dV = -C\ln\left[\frac{V_2}{V_1}\right]$$

Since pV = C then

$$W = -pV\ln\left[\frac{V_2}{V_1}\right]$$

$$\text{since } \frac{V_2}{V_1} = \frac{p_1}{p_2}$$

$$W = -pV\ln\left[\frac{p_1}{p_2}\right]$$

In the case of gas we can substitute pV = mRT and so

$$W = -mRT \ln\left[\frac{V_2}{V_1}\right] = -mRT \ln\left[\frac{p_1}{p_2}\right]$$

POLYTROPIC

In this case the expansion follows the law $pV^n = C$. The solution is as follows.

$$W = -\int_{V_1}^{V_2} pdV \quad \text{but} \quad p = CV^{-n}$$

$$W = -C\int_{V_1}^{V_2} V^{-n} dV$$

$$W = -C\frac{\left[V_2^{-n+1} - V_1^{-n+1}\right]}{-n+1}$$

$$\text{Since } C = p_1V_1 \text{ or } p_2V_2$$

$$W = \frac{\left[p_2V_2 - p_1V_1\right]}{n-1}$$

For gas only we may substitute pV = mRT and so $W = mR\frac{\left[T_2 - T_1\right]}{n-1}$

ADIABATIC

Since an adiabatic case is the special case of a polytropic expansion with no heat transfer, the derivation is identical but the symbol γ is used instead of n.

$$W = \frac{\left[p_2V_2 - p_1V_1\right]}{\gamma - 1}$$

For gas only we may substitute pV = mRT and so $W = mR\frac{\left[T_2 - T_1\right]}{\gamma - 1}$

This is the special case of the polytropic process in which Q=0. $Q = 0 \quad W = \frac{mR\Delta T}{\gamma - 1}$

Substituting for Q and ΔU in the NFEE we find

$$Q + W = \Delta U \qquad 0 + \frac{mR\Delta T}{\gamma - 1} = mCv\Delta T \qquad \frac{R}{\gamma - 1} = Cv$$

$$\text{Since } R = Cp - Cv \qquad Cp - Cv = Cv(\gamma - 1) \qquad \frac{Cp}{Cv} = \gamma$$

This shows that the ratio of the principal specific heat capacities is the adiabatic index. It was shown earlier that the difference is the gas constant R. These important relationships should be remembered.

Cp – Cv = R

γ = Cp/Cv

WORKED EXAMPLE No.15

Air at a pressure of 500 kPa and volume 50 cm^3 is expanded reversibly in a closed system to 800 cm^3 by the law pV$^{1.3}$ = C. Calculate the following.

a. The final pressure.

b. The work done.

SOLUTION

p1 = 500 kPa V_1 = 50 x 10^{-6} m^3 V_2 = 800 x 10^{-6} m^3

$$p_1V_1^{1.3} = p_2V_2^{1.3} \qquad 500x10^3(50x10^{-6})^{1.3} = p_2(800x10^{-6})^{1.3}$$

$$p_2 = 13.6x10^3 \quad \text{or} \quad 13.6kPa$$

$$W = \frac{(p_2V_2 - p_1V_1)}{n-1} = \left(\frac{13.6x10^3x800x10^{-6} - 500x10^3x50x10^{-6}}{1.3-1}\right)$$

$$W = -47 Joules$$

WORKED EXAMPLE No.16

Steam at 6 bar pressure and volume 100 cm^3 is expanded reversibly in a closed system to 2 dm^3 by the law pV$^{1.2}$ = C. Calculate the work done.

SOLUTION

p1 = 6 bar V_1 = 100 x 10^{-6} m^3 V_2 = 2 x 10^{-3} m^3

$$p_2 = \frac{p_1V_1^{1.2}}{V_2^{1.2}} = 6x\left(\frac{100x10^{-6}}{2x10^{-3}}\right)^{1.2} = 0.1648bar$$

$$W = \frac{(p_2V_2 - p_1V_1)}{n-1} = \frac{(0.1648x10^5x2x10^{-3} - 6x10^5x100x10^{-6})}{1.2-1}$$

$$W = -135.2 Joules$$

SELF ASSESSMENT EXERCISE No.8

1. 10 g of steam at 10 bar and 350°C expands reversibly in a closed system to 2 bar by the law $pV^{1.3}$=C . Calculate the following.

   i. The initial volume. (0.00282 m^3)

   ii. The final volume. (0.00974 m^3)

   iii. The work done. (-2.92 kJ)

2. 20 g of gas at 20°C and 1 bar pressure is compressed to 9 bar by the law $pV^{1.4}$ = C. Taking the gas constant R = 287 J/kg K calculate the work done. (Note that for a compression process the work will turn out to be positive if you correctly identify the initial and final conditions). (3.67 kJ)

3. Gas at 600 kPa and 0.05 dm^3 is expanded reversibly to 100 kPa by the law $pV^{1.35}$ = C. Calculate the work done. (-31.8 kJ)

4. 15 g of gas is compressed isothermally from 100 kPa and 20°C to 1 MPa pressure. The gas constant is 287 J/kg K. Calculate the work done. (2.9 kJ)

5. Steam at 10 bar with a volume of 80 cm^3 is expanded reversibly to 1 bar by the law pV=C. Calculate the work done. (-184.2 kJ)

6. Gas fills a cylinder fitted with a frictionless piston. The initial pressure and volume are 40 MPa and 0.05 dm^3 respectively. The gas expands reversibly and polytropically to 0.5 MPa and 1 dm^3 respectively. Calculate the index of expansion and the work done. (1.463 and -3.24 kJ)

7. An air compressor commences compression when the cylinder contains 12 g at a pressure is 1.01 bar and the temperature is 20°C. The compression is completed when the pressure is 7 bar and the temperature 90°C. (1.124 and 1944 J)

   The characteristic gas constant R is 287 J/kg K. Assuming the process is reversible and polytropic, calculate the index of compression and the work done.

WORKED EXAMPLE No.17

0.2 kg of gas at 100 °C is expanded isothermally and reversibly from 1 MPa pressure to 100 kPa. Take C_V = 718 J/kg K and R = 287 J/kg K.
Calculate

i. The work transfer.
ii. The change in internal energy.
iii. The heat transfer.

SOLUTION

$$W = -pV\ln\left(\frac{V_2}{V_1}\right) = -mRT\ln\left(\frac{V_2}{V_1}\right) = -mRT\ln\left(\frac{p_1}{p_2}\right)$$

$$W = -0.2x287x373\ln\left(\frac{1x10^6}{1x10^5}\right) = -49300\text{ J or } -49.3\text{ kJ}$$

The work is leaving the system so it is a negative work transfer.

Since T is constant ΔU = 0   Q - 49.3 = 0   Q = 49.3 kJ

Note that 49.3 kJ of heat is transferred into the gas and 49.3 kJ of work is transferred out of the gas leaving the internal energy unchanged.

WORKED EXAMPLE No.18

Repeat worked example 17 but for an adiabatic process with γ = 1.4
Calculate

SOLUTION

$$T_2 = 373\text{ x}\left(\frac{100\text{ x }10^3}{1\text{ x }10^6}\right)^{1-\frac{1}{\gamma}} = 193\text{ K}$$

$$\text{W} = -\text{mRT}(\text{T}_2 - \text{T}_1) = -0.2\text{ x }287\text{ x }\frac{(193-373)}{0.4}$$

$$\text{W} = -25\,830\text{ J}$$

For an adiabatic process Q = 0

Q + W = ΔU hence ΔU = -25 830 J

Check $\Delta U = mC_V\Delta T = 0.2\text{ x }718\text{ x }(193-373) = -25\,848\text{ J}$

WORKED EXAMPLE No.19

Repeat worked example 17 but for a polytropic process with n=1.25
Calculate

SOLUTION

$$T_2 = 373 \text{ x} \left(\frac{100 \text{ x } 10^3}{1 \text{ x } 10^6}\right)^{1-\frac{1}{n}} = 235.3 \text{ K}$$

$$W = -mRT(T_2 - T_1) = -0.2 \text{ x } 287 \text{ x } \frac{(235.3 - 373)}{0.4}$$

$$W = -31605 \text{ J}$$

$$\Delta U = mC_V \Delta T = 0.2 \text{ x } 718 \text{ x } (235.3 - 373) = -19773.7 \text{ J}$$

$$Q = \Delta U - W$$

$$Q = -19773.7 - (-31603) = 11831.3 \text{ J}$$

SELF ASSESSMENT EXERCISE No.9

Take C_V = 718 J/kg K and R = 287 J/kg K throughout.

1. 1 dm^3 of gas at 100 kPa and 20°C is compressed to 1.2 MPa reversibly by the law $pV^{1.2} = C$. Calculate the following.

   i. The final volume. (0.126 dm^3)
   ii. The work transfer. (257 J)
   iii. The final temperature. (170oC)
   iv. The mass. (1.189 g)
   v. The change in internal energy. (128 J)
   vi. The heat transfer. (-128 J)

2. 0.05 kg of gas at 20 bar and 1100°C is expanded reversibly to 2 bar by the law $pV^{1.3}$ =C in a closed system. Calculate the following.

   i. The initial volume. (9.85 dm^3)
   ii. The final volume. (58 dm^3)
   iii. The work transfer. (-27 kJ)
   iv. The change in internal energy. (-20.3 kJ)
   v. The heat transfer. (6.7 kJ)

3. 0.08 kg of air at 700 kPa and 800°C is expanded adiabatically to 100 kPa in a closed system. Taking γ = 1.4 calculate the following.

   i. The final temperature. (615.4 K)
   ii. The work transfer. (26.3 kJ)
   iii. The change in internal energy. (-26.3 J)

4. A horizontal cylinder is fitted with a frictionless piston and its movement is restrained by a spring as shown (Figure 16.)

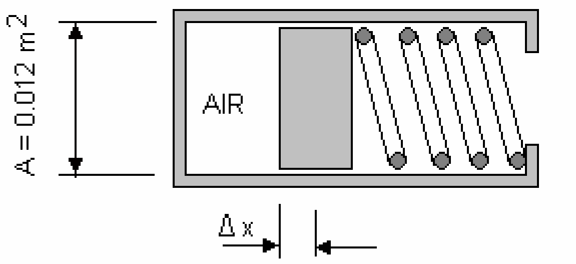


Figure 16

a. The spring force is directly proportional to movement such that $\Delta F/\Delta x = k$

Show that the change in pressure is directly proportional to the change in volume such that $\Delta p/\Delta V = k/A^2$

b. The air is initially at a pressure and temperature of 100 kPa and 300 K respectively. Calculate the initial volume such that when the air is heated, the pressure – volume graph is a straight line that extends to the origin. (0.5 ^{dm3})

c. The air is heated making the volume three times the original value. Calculate the following.

i. The mass. (0.58 g)
ii. The final pressure. (300 kPa)
iii. The final temperature. (2700 K)
iv. The work done. (-200 kJ)
v. The change in internal energy. (917 J)
vi. The heat transfer. (1.12 kJ)

4.5. CLOSED SYSTEM PROBLEMS INVOLVING VAPOUR

The solution of problems involving steam and other vapours is done in the same way as for gases with the important proviso that gas laws must not be used. Volumes and internal energy values should be obtained from tables and property charts. This is best illustrated with a worked example.

WORKED EXAMPLE No.20

1kg of steam occupies a volume of 0.2 m^3 at 9 bar in a closed system. The steam is heated at constant pressure until the volume is 0.3144 m^3. Calculate the following.

i. The initial dryness fraction.
ii. The final condition.
iii. The work transfer.
iv. The change in internal energy.
v. The heat transfer.

SOLUTION

First find the initial dryness fraction.

$V_1 = 0.2 = mx_1v_g$ at 9 bar $\quad x_1 = 0.2/(1 \times 0.2149)$

x_1= 0.931 (initial dryness fraction).

Now determine the specific volume after expansion.

$p_2 = 9$ bar (constant pressure) $\quad V_2 = 0.3144\ m^3$.

$V_2 = mv_2 \quad v_2 = 0.3144/1 = 0.3144 m^3/kg$

First, look in the superheat tables to see if this value exists for superheat steam. We find that at 9 bar and 350°C, the specific volume is indeed 0.3144 m^3/kg.

The final condition is superheated to 350°C .

Note that if v_2 was less than v_g at 9 bar the steam would be wet and x_2 would have to be found.

Next find the work.

$W = -p(V_2 - V_1) = -9 \times 10^5 (0.3144 - 0.2) = -102950$ J

W = -102.95 kJ (Energy leaving the system)

Next determine the internal energy from steam tables.

$U_1 = m\,u_1$ and $u_1 = u_f + x_1u_{fg}$ at 9 bar

u_{fg} at 9 bar = $u_g - u_f = 2581 - 742 = 1839$ kJ/kg

$U_1 = 1\{742 + 0.931(1839)\} = 2454$ kJ

$U_2 = m\,u_2$ and $u_2 = u$ at 9 bar and 350 ^oC = 2877 kJ/kg

$U_2 = m\,u_2 = 1(2877) = 2877$ kJ.

The change in internal energy = $U_2 - U_1$ = 423 kJ (increased)

Finally deduce the heat transfer from the NFEE

$Q + W = \Delta U$

hence $Q = \Delta U - W = 423 - (-102.95)$

Q = 526 kJ (energy entering the system)

SELF ASSESSMENT EXERCISE No.10

1. 0.2 kg of dry saturated steam at 10 bar pressure is expanded reversibly in a closed system to 1 bar by the law $pV^{1.2}$ =C. Calculate the following.

   i. The initial volume. (38.9 dm^3)

   ii. The final volume. (264 dm^3)

   iii. The work transfer. (-62 kJ)

   iv. The dryness fraction. (0.779)

   v. The change in internal energy. (-108 kJ)

   vi. The heat transfer. (-46 kJ)

2. Steam at 15 bar and 250°C is expanded reversibly in a closed system to 5 bar. At this pressure the steam is just dry saturated. For a mass of 1 kg calculate the following.

   i. The final volume. (0.375 m^3)
   ii. The change in internal energy. (-165 kJ)
   iii. The work done. (-187 kJ)
   iv. The heat transfer. (22.1 kJ)

EDEXCEL HIGHERS
ENGINEERING THERMODYNAMICS H2
NQF LEVEL 4

OUTCOME 2
INTERNAL COMBUSTION ENGINE PERFORMANCE

TUTORIAL No. 3 – HEAT ENGINE THEORY

Internal combustion engine performance

Second law of thermodynamics: statement of law; schematic representation of a heat engine to show heat and work flow

Heat engine cycles: Carnot cycle; Otto cycle; Diesel cycle; dual combustion cycle; Joule cycle; property diagrams; Carnot efficiency; air-standard efficiency

Performance characteristics: engine trials; indicated and brake mean effective pressure; indicated and brake power; indicated and brake thermal efficiency; mechanical efficiency; relative efficiency; specific fuel consumption; heat balance

Improvements: turbocharging; turbocharging and intercooling; cooling system and exhaust gas heat recovery systems

When you have completed this tutorial, you should be able to do the following.

- Explain the basic idea behind the Second Law of Thermodynamics.
- Define the property called Entropy
- Define an Isentropic process.
- Solve basic problems involving isentropic expansions.
- Explain the Carnot Principle.

1. THE SECOND LAW OF THERMODYNAMICS

The Second Law of Thermodynamics is not something that can be written as a simple statement or formulae. It is a set of observations concerning the way that things flow or run as time progresses forward. It encompasses many observations such as "water normally flows from high levels to low levels" and "heat normally flows from hot to cold". In this module, you must concern yourself only with how the second law relates to heat engines and the efficiency of a heat engine.

In the context of heat engines, the second law may be summed as :

"No heat engine can be 100% efficient".

This should become apparent in the following sections.

1.1 HEAT ENGINES

Nearly all motive power is derived from heat using some form of heat engine. Here are some examples.

- Steam Power Plant.
- Gas Turbines.
- Jet Engines.
- Internal Combustion Engines.

A heat engine requires a source of hot energy. We get this by burning fossil fuel or by nuclear fission. The main sources of natural heat are solar and geothermal. In order to understand the basic theory, it might help to draw an analogy with a hydraulic motor and an electric motor. All motors require a high level source of energy and must exhaust at a low level of energy.

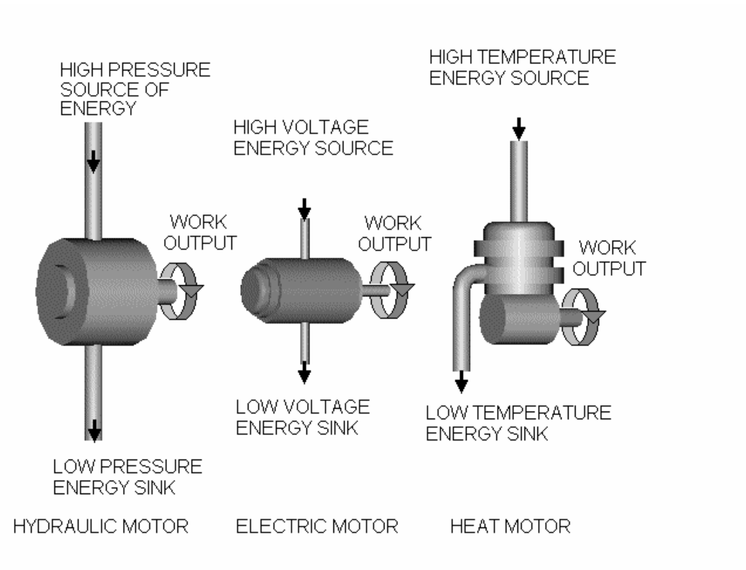


Figure 1

1.1.1 HYDRAULIC MOTOR

Fluid power is transported by the flow Q m^3/s. The energy contained in a volume Q m^3 of liquid at a pressure p is the flow energy given by the expression pQ. The hydraulic motor requires a source of liquid at a high pressure p_1 and exhausts at a lower pressure p_2.The energy supplied is p_1Q and some of this is converted into work. The energy in the low pressure liquid is p_2Q. For a perfect motor with no losses due to friction, the law of energy conservation gives the work output and efficiency as follows.

$$W_{out} = p_1Q - p_2Q = Q(p_1 - p_2)$$

$$\eta = \frac{W_{out}}{Energy\ input} = \frac{W_{out}}{p_1Q} = \frac{Q(p_1 - p_2)}{p_1Q} = \frac{(p_1 - p_2)}{p_1} = 1 - \frac{p_2}{p_1}$$

1.1.2 ELECTRIC MOTOR

Electric power is transported by the current. Electrical energy is the product of the charge Q Coulombs and the electric potential V Volts. The energy input at a high voltage is V_1Q and the energy exhausted at low voltage is V_2Q. For a perfect motor with no losses due to friction, the work output and efficiency are found from the law of energy conservation as follows.

$$W_{out} = V_1Q - V_2Q = Q(V_1 - V_2)$$

$$\eta = \frac{W_{out}}{Energy\ input} = \frac{W_{out}}{V_1Q} = \frac{Q(V_1 - V_2)}{V_1Q} = \frac{(V_1 - V_2)}{V_1} = 1 - \frac{V_2}{V_1}$$

1.1.3 HEAT MOTOR

Temperature is by analogy the equivalent of pressure and electric potential. In order to complete the analogy, we need something that is equivalent to volume and electric charge that transports the energy. It is not difficult to visualise a volume of liquid flowing through a hydraulic motor. It is not impossible to visualise a flow of electrons bearing electric charge through an electric motor. It is impossible to visualise something flowing through our ideal heat engine that transports pure heat but the analogy tells us there must be something so let us suppose a new property called ENTROPY and give it a symbol S. Entropy must have units of energy per degree of temperature or Joules per Kelvin. Entropy is dealt with more fully later on.

The energy supplied at temperature T_1 is T_1S and the energy exhausted is T_2S. For a perfect motor with no losses due to friction, the law of energy conservation gives the work output and efficiency as follows.

$$W_{out} = T_1S - T_2S = S(T_1 - T_2)$$

$$\eta = \frac{W_{out}}{Energy\ input} = \frac{W_{out}}{T_1S} = \frac{S(T_1 - T_2)}{T_1S} = \frac{(T_1 - T_2)}{T_1} = 1 - \frac{T_2}{T_1}$$

1.1.4 EFFICIENCY

In our perfect motors, the energy conversion process is 100% efficient but we may not have converted all the energy supplied into work and energy may be wasted in the exhaust. In the case of the electric motor, the lowest value for V_2 (so far as we know) is ground voltage zero, so theoretically we can obtain 100% efficiency by exhausting the electric charge with no residual energy.

In the case of the hydraulic motor, the lowest pressure we can exhaust to is atmospheric so we always waste some energy in the exhausted liquid.

In the case of the heat motor, the lowest temperature to which we can exhaust is ambient conditions, typically 300K, so there is a lot of residual energy in the exhaust. Only by exhausting to absolute zero, can we extract all the energy.

A model heat engine is usually represented by the following diagram. (Note that the word engine is usually preferred to motor).

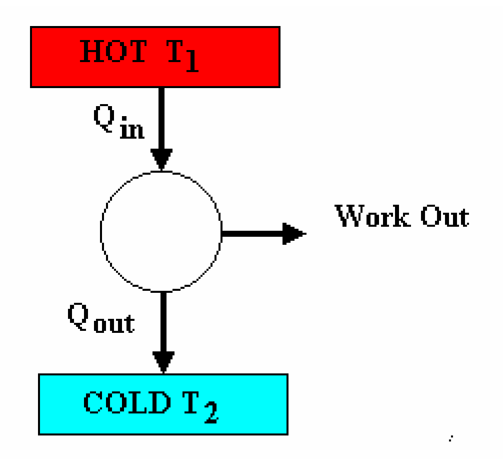


Fig. 2

The energy transfer from the hot source is Q_{in} Joules.
The energy transfer rate from the hot source is Φ_{in} Watts.

The energy transfer to the cold sink is Q_{out} Joules.
The energy transfer rate to the cold sink is Φ_{out} Watts.
The work output us W Joules.
The power output is P Watts.
By considering the total conservation of energy, it follows that the energy converted into work must be $W = Q_{in} - Q_{out}$ Joules or

$P = \Phi_{in} - \Phi_{out}$ Watts

The efficiency of any machine is the ratio Output/Input so the thermal efficiency of a heat engine may be developed as follows.

$$\eta_{th} = \frac{W}{Q_{in}} \qquad W = Q_{in} - Q_{out} \qquad \eta_{th} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{int}}$$

In terms of energy transfer rates in Watts this is written as

$$\eta_{th} = 1 - \frac{\Phi_{out}}{\Phi_{in}}$$

It follows from our analogy that $Q_{in} = ST_1$ and $Q_{out} = ST_2$ and confirms $\eta = 1 - \frac{T_2}{T_1}$

SELF ASSESSMENT EXERCISE No. 1

1. A heat engine is supplied with 60 MW of energy and produces 20 MW of power. What is the thermal efficiency and the heat lost?
(Answers 33.3% and 40 MW)

2. A heat engine is supplied with 40 kJ of energy that it converts into work with 25% efficiency. What is the work output and the heat lost?
(Answers 10 kJ and 30 kJ)

1.3. PRACTICAL HEAT ENGINE CONSIDERATIONS

Let us consider how we might design a practical heat engine with a piston, connecting rod and crank shaft mechanism. Figure 3 shows how heat may be passed to a gas inside a cylinder causing it to expand. This pushes a piston and makes it do some work. This at first looks like a good way of converting heat into work but the problem is that it works only once and cannot convert heat into work continuously.

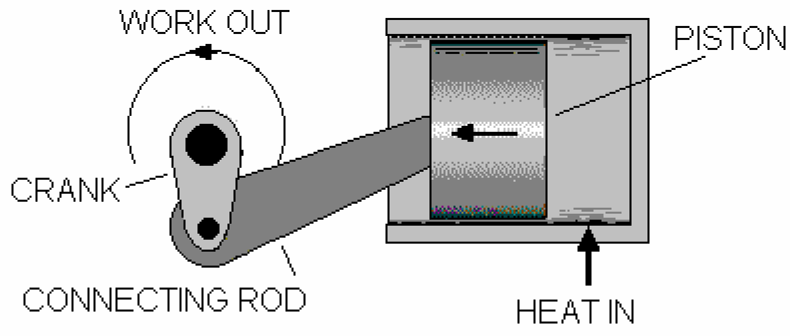


Figure 3

No practical heat engine has ever been invented that continuously converts heat directly into work as supposed in our ideal model. Practical heat engines use a working fluid such as gas or steam. A cycle of thermodynamic processes is conducted on the fluid with the end result being a conversion of heat into work.

First energy is given to the working fluid by use of a heat transfer at a hot temperature. Next we must convert as much of this energy as possible into work by allowing the fluid to expand. Our studies of polytropic expansions tell us that the pressure, volume and temperature all change as the gas or vapour gives up its energy as work. The pressure is vitally important to produce a motivating force on the piston.

Having extracted as much energy as possible from the working fluid, we must return it back to the starting condition in order to repeat the process. To do this, we must raise the pressure of the fluid back to the high level with some form of compression.

A simple reversal of the expansion process would return the fluid back to the original pressure and temperature. However, this would require us to give back all the work we got out so nothing is gained.

The only way we can return the fluid back to a high pressure with less work involves cooling it first. In fact, if it is to be heat engine, we must have a cooling process as indicated in our model.

We have deduced that a practical heat engine must meet the following criteria.

- It must produce work continuously.
- It must return the working fluid back to the same pressure and temperature at the beginning of every cycle.

A model of a practical engine is shown in Fig. 4. This indicates that we need four processes, heating, expansion, cooling and compression. This may be achieved practically using either closed system processes (as in a mechanism with a piston, connecting rod and crank shaft) or open system processes such as with a steam boiler, turbine, cooler and pump).

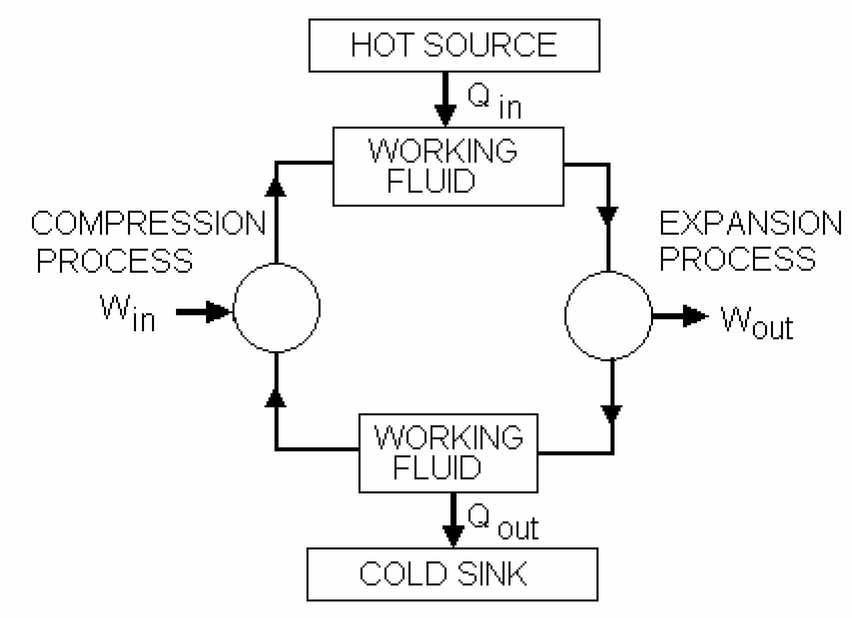


Figure 4

1.4 ENTROPY

We have just discovered that entropy is a property that governs the quantity of energy conveyed at a given temperature such that in our ideal heat engine, the energy is given by the expression Q = ST.

Entropy is a property that is closely associated with the second law of thermodynamics.

In thermodynamics there are two forms of energy transfer, work (W) and heat (Q). You should already be familiar with the theory of work laws in closed systems and know that the area under a pressure - volume diagram gives work transfer. By analogy there should be a property that can be plotted against temperature such that the area under the graph gives the heat transfer. This property is entropy and it is given the symbol S. This idea implies that entropy is a property that can be transported by a fluid. Consider a p-V and T-s graph for a reversible expansion (Fig. 5).

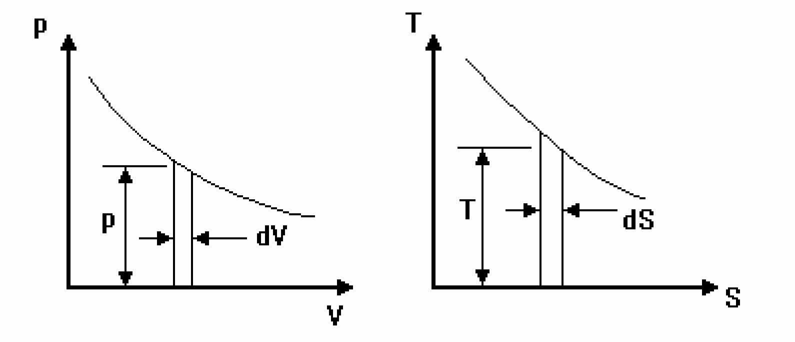


Fig.5

From the p-V graph we have $W = \int pdV$

From the T-S graph we have $Q = \int Tds$

This is the way entropy was developed for thermodynamics and from the above we get the following definition **dS = dQ/T**

The units of entropy are hence J/K. Specific entropy has a symbol s and the units are J/kg K

It should be pointed out that there are other definitions of entropy but this one is the most meaningful for thermodynamics. A suitable integration will enable you to solve the entropy change for a fluid process. For those wishing to do studies in greater depth, these are shown in appendix A.

Entropy values for steam may be found in your thermodynamic tables in the columns headed s_f, s_{fg} and s_g.

s_f is the specific entropy of saturated liquid.
s_{fg} is the change in specific entropy during the latent stage.
s_g is the specific entropy of dry saturated vapour.

1.4.1 ISENTROPIC PROCESSES

The word ISENTROPIC means constant entropy and this is a very important thermodynamic process. It occurs in particular when a process is reversible and adiabatic. This means that there is no heat transfer to or from the fluid and no internal heat generation due to friction. In such a process it follows that if dQ is zero then dS must be zero. Since there is no area under the T-S graph, the graph must be a vertical line as shown.

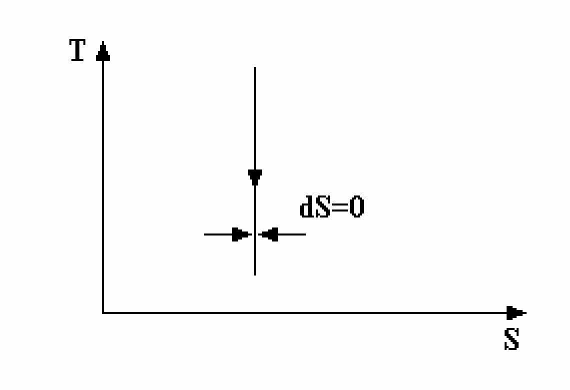


Fig. 6

There are other cases where the entropy is constant. For example, if there is friction in the process generating heat but this is lost through cooling, then the net result is zero heat transfer and constant entropy. You do not need to be concerned about this at this stage.

1.4.2 TEMPERATURE - ENTROPY (T-s) DIAGRAM FOR VAPOURS.

If you plot the specific entropy for saturated liquid (s_f) and for dry saturated vapour (s_g) against temperature, you would obtain the saturation curve. Lines of constant dryness fraction and constant pressure may be shown (Figure 7).

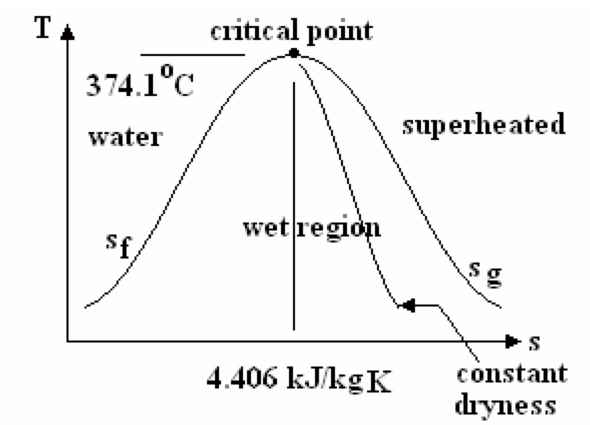


Fig. 7

1.4.3 SPECIFIC ENTHALPY-SPECIFIC ENTROPY (h-s) DIAGRAM.

This diagram is especially useful for steady flow processes (figure 8). The diagram is obtained by plotting h_g against s_g and h_f against s_f to obtain the characteristic saturation curve. The two curves meet at the critical point C. Lines of constant pressure, temperature and dryness are superimposed on the diagram. This is an extremely useful chart and it is available commercially. If any two coordinates are known, a point can be obtained on the chart and all other relevant values may be read off it. h –s charts are especially useful for solving isentropic processes because the process is a vertical line on this graph.

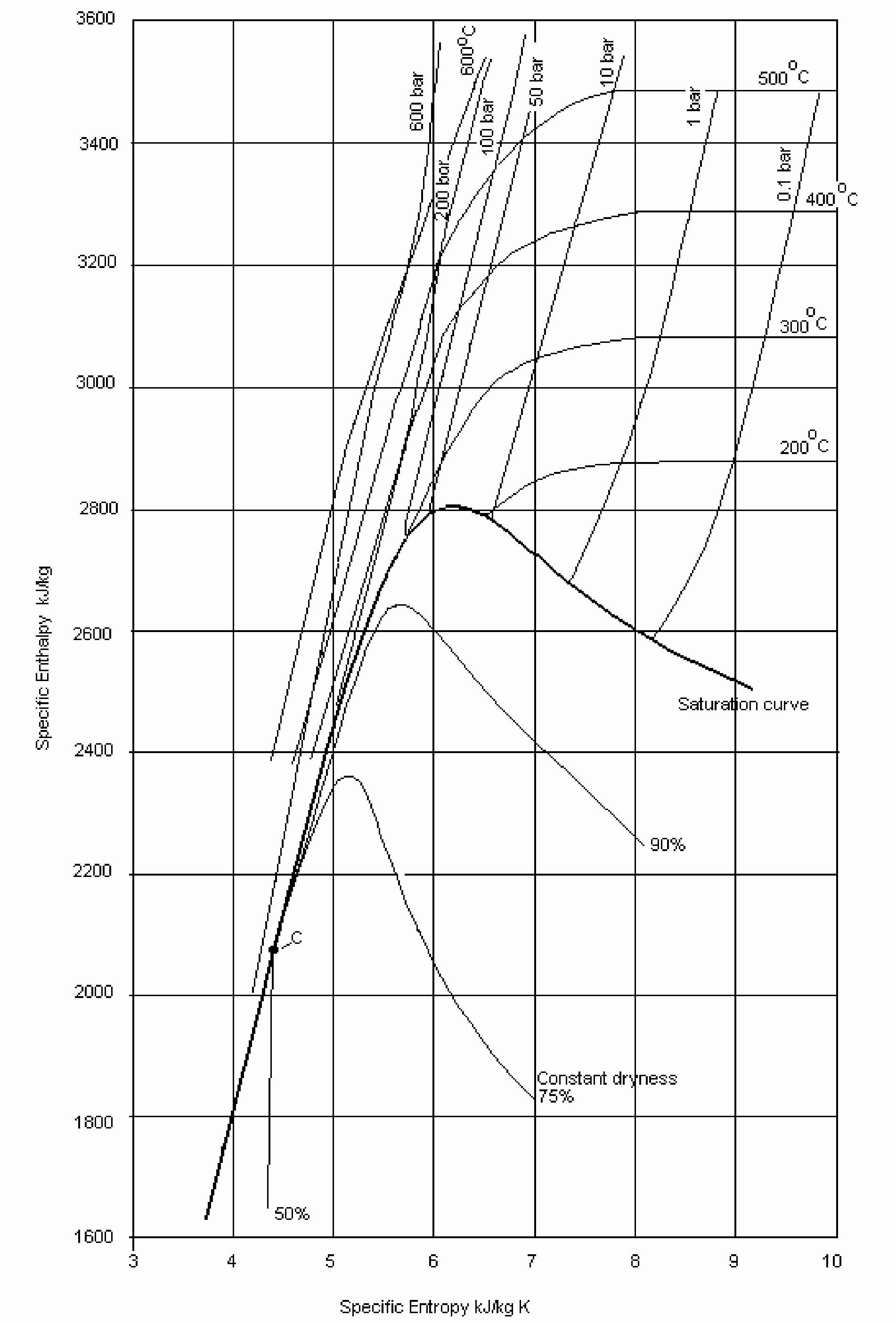


Fig. 8

Entropy values can be used to determine the dryness fraction following a steam expansion into the wet region when the process is isentropic. This is a very important point and you must master how to do this in order to solve steam expansion problems, especially in the following tutorials where steam cycles and refrigeration cycles are covered. The following examples show how this is done.

WORKED EXAMPLE No.1

Steam at 2 bar and 150^oC is expanded reversibly and adiabatically to 1 bar. Calculate the final dryness fraction and the enthalpy change.

SOLUTION

Let suffix (1) refer to the conditions before the expansion and (2) to the conditions after.

h_1 at 2 bar and 150^oC = 2770 kJ/kg

s_1 at 2 bar and 150^oC is 7.280 kJ/kg K.

Because the process is adiabatic and reversible, the entropy remains the same.

s_2 at 1 bar and assumed wet is $s_f + xs_{fg} = s_1$

$7.280 = 1.303 + x(6.056)$

$x= 0.987$

h_2 at 1 bar and 0.987 dry = $h_f + x\,h_{fg}$

$h_2 = 417 + 0.987(2258) = 2645.6$ kJ/kg

$\Delta h = 2645.6 - 2770 = -124.4$ kJ/kg

Being ale to solve the changes in enthalpy enable us to apply the first law of thermodynamics to solve problems with steam turbines. The next example shows you how to do this.

WORKED EXAMPLE No.2

A steam turbine expands 60 kg/s from 40 bar and 300°C to 4 bar reversibly and adiabatically (isentropic). Calculate the theoretical power output.

SOLUTION

$\dot{\Phi} + P = \Delta E$ per second (SFEE)

The process is adiabatic. $\dot{\Phi} = 0$ and the only energy term to use is enthalpy.

$P = \Delta H$ per second.

h_1 at 40 bar and 300°C = 2963 kJ/kg

s_1 at 40 bar and 300°C is 6.364 kJ/kg K.

s_2 at 4 bar and assumed wet is $s_f + xs_{fg} = s_1$

$6.364 = 1.776 + x(5.121)$

$x = 0.896$

h_2 at 4 bar and 0.896 dry = $h_f + x\,h_{fg}$

$h_2 = 605 + 0.896(2134) = 2517$ kJ/kg

$P = \Delta H$ per second = 60(2517-2963) = -26756 kW (out of system)

SELF ASSESSMENT EXERCISE No.2

1. A turbine expands 40 kg/s of steam from 20 bar and 250^{o}C reversibly and adiabatically to 0.5 bar. Calculate the theoretical power output.
   (Answer 25.2 MW)

2. A turbine expands 4 kg/s of steam from 50 bar and 300^{o}C reversibly and adiabatically to 0.1 bar. Calculate the theoretical power output.
   (Answer 3.8 MW)

3. A turbine expands 20 kg/s of steam from 800 bar and 400^{o}C reversibly and adiabatically to 0.2 bar. Calculate the theoretical power output.
   (Answer 11.2 MW)

4. A turbine expands 1 kg/s of steam reversibly and adiabatically. The inlet conditions are 10 bar and dry saturated. The outlet pressure is 3 bar. Calculate the theoretical power output.
   (Answer 218.5 MW)

2. THE CARNOT PRINCIPLE

A man called Sadi Carnot deduced that if the heat transfers from the hot reservoir and to the cold sump were done at constant temperature (isothermal processes), then the efficiency of the engine would be the maximum possible.

The reasoning behind this is as follows. Consider heat being transferred from a hot body A to a slightly cooler body B. The temperature of body A falls and the temperature of body B rises until they are at the same temperature.

If body B is now raised in temperature by heat transfer from the surroundings, it becomes the hotter body and the heat flow is reversed from B to A. If body A returns to its original temperature then the net heat transfers between A and B is zero. However body B is now hotter than its original temperature so there has been a net heat transfer from the surroundings. The heat transfer process is hence not reversible as external help was needed to reverse the process.

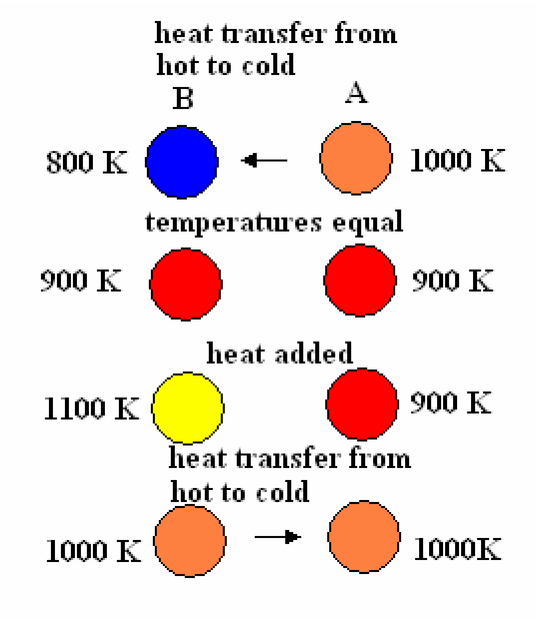


Figure 9

If it were possible to transfer heat with no temperature difference from A to B then it could be reversed with no external help. Such a process is an ISOTHERMAL process. Isothermal heat transfer is possible, for example evaporation of water in a boiler is isothermal.

Carnot devised a thermodynamic cycle using isothermal heat transfers only so by definition, the efficiency of this cycle is the most efficient any engine could be operating between two temperatures. Engine cycles are covered in the next tutorial but the following shows how the Carnot cycle might be conducted. In practice, it is not possible to make this cycle work.

2.1.1 CLOSED SYSTEM CARNOT CYCLE.

The cycle could be conducted on gas or vapour in a closed or open cycle. The cycle described here is for gas in a cylinder fitted with a piston. It consists of four closed system processes as follows.

1 to 2. The fluid is compressed isentropically. Work is put in and no heat transfer occurs.

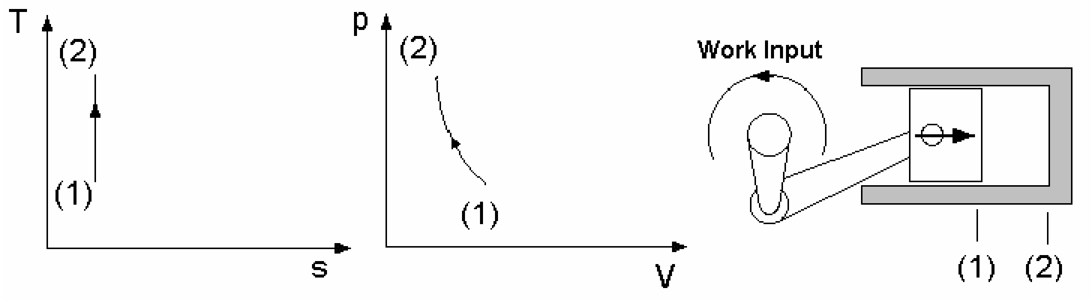


Fig. 10

2 to 3. The fluid is heated isothermally. This could only occur if it is heated as it expands so there is work taken out and heat put in.

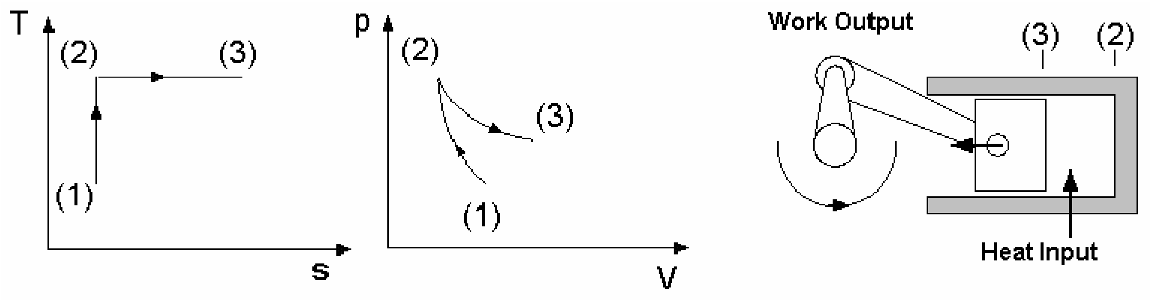


Fig. 11

3 to 4. The fluid continues to expand isentropically with no heat transfer. Work output is obtained.

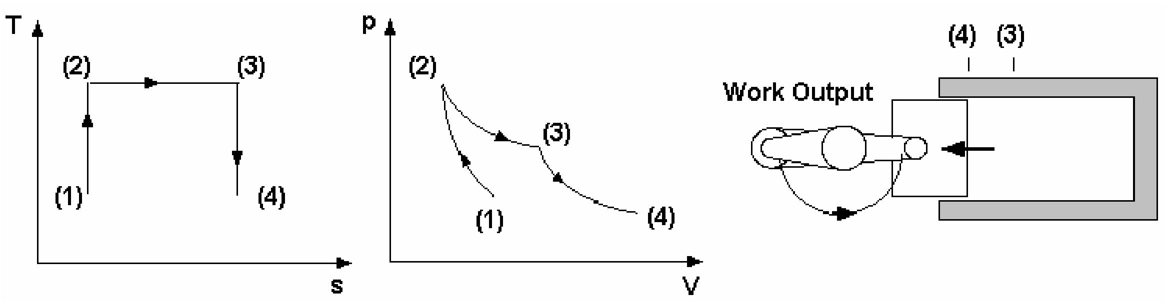


Fig.12

4 to 1 The fluid is cooled isothermally. This can only occur if it cooled as it is compressed, so work is put in and heat is taken out. At the end of this process every thing is returned to the initial condition.

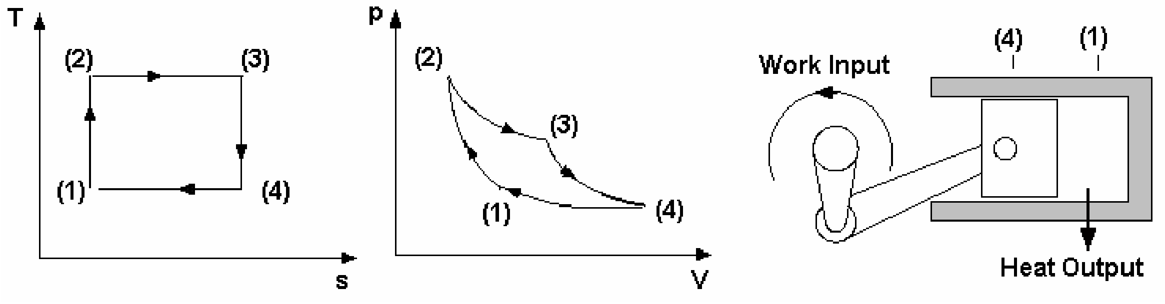


Fig.13

The total work taken out is W_{out} and the total work put in is W_{in}.

To be an engine, W_{out} must be larger than W_{in} and a net amount of work is obtained from the cycle. It also follows that since the area under a p-V graph represents the work done, then the area enclosed by the p-V diagram represents the net work transfer. It also follows that since the area under the T-s graph is represents the heat transfer, and then the area enclosed on the T-s diagram represents the net heat transfer. This is true for all cycles and also for real engines.

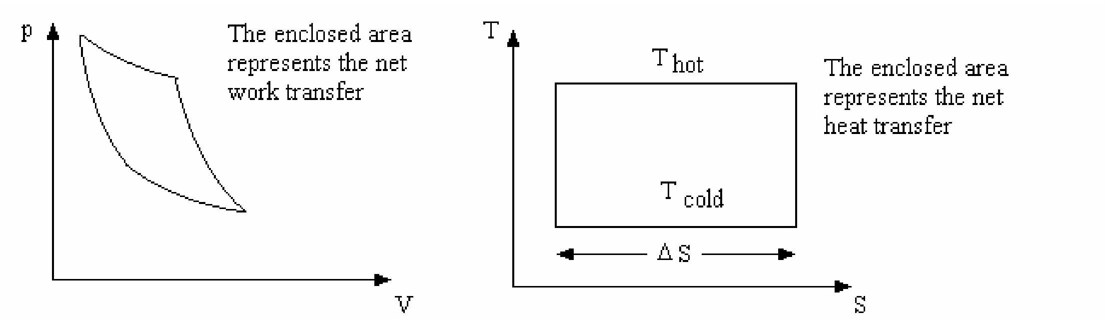


Fig.14

Applying the first law, it follows **$Q_{nett} = W_{nett}$**

For isothermal heat transfers $Q = \int T\,ds = T\Delta S$ since T is constant.

The efficiency would then be given by $\eta_{th} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{T_{cold}\Delta s_{cold}}{T_{hot}\Delta s_{hot}}$

It is apparent from the T-s diagram that the change in entropy Δs is the same at the hot and cold temperatures. It follows that $\eta_{th} = 1 - \frac{T_{cold}}{T_{hot}}$

This expression, which is the same as that used for the ideal model, gives the CARNOT EFFICIENCY and it is used as a target figure that cannot be surpassed (in fact not even attained).

WORKED EXAMPLE No.3

A heat engine draws heat from a combustion chamber at 300°C and exhausts to atmosphere at 10°C. What is the maximum possible thermal efficiency that could be achieved?

SOLUTION

The maximum efficiency possible is the Carnot efficiency. Remember to use absolute temperatures.

$$\eta_{th} = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{273 + 10}{273 + 300} = 1 - \frac{283}{573} = 0.505 \text{ or } 50.6\%$$

SELF ASSESSMENT EXERCISE No.3

1. A heat engine works between temperatures of 1100° C and 120°C. It is claimed that it has a thermal efficiency of 75%. Is this possible?
(Answer the maximum efficiency cannot exceed 71%)

2. Calculate the efficiency of a Carnot Engine working between temperatures of 1200°C and 200° C.
(Answer 67.9%)

EDEXCEL HIGHERS
ENGINEERING THERMODYNAMICS H2
NQF LEVEL 4

OUTCOME 2
INTERNAL COMBUSTION ENGINE PERFORMANCE

TUTORIAL No. 4 – HEAT ENGINE CYCLES

Internal combustion engine performance

Second law of thermodynamics: statement of law; schematic representation of a heat engine to show heat and work flow

Heat engine cycles: Carnot cycle; Otto cycle; Diesel cycle; dual combustion cycle; Joule cycle; property diagrams; Carnot efficiency; air-standard efficiency

Performance characteristics: engine trials; indicated and brake mean effective pressure; indicated and brake power; indicated and brake thermal efficiency; mechanical efficiency; relative efficiency; specific fuel consumption; heat balance

Improvements: turbocharging; turbocharging and intercooling; cooling system and exhaust gas heat recovery systems

- Define AIR STANDARD CYCLES.
- Identify the ideal cycle for a given type of engine.
- Explain and solve problems for the OTTO cycle
- Explain and solve problems for the DIESEL cycle
- Explain and solve problems for the Dual Combustion cycle
- Explain and solve problems for the JOULE cycle

1. THEORETICAL CYCLES FOR ENGINES

Internal combustion engines fall into two groups, those that use a sparking plug to ignite the fuel (spark ignition engines) and those that use the natural temperature of the compressed air to ignite the fuel (compression ignition engines).

Another way to group engines is into those that use non-flow processes and those that use flow processes. For example, non-flow processes are used in piston engines. Flow processes are used in gas turbine engines.

Theoretical cycles are made up of ideal thermodynamic processes to resemble those that occur in a real engine as closely as possible. Many of these cycles are based on air as the working fluid and are called ***AIR STANDARD CYCLES***. Before looking at air standard cycles, we should briefly revise the Carnot Cycle from tutorial 3.

1. THE CARNOT CYCLE

The most efficient way of transferring heat into or out of a fluid is at constant temperature. All the heat transfer in the Carnot cycle is at constant temperature so it follows that the Carnot cycle is the most efficient cycle possible. The heat transfer into the cycle occurs at a hot temperature T_{hot} and the heat transfer out of the cycle occurs at a colder temperature T_{cold}. The thermodynamic efficiency was sown to be given as follows.

$$\eta_{th} = 1 - \frac{T_{cold}}{T_{hot}}$$

None of the following cycles can have an efficiency greater than this when operating between the same temperatures limits.

2 SPARK IGNITION ENGINE

2.1 THE OTTO CYCLE

The ideal cycle is named after Count N.A.Otto. It represents the ideal cycle for a spark ignition engine. In an ideal spark ignition engine, there are four processes as follows.

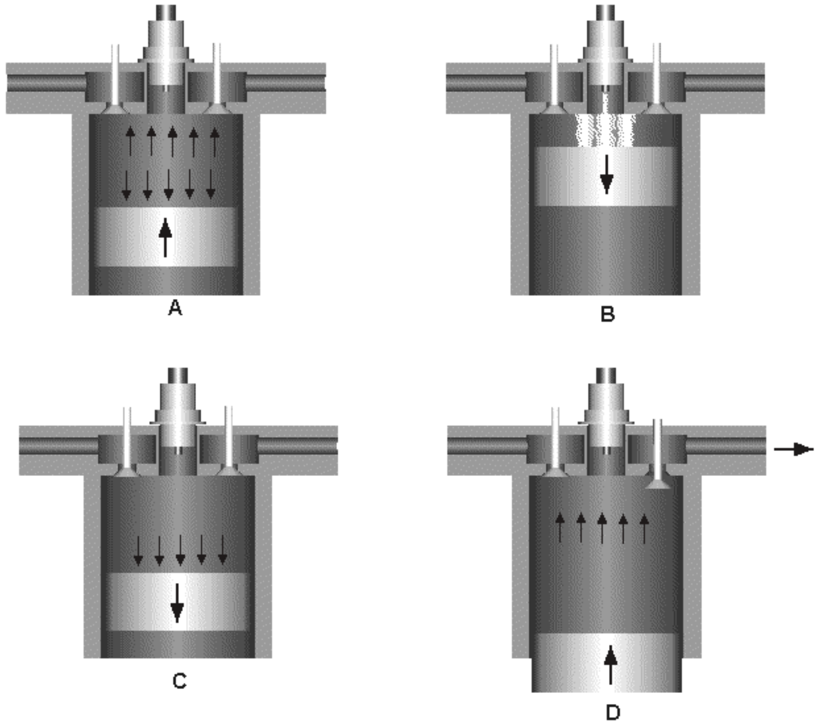


Fig.1

COMPRESSION STROKE

Air and fuel are mixed and compressed so rapidly that there is no time for heat to be lost. (Figure A) In other words the compression is adiabatic. Work must be done to compress the gas.

IGNITION

Just before the point of maximum compression, the air is hot and a spark ignites the mixture causing an explosion (Figure B). This produces a rapid rise in the pressure and temperature. The process is idealised as a constant volume process in the Otto cycle.

EXPANSION OR WORKING STROKE

The explosion is followed by an adiabatic expansion pushing the piston and giving out work. (Figure C)

EXHAUST

At the end of the working stroke, there is still some pressure in the cylinder. This is released suddenly by the opening of an exhaust valve. (Figure D) This is idealised by a constant volume drop in pressure in the Otto cycle. In 4 stroke engines a second cycle is performed to push out the products of combustion and draw in fresh air and fuel. It is only the power cycle that we are concerned with.

The four ideal processes that make up the Otto cycle are as follows.

1 to 2 The air is compressed reversibly and adiabatically. Work is put in and no heat transfer occurs.

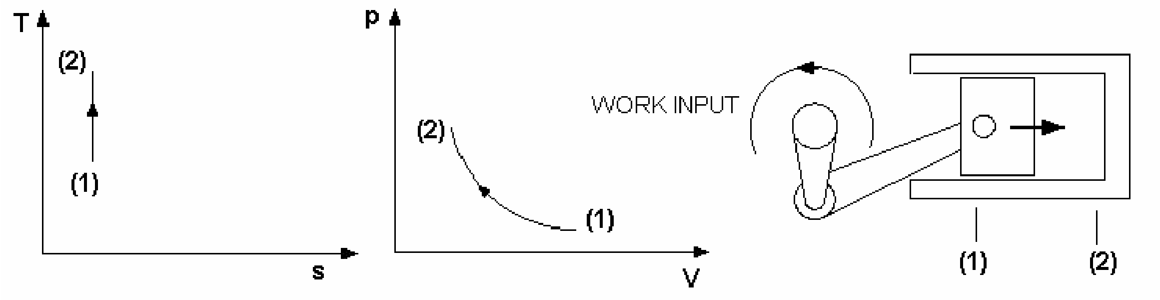


Fig.2

2 to 3 The air is heated at constant volume. No work is done. $Q_{in} = mc_V(T_3-T_2)$

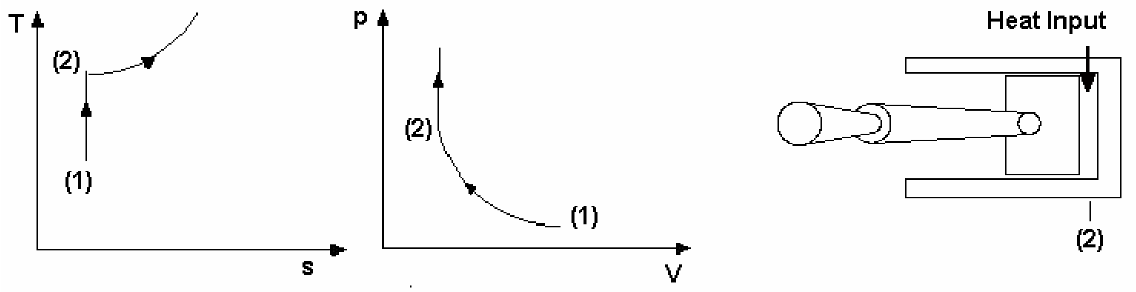


Fig.3

3 to 4 The air expands reversibly and adiabatically with no heat transfer back to its original volume. Work output is obtained.

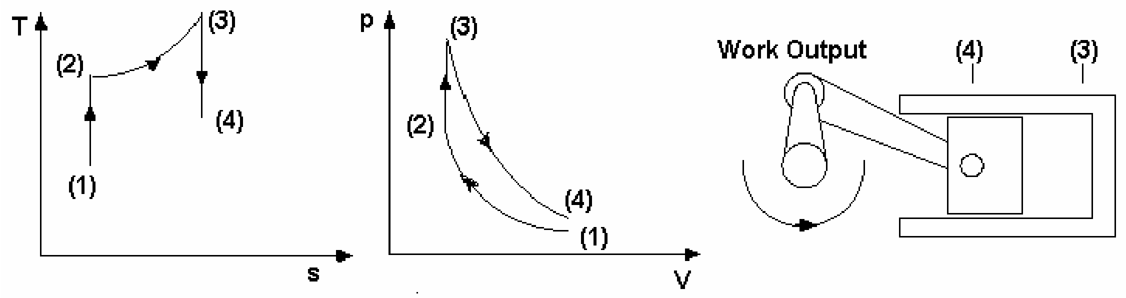


Fig.4

4 to 1 The air is cooled at constant volume back to its original pressure and temperature. No work is done $Q_{out} = mc_V(T_4-T_1)$

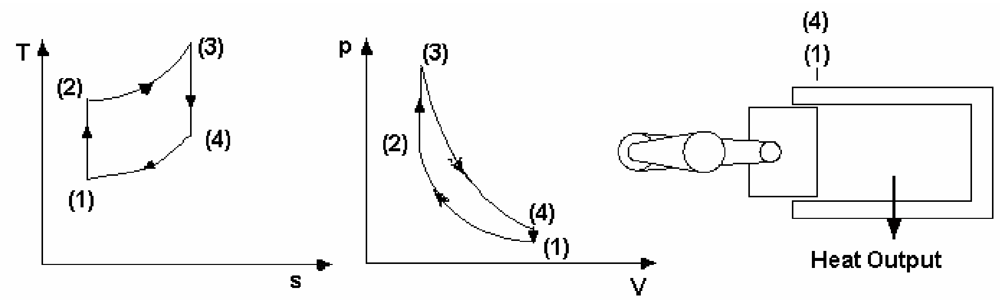


Fig.5

The total heat transfer into the system during one cycle is $Q_{nett} = Q_{in} - Q_{out}$

The total work output per cycle is W_{nett}

From the 1st. Law of thermodynamics $Q_{nett} = W_{nett}$

EFFICIENCY

$$\eta = \frac{W_{nett}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_4 - T_1)}{mc_v(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

For the process (1) to (2) we may use the rule $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r_v^{\gamma-1}$

For the process (3) to (4) we may similarly write $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = r_v^{\gamma-1}$

where r_v is the volume compression ratio $r_v = \frac{V_1}{V_2} = \frac{V_4}{V_3}$

It follows that $\frac{T_2}{T_1} = \frac{T_3}{T_4}$ and $\frac{T_4}{T_1} = \frac{T_3}{T_2}$

and that $\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{\frac{T_3 T_1}{T_2} - T_1}{\frac{T_2 T_4}{T_1} - T_2} = 1 - \frac{T_1\left(\frac{T_3}{T_2} - 1\right)}{T_2\left(\frac{T_4}{T_1} - 1\right)}$

$\frac{T_4}{T_1} = \frac{T_3}{T_2}$ then $\frac{T_4}{T_1} - 1 = \frac{T_3}{T_2} - 1$

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} = 1 - \frac{1}{r_v^{\gamma-1}} = 1 - r_v^{1-\gamma}$$

Since this theoretical cycle is carried out on air for which $\gamma = 1.4$ then the efficiency of an Otto Cycle is given by $\eta = 1 - r_v^{0.4}$

This shows that the thermal efficiency depends only on the compression ratio. If the compression ratio is increased, the efficiency is improved. This in turn increases the temperature ratios between the two isentropic processes and explains why the efficiency is improved.

WORKED EXAMPLE No.1

An Otto cycle is conducted as follows. Air at 100 kPa and 20ºC is compressed reversibly and adiabatically. The air is then heated at constant volume to 1500ºC. The air then expands reversibly and adiabatically back to the original volume and is cooled at constant volume back to the original pressure and temperature. The volume compression ratio is 8. Calculate the following.

i. The thermal efficiency.

ii. The heat input per kg of air.

iii. The net work output per kg of air.

iv. The maximum cycle pressure.

c_V = 718 kJ/kg γ= 1.4. R = 287 J/kg K

SOLUTION

Remember to use absolute temperatures throughout. Solve for a mass of 1 kg.

T_1=20 +273=293K T_3=1500+273=1773K r_v=8

$$\eta = 1 - r^{1-\gamma} = 1 - 8^{\ 0.4} = 0.565 \quad \text{or } 56.5\%$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 293\left(8^{0.4}\right) = 673.1\,K$$

$$Q_{in} = mc_v(T_3 - T_2) = 1x718(1773 - 673.1) = 789700\ J/kg = 789.7\ kJ/kg$$

$$W_{nett} = \eta Q_{in} = 0.56x789.7 = 446.2\ kJ/kg$$

From the gas law we have

$$p_3 = \frac{p_1 V_1 T_3}{T_1 V_3} = \frac{100000 \text{ x } V_1 \text{ x } 1773}{293 \text{ x } V_3}$$

$$\frac{V_1}{V_3} = 8$$

$$p_3 = \frac{100000 \text{ x } 1773}{293} \text{ x } 8 = 4.84 \text{ MPa}$$

If you have followed the principles used here you should be able to solve any cycle.

SELF ASSESSMENT EXERCISE No.1

Take Cv = 0.718 kJ/kg K, R = 287 J/kg K and γ= 1.4 throughout.

1. An Otto cycle has a volume compression ratio of 9/1. The heat input is 500kJ/kg. At the start of compression the pressure and temperature are 100 kPa and 40°C respectively. Calculate the following.

   i. The thermal efficiency. (58.5%)
   ii. The maximum cycle temperature. (1450 K).
   iii. The maximum pressure. (4.17 MPa).
   iv. The net work output per kg of air. (293 kJ/kg).

2. Calculate the volume compression ratio required of an Otto cycle which will produce an efficiency of 60%. (9.88/1)

   The pressure and temperature before compression are 105 kPa and 25°C respectively. The net work output is 500 kJ/kg). Calculate the following.

   i. The heat input. (833 kJ/kg).
   ii. The maximum temperature. (1 906 K)
   iii. The maximum pressure. (6.64 MPa).

3. An Otto cycle uses a volume compression ratio of 9.5/1. The pressure and temperature before compression are 100 kPa and 40°C respectively. The mass of air used is 11.5 grams/cycle. The heat input is 600 kJ/kg. The cycle is performed 3 000 times per minute. Determine the following.

   i. The thermal efficiency. (59.4%).
   ii. The net work output. (4.1 kJ/cycle)
   iii. The net power output. (205 kW).

4. An Otto cycle with a volume compression ratio of 9 is required to produce a net work output of 450 kJ/cycle. Calculate the mass of air to be used if the maximum and minimum temperatures in the cycle are 1300°C and 20°C respectively. (1.235 kg).

5. The working of a petrol engine can be approximated to an Otto cycle with a compression ratio of 8 using air at 1 bar and 288 K with heat addition of 2 MJ/kg. Calculate the heat rejected and the work done per kg of air. (871 kJ/kg and 1129 kJ/kg).

Now let's move on to study engines with compression ignition.

3 COMPRESSION IGNITION ENGINES

The invention of compression ignition engines, commonly known as diesel engines, was credited to Rudolf Diesel, although many other people worked on similar engines. The basic principle is that when high compression ratios are used, the air becomes hot enough to make the fuel detonate without a spark. Diesel's first engine used coal dust blasted into the combustion chamber with compressed air. This developed into blasting in oil with compressed air. In modern engines the fuel oil is injected directly into the cylinder as fine droplets. There are two ideal cycles for these engines, the Diesel Cycle and the Dual Combustion Cycle.

3.1 DUAL COMBUSTION CYCLE

This is the air standard cycle for a modern fast running diesel engine. First the air is compressed isentropically making it hot. Fuel injection starts before the point of maximum compression. After a short delay in which fuel accumulates in the cylinder, the fuel warms up to the air temperature and detonates causing a sudden rise in pressure. This is ideally a constant volume heating process. Further injection keeps the fuel burning as the volume increases and produces a constant pressure heating process. After cut off, the hot air expands isentropically and then at the end of the stroke, the exhaust valve opens producing a sudden drop in pressure. This is ideally a constant volume cooling process. The ideal cycle is shown in figure 6.

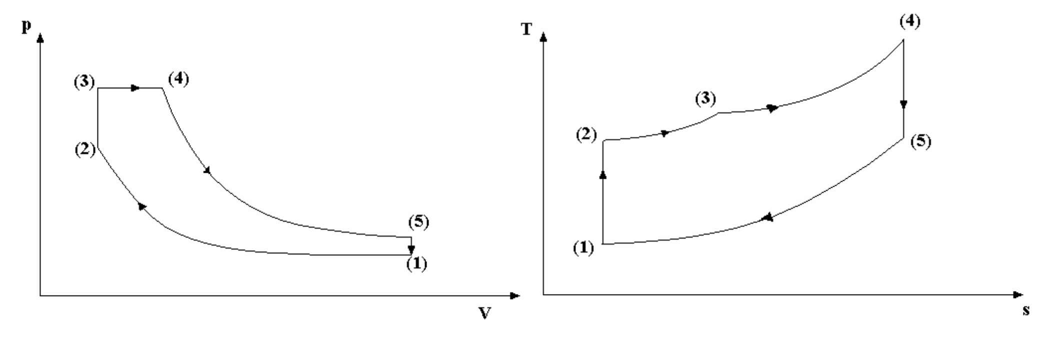


Fig. 6

The processes are as follows.

1 - 2 reversible adiabatic (isentropic) compression.

2 - 3 constant volume heating.

3 - 4 constant pressure heating.

4 - 5 reversible adiabatic (isentropic) expansion.

5 - 1 constant volume cooling.

The analysis of the cycle is as follows.

The heat is supplied in two stages hence $Q_{in} = mC_p(T_4 - T_3) + mC_V (T_3 - T_2)$

The heat rejected is $Q_{out} = mC_V (T_5 - T_1)$

The thermal efficiency may be found as follows.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_5 - T_1)}{mc_v(T_3 - T_2) + mc_p(T_4 - T_3)} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

The formula can be further developed to show that

$$\eta = 1 - \frac{k\beta^{\gamma} - 1}{[(k-1) + \gamma k(\beta - 1)]r_v^{\gamma - 1}}$$

r_v is the VOLUME COMPRESSION RATIO. $r_v = V_1/V_2$
β is the CUT OFF RATIO. $\beta = V_4/V_3$
k is the ratio p_3/p_2.

Most students will find this adequate to solve problems concerning the dual combustion cycle. Generally, the method of solution involves finding all the temperatures by application of the gas laws.

Those requiring a detailed analysis of the cycle should study the following derivation.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

Obtain all the temperatures in terms of T_2

Isentropic compression 1 to 2

$$T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{T_2}{r_v^{\gamma-1}}$$

Constant volume heating 2 to 3 note $V_3 = V_2$

$$T_3 = \frac{p_3 V_3 T_2}{p_2 V_2} = \frac{p_3 T_2}{p_2} = kT_2$$

Constant pressure heating 3 to 4 note $p_3 = p_4$

$$T_4 = \frac{p_4 V_4 T_3}{p_3 V_3} = \frac{V_4 T_3}{V_3} = \beta T_3 = \beta k T_2$$

Isentropic expansion 4 to 5

$$T_5 = T_4\left(\frac{V_4}{V_5}\right)^{\gamma-1} = T_4\left(\frac{V_4 V_2}{V_5 V_2}\right)^{\gamma-1} = T_4\left(\frac{\beta}{r_v}\right)^{\gamma-1} = \frac{k\beta^{\gamma} T_2}{r_r^{\gamma-1}}$$

Substitute for all temperatures in the efficiency formula.

$$\eta = 1 - \frac{\dfrac{k\beta^{\gamma} T_2}{r_r^{\gamma\ 1}} - \dfrac{T_2}{r_v^{\gamma\ 1}}}{(kT_2 - T_2) + \gamma(\beta k T_2 - kT_2)} = 1 - \frac{\dfrac{k\beta^{\gamma}}{r_r^{\gamma-1}} - \dfrac{1}{r_v^{\gamma-1}}}{(k-1) + \gamma(\beta k - k)}$$

$$\eta = 1 - \frac{k\beta^{\gamma} - 1}{[(k-1) + \gamma k(\beta - 1)]r_v^{\gamma-1}}$$

Note that if β=1, the cycle becomes an Otto cycle and the efficiency formulae becomes the same as for an Otto cycle.

WORKED EXAMPLE No. 2

In a dual combustion cycle, the compression starts from 1 bar and 20^oC. The compression ratio is 18/1 and the cut off ratio is 1.15. The maximum cycle pressure is 1360 K. The total heat input is 1 kJ per cycle. Calculate the following.

i. The thermal efficiency of the cycle.

ii. The net work output per cycle.

Check that the efficiency does not contravene the Carnot principle.

SOLUTION

Known data.

T_1 = 20 +273 = 293 K The hottest temperature is T_4 = 1360K.

$\beta=1.15$ $r_v = 18$ $\gamma=1.4$

$$T_2 = T_1 r_v^{\gamma-1} = 293 \text{ x } 18^{0.4} = 931\text{ K}$$

$$T_3 = \frac{V_3 T_4}{V_4} = \frac{T_4}{\beta} = \frac{1360}{1.15} = 1183\text{ K}$$

$$\frac{p_3}{p_2} = k = \frac{T_3}{T_2} = 1.27$$

$$\eta = 1 - \frac{k\beta^\gamma - 1}{[(k-1) + \gamma k(\beta - 1)]r_v^{\gamma-1}} = 1 - \frac{1.27 \text{ x } 1.15^{1.4} - 1}{[(1.27-1) + (1.4 \text{ x } 1.27 \text{ x } (1.15-1))] \text{ x } 18^{0.4}}$$

$$\eta = 0.68 \text{ or } 68\%$$

$W_{nett} = \eta \text{ x } Q_{in}$ = 0.68 x 1 = 0.68 kJ per cycle.

The Carnot efficiency should be higher.

$$\eta = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{293}{1360} = 0.785$$

The figure of 0.68 is lower so the Carnot principle has not been contravened.

WORKED EXAMPLE No.3

A dual combustion cycle has a compression ratio of 18/1. The maximum pressure in the cycle is 9 MPa and the maximum temperature is 2000°C. The pressure and temperature before compression is 115 kPa and 25°C respectively. Calculate the following.

i. the cut off ratio.
ii. the cycle efficiency.
iii. the net work output per kg of air.

Assume γ =1.4 C_p = 1.005 kJ/kgK C_V = 0.718 kJ/kg K.

SOLUTION

Known data.

$T_1 = 298$ K $\quad T_4 = 2273$ K $\quad p_3 = p4 = 9$ MPa $\quad p_1 = 115$ kPa

$V_1/V_2 = V_1/V_3 = 18 \qquad V_2 = V_3$

$T_2 = 298 \text{ x } 18^{(\gamma-1)} = 947$ K

$$T_3 = \frac{p_3 T_1 V_3}{p_1 V_1} = \frac{9 \text{ x } 10^6 \text{ x } 298}{115 \text{ x } 10^3} \text{ x } \frac{V_3}{V_1} = \frac{9 \text{ x } 10^6 \text{ x } 298}{115 \text{ x } 10^3} \text{ x } \frac{1}{18} = 1296\text{K}$$

$$\text{Cut off ratio} = \beta = \frac{V_4}{V_3} = \frac{p_3 T_4}{p_4 T_3} \quad \text{but } p_4 = p_3 \text{ so } \beta = \frac{T_4}{T_3}$$

$$\beta = \frac{2273}{1296} = 1.75$$

$$T_5 = T_4 \left(\frac{V_4}{V_5}\right)^{\gamma-1} \quad \text{but } \frac{V_4}{V_5} = \frac{V_4}{V_3} \text{ x } \frac{V_3}{V_5} = \frac{1.75}{18} = 0.0974$$

$$T_5 = 2273 \text{ x } 0.0974^{0.4} = 895.6 \text{ K}$$

$Q_{in} = mC_p(T4 - T3) + mC_V (T3 - T2)$ m= 1 kg

Q_{in} =1.005(2274 -1296) + 0.718(1296-947) = 1232.5 kJ/kg

$Q_{out} = mC_V (T_5 - T_1)$

Q_{out} = 0.718(895.6 - 298) = 429 kJ/g

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{429}{1232} = 0.65 \text{ or } 65\%$$

$$W_{nett} = Q_{in} \quad Q_{out} = 1232 \quad 428.6 = 803.5 \text{ kJ/kg}$$

3.2 THE DIESEL CYCLE

The Diesel Cycle precedes the dual combustion cycle. The Diesel cycle is a reasonable approximation of what happens in slow running engines such as large marine diesels. The initial accumulation of fuel and sharp detonation does not occur and the heat input is idealised as a constant pressure process only.

Again consider this cycle as being carried out inside a cylinder fitted with a piston. The p-V and T-s cycles diagrams are shown in figure 7

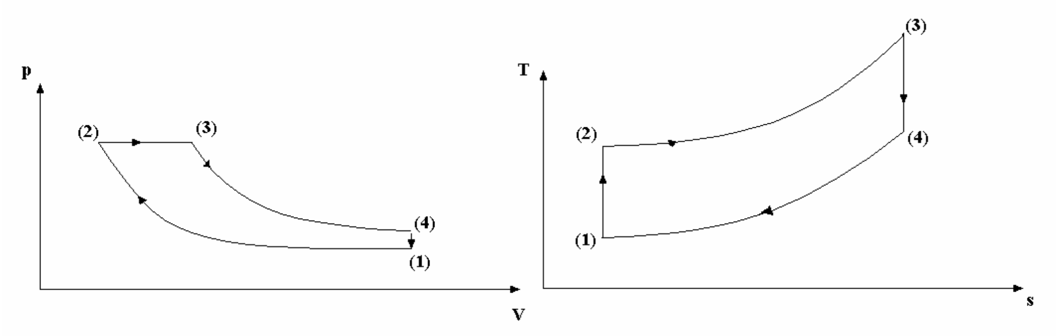


Fig. 7

1 - 2 reversible adiabatic (isentropic) compression.

2 - 3 constant pressure heating.

3 - 4 reversible adiabatic (isentropic) expansion.

4 - 1 constant volume cooling.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)}$$

The cycle is the same as the dual combustion cycle without the constant volume heating process. In this case since k=1 the efficiency is given by the following formula.

$$\eta = 1 - \frac{\beta^{\gamma} - 1}{(\beta - 1)\gamma r_v^{\gamma-1}}$$

WORKED EXAMPLE No.4

An engine using the Diesel Cycle has a compression ratio of 20/1 and a cut off ratio of 2. At the start of the compression stroke the air is at 1 bar and 15°C. Calculate the following.

i. The air standard efficiency of the cycle.
ii. The maximum temperature in the cycle.
iii. The heat input.
iv. The net work output.

SOLUTION

Initial data.

β=2 r_v=20 γ=1.4 c_v = 718 J/kg K for air T_1=288 K p_1=1 bar.

The maximum temperature is T_3 and the maximum pressure is p_3 and p_2.

$$\eta = 1 - \frac{\beta^{\gamma} - 1}{(\beta - 1)\gamma r_v^{\gamma-1}}$$

$$\eta = 1 - \frac{2^{1.4} - 1}{(2-1) \text{ x } 1.4 \text{ x } 20^{0.4}}$$

$$\eta = 1 - \frac{1.639}{1 \text{ x } 1.4 \text{ x } 3.314} = 0.647 \text{ or } 64.7\%$$

$$T_2 = T_1 r_v^{\gamma-1} = 288 \text{ x } 20^{0.4} = 954.5 \text{ K}$$

$$T_3 = \frac{V_2}{V_3} T_2 = \beta T_2 = 954.3 \text{ x } 2 = 1909 \text{ K}$$

$$Q_{in} = mc_p (T_3 - T_2)$$

$$Q_{in} = 1.005(1909 - 954.5) = 959.3 \text{ kJ}$$

$$\eta = \frac{W_{nett}}{Q_{in}}$$

$$W_{nett} = \eta Q_{in} = 0.647 \text{ x } 959.3 = 620.6 \text{ kJ}$$

SELF ASSESSMENT EXERCISE No.2

Use c_v = 0.718 kJ/kg K , c_p = 1.005 kJ/kg K and γ=1.4 throughout.

1. Draw a p - V and T - s diagram for a Diesel Cycle.

   The performance of a compression ignition engine is to be compared to the Diesel cycle. The compression ratio is 16. The pressure and temperature at the beginning of compression are 1 bar and 15°C respectively. The maximum temperature in the cycle is 1200 K.

   Calculate the following.

   i. The cut off ratio.(1.374)

   ii. The air standard efficiency. (66%)

2. A Dual Combustion Cycle uses a compression ratio of 12/1. The cut off ratio is 2/1. The temperature and pressure before compression is 280 K and 1 bar respectively. The maximum temperature 2000 K. Calculate the following.

   i. The net work output per cycle. (680 kJ/kg).
   ii. The thermal efficiency. (57.6 %).

3. A Dual Combustion Cycle uses a compression ratio of 20/1. The cut off ratio is 1.6/1. The temperature and pressure before compression is 30°C and 1 bar respectively. The maximum cycle pressure is 100 bar. Calculate the following.

   i. The maximum cycle temperature. (2424 K).
   ii. The net work output per cycle. (864 kJ/kg).
   iii. The thermal efficiency. (67.5 %).

4 GAS TURBINES

A gas turbine engine normally burns fuel in the air that it uses as the working fluid. From this point of view it is an internal combustion engine that uses steady flow processes. Figure 8 shows a basic design.

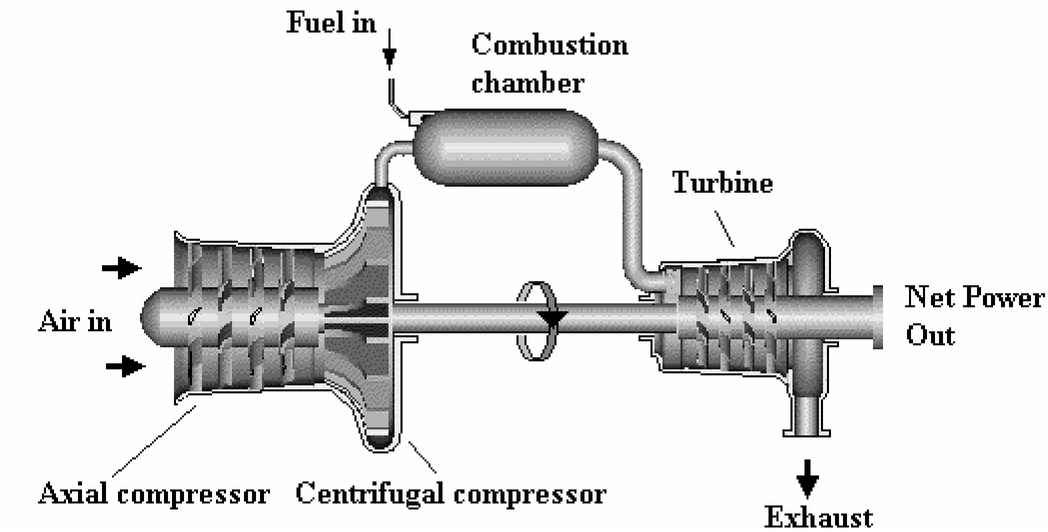


Fig.8

The air is drawn in from atmosphere and compressed. This makes it hotter. The compressed air is blown into a combustion chamber and fuel is burned in it making it even hotter. This makes the volume increase. The hot air expands out of the chamber through a turbine forcing it to revolve and produce power. The air becomes colder as it expands and eventually exhausts to atmosphere. The temperature drop over the turbine is larger than the temperature increase over the compressor. The turbine produces more power than is needed to drive the compressor. Net power output is the result. In the basic system, the turbine is coupled directly to the compressor and the power output is taken from the same shaft. The ideal air standard cycle is the Joule Cycle.

4.1 THE JOULE CYCLE

The Joule Cycle is also known as the constant pressure cycle because the heating and cooling processes are conducted at constant pressure. The cycle is that used by a gas turbine engine but could conceivably be used in a closed system.

We may draw the layout in block diagram form as shown in figure 9

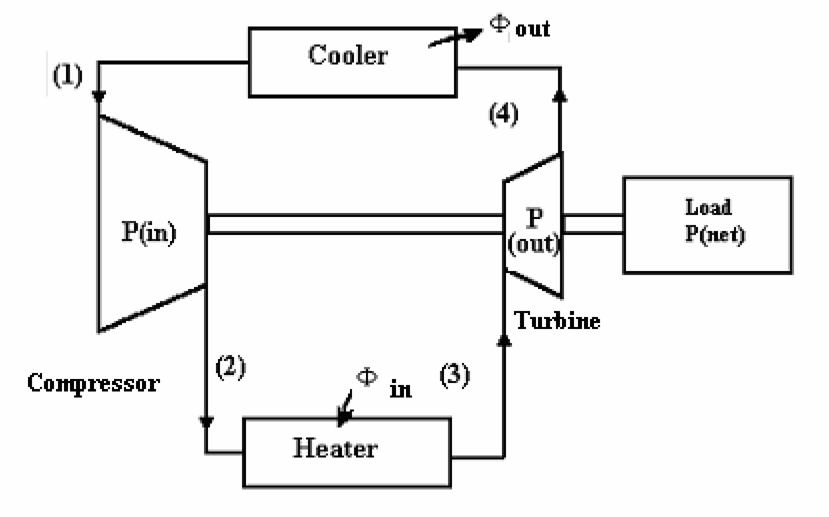


Figure 9

There are 4 ideal processes in the cycle.

1 - 2 Reversible adiabatic (isentropic) compression requiring power input.
$P_{in} = \Delta H/s = mC_p(T_2 - T_1)$

2 - 3 Constant pressure heating requiring heat input.
$\Phi_{in} = \Delta H/s = mC_p(T_3 - T_2)$

3 - 4 Reversible adiabatic (isentropic) expansion producing power output.
$P_{out} = \Delta H/s = mC_p(T_3 - T_4)$

4 - 1 Constant pressure cooling back to the original state requiring heat removal.
$\Phi_{out} = \Delta H/s = mC_p(T_4 - T_1)$

The pressure – volume, pressure - enthalpy and temperature-entropy diagrams are shown in figure 10

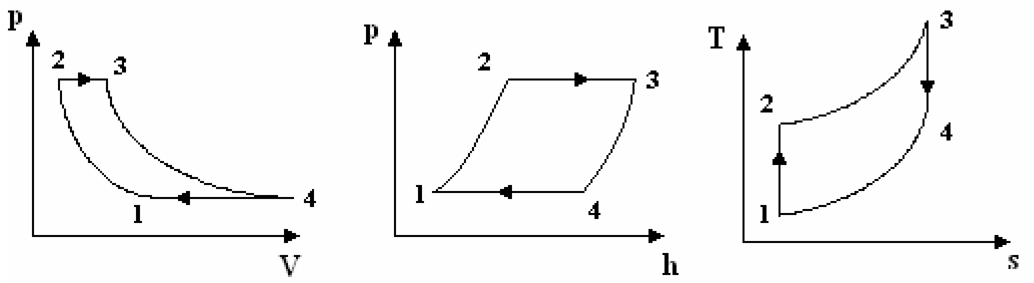


Fig. 10

The efficiency is found by applying the first law of thermodynamics.

$$\Phi_{nett} = P_{nett}$$

$$\Phi_{in} - \Phi_{out} = P_{out} - P_{in}$$

$$\eta_{th} = \frac{P_{nett}}{\Phi_{in}} = 1 - \frac{\Phi_{out}}{\Phi_{in}} = 1 - \frac{mc_p(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

It assumed that the mass and the specific heats are the same for the heater and cooler.

It is easy to show that the temperature ratio for the turbine and compressor are the same.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}} = r_p^{1-\frac{1}{\gamma}} \qquad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{1-\frac{1}{\gamma}} = r_p^{1-\frac{1}{\gamma}} \qquad \frac{T_3}{T_4} = \frac{T_2}{T_1}$$

r_p is the pressure compression ratio for the turbine and compressor.

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{\left(\frac{T_3 T_1}{T_2} - T_1\right)}{\left(\frac{T_2 T_4}{T_1} - T_2\right)} = 1 - \frac{T_1\left(\frac{T_3}{T_2} - 1\right)}{T_2\left(\frac{T_4}{T_1} - 1\right)}$$

$$\frac{T_3}{T_2} = \frac{T_4}{T_1} \qquad \frac{T_3}{T_2} - 1 = \frac{T_4}{T_1} - 1$$

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} = 1 - \frac{1}{r_p^{1-\frac{1}{\gamma}}} = 1 - r_p^{-0.286} \quad \text{since } \gamma = 1.4$$

This shows that the efficiency depends only on the pressure ratio which in turn affects the hottest temperature in the cycle.

WORKED EXAMPLE No. 5

A gas turbine uses the Joule cycle. The pressure ratio is 6/1. The inlet temperature to the compressor is 10°C. The flow rate of air is 0.2 kg/s. The temperature at inlet to the turbine is 950°C. Calculate the following.

i. The cycle efficiency.

ii. The heat transfer into the heater.

iii. The net power output.

$\gamma = 1.4$ $C_p = 1.005$ kJ/kg K

SOLUTION

$$\eta_{th} = 1 - r_p^{-0.286} = 1 - 6^{-0.286} = 0.4 \text{ or } 40\%$$

$$T_2 = T_1 r_p^{0.286} = 283 \text{ x } 6^{0.286} = 472.4K$$

$$\Phi_{in} = mc_p(T_3 - T_2) = 0.2 \text{ x } 1.005 \text{ x } (1223 - 472.4) = 150.8 \text{ kW}$$

$$\eta_{th} = \frac{P_{nett}}{\Phi_{in}}$$

$$P_{nett} = 0.4 \text{ x } 150.8 = 60.3 \text{ kW}$$

SELF ASSESSMENT EXERCISE No.3

γ= 1.4 and C_p = 1.005 kJ/kg K throughout.

1. A gas turbine uses the Joule cycle. The inlet pressure and temperature to the compressor are respectively 1 bar and -10°C. After constant pressure heating, the pressure and temperature are 7 bar and 700°C respectively. The flow rate of air is 0.4 kg/s. Calculate the following.

   i. The cycle efficiency.

   ii. The heat transfer into the heater.

   iii. The net power output.

   (Answers 42.7 % , 206.7 kW and 88.26 kW)

2. A gas turbine expands draws in 3 kg/s of air from atmosphere at 1 bar and 20°C. The combustion chamber pressure and temperature are 10 bar and 920°C respectively. Calculate the following.

   i. The Joule efficiency.

   ii. The exhaust temperature.

   iii. The net power output.

   (Answers 48.2 % , 617.5 K and 911 kW)

3. A gas turbine draws in 7 kg/s of air from atmosphere at 1 bar and 15°C. The combustion chamber pressure and temperature are 9 bar and 850°C respectively. Calculate the following.

   i. The Joule efficiency.

   ii. The exhaust temperature.

   iii. The net power output.

   (Answers 46.7 % , 599 K and 1.916 MW)

EDEXCEL HIGHERS
ENGINEERING THERMODYNAMICS H2
NQF LEVEL 4

OUTCOME 2
INTERNAL COMBUSTION ENGINE PERFORMANCE

TUTORIAL No. 5 – PERFORMANCE CHARACTERISTICS

Internal combustion engine performance

Second law of thermodynamics: statement of law; schematic representation of a heat engine to show heat and work flow

Heat engine cycles: Carnot cycle; Otto cycle; Diesel cycle; dual combustion cycle; Joule cycle; property diagrams; Carnot efficiency; air-standard efficiency

Performance characteristics: engine trials; indicated and brake mean effective pressure; indicated and brake power; indicated and brake thermal efficiency; mechanical efficiency; relative efficiency; specific fuel consumption; heat balance

Improvements: turbocharging; turbocharging and intercooling; cooling system and exhaust gas heat recovery systems

On completion of this tutorial you should be able to do the following.

- Calculate the fuel power of an engine.
- Calculate the brake power of an engine.
- Calculate the indicated power of an engine.
- Calculate the Mean Effective Pressure of an engine.
- Calculate the various efficiencies of an engine.
- Examine how the performance of real engines may be improved.
- Explain the purpose f a turbo charger.
- Explain the purpose of a supercharger
- Explain the advantages of inter-cooling.
- Discuss the advantages of using waste heat recovery.
- Discuss the use of waster heat boilers.
- Discuss combined heating and power systems.

1 FUEL POWER (F.P.)

Fuel power is the thermal power released by burning fuel inside the engine.

F.P. = mass of fuel burned per second x calorific value of the fuel.

F.P. = m_f x C.V.

All engines burn fuel to produce heat that is then partially converted into mechanical power. The chemistry of combustion is not dealt with here. The things you need to learn at this stage follow.

1.1 AIR FUEL RATIO

This is the ratio of the mass of air used to the mass of fuel burned.

Air Fuel Ratio = m_a/m_f

The ideal value that just completely burns all the fuel is called the *STOICHIOMETRIC RATIO.*

In reality, the air needed to ensure complete combustion is greater than the ideal ratio. This depends on how efficient the engine is at getting all the oxygen to meet the combustible elements. The volume of air drawn into the engine is theoretically equal to the capacity of the engine (the swept volumes of the cylinders). The mass contained in this volume depends upon the pressure and temperature of the air. The pressure in particular depends upon the nature of any restrictions placed in the inlet flow path.

1.2 CALORIFIC VALUE

This is the heat released by burning 1 kg of fuel. There is a higher and lower value for fuels containing hydrogen. The lower value is normally used because water vapour formed during combustion passes out of the system and takes with it the latent energy.

WORKED EXAMPLE No.1

An engine consumes 0.01573 kg/s of air. The air fuel ratio is 12/1. The calorific value is 46 MJ/kg. Calculate the Fuel Power.

SOLUTION

Air consumed m_a= 0.01573 kg/s.
Mass of fuel m_f = 0.01573/12 = 0.00131 kg/s

Heat released F.P.= calorific value x m_f = 46 000 kJ/kg x 0.00131 kg/s
F.P.= 60.3 KW

SELF ASSESSMENT EXERCISE No.1

1. An engine consumes 43.1 g/s of air with an air/fuel ratio of 13/1. The calorific value is 45 MJ/kg. Calculate the heat released by combustion. (149 kW)

2. An engine requires 120 kW of fuel power by burning fuel with a calorific value of 37 MJ/kg. The air fuel/ratio required is 14/1. Calculate the mass flow rate of air required. (45.4 g/s)

2 BRAKE POWER (B.P.)

Brake power is the output power of an engine measured by developing the power into a brake dynamometer on the output shaft. Dynamometers measure the speed and the Torque of the shaft. The Brake Power is calculated with the formula

B.P. = $2\pi NT$ where N is the shaft speed in rev/s
T is the torque in N m

You may need to know how to work out the torque for different types of dynamometers. In all cases the torque is **T = net brake force x radius**

The two main types are shown below.

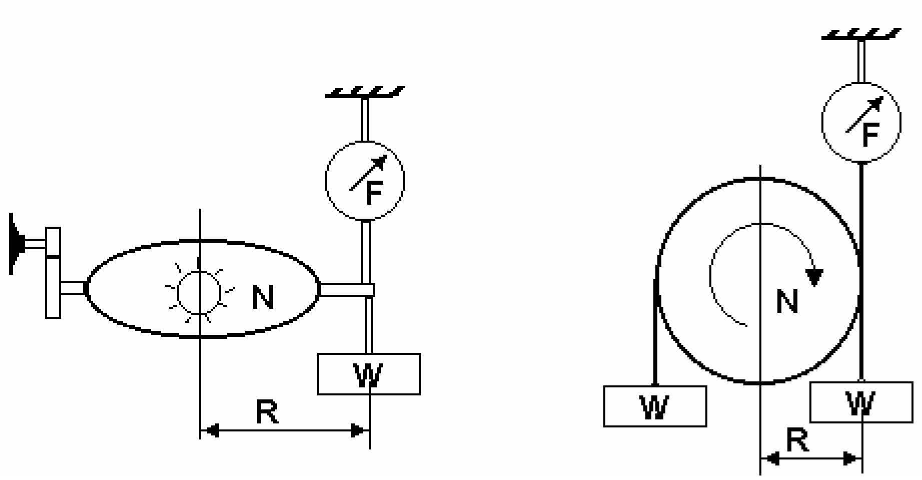


Figure 1 Figure 2

Figure 1 shows a hydraulic dynamometer which absorbs the engine power with an impeller inside a water filled casing. Basically it is a pump with a restricted flow. The power heats up the water and produces a torque on the casing. The casing is restrained by a weight pulling down and a compression spring balance pushing down also. The torque is then (F + W) x R.

Figure 2 shows a friction drum on which a belt rubs and absorbs the power by heating up the drum which is usually water cooled. The belt is restrained by a spring balance and one weight. the second equal weight acts to cancel out the other so the torque is given by T = F R.

3 INDICATED POWER

This is the power developed by the pressure of the gas acting on the pistons. It is found by recording the pressure against volume inside the piston. Such diagrams are called indicator diagrams and they are taken with engine indicators. The diagram shows a typical indicator diagram for an internal combustion engine.

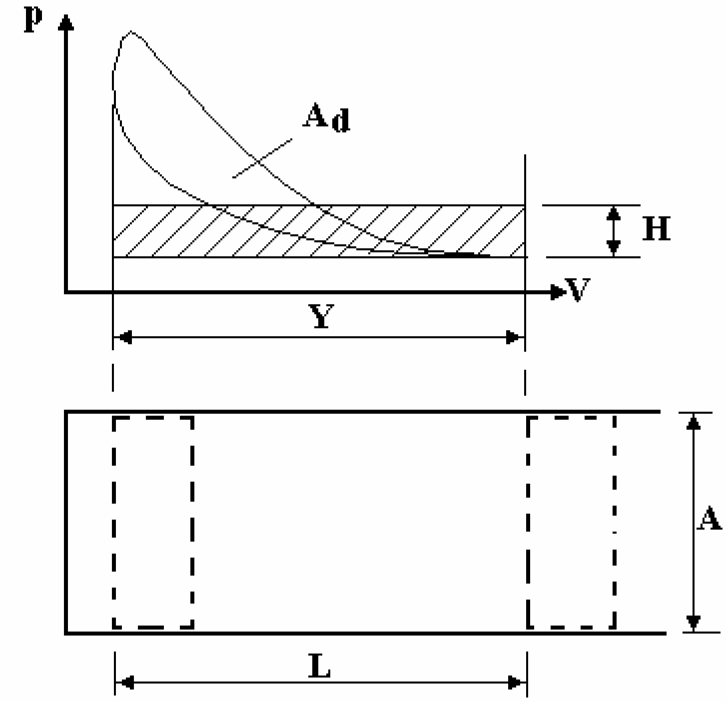


Figure 3

The average force on the piston throughout one cycle is F where

F = MEP x Area of piston = pA

The Mean Effective Pressure p is the mean pressure during the cycle.

The work done during one cycle is

W = F L = pAL

L is the stroke.

The number of cycles per second is N. The Indicated Power is then

I.P. = pLAN per cylinder.

Note for a 4 stroke engine N = ½ the shaft speed.

The mean effective pressure is found from the indicator diagram as follows.

The area enclosed by the indicator diagram represents the work done per cycle per cylinder. Let this area be A_d mm^2. The average height of the graph is H mm. The length of the diagram is Y mm. The hatched area is equal to A_d and so

$$A_d = YH$$
$$H = A_d/Y$$

In order to convert H into pressure units, the pressure scale (or spring rate) of the indicator measuring system must be known. Let this be S_p kPa/mm. The MEP is then found from

$$MEP = S_p\ H$$

This is also known as the Indicated Mean Effective Pressure because it is used to calculate the Indicated Power. There is also a Brake Mean Effective Pressure (BMEP) which is the mean pressure which would produce the brake power.

$$BP = (BMEP)\ LAN$$

The BMEP may be defined from this as

$$\mathbf{BMEP = BP/LAN}$$

4 EFFICIENCIES

4.1 BRAKE THERMAL EFFICIENCY

This tells us how much of the fuel power is converted into brake power.

$$\eta_{BTh} = \mathbf{B.P./F.P.}$$

4.2 INDICATED THERMAL EFFICIENCY

This tells us how much of the fuel power is converted into brake power.

$$\eta_{ITh} = \mathbf{I.P./F.P.}$$

4.3 MECHANICAL EFFICIENCY

This tells us how much of the indicated power is converted into brake power. The difference between them is due to frictional losses between the moving parts and the energy taken to run the auxiliary equipment such as the fuel pump, water pump, oil pump and alternator.

$$\eta_{mech} = \mathbf{B.P./I.P.}$$

WORKED EXAMPLE No.2

A 4 cylinder, 4 stroke engine gave the following results on a test bed.

Shaft Speed	N = 2 500 rev/min
Torque arm	R = 0.4 m
Net Brake Load	F = 200 N
Fuel consumption	m_f = 2 g/s
Calorific value	C.V.= 42 MJ/kg
Area of indicator diagram	A_d = 300 mm^2
Pressure scale	S_p = 80 kPa/mm
Stroke	L = 100 mm
Bore	D = 100 mm
Base length of diagram	Y = 60 mm.

Calculate the B.P., F.P., I.P., MEP, η_{BTh}, η_{ITh}, and η_{mech},

SOLUTION

BP = 2 πNT = 2π x (2500/60)x (200 x 0.4) = 20.94 kW

FP = mass/s x C.V. = 0.002 kg/s x 42 000 kJ/kg = 84 kW

IP = pLAN

p = MEP = A_d/Y x S_p = (300/60) x 80 = 400 kPa
IP = 400 x 0.1 x (π x 0.1^2/4) x (2500/60)/2 per cylinder
IP = 6.54 kW per cylinder.
For 4 cylinders IP = 6.54 x 4 = 26.18 kW

η_{BTh}= 20.94/84 = 24.9%
η_{ITh}= 26.18/84 = 31.1 %
η_{mech}= 20.94/26.18 = 80%

SELF ASSESSMENT EXERCISE No.2

1. A 4 stroke spark ignition engine gave the following results during a test.

Number of cylinders	6
Bore of cylinders	90 mm
Stroke	80 mm
Speed	5000 rev/min
Fuel consumption rate	0.225 kg/min
Calorific value	44 MJ/kg
Net brake load	180 N
Torque arm	0.5 m
Net indicated area	720 mm^2
Base length of indicator diagram	60 mm
Pressure scale	40 kPa/mm

Calculate the following.

i. The Brake Power. (47.12 kW)
ii. The Mean effective Pressure. (480 kPa)
iii. The Indicated Power. (61 kW)
iv. The Mechanical Efficiency. (77.2%)
v. The Brake Thermal efficiency. (28.6%)

2. A two stroke spark ignition engine gave the following results during a test.

Number of cylinders	4
Bore of cylinders	100 mm
Stroke	100 mm
Speed	2000 rev/min
Fuel consumption rate	5 g/s
Calorific value	46 MJ/kg
Net brake load	500 N
Torque arm	0.5 m
Net indicated area	1 500 mm^2
Base length of indicator diagram	66 mm
Pressure scale	25 kPa/mm

Calculate the following.

i. The Indicated thermal efficiency. (25.9%)
ii. The Mechanical Efficiency. (88%)
iii. The Brake Thermal efficiency. (22.8%)

3. A two stroke diesel engine gave the following results during a test.

Number of cylinders	4
Bore of cylinders	80 mm
Stroke	80 mm
Speed	2 200 rev/min
Fuel consumption rate	1.6 cm^3/s
Fuel density	750 kg/m^3
Calorific value	60 MJ/kg
Nett brake load	195 N
Torque arm	0.4 m
Nett indicated area	300 mm^2
Base length of indicator diagram	40.2 mm
Pressure scale	50 kPa/mm

Calculate

i. The Indicated thermal efficiency. (30.5%)
ii. The Mechanical Efficiency. (81.7%)
iii. The Brake Thermal efficiency. (25%)

4. A four diesel engine gave the following results during a test.

Number of cylinders	4
Bore of cylinders	90 mm
Stroke	80 mm
Speed	5 000 rev/min
Fuel consumption rate	0.09 kg/min
Calorific value	44 MJ/kg
Nett brake load	60 N
Torque arm	0.5 m
MEP	280 kPa

Calculate the following.

i. The Mechanical Efficiency. (66.1%)
ii. The Brake Thermal efficiency. (23.8%)
iii. The Indicated Thermal Efficiency. (36%)

5.

PERFORMANCE IMPROVEMENT

5.1 INTERNAL COMBUSTION ENGINES

In the preceding work it has been shown that the efficiency of internal combustion engines depends upon the volume compression ratio and for gas turbines it depends upon the pressure compression ratio. This section debates some of the practical problems and solutions for improvement of performance.

5.1.1 ENGINE MANAGEMENT

High compression ratios in spark ignition engines leads to pre-ignition as the fuel detonates without the aid of a spark before the point of maximum compression. This produces very high peaks of pressure and damages the engine. Reducing this problem involves the use of fuels that are less prone to detonate (high octane ratings). Timing of the spark ignition is also vital. The correct timing depends upon many factors such as air/fuel ratio and engine load. Modern engines use fuel management systems in which the timing and the air/fuel ratio are controlled by a computer connected to sensors. This allows greater compression ratios and hence efficiency.

Fuel injection gives a measure of control over the combustion process and this is now possible with petrol engines as well as diesel engines.

5.1.2 TURBO CHARGING

The power produced by an engine basically depends on the amount of fuel burned. This is limited by the mass of air in the cylinder. To burn more fuel requires more air. Blowing air into the cylinders under pressure may do this and requires an air blower. This is a successful process in compression ignition engines but increases the problem of pre-ignition on spark ignition engines. The blower may be driven by a mechanical connection direct to the engine crankshaft. The Lobe compressor shown in figure 4 is commonly used. This arrangement is called SUPERCHARGING. On large engines, the blower is driven by a small gas turbine that uses the exhaust from the engine to power it. This is called TURBO CHARGING. Figure 5 shows a turbo charger.

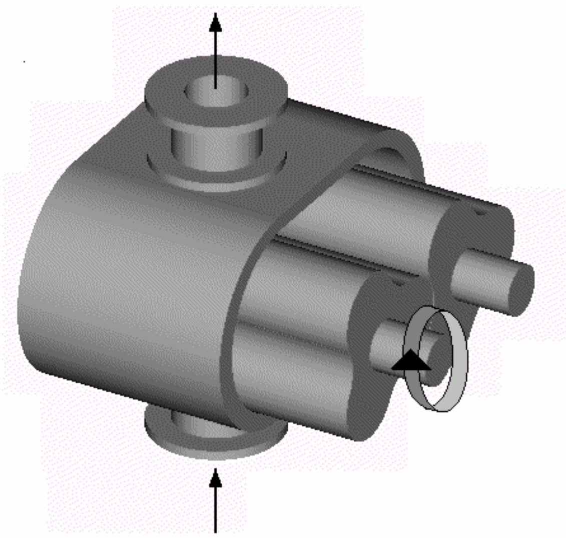


Fig. 4

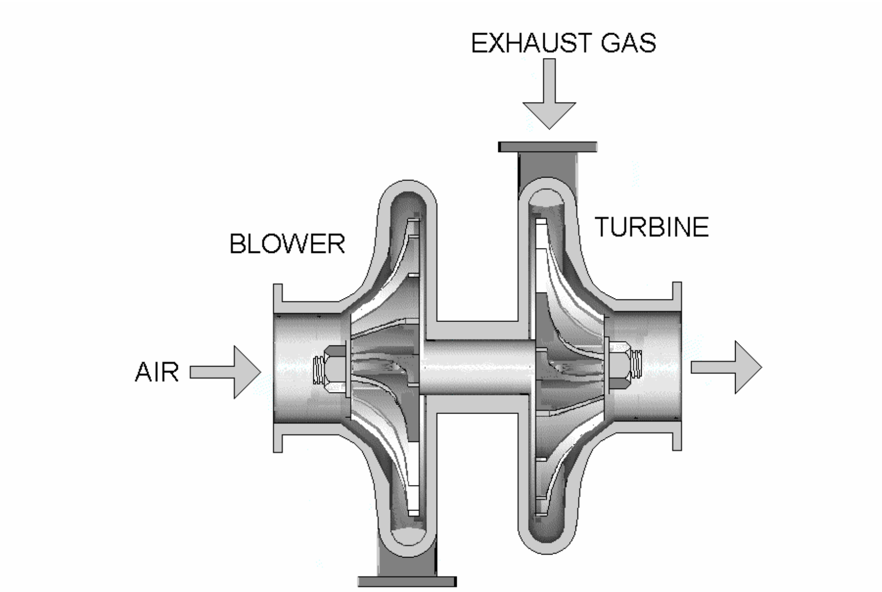


Fig. 5

5.1.3 INTER-COOLING

The mass contained in a volume of air depends upon the temperature. The colder the air, the more mass it contains. Compressed air is naturally hot so if it can be cooled after compression, a greater mass of air may be supplied to the cylinder.

Turbo charging and inter-cooling on large compression ignition engines leads to improved efficiency as well as increased power. Fig.6 shows an intercooler designed to fit under a car radiator.

Fig. 6

6 EXHAUST GAS HEAT RECOVERY

When large amounts of hot exhaust gas is produced, by either gas turbines or large diesel engines, the heat in the exhaust gas may be recovered for useful applications such as using it to produce hot water or steam in a boiler. A factory might well use a gas turbine to produce electric power and hot water or steam. This is more economical than buying electricity.

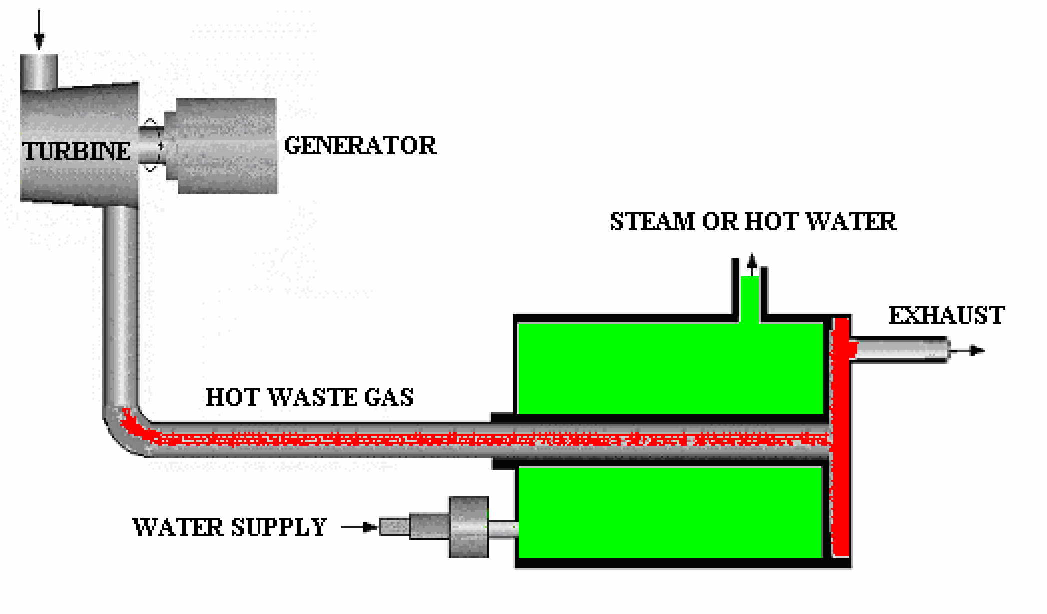


Fig. 7

When the generation of electric power is combined with a waste heat system, it is called COMBINED POWER AND HEATING. On a large scale, this is sometimes done on major power stations. The enormous quantities of waste heat produced in the form of hot water from the condensers may be pumped through a hot water pipe system to heat buildings or large greenhouses.

SELF ASSESSMENT EXERCISE No.3

A factory is to be built that uses both electricity and steam. There are two proposals to be considered.

PROPOSAL 1

Produce steam in an oil fired boiler and purchase electricity.

PROPOSAL 2

Generate electric power with a gas turbine and produce steam in a waste heat boiler using the exhaust gas.

OPERATING DATA FOR STEAM BOILER

Mass Flow rate	1 kg/s
Steam condition	5 bar and dry saturated.
Feed water temperature	15°C.

When burning fuel, the combustion efficiency is typically 85%

When using exhaust gas, the heat transfer from the gas may be assumed to be equal to the heat gained by the water and steam. The exhaust gas is cooled to 100°C before leaving the boiler.

GAS TURBINE DATA

Pressure ratio	7
Inlet air pressure	1 bar
Inlet air temperature	15°C
Combustion chamber temperature	1500°C

FUEL DATA

Any fuel to be burned in either the gas turbine or the boiler will be light oil with a calorific value of 42 MJ/kg. The cost of fuel is 12.7 pence per kg.

Electricity cost 2.5 pence per kWhr (1 kWhr = 3600 kJ)

PROPERTIES

AIR	BURNED GAS
Cp=1.005 kJ/kg K	Cp=1.1 kJ/kg K
γ=1.4	γ=1.3

Produce a report comparing the costs for both schemes. You will need to do the following tasks.

GUIDANCE

In the course of your work you will need to do the following.

STEAM BOILER

You will need to determine the following.

i. The energy required to make the steam.
ii. The fuel required in kg/s.
iii. The mass of exhaust gas required to produce the same steam in kg/s.

GAS TURBINE

You will need to equate the heat transfer from burning fuel to the energy required to raise the temperature in the combustion chamber.

You will need to determine the following.

vi. The mass flow of air.
v. The fuel burned in kg/s.
vi. The Power input of the compressor.
vii. The power output of the turbine.
viii. The net power for generating electricity.

COSTING

Base the cost of option 1 on the cost of fuel plus the cost of buying the same electricity as for option 2.

Base the cost on the cost of fuel only.

What other factors would you consider when making a decision on which option take?

EDEXCEL HIGHERS
ENGINEERING THERMODYNAMICS H2
NQF LEVEL 4

OUTCOME 3
TUTORIAL No. 6 - RECIPROCATING AIR COMPRESSORS

Air compressors

Property diagrams: theoretical pressure-volume diagrams for single and multistage compressors; actual indicator diagrams; actual, isothermal and adiabatic compression curves; induction and delivery lines; effects of clearance volume

Performance characteristics: free air delivery; volumetric efficiency; actual and isothermal work done per cycle; isothermal efficiency

First law of thermodynamics: input power; air power; heat transfer to intercooler and aftercooler; energy balance

Faults and hazards: effects of water in compressed air; causes of compressor fires and explosions

In order to complete this section you should be familiar with gas laws and polytropic gas processes. You will study the principles of reciprocating compressors in detail and some principles of rotary compressors. On completion you should be able to do the following.

- Describe the working principles of reciprocating compressors.
- Describe the basic design of various other compressors.
- Define and calculate swept volume.
- Define and calculate volumetric efficiency.
- Define and calculate isothermal efficiency.
- Define and calculate indicated power.
- State the benefits of cooling.
- Calculate the heat rejected through cooling.
- Define and calculate the inter-stage pressures for multiple compressors.

INTRODUCTION

Air is an expansive substance and dangerous when used at high pressures. For this reason, most applications are confined to things requiring low pressures (10 bar or lower) but there are industrial uses for high pressure air up to 100 bar.

The common source of the air is the compressor. There are many types of compressors with different working principles and working conditions. This is a list of the main types.

- Reciprocating.
- Sliding vane compressors.
- Lobe compressors.
- Helical screws.
- Centrifugal.
- Axial turbine compressors.

The function of all of them is to draw in air from the atmosphere and produce air at pressures substantially higher. Usually a storage vessel or receiver is used with the compressor.

Compressed air has many applications. It is also used for powering pneumatically operated machines. It is used as a power medium for workshop tools such as shown in Fig.1.

Fig. 1

1. COMPRESSED AIR

1.1 ATMOSPHERIC VAPOUR

Water vapour in the atmosphere has important consequences on compressors. Atmospheric air contains *WATER VAPOUR* mixed with the other gases. The ratio of the mass of water vapour in the air to the mass of the air is called the *ABSOLUTE HUMIDITY*. The quantity of water that can be absorbed into the air at a given pressure depends upon the temperature. The hotter the air, the more water it can absorb. When the air contains the maximum possible amount of vapour it is at its dew point and rain or fog will appear. The air is then said to have 100% humidity. When the air contains no water vapour at all (dry air), it has 0% humidity. This is called the *RELATIVE HUMIDITY*. For example if the air has 40% relative humidity it means that it contains 40% of the maximum that it could contain. There are various ways to determine the humidity of air and instruments for doing this are called *HYGROMETERS*.

The importance of humidity to air compressors is as follows. When air is sucked into the compressor, it brings with it water vapour. When the air is compressed the pressure and the temperature of the air goes up and the result is that the compressed air will have a relative humidity of about 100% and it will be warm. When the air leaves the compressor it will cool down and the water vapour will condense. Water will then clog the compressor, the receiver and the pipes.

Water damages air tools; ruins paint sprays, and corrode pipes and equipment. For this reason the water must be removed and the best way is to use a well designed compressor installation and distribution network.

1.2 TYPICAL RECIPROCATING COMPRESSOR LAYOUT

Figure 2 shows the layout of a two stage reciprocating compressor typically for supplying a workshop.

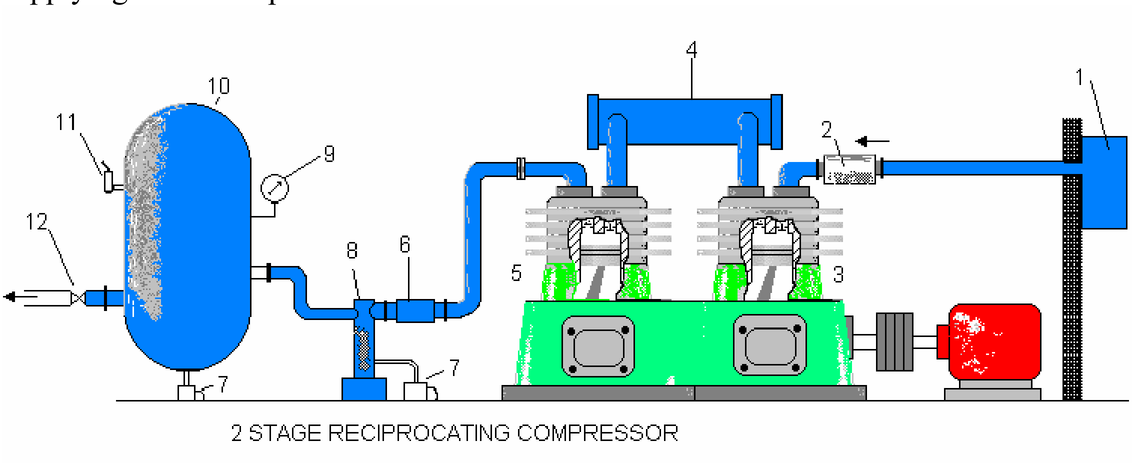


Fig.2

1. Induction box and silencer on outside of building with course screen.
2. Induction filter.
3. Low pressure stage.
4. Intercooler.
5. High pressure stage.
6. Silencer.
7. Drain trap.
8. After cooler
9. Pressure gauge.
10. Air receiver.
11. Safety pressure relief valve.
12. Stop valve

1.3 HAZARDS AND SAFETY

Dangers associated with air compressors are as follows.

- Pressure vessels may rupture.
- Oil leaks may burn or cause other accidents.
- Oil in the compressed air may explode.
- Water in the compressed air may damage equipment.

There are many regulations concerning the use, maintenance and inspection of pressure vessels. Vessels must have the safe working pressure marked on them. They must have a pressure gauge and be fitted with an isolating valve. They must also be fitted with a pressure release valve to prevent overpressure.

In particular, with reciprocating compressors, if water accumulates in the cylinder, it may fill the space so completely that it prevents the piston reaching the end of its travel and cause damage to the piston and head.

Oil in the cylinder can explode during compression. Normally the operating pressure is not high enough to produce the temperature required. However, if the outlet becomes blocked (e.g. the valve sticks or the outlet pipe is closed with an isolating valve), then the danger exists.

Oil or water in the air can also cause damage when supplied to some kinds of tools. For this reason a good installation fully conditions the air to remove water, dirt and oil.

The following is a list of precautions to be taken against fires and explosions.

- Avoid overheating.
- Keep discharge temperatures within the recommended limits.
- Keep the deposit formation to a minimum by using the correct lubricant.
- Ensure efficient filtration of the air. This reduces wear on the valves and pistons and reduces deposits of carbonised particles.
- Avoid over-feeding oil to the cylinders.
- Minimise the carry over of oil between stages.
- Avoid high temperatures and low airflow when idling.
- Keep the coolers in good condition.
- Do not use solvents for cleaning or use anywhere near to an installation as the vapour given off can ignite.
- Do not allow naked flames (e.g. smoking) near to an installation when it is opened.

1.4 FREE AIR DELIVERY

When a gas such as air flows in a pipe, the mass of the air depends upon the pressure and temperature. It would be meaningless to talk about the volume of the air unless the pressure and temperature are considered. For this reason the volume of air is usually stated as FREE AIR DELIVERY or FAD. In other words FAD refers to the volume the air would have if let out of the pipe and returned to atmospheric pressure and temperature.

The FAD is also the volume of air drawn into a compressor from the atmosphere. After compression and cooling the air is returned to the original temperature but it is at a higher pressure. Suppose atmospheric conditions are $p_a T_a$ and V_a (the FAD) and the compressed conditions are p, V and T. Applying the gas law we have

$$\frac{pV}{T} = \frac{p_a V_a}{T_a} \qquad V_a = \frac{pVT_a}{Tp_a} = F.A.D$$

2. CYCLE FOR RECIPROCATING COMPRESSOR

2.1 THEORETICAL CYCLE

Figure 3 shows the basic design of a reciprocating compressor.

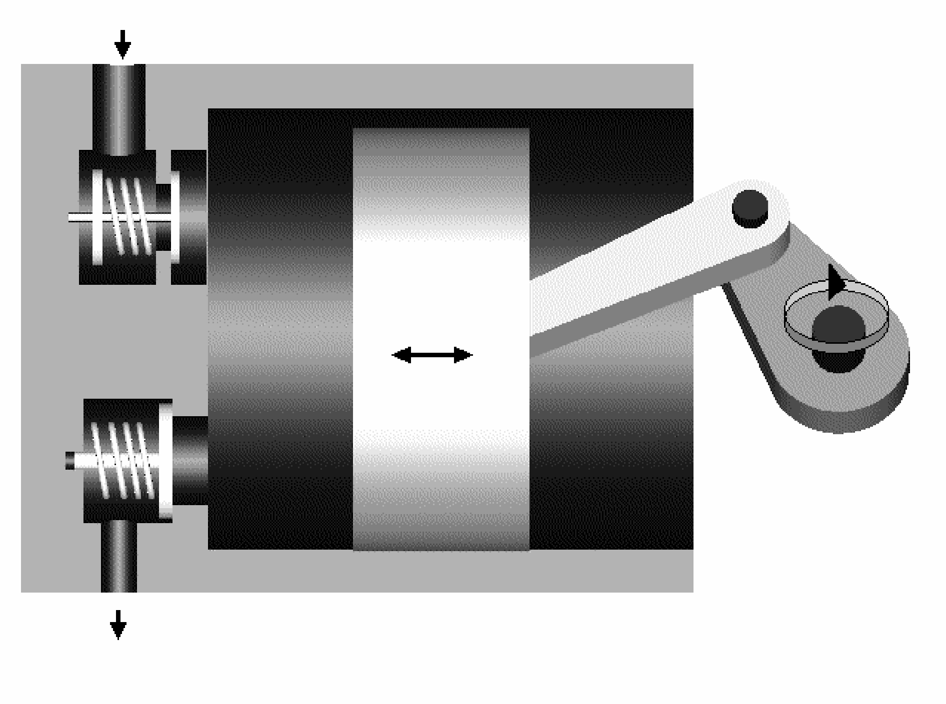


Fig. 3

Figure 4 shows the pressure – volume diagram for an ideal reciprocating compressor.

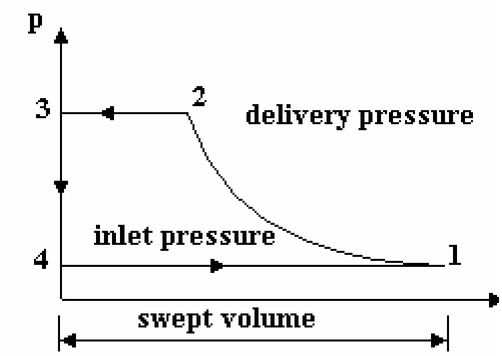


Fig.4

The piston reciprocates drawing in gas, compressing it and expelling it. If the piston expels all the air and there is no restriction at the valves, the pressure - volume cycle is as shown. Gas is induced from 4 to 1 at the inlet pressure. It is then trapped inside the cylinder and compressed according the law $pV^n = C$. At point 2 the pressure reaches the same level as that in the delivery pipe and the outlet valve pops open. Air is then expelled at the delivery pressure. The delivery pressure might rise very slightly during expulsion if the gas is being compacted into a fixed storage volume. This is how pressure builds up from switch on.

2.2 VOLUMETRIC EFFICIENCY

In reality, the piston cannot expel all the gas and a clearance volume is needed between the piston and the cylinder head. This means that a small volume of compressed gas is trapped in the cylinder at point 3. When the piston moves away from the cylinder head, the compressed gas expands by the law $pV^n = C$ until the pressure falls to the level of the inlet pressure. At point 4 the inlet valve opens and gas is drawn in. The volume drawn in from 4 to 1 is smaller than the swept volume because of this expansion.

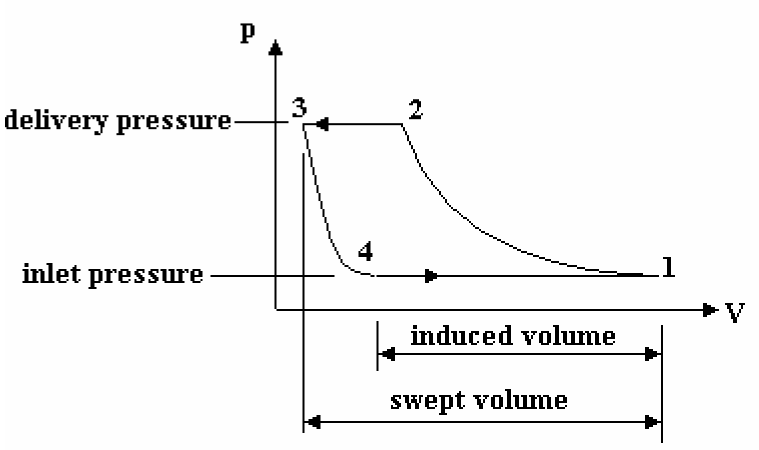


Fig. 5

The volumetric efficiency is defined by the following equation.

$$\eta_{vol} = \frac{\text{Induced Volume}}{\text{Swept Volume}}$$

It may be shown that this reduces to $\eta_{vol} = 1 - c\left[\left(\frac{p_2}{p_1}\right)^{(1/n)} - 1\right]$

This efficiency is made worse if leaks occur past the valves or piston.

The clearance ratio is defined as c = Clearance volume/Swept volume.

Ideally the process 2 to 3 and 4 to 1 are isothermal. That is to say, there is no temperature change during induction and expulsion.

WORKED EXAMPLE No.1

Gas is compressed in a reciprocating compressor from 1 bar to 6 bar. The FAD is 13 dm^3/s. The clearance ratio is 0.05. The expansion part of the cycle follows the law $pV^{1.2}$ =C. The crank speed is 360 rev/min. Calculate the swept volume and the volumetric efficiency.

SOLUTION

Swept Volume = V   Clearance volume = 0.05 V

Consider the expansion from 3 to 4 on the p-V diagram.

$p_4 = 1$ bar $p_3 = 6$ bar. $p_3V_3^{1.2} = p_4V_4^{1.2}$

$6(0.05V)^{1.2} = 1(V_4^{1.2})$

$V_4 = 0.222V$ or 22.2%% of V

F.A.D. = 0.013 m^3/s.

$V_1 = V + 0.05V = 1.05V$

Induced volume = $V_1 - V_4 = 1.05V - 0.222V = 0.828V$

Induced volume = 0.013 m^3/s

V = 0.013/0.828 = 0.0157 m^3/s

Crank speed = 6 rev/s so the swept volume = 0.0157/6 = 2.62 dm^3.

$$\eta_{vol} = \frac{\text{Induced Volume}}{\text{Swept Volume}}$$

$$\eta_{vol} = \frac{0.828V}{V} = 82.8\,\%$$

WORKED EXAMPLE No.2

Show that the volumetric efficiency of an ideal single stage reciprocating compressor with a clearance ratio is c is given by the expression below.

$$\eta_{vol} = 1 - c\left[\left(\frac{p_H}{p_L}\right)^{(1/n)} - 1\right]$$

p_L is the inlet pressure and p_H the outlet pressure.

SOLUTION

Swept volume = $V_1 - V_3$ Induced volume = $V_1 - V_4$

Clearance volume = V_3

$$c = \frac{V_3}{V_1 - V_3}$$

$$V_1 - V_3 = \frac{V_3}{c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(A)$$

$$\frac{V_1}{V_3} - 1 = \frac{1}{c}$$

$$\frac{V_1}{V_3} = \frac{1}{c} + 1 = \frac{(1+c)}{c} \ldots\ldots\ldots\ldots\ldots\ldots(B)$$

$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3} \quad \text{Substitute (A) for the bottom line.}$$

$$\eta_{vol} = \frac{c(V_1 - V_4)}{V_3} = c\left\{\frac{V_1}{V_3} - \frac{V_4}{V_3}\right\} \quad \text{Substitute (B)}$$

$$\eta_{vol} = c\left\{\left(\frac{(1+c)}{c}\right) - \frac{V_4}{V_3}\right\} = 1 + c - c\frac{V_4}{V_3}$$

$$\frac{V_4}{V_3} = \left(\frac{p_3}{p_4}\right)^{1/n} = \left(\frac{p_H}{p_L}\right)^{1/n}$$

$$\eta_{vol} = 1 + c - c\left(\frac{p_H}{p_L}\right)^{1/n}$$

$$\eta_{vol} = 1 - c\left\{\left(\frac{p_H}{p_L}\right)^{1/n} - 1\right\}$$

WORKED EXAMPLE No.3

A single stage reciprocating compressor has a clearance volume of 20 cm^3. The bore and stroke are 100 mm and 80 mm respectively. The compression and expansion processes have a polytropic index of 1.25. The inlet and outlet pressures are 1 and 6 bar respectively.

Determine the volumetric efficiency.

SOLUTION

The swept volume is the product of stroke and bore area.

$$\text{S.V.} = 80 \text{ x } \frac{\pi \text{ x } 100^2}{4} = 628.3 \text{ x } 10^3 \text{ mm}^3 \text{ or } 628.3 \text{ cm}^3$$

The Clearance volume is 20 cm^3.

$$c = \frac{20}{628.3} = 0.03183$$

$$\eta_{vol} = 1 - c\left\{\left(\frac{p_H}{p_L}\right)^{1/n} - 1\right\} = 1 - 0.03183\left\{\left(\frac{6}{1}\right)^{1/1.25} - 1\right\}$$

$$\eta_{vol} = 1 - 0.03183\{4.192 - 1\} = 1 - 0.1016 = 0.898 \text{ or } 89.8\%$$

2.3 REAL p-V DIAGRAMS

In real compressors the warm cylinder causes a slight temperature rise over the induction from 4 to 1. The gas is restricted by the valves and p_1 is slightly less than p_4. The valves also tend to move so the real cycle looks more like figure 6

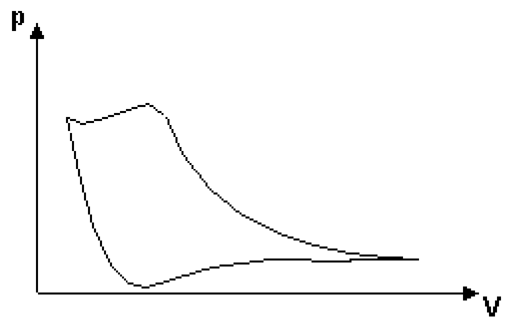


Fig. 6

WORKED EXAMPLE No.4

A single stage reciprocating compressor produces a FAD of 2 dm^3/s at 420 rev/min. The inlet pressure is 1 bar. The polytropic index is 1.2 for the compression and expansion. The outlet pressure is 8 bar. The clearance volume is 10 cm^3.

Determine the volumetric efficiency.

SOLUTION

First find the induced volume. This is the free air drawn in for each revoution.

$$F.A.D.\text{ per rev.} = \frac{2x60}{420} = 0.2857\ \text{dm}^3/\text{rev}$$

This is the induced volume $V_1 - V_4$

The clearance volume is $V_3 = 10\ \text{cm}^3$ or $0.01\ \text{dm}^3$.

Next we need to find V_4

$$p_3V_3^n = p_4V_4^n$$

$$8 \text{ x } 0.01^{1.2} = 1xV_4^{1.2} \quad \text{hence } V_4 = 0.0566\ \text{dm}^3$$

$$V_1 = 0.2857 + 0.0566 = 0.3423\text{m}^3$$

$$\text{Swept volume} = V_1 - V_3 = 0.3323\ \text{dm}^3$$

$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3} = \frac{0.2857}{0.3323} = 0.859 \text{ or } 85.9\%$$

2.4 INDICATED POWER

The indicated work per cycle is the area enclosed by the p - V diagram. The easiest way to find this is by integrating with respect to the pressure axis.

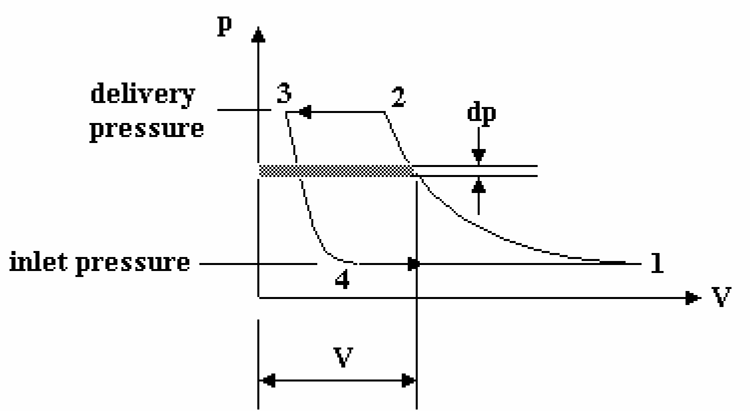


Fig.7

2.4.1 POLYTROPIC PROCESSES

If the processes 1 to 2 and 3 to 4 are polytropic $pV^n = C$. $V = C^{1/n} p^{-1/n}$

The work done is given by $W = \int V dp$

Consider the expression

$$\int V dp = C^{1/n} \int p^{-1/n} dp = \left[\frac{C^{1/n} p^{1-1/n}}{(1-1/n)} \right] = \frac{n}{n-1}\left(C^{1/n} p^{1-1/n}\right)$$

$$C = pV^n \qquad c^{\frac{1}{n}} = p^{\frac{1}{n}} V \quad \text{substitute in and}$$

$$\int V dp = \left[\frac{nVp^{1/n} p^{1-1/n}}{(n-1)} \right] = \left[\frac{npV}{(n-1)} \right]$$

Between the limits of p2 and p1 this becomes $W_{1\text{-}2} = \dfrac{n[p_2V_2 - p_1V_1]}{(n-1)}$

Between the limits p4 and p3 this becomes $W_{4\text{-}3} = \dfrac{n[p_4V_4 - p_3V_3]}{(n-1)}$

Subtract one from the other to find the indicated work.

$$W = \frac{n[p_2V_2 - p_1V_1]}{(n-1)} - \frac{n[p_3V_3 - p_4V_4]}{(n-1)}$$

$$W = \left(\frac{n}{n-1}\right)\left[p_1V_1\left\{\left(\frac{p_2V_2}{p_1V_1}\right) - 1\right\} - p_4V_4\left\{\left(\frac{p_3V_3}{p_4V_4}\right) - 1\right\}\right]$$

Substitute the relationships $\dfrac{V_2}{V_1} = \left(\dfrac{p_2}{p_1}\right)^{-\frac{1}{n}}$ and $\dfrac{V_3}{V_{41}} = \left(\dfrac{p_3}{p_4}\right)^{-\frac{1}{n}}$

$$W = \left(\frac{n}{n-1}\right)\left[p_1V_1\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} - p_4V_4\left\{\left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} - 1\right\}\right]$$

$$W = \left(\frac{n}{n-1}\right)\left[p_1V_1\left\{r_p^{\frac{n-1}{n}} - 1\right\} - p_4V_4\left\{r_p^{\frac{n-1}{n}} - 1\right\}\right]$$

$$W = \left(\frac{n}{n-1}\right)\left\{r_p^{\frac{n-1}{n}} - 1\right\}[p_1V_1 - p_4V_4]$$

since $p_1 = p_4$

$$W = p_1\left(\frac{n}{n-1}\right)\left\{r_p^{\frac{n-1}{n}} - 1\right\}[V_1 - V_4]$$

$W = p_1\left(\dfrac{n}{n-1}\right)\left\{r_p^{\frac{n-1}{n}} - 1\right\}[\Delta V]$ where ΔV is the induced volume.

The corresponding induced mass is $m = \dfrac{p_1 \Delta V}{RT_1}$

$$W = mRT_1\left(\frac{n}{n-1}\right)\left\{r_p^{\frac{n-1}{n}} - 1\right\}$$

If the clearance volume is ignored $\Delta V = V_1$.

$$W = \left(\frac{n}{n-1}\right) p_1 V_1 \left\{ r_p^{\frac{n-1}{n}} - 1 \right\} \quad \text{or} \quad W = mRT_1 \left(\frac{n}{n-1}\right) \left\{ r_p^{\frac{n-1}{n}} - 1 \right\}$$

2.4.2 ISOTHERMAL PROCESSES

If the processes 1 to 2 and 3 to 4 are isothermal then $pV = C$. $V = C^1 p^{-1}$

The work done is given by $W = \int V dp$

Consider the expression

$$\int V dp = C \int p^{-1} dp = C \ln p$$

Between the limits of p2 and p1 this becomes $p_1 V_1 \ln \frac{p_2}{p_1}$

Between the limits p4 and p3 this becomes $p_4 V_4 \ln \frac{p_3}{p_3}$

The indicated work (input) is then

$$W = p_1 V_1 \ln\left(\frac{p_2}{p_1}\right) - p_4 V_4 \ln\left(\frac{p_3}{p_4}\right)$$
$$W = p_1 V_1 \ln(r_p) - p_4 V_4 \ln(r_p)$$
$$W = \ln(r_p)(p_1 V_1 - p_4 V_4)$$
since $p_1 = p_2$ we get the following.
$$W = p_1 \ln(r_p)(V_1 - V_4) = p_1 \ln(r_p)\Delta V$$
$$W = \ln(r_p) mRT_1$$

If the clearance volume is neglected $\Delta V = V_1$

$$W = \ln(r_p)(p_1 V_1)$$
$$W = \ln(r_p) mRT_1$$

m is the mass induced and expelled each cycle and W is the indicated work per cycle. The indicated power is found by multiplying W by the strokes per second.

I.P. = W x N where N is the shaft speed in Rev/s

2.5 ISOTHERMAL EFFICIENCY

The minimum indicated power is obtained when the index n is a minimum. The ideal compression is hence isothermal with n=1. The isothermal efficiency is defined as

$$\eta_{iso} = \frac{\text{ISOTHERMAL WORK}}{\text{POLYTROPIC WORK}}$$

$$\eta_{iso} = \frac{p_1 \ln(r_p)(\Delta V)}{p_1\left(\frac{n}{n-1}\right)\left\{r_p^{\frac{n-1}{n}} - 1\right\}[\Delta V]}$$

$$\eta_{iso} = \frac{(n - 1)\ln(r_p)}{n\left\{r_p^{\frac{n-1}{n}} - 1\right\}}$$

WORKED EXAMPLE No.5

A single stage reciprocating compressor draws in air at atmospheric pressure of 1.01 bar and delivers it at 9.5 bar. The polytropic index is 1.18 for the compression and expansion.

The swept volume is 1.5 dm^3 and the clearance volume is 0.10 dm^3. The speed is 500 rev/min.

Determine the following.

i. The volumetric efficiency.
ii. The free air delivery.
iii. The indicated power.
iv. The isothermal efficiency.

SOLUTION

$$\text{The clearance ratio } c = \frac{\text{clearance volume}}{\text{swept volume}} = \frac{0.1}{1.5} = 0.0667$$

$$\text{Pressure ratio } r_p = \frac{p_2}{p_1} = \frac{9.5}{1.01} = 9.406$$

$$\eta_v = 1 - c\left[\left(r_p^{\frac{1}{n}}\right) - 1\right]$$

$$\eta_v = 1 - 0.0667\left[\left(9.406^{\frac{1}{1.18}}\right) - 1\right] = 0.621$$

$$\text{Induced Volume} = \eta_v \text{ x Swept Volume} = 0.621 \text{ x } 1.5 = 0.9315 \text{ dm}^3$$

$$\text{FAD per stroke} = \text{Induced volume} = 0.9318 \text{ dm}^3$$

$$FAD \text{ per minute} = 0.9315 \text{ x } 500 = 465.8 \text{ dm}^3/\text{min or } 0.4658 \text{ m}^3/s$$

$$\text{Indicated Power} = p_1\left(\frac{n}{n-1}\right)\left[r_p^{\frac{n-1}{n}} - 1\right] \text{ x FAD}$$

$$\text{I.P.} = 1.01 \text{ x } 10^5\left(\frac{1.18}{0.18}\right)\left[9.406^{\frac{0.18}{1.18}} - 1\right] \text{ x } \frac{0.4658}{60}$$

$$I.P. = 5141.3\left[9.406^{0.1525} - 1\right]$$

$$I.P. = 5141.3[1.407 - 1] = 2096 \text{ Watt}$$

$$\eta_{iso} = \frac{(n-1)\ln(r_p)}{n\left\{r_p^{\frac{n-1}{n}} - 1\right\}} = \frac{0.18\ln 9.406}{1.18\left\{9.406^{\frac{0.18}{1.18}} - 1\right\}} = \frac{0.3419}{0.407} = 0.84$$

SELF ASSESSMENT EXERCISE No.1

1. A reciprocating air compressor operates between 1 bar and 8 bar. The clearance volume is 15 cm^3 and the swept volume is 900 cm^3. The index of compression and expansion is 1.21. Calculate the following.

    i. The ideal volumetric efficiency. (92.4%)

    ii. The ideal indicated work per cycle. (208.2 J)

    iii. The Isothermal work per cycle. (172.9 J)

    iv. The Isothermal efficiency. (83%)

2. A reciprocating air compressor following the ideal cycle has a free air delivery of 60 dm^3/s. The clearance ratio is 0.05. The inlet is at atmospheric pressure of 1 bar. The delivery pressure is 7 bar and the compression is polytropic with an index of 1.3. Calculate the following.

    i. The ideal volumetric efficiency. (82.7%)

    ii. The ideal indicated power. (14.74 KW)

    iii. The Isothermal efficiency. (79.2 %)

3. A single stage reciprocating compressor draws in air at atmospheric pressure of 1.0 bar and delivers it at 12 bar. The polytropic index is 1.21 for the compression and expansion. The swept volume is 2.0 dm^3 and the clearance volume is 0.16 dm^3. The speed is 600 rev/min. Determine the following.

    i. The ideal volumetric efficiency. (45.6%)

    ii. The free air delivery. (547.6 dm^3/min)

    iii. The ideal indicated power. (2.83 kW)

    iv. The Isothermal efficiency. (80%)

3 MULTIPLE STAGE COMPRESSORS

The main advantage to compressing the air in stages is that the air may be cooled between each stage and the overall compression is nearer to being isothermal. This reduces the power requirement and allows removal of water from the air. Two stage compressions are common but when very high pressure is required, more stages may be used.

3.1 THE EFFECT OF INTER-COOLING ON THE INDICATED WORK

Consider the p - V diagram for a compressor with two stages.

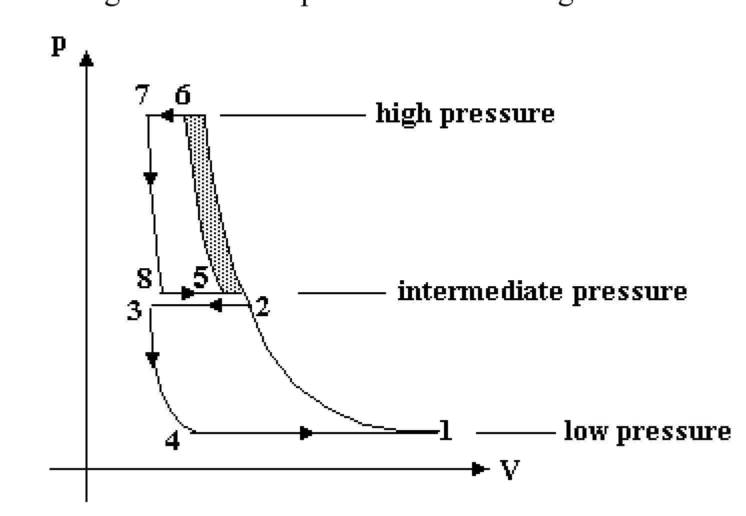


Fig.8

The cycle 1 to 4 is a normal cycle conducted between p_L and p_M. The air is expelled during process 3 to 4 at p_M and constant temperature. The air is then cooled at the intermediate pressure and this causes a contraction in the volume so that the induced volume V_8 to V_5 is smaller than the expelled volume V_2 to V_3. The high pressure cycle is then a normal cycle conducted between p_M and p_H.

The shaded area of the diagram represents the work saved by using the intercooler. The optimal saving is obtained by choosing the correct intermediate pressure. This may be found as follows.

3.2 OPTIMAL INTER-STAGE PRESSURE

$W = W_1 + W_2$ where W_1 is the work done in the low pressure stage and W_2 is the work done in the high pressure stage.

$$W = \frac{mRn(T_2 - T_1)}{(n-1)} + \frac{mRn(T_6 - T_5)}{(n-1)}$$

Since $T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(1-1/n)}$ *and* $T_6 = T_5\left(\frac{p_6}{p_5}\right)^{(1-1/n)}$

then assuming the same value of n for each stage

$$W = mR\left[\left\{\frac{nT_1}{(n-1)}\right\}\left\{\frac{p_2}{p_1}\right\}^{1-(1/n)} - 1\right] + mR\left[\left\{\frac{nT_6}{(n-1)}\right\}\left\{\frac{p_6}{p_5}\right\}^{1-(1/n)} - 1\right]$$

Since $p_2 = p_5 = p_m$ and $p_6 = p_H$ and $p_1 = p_L$

$$W = mR\left[\left\{\frac{nT_1}{(n-1)}\right\}\left\{\frac{p_M}{p_L}\right\}^{1-(1/n)} - 1\right] + mR\left[\left\{\frac{nT_6}{(n-1)}\right\}\left\{\frac{p_H}{p_M}\right\}^{1-(1/n)} - 1\right]$$

For a minimum value of W we differentiate with respect to p_M and equate to zero.

$$\frac{dW}{dp_M} = mRT_1 p_L^{(1-n)/n} p_M^{-1/n} - mRT_5 p_H^{(n-1)/n} p_M^{(1-2n)/n}$$

If the intercooler returns the air to the original inlet temperature so that $T_1 = T_5$, then equating to zero reveals that for minimum work

$$\mathbf{p_M = (p_L p_H)^{1/2}}$$

It can further be shown that when this is the case, the work done by both stages is equal.

When K stages are used, the same process reveals that the minimum work is done when the pressure ratio for each stage is $\mathbf{(p_L/p_H)^{1/K}}$

4 ROTARY COMPRESSORS

Figure 9 shows three types of rotary compressors. From left to right - the vane type, centrifugal type and axial flow type. Figure 10 shows a screw compressor.

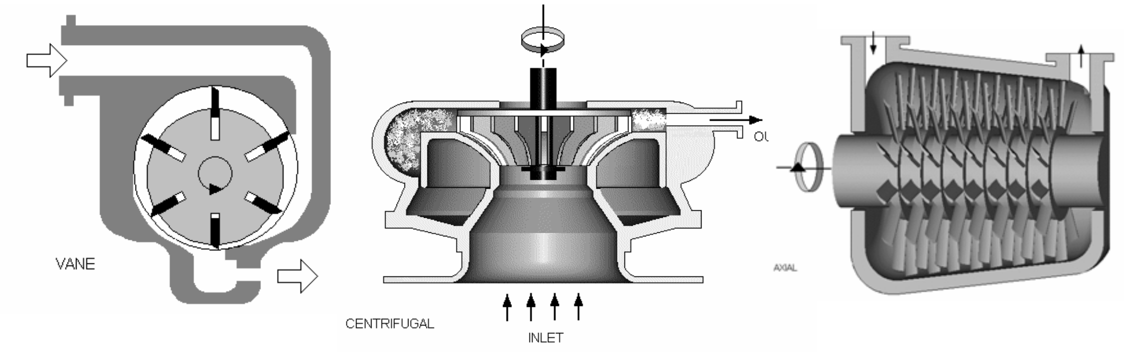


Fig. 9

4.1 VANE TYPE

The vanes fit in slots in the rotor. The rotor is eccentric to the bore of the cylinder. When the rotor is turned, centrifugal force throws the vanes out against the wall of the cylinder. The space between the vanes grows and shrinks as the rotor turns so if inlet and outlet passages are cut in the cylinder at the appropriate point, air is drawn in, squeezed and expelled. This type of compressor is suitable for small portable applications and is relatively cheap. Vane compressors often use oil to lubricate and cool the air and a system similar to that shown for the screw compressor (figure 10) is used.

4.2 CENTRIFUGAL TYPE

The rotor has a set of vanes shaped as shown in the diagram. When the rotor spins, the air between the vanes is thrown outwards by centrifugal force and gathered inside the casing. As the air slows down in the casing, the kinetic energy is converted into pressure. The shape of the casing is important and is basically an eccentric passage surrounding the rotor edge. Fresh air is drawn in from the front of the rotor.

These types of compressor are suitable for medium and large flow rates. Pressure up to 25 bar may be obtained by using several stages or using them as the second stage of an axial flow type. Very large compressors are used to supply large volumes of air at low pressure to combustion chambers and blast furnaces.

4.3 AXIAL FLOW TYPE

The axial flow compressor is basically many rows of fan blades arranged along the axis. Each row gives the air kinetic energy. Not shown on the diagram are fixed vanes in between each row that slows the air down again and raises the pressure. In this way the pressure gradually increases as the air flows along the axis from inlet to outlet. Often a centrifugal stage is situated at the end in order to give it a boost and change the direction of flow to the side. Typical industrial compressors can provide 70 m^3/s at 15 bar. They are not suitable for small flow rates (below 15 m^3/s).

Axial flow compressors are commonly used in jet and gas turbine engines but they have many applications where large flow rates and medium to high pressure are required.

4.4 SCREW TYPES

Two rotors have helical lobes cut on them in such a way that when they mesh and rotate in opposite directions, air is drawn along the face of the lobes from input to output. Oil is used liberally to seal the air. The oil also acts as a coolant and the diagram shows how the oil and air are separated and then cooled in a radiator. The oil is re-circulated.

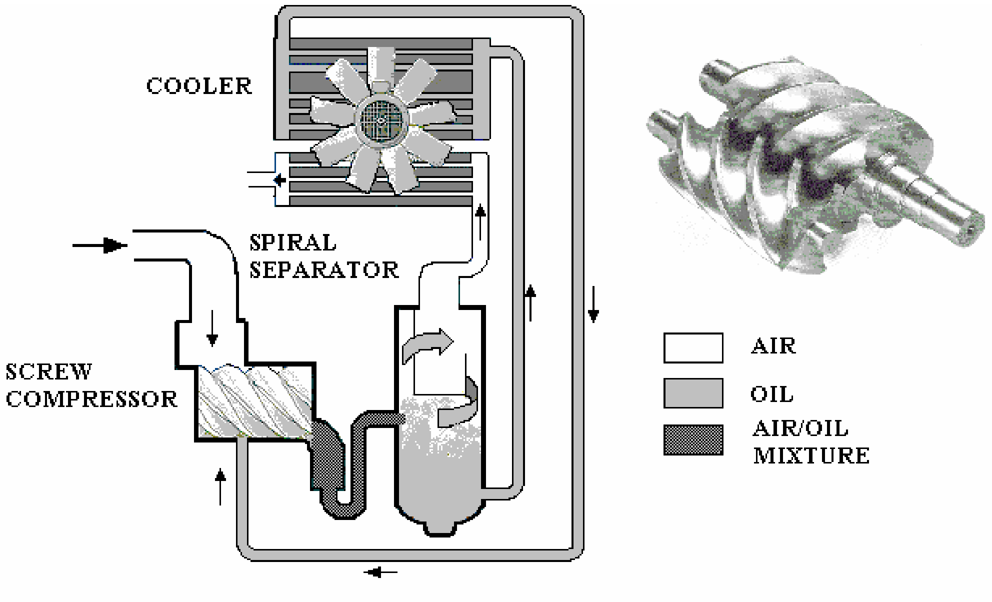


Fig. 10

4.5 LOBE TYPES

Lobe compressors are commonly used as superchargers on large engines. Figure 11 shows the basic design. Air is carried around between the lobes and the outer wall and is expelled when the lobes come together. These compressors are not suitable for high pressures but flow rates around 10 000 m^3/hr are achievable.

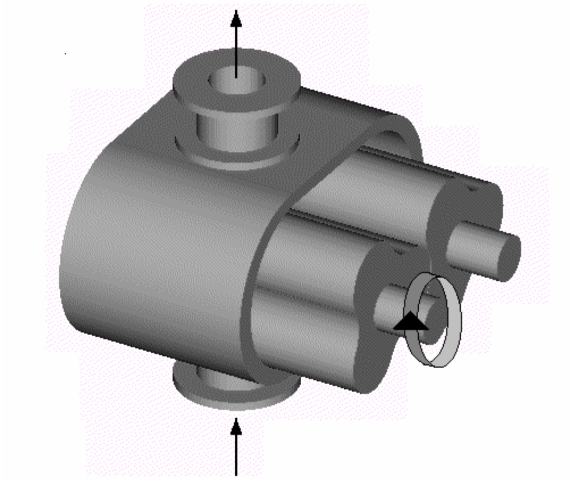


Figure 11

5. COOLERS

Coolers are used for the following reasons.

- To reduce the indicated work in multiple stage compressors.
- To condense water from the air.

For reciprocating compressors, the cooling takes place in the following places.

- The cylinder.
- Between stages.
- After final compression.

5.1 CYLINDER COOLING

The cylinders may be cooled with air and are designed with cooling fins on the outside. Circulating water through a cooling jacket produces more effective cooling.

5.2 INTER COOLING

These are usually simple heat exchangers with water-cooling. Drain traps are fitted to them to remove the water that condenses out of the air. Indicated work is saved as explained in section 3.

Assuming that no heat is lost to the surroundings, the 1st. Law may be applied to the air and water side to produce the following heat balance.

$$\Phi = m_a c_a \Delta T_a = m_w c_w \Delta T_w$$

m_a is the mass flow rate of air.
m_w is the mass flow rate of water.
ΔT_a is the temperature change of the air.
ΔT_w is the temperature change of the water.
c_a is the specific heat capacity of air.
c_w is the specific heat capacity of water.

5.3 AFTER COOLING

The only purpose of an after cooler is to cool the air to around ambient conditions and condense water from the air. This is usually another water-cooled heat exchanger and the same heat balance may be applied.

WORKED EXAMPLE No.6

A single acting reciprocating compressor runs at 360 rev/min and takes in air at 1 bar and 15^oC and compresses it in 3 stages to 64 bar. The free air delivery is 0.0566 m^3/s. There is an intercooler between each stage, which returns the air to 15^oC. Each stage has one piston with a stoke of 100 mm. Calculate the following.

i. The ideal pressure between each stage.
ii The ideal indicated power per stage.
iii The heat rejected from each cylinder.
iv The heat rejected from each intercooler.
v The isothermal efficiency.
vi The swept volume of each stage.
vii The bore of each cylinder.

Ignore leakage and the effect of the clearance volume. The index of compression is 1.3 for all stages.

SOLUTION

Pressure ratio for each stage = $(64/1)^{1/3} = 4$

Hence the pressure after stage 1 is 1 x 4 = 4 bar.

The pressure after the second stage is 4 x 4 = 16 bar

The final pressure is 16 x 4 = 64 bar.

$T_1 = 288$ K. $m = p_1V/RT_1 = 1 \times 10^5 \times 0.0566/(287 \times 288) = 0.06847$ kg/s

$T_2 = 288(4)^{0.3/1.3} = 396.5$ K

The indicated power for each stage is the same so it will be calculated for the 1st. stage.

I.P. = $mRn\,T_1\{(p_2/p_1)^{(1-1/n)} - 1\}/(n-1)$ since m is the mass compressed.

I.P. = $0.06847 \times 287 \times 1.3 \times 288\{4^{0.3/1.3} - 1\}/(1.3-1) = 9.246$ kW

CYLINDER COOLING

Consider the energy balance over the first stage.

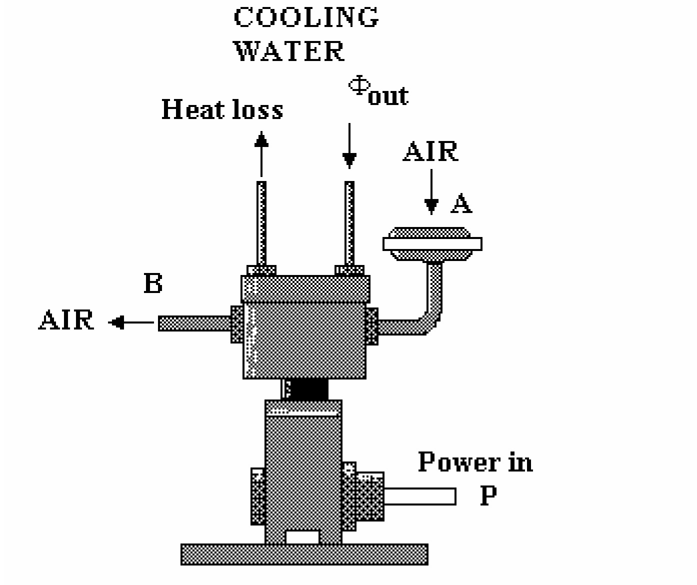


Fig. 12

Balancing the energy we have

$H_A + P(in) = H_B + \Phi\ (out)$

$\Phi\ (out) = P(in) - mC_p(T_B - T_A)$

$\Phi\ (out) = 9.246 - 0.06847 \times 1.005\ (396.5 - 288)$

$\Phi\ (out) = 1.78$ kW (rejected from each cylinder)

INTERCOOLER

Now consider the Intercooler. No work is done and the temperature is cooled from T_2 to T_5.

$\Phi\ (out) = mc_p(T_C - T_D) = 0.0687 \times 1.005\ (396.5 - 288) = 7.49$ kW

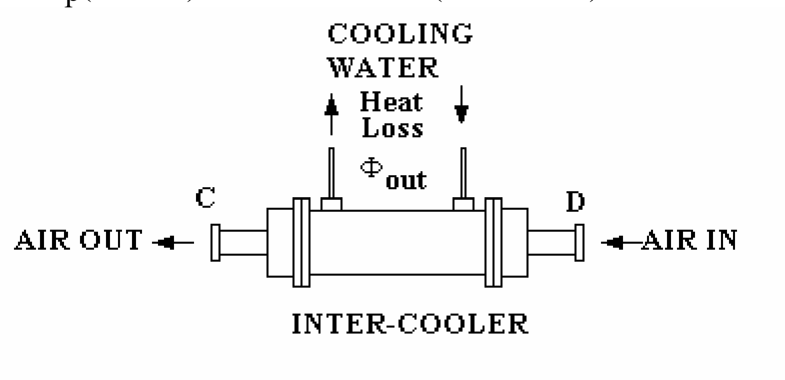


Fig.13

ISOTHERMAL EFFICIENCY

The ideal isothermal power = $mRT_1 ln(p_1/p_2)$ per stage.

P(isothermal) = 0.06847 x 287 x 288 ln4 = 7.846 kW

$\eta_{(iso)}$ = 7.846/9.246 = 84.9 %

SWEPT VOLUMES

Consider the first stage.

The F.A.D. is 0.0566 m^3/s. In the ideal case where the air is drawn in at constant temperature and pressure from the atmosphere, the FAD is given by

FAD = Swept Volume x Speed and the speed is 6 rev/s

If the clearance volume is ignored, the FAD gives the swept volume.

S.V. (1st. Stage) = 0.0566/6 = 0.00943 m^3

S.V. = Bore Area x Stroke

$0.00943 = \pi D^2/4 \times 0.1 \quad D_1 = 0.347$ m.

Now consider the second stage. The air is returned to atmospheric temperature at inlet with a pressure of 4 bar. The volume drawn is hence 1/4 of the original FAD.

The swept volume of the second stage is hence 0.00943/4 = 0.00236 m^3.

$0.00236 = \pi D^2/4 \times 0.1$ hence $D_2 = 0.173$ m

By the same reasoning the swept volume of the third stage is

SV(3rd stage) = 0.00943/16 = 0.000589 m^3.

$0.000589 = \pi D^2/4 \times 0.1 \quad D_3 = 0.0866$ m

SELF ASSESSMENT EXERCISE No.2

1. A two stage compressor draws in 8 m^3/min from atmosphere at 15°C and 1.013 bar. The air is compressed with an index of compression of 1.27 to the inter-stage pressure of 6 bar. The intercooler must cool the air back to 15 °C.

   Look up the appropriate mean value of c_p in your tables.

   Calculate the heat that must be extracted from the air in kW. (21.8 kW)

2. A single acting 2 stage compressor draws in 8.5 m^3/min of free air and compresses it to 40 bar. The compressor runs at 300 rev/min. The atmospheric conditions are 1.013 bar and 15°C. There is an intercooler between stages that cools the air back to 15°C. The polytropic index for all compressions is 1.3. The volumetric efficiency is 90% for the low pressure stage and 85% for the high pressure stage. Calculate the following.

   i. The intermediate pressure for minimum indicated work. (6.37 bar)
   ii. The theoretical indicated power for each stage. (32.8 kW)
   iii. The heat rejected in each cylinder. (6.3 kW for both)
   iv. The heat rejected by the intercooler. (26.5 kW)
   v. The swept volumes of both stages. (31.1 and 5.24 dm^3)

3. A two stage reciprocating air compressor works between pressure limits of 1 and 20 bar. The inlet temperature is 15°C and the polytropic index is 1.3. Inter-cooling between stages reduces the air temperature back to 15°C. Both stages have the same stroke. Neglect the effect of the clearance volume.

   Calculate he following.

   i. The free air delivery for each kWh of indicated work. (10.06 m^3)
   ii. The mass of air that can be compressed for each kW h of indicated work. (12.17 kg)
   iii. The ratio of the cylinder diameters. (2.115)

   (Note 1 kW h is 3.6 MJ)

4. A single acting compressor draws in atmospheric air at 1 bar and 15°C. The air is compressed in two stages to 9 bar. The compressor runs at 600 rev/min.

The installation has an inter-cooler that reduces the air temperature to 30°C at inlet to the second stage. The polytropic index for all compressions is 1.28.

The clearance volume for each stage is 4% of the swept volume. The low pressure cylinder is 300 mm diameter and the stroke for both stages is 160 mm.

Calculate the following.

i. The optimal inter-stage pressure. (3 bar)
ii. The volumetric efficiency of each stage. (98.9)%
iii. The free air delivery. (7.77 m^3/min)
iv. The induced volume of the high pressure stage. (4.545 dm^3)
v. The diameter of the high pressure cylinder. (191.2 mm)
vi. The indicated power for each stage. (16 and 16.93 kW)

EDEXCEL HIGHERS
ENGINEERING THERMODYNAMICS H2
NQF LEVEL 4

OUTCOME 4
STEAM AND GAS TURBINE POWER PLANT

TUTORIAL No. 7 - TURBINE THEORY

Steam and gas turbine

Principles of operation: impulse and reaction turbines; condensing; pass-out and back pressure steam turbines; single and double shaft gas turbines; regeneration and re-heat in gas turbines; combined heat and power plants

Circuit and property diagrams: circuit diagrams to show boiler/heat exchanger; superheater; turbine; condenser; condenser cooling water circuit; hot well; economiser/feedwater heater; condensate extraction and boiler feed pumps; temperature - entropy diagram of Rankine cycle

Performance characteristics: Carnot, Rankine and actual cycle efficiencies; turbine isentropic efficiency; power output; use of property tables and enthalpy-entropy diagram for steam

When you have completed this short tutorial, you should be able to explain the basic theory and design principles of turbines used in steam and gas turbine cycles.

1. TURBINES DESIGN

1.1 A BRIEF HISTORY

120 B.C. Hero of Alexandria constructs a simple reaction turbine.

This was constructed from a spherical vessel with two spouts as shown. Heat turned the water inside into steam that escaped through the spouts and made the vessel rotate.

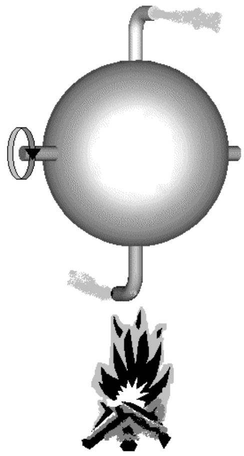


Figure 1 Hero's Turbine

1629 Branca, an Italian, created the first impulse turbine.

Steam issuing from a nozzle struck the vanes on a wheel and made it revolve.

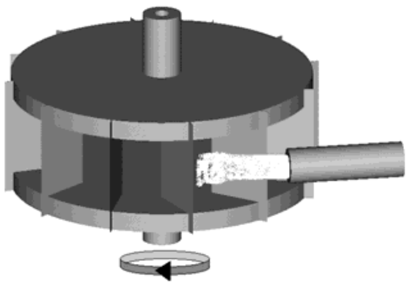


Figure 2 Branca's Turbine

Windmills, developed in medieval times formed the main source of power for centuries.

1884 Charles Parsons developed the first practical reaction turbine. This machine developed around 7 kW of power.

1889 De Laval developed the first practical impulse turbine capable of producing around 2 kW of power.

Others who developed the impulse turbine were ***Rateau*** in France and ***Curtis*** in the U.S.A.

1.2 IMPULSE THEORY

Turbines are generally classified as either impulse or reaction. This refers to the type of force acting on it and causing it to rotate.

IMPULSIVE FORCES are exerted on an object when it diverts or changes the flow of a fluid passing over it.

A very basic impulse turbine is the windmill and this converts the kinetic energy of the wind into mechanical power.

Consider a rotor with vanes arranged around the edge. Fluid is directed at the vanes by a set of nozzles.

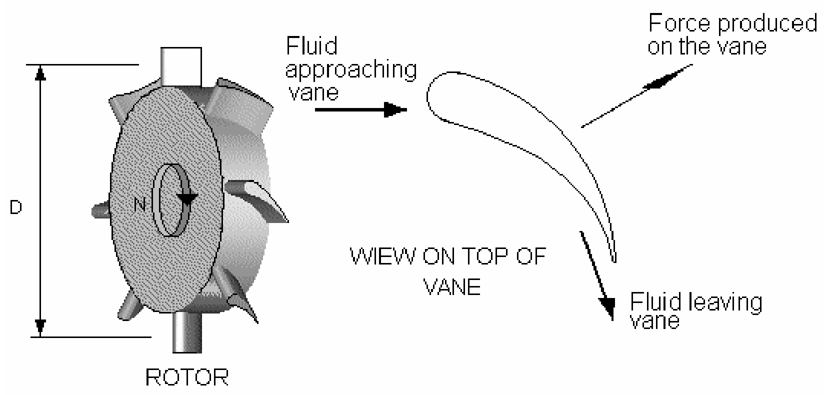


Fig. 3

If there is no pressure drop in the process, the resulting force on the vane is entirely due to the change in the momentum of the fluid and the force is entirely impulsive. It is of interest to note that the name impulsive comes from Newton's second law of motion.

Impulse = change in momentum

Impulsive force = rate of change in momentum.

$F = m\ \Delta v$

m is the mass flow rate in kg/s and Δv is the change in velocity of the fluid. This is a vector quantity and may be applied to any direction. If we make Δv the change in velocity in the direction of motion we obtain the force making the rotor turn. This direction is usually called the whirl direction and Δv_w means the change in velocity in the whirl direction.

$$F = m\Delta v_w$$

Suppose the vanes to be rotating on a mean circle of diameter D at N rev/s. The linear velocity of the vanes is u m/s. This is given by the following equation.

$$u = \pi DN$$

The power produced by any moving force is the product of force and velocity. The power of the ideal rotor is given by the following equation.

$$P = m\Delta v_w\ u = m\ \Delta v_w \pi ND$$

This is the fundamental way of finding the power produced by fluids passing over moving vanes. In order to find the vector quantity Δv_w, we draw vector diagrams for the velocities. For this reason, the power is called ***DIAGRAM POWER***.

$$\text{DIAGRAM POWER} = m\,\Delta v_w \pi N D$$

The construction of the vector diagrams for fluids flowing over vanes is not covered in this book and you should refer to more advanced text if you wish to study it at a deeper level.

WORKED EXAMPLE No.1

The vanes on a simple steam turbine are mounted on a rotor with a mean diameter of 0.6 m. The steam flows at a rate of 0.8 kg/s and the velocity in the whirl direction is changed by 80 m/s. The turbine rotates at 600 rev/min. Calculate the diagram power.

SOLUTION

Rotor Speed $N = 600/60 = 10$ rev/s

Velocity of the vanes $u = \pi ND = \pi \times 10 \times 0.6 = 18.85$ m/s

Diagram Power $DP = m\,u\,\Delta v_w = 0.8 \times 18.85 \times 80 = 1206.5$ W

A practical impulse turbine needs several sets of moving vanes and fixed vanes as shown in figure 4. The fixed vanes act as nozzles that convert pressure into velocity. The steam from the nozzles is deflected by the moving row. There is a pressure drop over each fixed row.

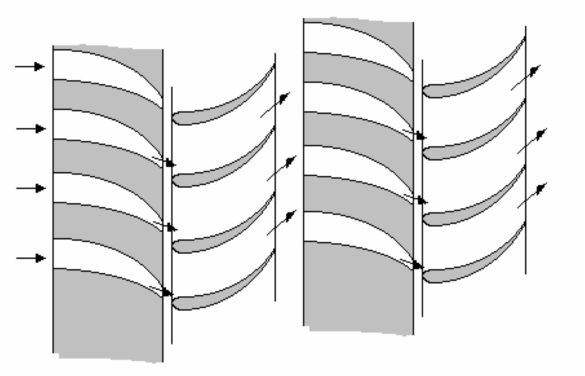


Fig.4

1.3 REACTION THEORY

REACTION FORCES are exerted on an object when it causes the velocity of the fluid to change. Consider a simple nozzle in which the fluid accelerates due to the change in the cross sectional area. The kinetic energy of the fluid increases and since energy is conserved, the pressure of the fluid drops. In other words, the pressure behind the fluid forces it through the nozzle causing it to speed up. The force required to accelerate the fluid is in the direction of the acceleration. Every force has an equal and opposite reaction so an equal and opposite force is exerted on the nozzle. This is the principle used in rockets.

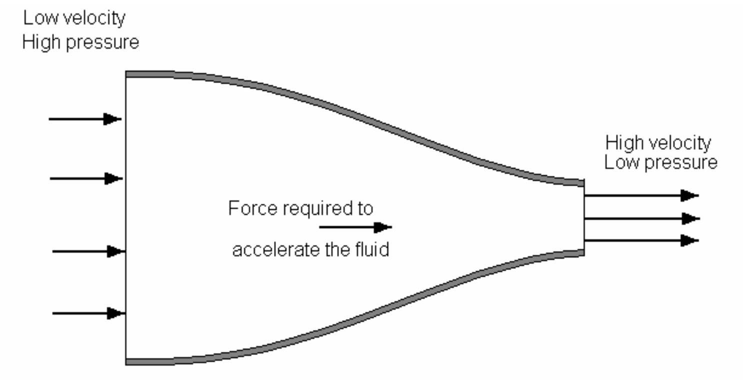


Fig. 5

It should be borne in mind that steam and gas, unlike liquids, undergoes a volume increase when the pressure falls. It is thus possible to accelerate steam and gas without narrowing the flow passage. The force required to accelerate the fluid is given by the following equation.

$$F = m\,\Delta v$$

The reaction force acting on the nozzle is equal and opposite in direction.

Figure 6 shows the layout of the blades for a turbine that uses both reaction and impulse. The fixed rows accelerate the steam and there is a pressure drop over the row. The moving row also accelerates the steam and there is a further pressure drop over the moving row. The moving blades are thus moved by both impulse and reaction forces. If the rows of blades are identical, the pressure drop over each is the same and there is 50% impulse and 50% reaction.

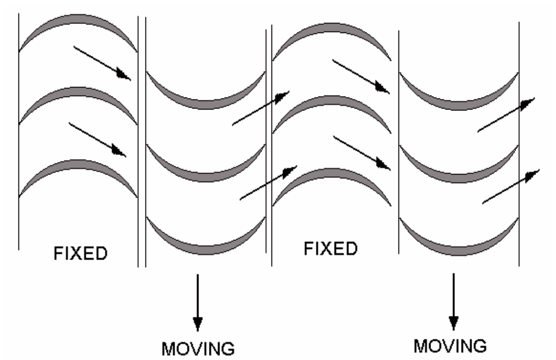


Fig.6

The moving vanes experience both reaction and impulsive forces and the two together is given by the change in momentum. The formula developed in section 1.2 applies to any kind of turbine.

$$\text{DIAGRAM POWER} = m\,\Delta v_w \pi N D$$

<u>AXIAL FORCE</u>

The change in momentum that produces the force on the blade is not only in the direction of rotation. There is also a change of velocity and hence momentum in the direction of the axis of rotation and this pushes the turbine rotor in that direction. This would require a large thrust bearing in the turbine design. This can be avoided by placing two identical rotors back to back so the axial thrust cancels out. Figure 7 shows the schematic for such an arrangement.

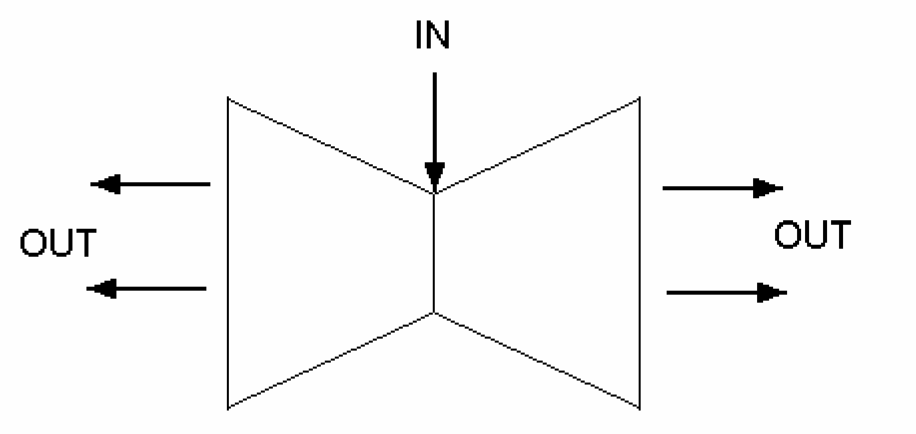


Fig. 7

Because the volume of the steam or gas increases greatly as it progresses along the axis, the height of the blades increases in order to accommodate it. Figure 8 shows a turbine with the casing removed. There are three sets or cylinders each with double flow. The exhaust steam has such a large volume that entry to the condenser is through the large passages underneath. The condenser occupies the space below the turbine hall.

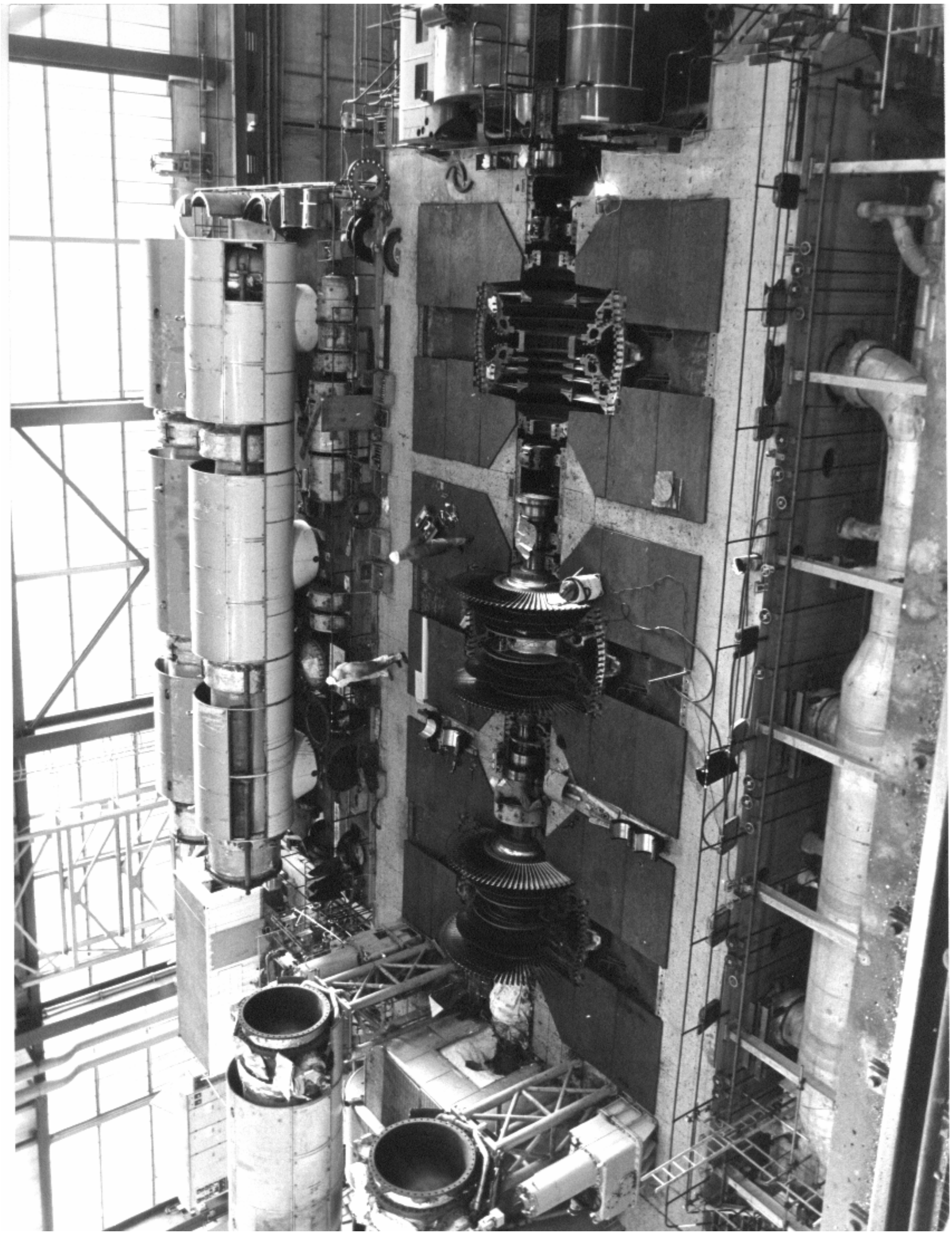


Fig. 8

Figure 9 is another picture showing the rotor of a large steam turbine.

Fig. 9

SELF ASSESSMENT EXERCISE No.1

1. A steam turbine has its vanes on a mean diameter of 1.2 m and rotates at 1500 rev/min. The change in the velocity of whirl is 65 m/s and the change in the axial velocity is 20 m/s. The flow rate is 1 kg/s. Calculate the following.

   i. The diagram power. (6.12 kW)

   ii. The axial force. (20 N)

2. A steam turbine is to be designed to rotate at 3000 rev/min and produce 5 kW of power when 1 kg/s is used. The vanes will be placed on a mean diameter of 1.4 m. Calculate the change in the velocity of whirl that will have to be produced.
   (22.7 m/s)

3. A gas turbine has rotor blades on a mean diameter of 0.5 m and the rotor turns at 2000 rev/min. The change in the whirl velocity is 220 m/s and the diagram power is 2 MW. Calculate the mass flow rate of gas.
   (173.6 kg/s)

EDEXCEL HIGHERS
ENGINEERING THERMODYNAMICS H2
NQF LEVEL 4

OUTCOME 4
STEAM AND GAS TURBINE POWER PLANT

TUTORIAL No. 8 – STEAM CYCLES

Steam and gas turbine

Principles of operation: impulse and reaction turbines; condensing; pass-out and back pressure steam turbines; single and double shaft gas turbines; regeneration and re-heat in gas turbines; combined heat and power plants

Circuit and property diagrams: circuit diagrams to show boiler/heat exchanger; superheater; turbine; condenser; condenser cooling water circuit; hot well; economiser/feedwater heater; condensate extraction and boiler feed pumps; temperature entropy diagram of Rankine cycle

Performance characteristics: Carnot, Rankine and actual cycle efficiencies; turbine isentropic efficiency; power output; use of property tables and enthalpy-entropy diagram for steam

When you have completed tutorial 8 you should be able to do the following.

- Explain the Carnot steam cycle.
- Explain the Rankine steam power cycle.
- Describe improvements to the Rankine cycle.

1. STEAM CYCLES

1.1 THE CARNOT STEAM CYCLE

In previous tutorials you learned that a Carnot cycle gave the highest thermal efficiency possible for an engine working between two temperatures. The cycle consisted of isothermal heating and cooling and reversible adiabatic expansion and compression.

Consider a cycle that uses vapour throughout. Evaporation and condensation at constant pressure is also constant temperature. Isothermal heating and cooling is theoretically possible. The cycle would consist of the same 4 processes as before only this time each process would be carried out in a separate steady flow plant item with the vapour flowing from one to the other in a closed loop as shown below.

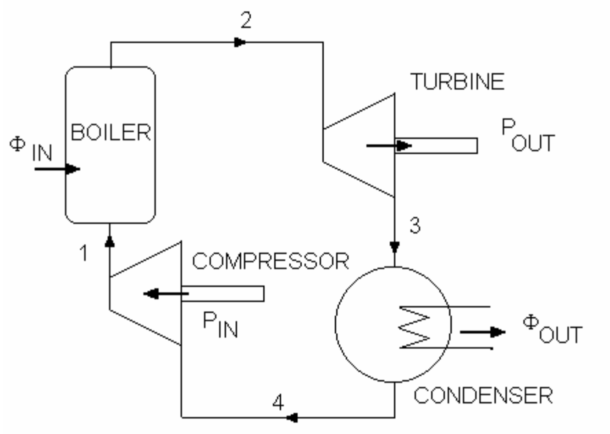


Fig. 1

The four processes are:

1 - 2 Evaporation at constant pressure and temperature requiring heat input.

2 - 3 Reversible adiabatic expansion in the turbine giving power output.

3 - 4 Cooling and condensing at constant pressure and temperature in the condenser requiring heat output.

4 - 1 Reversible adiabatic compression requiring power input.

In order that no temperature changes occur in the evaporator and condenser, the vapour must be wet at inlet and outlet. Over-cooling will produce liquid at temperatures below the saturation temperature and over-heating will superheat it beyond the saturation temperature. The cycle will be a rectangle on the T-s diagram and as shown on the h-s diagram.

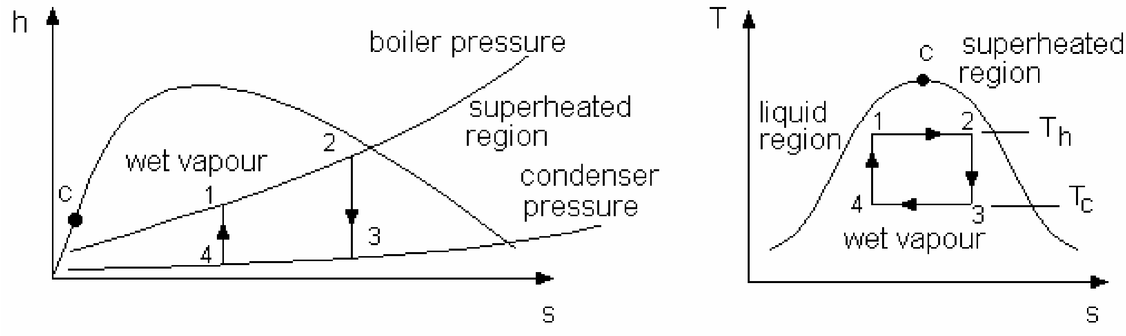


Fig.2

The limits are that at point (2) it may be dry saturated vapour but not superheated. At point 1 it may be saturated water but not under-cooled. If these limits are not used, then the vapour has a dryness fraction at each point. Since heat transfer only occurs at the evaporator and condenser the heat transfer rates are given by the following expressions.

$\Phi_{in} = m(h_2 - h_1) = T_h\ \Delta S$ (Boiler)

$\Phi_{out} = m(h_3 - h_4) = T_c\ \Delta S$ (Condenser)

T_h is the boiler temperature and T_c is the condenser temperature.

The thermal efficiency may be found from the 1st. Law.

$\eta_{th} = 1 - \Phi_{out}/\Phi_{in} = 1 - T_c / T_h$

This expression is the same as for the gas version.

WORKED EXAMPLE No. 1

A Carnot cycle is conducted on steam as follows. The evaporator produces dry saturated steam at 10 bar. The steam is expanded reversibly and adiabatically in a turbine to 1 bar. The exhaust steam is partially condensed and then compressed back to 10 bar. As a result of the compression, the wet steam is changed completely into saturated water.

Assuming a flow rate of 1 kg/s throughout determine the condition and specific enthalpy at each point in the cycle.

Calculate the energy transfers for each stage.

Show that the efficiency is correctly predicted by the expression

$\eta_{th} = T(cold)/T(hot)$

SOLUTION We will refer to the previous diagrams throughout.

EVAPORATOR

$h_2 = h_g$ at 10 bar (since it is dry saturated) = 2778 kJ/kg.
$s_2 = s_g$ at 10 bar (since it is dry saturated) = 6.586 kJ/kg K.
$h_1 = h_f$ at 10 bar (since it is saturated water) = 763 kJ/kg.

Φ_{in} = 1 (2778 - 763) = 2015 kW

TURBINE

Since the expansion is isentropic then $s_2 = s_3$ = 6.586 kJ/kg K
$s_3 = 6.586 = s_f + x_3 s_{fg}$ at 1 bar
$6.586 = 1.303 + x_3(6.056)$ hence $x_3 = 0.872$
$h_3 = h_f + x_3 h_{fg}$ at 1 bar = 417 + (0.872)(2258) = 2387 kJ/kg

P(output) = 1(2778 - 2387) = 391.2 kW

COMPRESSOR

Since the compression is isentropic then $s_4 = s_1$
$s_1 = s_f$ at 10 bar (since it is saturated water) = 2.138 kJ/kg K.
$s_4 = s_1 = 2.138 = s_f + x_4 s_{fg}$ at 1 bar
$2.138 = 1.303 + x_4(6.056)$ hence $x_4 = 0.138$
$h_4 = h_f + x_4 h_{fg}$ at 1 bar = 417 + (0.139)(2258) = 728.3 kJ/kg

Power Input = 1(763 - 728.3) = 34.7 kW

CONDENSER

Heat output = 1(2387 - 728.3) = 1658.7 kW
Energy Balances rounded off to nearest kW.
Total energy input = 34.7 + 2015 = 2050 kW

Total energy output = 391.2 + 1658.7 = 2050 kW

Net Power output = 391.2 - 34.7 = 356 kW
Net Heat input = 2015 - 1658.7 = 356 kW

Thermal efficiency = P_{nett}/Φ_{in} = 356/2015 = 17.7%
Thermal Efficiency = $1 - \Phi_{out} / \Phi_{in}$ = 1 - 1658.7/2015 = 17.7%

The hottest temperature in the cycle is t_s at 10 bar = 179.9 ^oC or 452.9 K
The coldest temperature in the cycle is t_s at 1 bar = 99.6 ^oC or 372.6 K

The Carnot efficiency = 1 - 372.6/452.9 = 17.7%

SELF ASSESSMENT EXERCISE No.1

1. A steam power plant uses the Carnot cycle. The boiler puts 25 kW of heat into the cycle and produces wet steam at 300^{o}C. The condenser produces wet steam at 50^{o}C.

   Calculate the following.

   i. The efficiency of the plant. (43.6%)
   ii. The net power output. (10.9 kW)
   iii. The heat removed by the condenser. (14 kW)

2. A steam power plant is based on the Carnot cycle. The boiler is supplied with saturated water at 20 bar and produces dry saturated steam at 20 bar. The condenser operates at 0.1 bar. Assuming a mass flow rate of 1 kg/s calculate the following.

   i. The thermal efficiency. (34.3%)
   ii. The power output of the turbine. (792 kW)
   iii. The heat transfer rate into the boiler. (1.89 MW)

1.2 THE RANKINE CYCLE

The Rankine Cycle is a practical cycle and most steam power plants are based on it. The problems with the Carnot Cycle are as follows.

- It produces only small net power outputs for the plant size because dry saturated steam is used at inlet to the turbine.
- It is impractical to compress wet steam because the water content separates out and fills the compressor.
- It is impractical to control the condenser to produce wet steam of the correct dryness fraction.

In order to get around these problems, the Rankine Cycle uses superheated steam from the boiler to the turbine. The condenser completely condenses the exhaust steam into saturated water. The compressor is replaced with a water (feed) pump to return the water to the boiler. The result of this is reduced efficiency but greater quantities of power.

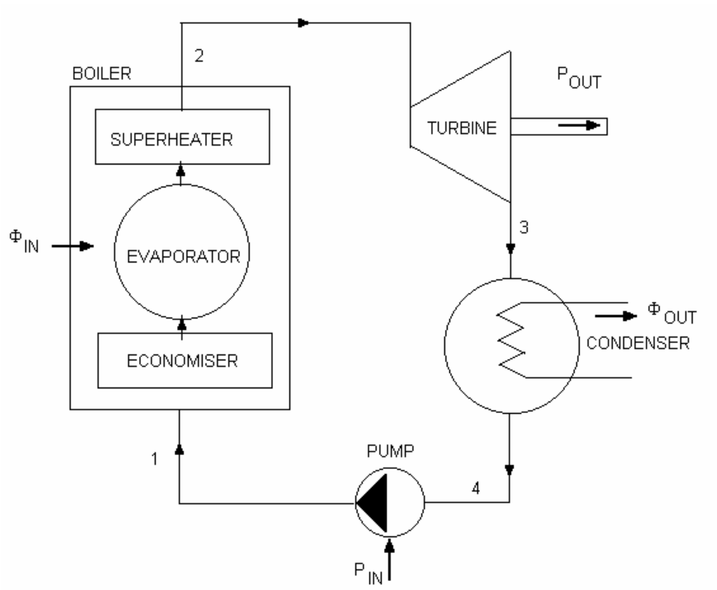


Fig.3

The plant layout is shown above. First let's briefly examine the boiler.

BOILER
For reasons of combustion efficiency (which you do not have to study), a practical boiler is made up of three sections.

a) Economiser
This is a water heater inside the boiler that raises the water temperature at the boiler pressure to just below the saturation temperature at that pressure.

b) Evaporator
This is a unit usually consisting of a drum and tubes in which the water is evaporated and the steam driven off.

c) Super-heater
This is a heater placed in the hottest part of the boiler that raises the temperature of the steam well beyond the saturation temperature.

There are many boiler designs and not all of them have these features. The main point is that a heat transfer rate is needed into the boiler unit in order to heat up the water, evaporate it and superheat it. The overall heat transfer is

$$\Phi_{in} = m\,(h_2 - h_1)$$

Next let's look at some other practical aspects of a steam power plant.

EXTRACTION PUMP AND HOTWELL.

In a practical steam cycle the condensate in the condenser is extracted with an extraction pump and the water produced is the coldest point in the steam cycle. This is usually placed into a vessel where it can be treated and extra added to make up for leaks. This point is called the HOTWELL because it contains hot water. The main feed pump returns this water to the boiler at high pressure. In the following work, extraction pumps and hotwells are not shown.

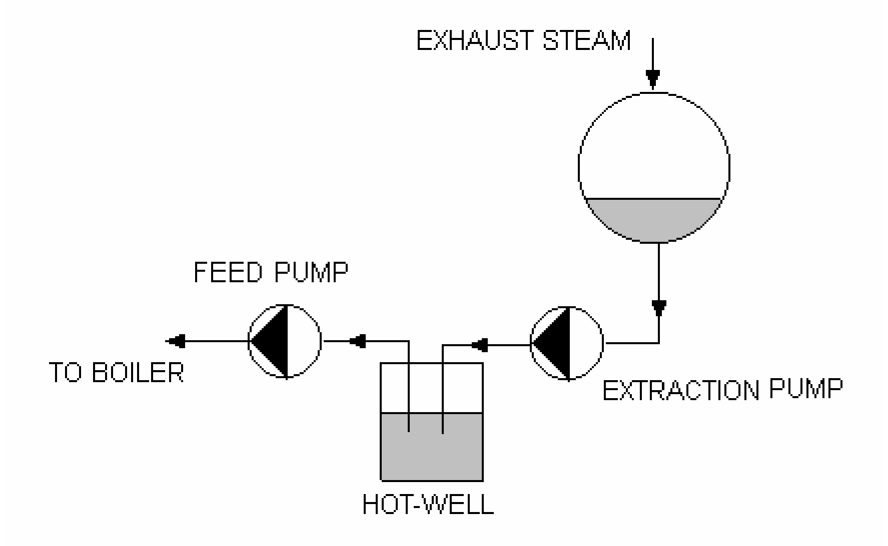


Fig.4

Now let's examine the cycle with the aid of property diagrams.

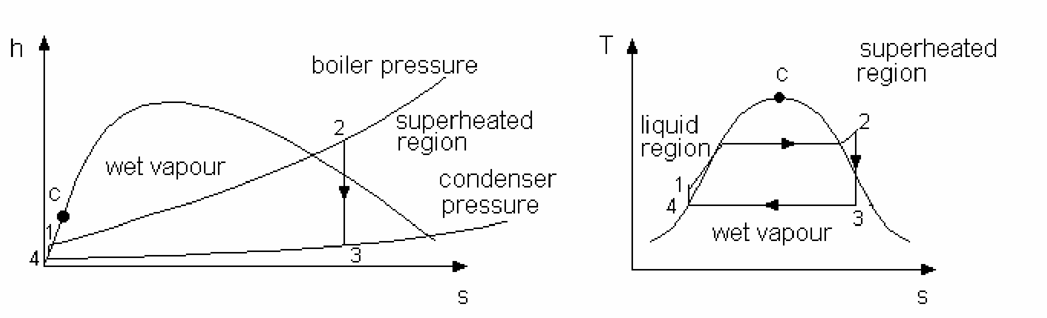


Fig.5

The process 4 to 1 is cramped into the corner of the h-s diagram and is not clear.

BOILER PROCESS (1) to (2) HEAT INPUT

The water at point 1 is below the saturation temperature at the boiler pressure. The economiser first heats it up raising the temperature, enthalpy and entropy until it reached the saturation curve. The water is then evaporated and finally, the temperature is raised by superheating the steam to point 2.

$$\Phi_{in} = m\,(h_2 - h_1)$$

TURBINE PROCESS (2) to (3) POWER OUTPUT

The second process is the expansion in the turbine and this is ideally reversible and adiabatic and is represented by a vertical line on the diagrams.

$$P_{out} = m(h_2 - h_3)$$

Turbines in real plant are often in several stages and the last stage is specially designed to cope with water droplets in the steam that becomes wet as it gives up its energy. You must use the isentropic expansion theory in order to calculate the dryness fraction and enthalpy of the exhaust steam.

CONDENSER PROCESS (3) to (4) HEAT OUTPUT

The third process is the condenser where the wet steam at point 3 is ideally turned into saturated water at the lower pressure (point 4). Condensers usually work at very low pressures (vacuums) in order to make the turbine give maximum power. The heat removed is given by

$$\Phi_{out} = m\,(h_3 - h_4)$$

Since the condenser produces condensate (saturated water) then $h_4 = h_f$ at the condenser pressure.

PUMP **PROCESS (4) to (1) POWER INPUT**

The final process which completes the cycle is the pumping of the water (point 4) from the low condenser pressure to the boiler at high pressure (point 1). In reality there are many things which are done to the feed water before it goes back into the boiler and the pressure is often raised in several stages. For the Rankine Cycle we assume one stage of pumping which is adiabatic and the power input to the pump is

$$P_{in} = m (h_1 - h_4)$$

The power required to pump the water is much less than that required to compress the vapour (if it was possible). The power input to the feed pump is very small compared to the power output of the turbine and you can often neglect it altogether. In this case we assume $h_1=h_4$.

If you are not ignoring the power input, then you need to find h_1. If you know the exact temperature of the water at inlet to the boiler (outlet from the pump) then you may be able to look it up in tables. The nearest approximation is to look up h_f at the water temperature. Since the water is at high pressure, this figure will not be very accurate and you may correct it by adding the flow energy. We will look at this in greater detail later. Lets first do a simple example with no great complications.

WORKED EXAMPLE No.2

A steam power plant is based on the Rankine cycle. The steam produced by the boiler is at 40 bar and 400^oC. The condenser pressure is 0.035 bar. Assume isentropic expansion. Ignore the energy term at the feed pump.

Calculate the Rankine cycle efficiency and compare it to the Carnot efficiency for the same upper and lower temperature limits.

SOLUTION

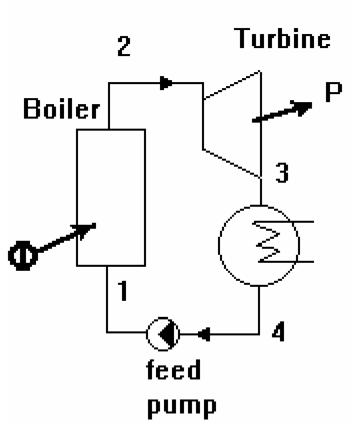


Figure 6

Turbine

h_2 = 3214 kJ/kg at 40 bar and 400^oC.
Since the expansion is isentropic then
s_2 = 6.769 kJ/kg K = s_3 = 0.391 + 8.13 x   x = 0.785
$h_3 = h_f + x\, h_{fg}$ = 112 + 0.785(2438) = 2024.6 kJ/kg

Condenser

$h_4 = h_f$ at 0.035 bar = 112 kJ/kg

Boiler

If the power input to the pump is neglected then
$h_4 = h_1$ = 112 kJ/kg

$\Phi_{in} = h_2 - h_1$ = 3102 kJ/kg.

P(output) = $h_2 - h_3$ = 1189.4 kJ/kg

$\eta = P/\Phi_{in}$ = 38.3 %

Carnot Efficiency

The hottest temperature in the cycle is 400°C (673 K) and the coldest temperature is t_s at 0.035 bar and this is 26.7 °C(299.7 K).

The Carnot efficiency is 1 - 299.7/673 = 55.5 % which is higher as expected.

Now let's examine the feed pump in more detail.

FEED PUMP

When water is compressed its volume hardly changes. This is the important factor that is different from the compression of a gas. Because the volume hardly changes, the temperature should not increase and the internal energy does not increase. The Steady flow Energy equation would then tell us that the power input to the pump is virtually equal to the increase in flow energy. We may write

$$P_{in} = m\, v\Delta p$$

Since the volume of water in nearly all cases is 0.001 m^3/kg then this becomes

$$P_{in} = 0.001\, m\, \Delta p = 0.001\, m\, (p_1 - p_2)$$

If we use pressure units of bars then

$$P_{in} = 0.001\, m(p_1 - p_2) \times 10^5 \text{ Watts}$$

Expressed in kilowatts this is

$$P_{in} = m(p_1 - p_2) \times 10^{-1} \text{ kW}$$

From this we may also deduce the enthalpy of the water after the pump.

$$P_{in} = m\,(h_1 - h_4)$$

Hence h_1 may be deduced.

WORKED EXAMPLE No.3

Repeat example 3, but this time do not ignore the feed pump and assume the boiler inlet condition is unknown.

SOLUTION

P_{in} = 1kg/s(40 - 0.035) x 10^{-1} = 4 kW
4 = 1 kg/s(h_1 - h_4) = (h_1 - 112)
h_1 = 116 kJ/kg

Reworking the energy transfers gives

$\Phi_{in} = h_2 - h_1 = 3214 - 116 = 3098$ kJ/kg.

$P_{nett} = P_{out} - P_{in} = 1189.4 - 4 = 1185.4$ kJ/kg

$\eta = P_{nett}/\Phi_{in} = 1185.4/3098 = 38.3$ %

Notice that the answers are not noticeably different from those obtained by ignoring the feed pump.

WORKED EXAMPLE No.4

A steam power plant uses the Rankine Cycle. The details are as follows.

Boiler pressure	100 bar
Condenser pressure	0.07 bar
Temperature of steam leaving the boiler	400°C
Mass flow rate	55 kg/s

Calculate the cycle efficiency, the net power output and the specific steam consumption.

SOLUTION

Turbine

h_2 = 3097 kJ/kg at 100 bar and 400^{o}C.

For an isentropic expansion we find the ideal condition at point 3 as follows.

s_2 = 6.213 kJ/kg K =s_3 = 0.559 + 7.715 x_3 x_3 = 0.733

$h_3 = h_f + x_3 h_{fg}$ = 163 + 0.733(2409) = 1928 kJ/kg

$P_{out} = m(h_2-h_3)$ = 55(3097 - 1928) = 64.3 MW

Condenser

$h_4 = h_f$ at 0.07 bar = 163 kJ/kg

$\Phi_{out} = m(h_3 - h_4)$ = 55(1928 - 163) = 97.1 MW

PUMP

Ideal power input = Flow Energy change = mv(Δp)

P_{in} = 55(0.001)(100-0.07) x 10^5 = 550 kW

$P_{in} = m(h_1 - h_4)$= 55(h_1-163) hence h_1 = 173 kJ/kg

Boiler

$\Phi_{in} = m(h_2 - h_1)$ = 55(3097 - 173) = 160.8 MW

EFFICIENCY

$P_{nett} = P_{out} - P_{in}$ = 64.3 - 0.55 = 63.7 MW

$\eta = P_{nett} / \Phi_{in}$ = 63.7/160.8 = 39.6 %

Alternatively $P_{nett} = \Phi_{in} - \Phi_{out}$ = 160.8-97.1 = 63.7 MW

This should be the same as P_{nett} since the net energy entering the cycle must equal the net energy leaving.

$\eta = 1 - \Phi_{out} / \Phi_{in}$ = 1 - 97.1/160.8 = 39.6%

SPECIFIC STEAM CONSUMPTION

This is given by

S.S.C. = P_{nett} /mass flow = 63.78/55 = 1.159 MW/kg/s or MJ/kg

SELF ASSESSMENT EXERCISE No.2

1. A simple steam plant uses the Rankine Cycle and the data for it is as follows.

Flow rate	45 kg/s
Boiler pressure	50 bar
Steam temperature from boiler	300°C
Condenser pressure	0.07 bar

Assuming isentropic expansion and pumping, determine the following.

i. The power output of the turbine. (44.9 MW)
ii. The power input to the pump. (225 kW)
iii. The heat input to the boiler. (124 MW)
iv. The heat rejected in the condenser. (79 MW)
v. The thermal efficiency of the cycle. (36%)

2. A simple steam power plant uses the Rankine Cycle. The data for it is as follows.

Flow rate	3 kg/s
Boiler pressure	100 bar
Steam temperature from boiler	600°C
Condenser pressure	0.04 bar

Assuming isentropic expansion and pumping, determine the following.

i. The power output of the turbine. (4.6 MW)
ii. The power input to the pump. (30 kW)
iii. The heat input to the boiler. (10.5 MW)
iv. The heat rejected in the condenser. (5.9 MW)
v. The thermal efficiency of the cycle. (44%)

3.
a) Explain why practical steam power plants are based on the Rankine Cycle rather than the Carnot Cycle.

b) A simple steam power plant uses the Rankine Cycle. The data for it is

Boiler pressure	15 bar
Steam temperature from boiler	300°C
Condenser pressure	0.1 bar
Net Power Output	1.1 MW

Calculate the following.

i. The cycle efficiency. (29.7 %)

ii. The steam flow rate. (1.3 kg/s)

2. BACK-PRESSURE AND PASS-OUT TURBINES

If an industry needs sufficient quantities of process steam (e.g. for sugar refining) and electric power, it becomes economical to use the steam for both purposes. This is done with a steam turbine and generator and the process steam is obtained in two ways as follows.

A) By exhausting the steam at the required pressure (typically 2 bar) to the process instead of to the condenser.

A turbine designed to do this is called a **BACK-PRESSURE TURBINE.**

B) By bleeding steam from an intermediate stage in the expansion process.

A turbine designed to do this is called a **PASS-OUT TURBINE.**

The steam cycle is standard except for these modifications.

2.1. BACK-PRESSURE TURBINES

Back-pressure turbines are designed to operate with a back pressure, unlike normal turbines that operate with a vacuum at the exhaust. The diagram shows the basic circuit.

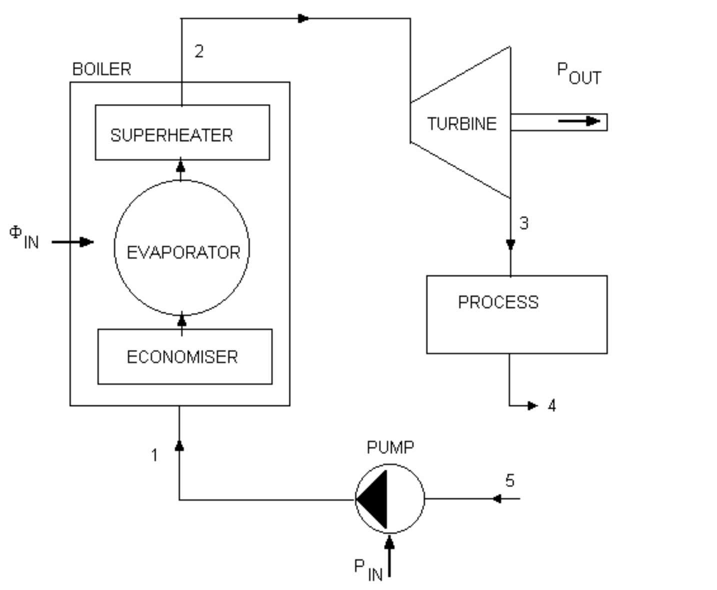


Fig.7

WORKED EXAMPLE No.5

For a steam circuit as shown previously, the boiler produces superheated steam at 50 bar and 400°C. This is expanded isentropically to 3 bar. The exhaust steam is used for a process. The returning feed water is at 1 bar and 40°C. This is pumped to the boiler. The water leaving the pump is at 40°C and 50 bar. The net power output of the cycle is 60 MW. Calculate the mass flow rate of the steam.

SOLUTION

Referring to the previous cycle sketch for location points we find

h_2= 3196 kJ/kg s_2 = 6.646 kJ/kg K

For an ideal expansion
$s_1 = s_2 = 6.646 = s_f + x s_{fg}$ at 3 bar
6.646 = 1.672 + x (5.321)
x = 0.935
$h_4 = h_f + x h_{fg}$ at 3 bar
h_4 = 561 + 0.935(2164)
h_4 = 2584 kJ/kg

Change in enthalpy = 2584 - 3196 = -612 kJ/kg

The power output of the turbine is found from the steady flow energy equation so
P = m(-612) kW
P = -612 m kW (output)

Next we examine the enthalpy change at the pump.
h_1 = 168 kJ/kg at 1 bar and 40°C
h_2= 172 kJ/kg at 50 bar and 40°C.

Change in enthalpy = 172 - 169 = 3 kJ/kg

The power input to the pump is found from the steady flow energy equation so :
P = -m(3) kW
P = -3 m kW(input)
Net Power output of the cycle = 60 MW hence
60 000 = 612 m - 3 m
m = 98.5 kg/s

SELF ASSESSMENT EXERCISE No.3

1. A steam cycle is performed as follows. The boiler produces 3 kg/s of superheated steam at 60 bar and 400°C. The steam is supplied to a turbine that it expands it isentropically and with no friction to 1.5 bar. The exhaust steam is supplied to a process. The feed water is supplied to the pump at 1.013 bar and 100°C and delivered to the boiler at 60 bar. The pump may be considered as ideal.

   Calculate the following.

   i. The power output of the turbine. (2.2 MW)
   ii. The heat input to the boiler. (8.3 MW)
   iii. The power input to the pump. (17.7 kW)
   iv. The thermal efficiency of the cycle. (26.3%)

2. A back pressure steam cycle works as follows. The boiler produces 8 kg/s of steam at 40 bar and 500°C. This is expanded isentropically to 2 bar. The pump is supplied with feed water at 0.5 bar and 30°C and delivers it to the boiler at 31°C and 40 bar.

   Calculate the following.

   i. The net power output. (6 MW)
   ii. The heat input to the boiler. (26.5 MW)
   iii. The thermal efficiency of the cycle. (22.5%)

2.2 PASS-OUT TURBINES

The circuit of a simple pass-out turbine plant is shown below. Steam is extracted between stages of the turbine for process use. The steam removed must be replaced by make up water at point 6.

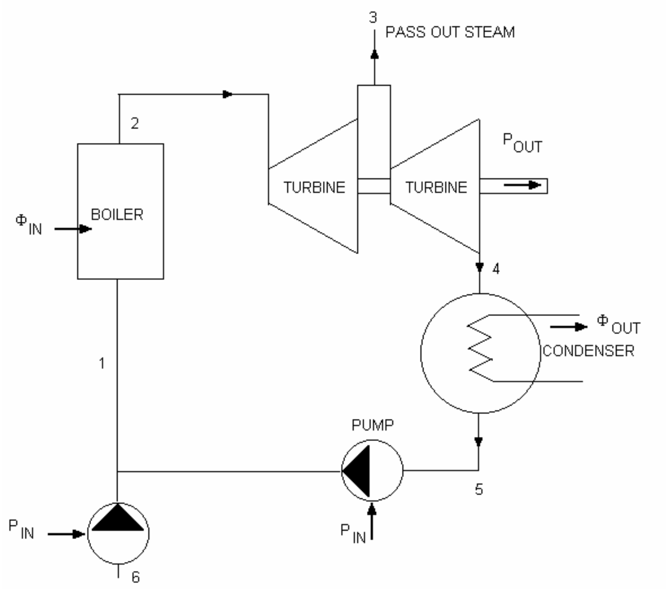


Fig. 8

In order to solve problems you need to study the energy balance at the feed pumps more closely so that the enthalpy at inlet to the boiler can be determined. Consider the pumps on their own as below.

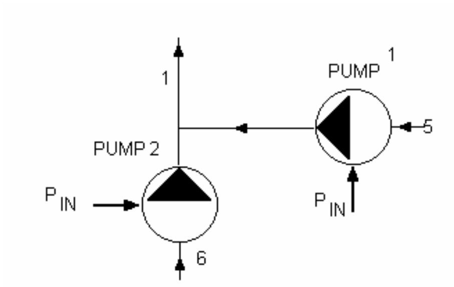


Fig. 9

The balance of power is as follows. $P_1 + P_2$ = increase in enthalpy per second.

$$P_1 + P_2 = m_1h_1 - m_6h_6 - m_5h_5$$

From this the value of h_1 or the mass m may be determined. This is best shown with a worked example.

WORKED EXAMPLE No.6

The circuit below shows the information normally available for a feed pump circuit. Determine the enthalpy at entry to the boiler.

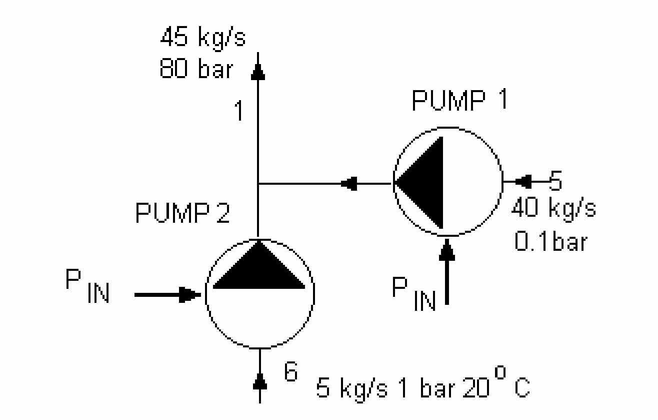


Fig. 10

SOLUTION

If no other details are available the power input to the pump is m v Δp
Assume v = 0.001 m^3/kg

P= m v Δp = m x 0.001 x Δp (bar) x 10^5 x 10^{-3} = mΔp x 10^{-1} kW

PUMP1 P_1= m v Δp = (40)(80 - 0.1)(10^{-1}) =319.6 kW

PUMP2 P_2 = m v Δp = (5)(80-1)(10^{-1}) = 39.5 kW

Total power input = 319.6 + 39.5 = 359.1 kW

$h_5 = h_f$ = 192 kJ/kg at 0.1 bar

h_6 = 84 kJ/kg (from water tables or approximately h_f at 20°C)

Balancing energy we have the following.

$359.1 = 45\ h_1 - 40\ h_5 - 5h_6$

$359.1 = 45\ h_1 - 40(192) - 5(84)$

h_1= 188 kJ/kg

WORKED EXAMPLE No.7

A pass-out turbine plant works as shown in fig. 11 The boiler produces steam at 60 bar and 500°C and this is expanded through two stages of turbines. The first stage expands to 3 bar where 4 kg/s of steam is removed. The second stage expands to 0.09 bar. Assume that the expansion is a straight line on the h - s chart.

The condenser produces saturated water. The make up water is supplied at 1bar and 20°C. The net power output of the cycle is 40 MW. Calculate the following.

i. The flow rate of steam from the boiler.
ii The heat input to the boiler.
iii. The thermal efficiency of the cycle.

SOLUTION

TURBINE EXPANSION

h_2 = 3421 kJ/kg from tables.
h_3 = 2678 kJ/kg using isentropic expansion and entropy.
h_4 = 2166 kJ/kg using isentropic expansion and entropy.

These results may be obtained from the h-s chart.

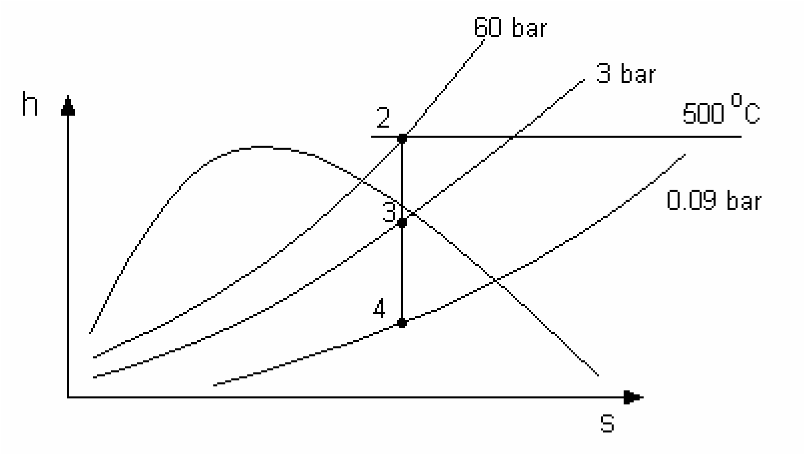


Fig. 11

POWER OUTPUT

$P_{out} = m(h_3 - h_2) + (m - 4)(h_3 - h_4)$

$P_{out} = m(3421 - 2678) + (m - 4)(2678 - 2166)$

$P_{out} = 743\,m + 512\,m - 2048$

POWER INPUT

The power input is to the two feed pumps.

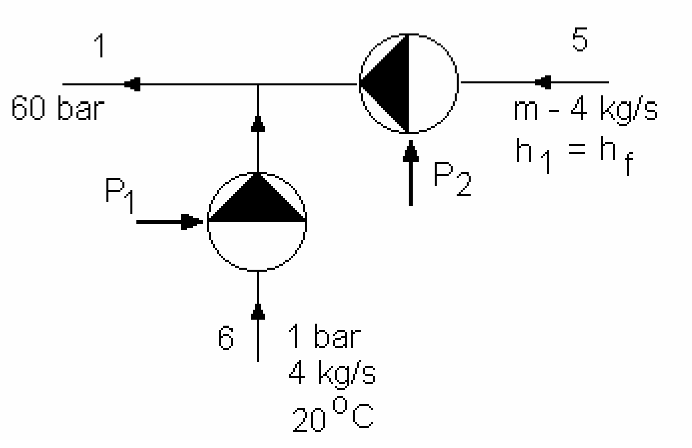


Fig. 12

h_6 = 84 kJ/kg (water at 1 bar and 20°C)

h_5 = hf at 0.09 bar = 183 kJ/kg.

P_1 =mvΔp = 4 x (60 - 1) x 10^{-1} = 23.6 kW

P_2 = =mvΔp = (m-4) x (60 - 0.09) x 10^{-1} = 5.99m - 23.96 kW

NET POWER

40 000 kW = P_{out} - P_1 - P_2

40 000 = 743 m + 512 m - 2048 - 23.6 - 5.99 m + 23.96

40000 = 1249 m - 2048 hence m = 33.66 kg/s

ENERGY BALANCE ON PUMPS

P_1 = 23.6 kW

P_2 = 177.7 kW (using the value of m just found)

m h_1 = (m - 4) h_5 + P_1 + P_2

33.66 h_1 = 29.66 x 183 + 23.6 + 177.7

h_1 = 167.2 kJ/kg

HEAT INPUT Φ_{in}= m(h_2 - h_1) = 109523 kW

EFFICIENCY Efficiency = η = 40/109.523 = 36.5 %

SELF ASSESSMENT EXERCISE No.4

1. A steam turbine plant is used to supply process steam and power. The plant comprises an economiser, boiler, super-heater, turbine, condenser and feed pump.

   The process steam is extracted between intermediate stages in the turbine at 2 bar pressure. The steam temperature and pressure at outlet from the super-heater are $500^{o}C$ and 70 bar respectively. The turbine exhausts at 0.1 bar.

   The make-up water is at $15^{o}C$ and 1 bar and it is pumped into the feed line to replace the lost process steam.

   Assume that the expansion is isentropic.

   If due allowance is made for the feed pump-work, the net mechanical power delivered by the plant is 30 MW when the process steam load is 5 kg/s.

   Sketch clear T- s and h-s and flow diagrams for the plant.

   Calculate the following.

   i. The flow rate of steam flow leaving the super-heater. (25.6 kg/s)
   ii. The rate of heat transfer to the boiler. (83.2 MW)

2. An industrial plant requires 60 MW of process heat using steam at 2.6 bar which it condenses to saturated water. This steam is derived from the intermediate stage of a turbine that produces 30 MW of power to drive the electric generators.

   The steam is raised in boilers and supplied to the turbines at 80 bar pressure and $600^{o}C$.
   The steam is expanded isentropically to the condenser pressure of 0.05 bar.

   Calculate the following.

   i. The flow rate of the steam bled to the process. (27.7 kg/s)
   ii. The flow rate of the steam from the boiler. (30.5 kg/s)

2.3 REHEATING

Steam reheating is another way of improving the thermodynamic efficiency by attempting to keep the steam temperature more constant during the heating process.

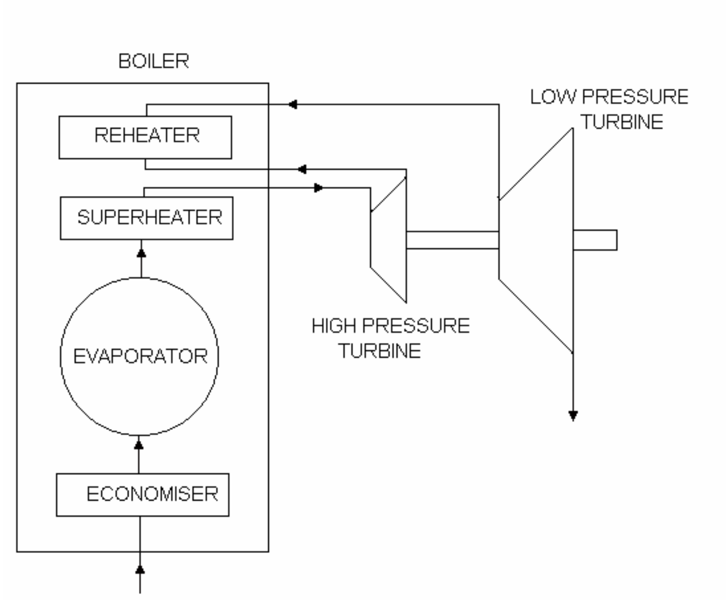


Fig.13

Superheated steam is first passed through a high pressure turbine. The exhaust steam is then returned to the boiler to be reheated almost back to its original temperature. The steam is then expanded in a low pressure turbine. In theory, many stages of turbines and reheating could be done thus making the heat transfer in the boiler more isothermal and hence more reversible and efficient.

2.4. FEED HEATING

Feed heating is another way of improving the efficiency of a steam power plant. The feed water returning to the boiler is heated nearer to the saturation temperature with steam bled from the turbines at appropriate points. There are two types.

- Indirect contact.
- Direct contact.

INDIRECT CONTACT

The steam does not mix with the feed water at the point of heat exchange. The steam is condensed through giving up its energy and the hot water resulting may be inserted into the feed system at the appropriate pressure.

DIRECT CONTACT

The bled steam is mixed directly with the feed water at the appropriate pressure and condenses and mixes with the feed water. The diagram shows a basic plant with a single feed heater added.

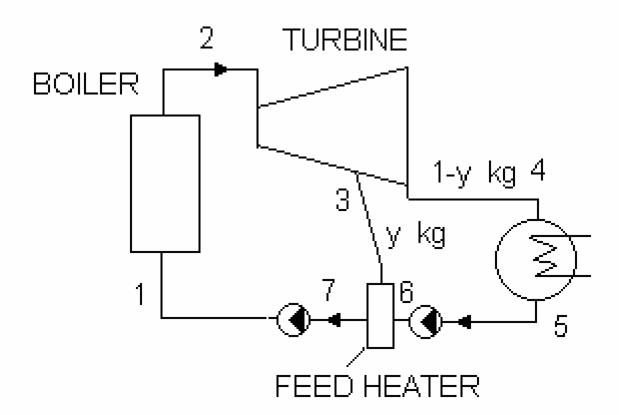


Fig.14

If a steam cycle used many stages of regenerative feed heating and many stages of reheating, the result would be an efficiency similar to that of the Carnot cycle. Although practicalities prevent this happening, it is quite normal for an industrial steam power plant to use several stages of regenerative feed heating and one or two stages of reheating. This produces a significant improvement in the cycle efficiency.

There are other features in advanced steam cycles that further improve the efficiency and are necessary for practical operation. For example air extraction at the condenser, steam recovery from turbine glands, de-super-heaters, de-aerators and so on. These can be found in detail in textbooks devoted to practical steam power plant.

SELF ASSESSMENT EXERCISE No.5

Draw the plant cycle of a steam plant with the following features.

1. Economiser.
2. Evaporator.
3. Super-heater
4. Re-heater.
5. High pressure turbine.
6. Low pressure turbine.
7. Condenser.
8. Direct contact feed heaters fed from the l.p. turbine.
9. Direct contact feed heaters fed from the h.p. turbine.
10. Hot well.
11. High pressure feed pump.
12. Low pressure feed pump.
13. Extraction pump.

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OUTCOME 4
STEAM AND GAS TURBINE POWER PLANT

TUTORIAL No. 9 – GAS TURBINE THEORY

Steam and gas turbine

Principles of operation: impulse and reaction turbines; condensing; pass-out and back pressure steam turbines;
single and double shaft gas turbines; regeneration and re-heat in gas turbines; combined heat and power plants

Circuit and property diagrams: circuit diagrams to show boiler/heat exchanger; superheater; turbine; condenser; condenser cooling water circuit; hot well; economiser/feedwater heater; condensate extraction and boiler feed pumps; temperature entropy diagram of Rankine cycle

Performance characteristics: Carnot, Rankine and actual cycle efficiencies; turbine isentropic efficiency; power output; use of property tables and enthalpy-entropy diagram for steam

When you have completed tutorial 9 you should be able to do the following.

- Describe the basic gas turbine power cycle.
- Describe advanced gas turbine power cycles.
- Solve problems concerning gas turbine power plant.

1 GAS TURBINE ENGINES

In this section we will examine how practical gas turbine engine sets vary from the basic Joule cycle.

The efficiency of gas turbine engines increases with pressure compression ratio. In practice this is limited, as the type of compressor needed to produce very large flows of air cannot do so at high pressures. 6 bar is a typical pressure for the combustion chamber.

The efficiency of gas turbines may be improved by the use of inter-cooling and heat exchangers.

1.1 GAS CONSTANTS

The first point is that in reality, although air is used in the compressor, the gas going through the turbine contains products of combustion so the adiabatic index and specific heat capacity is different in the turbine and compressor.

1.2 FREE TURBINES

Most designs used for gas turbine sets use two turbines, one to drive the compressor and a free turbine. The free turbine drives the load and it is not connected directly to the compressor. It may also run at a different speed to the compressor. Fig.1 shows the layouts for parallel turbines.

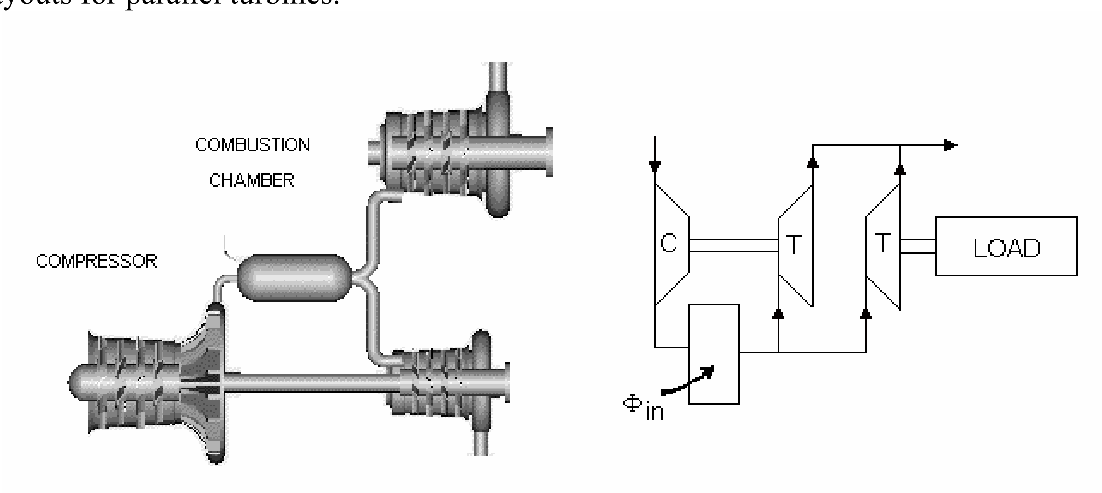

Fig. 1 PARALLEL TURBINES

Figure 2 shows the layout for series turbines.

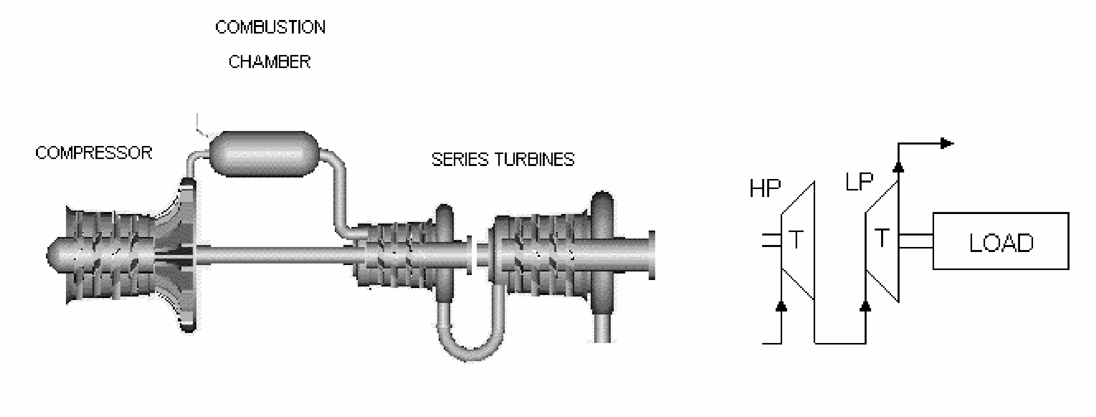

Figure 2 SERIES TURBINES

1.3 INTER-COOLING AND REHEATING

Basically, if the air is compressed in stages and cooled between each stage, then the work of compression is reduced and the efficiency increased.

The reverse theory also applies. If several stages of turbine expansions are used and the gas reheated between stages, the power output and efficiency is increased.

Figure 3 shows the layout for intercoolers and re-heaters.

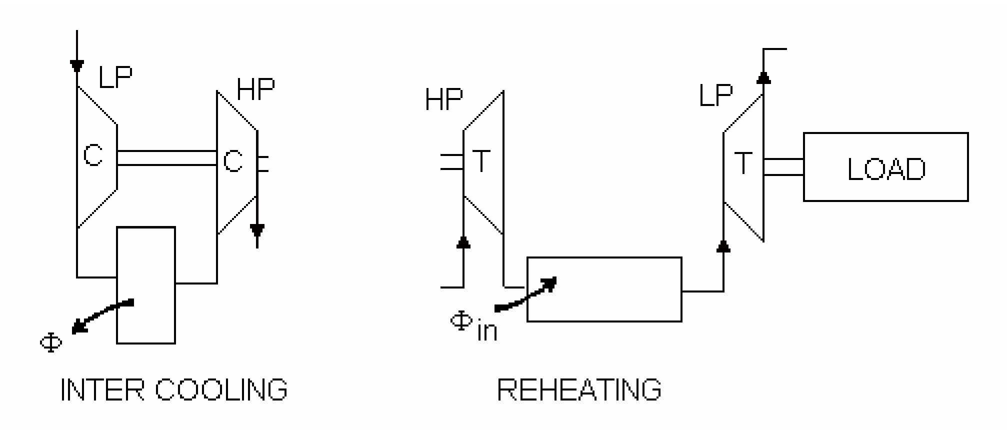

Fig.3 INTER-COOLER and RE-HEATER

WORKED EXAMPLE No.1

A gas turbine draws in air from atmosphere at 1 bar and 10^oC and compresses it to 5 bar. The air is heated to 1200 K at constant pressure and then expanded through two stages in series back to 1 bar. The high pressure turbine is connected to the compressor and produces just enough power to drive it. The low pressure stage is connected to an external load and produces 80 kW of power.

Draw the circuit.

Calculate the mass flow of air, the inter-stage pressure of the turbines and the thermal efficiency of the cycle.

Assume γ = 1.4 and c_p=1.005 kJ/kg K for both the turbines and the compressor.

Neglect the increase in mass due to the addition of fuel for burning.

Compare the efficiency to the air standard efficiency.

SOLUTION

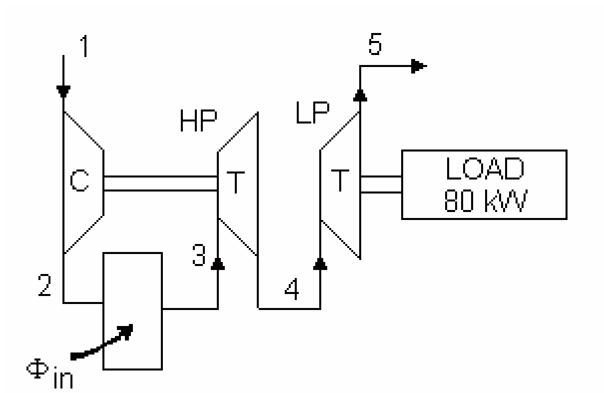

Fig. 4

COMPRESSOR

$$T_2 = T_1 r_p^{\left(1-\frac{1}{\gamma}\right)} = 283 \text{ x } 5^{0.286} = 448.4 \text{ K}$$

Power input to compressor = $mc_p(T_2-T_1)$
Power output of h.p. turbine = $mc_p(T_3-T_4)$
Since these are equal we may equate them.

1.005(448.4-283)=1.005(1200-T_4) **T_4=1034.6 K**

HIGH PRESSURE TURBINE

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_5}\right)^{1-\frac{1}{\gamma}}$$

$$\frac{T_4}{T_5} = \frac{1034.6}{1200} = 0.595 = \left(\frac{p_4}{5}\right)^{0.286} \qquad p_4 = 5 \text{ x } 0.595^{\frac{1}{0.286}} = 2.977 \text{ bar}$$

LOW PRESSURE TURBINE

$$\frac{T_5}{T_4} = \left(\frac{1}{2.977}\right)^{1-\frac{1}{\gamma}} = (0.336)^{0.286} = 0.732 \qquad T_5 = 757.3 \text{ K}$$

NET POWER

The net power is 80 kW.

$80 = mc_p(T_4-T_5)= m \text{ x } 1.005(1034.6 - 757.3)$

m = 0.288 kg/s

HEAT INPUT

$\Phi(in) = mc_p(T_3-T_2) = 0.288 \text{ x } 1.005 (1200 - 448.4) = 217.5$ kW

THERMAL EFFICIENCY

$\eta_{th} = P(nett)/\Phi(in) = 80/219.9 = 0.367$or 36.7%

The air standard efficiency is the Joule efficiency.

$\eta = 1-r_p^{-0.286} = 1 - 5^{-0.286} = 0.369$ or 36.9%

SELF ASSESSMENT EXERCISE No.1

1. A gas turbine draws in air from atmosphere at 1 bar and 15°C and compresses it to 4.5 bar. The air is heated to 1100 K at constant pressure and then expanded through two stages in series back to 1 bar. The high pressure turbine is connected to the compressor and produces just enough power to drive it. The low pressure stage is connected to an external load and produces 100 kW of power.

   For the compressor $\gamma = 1.4$ and for the turbines $\gamma = 1.3$.
   The gas constant is 0.287 kJ/kg K for both.

   Neglect the increase in mass due to the addition of fuel for burning. Assume the specific heat of the gas in the combustion chamber is the same as that for the turbines.

   Calculate the following.

   i. The specific heat c_p of the air and the burned mixture. (1.005 and 1.243)

   ii. The mass flow of air. (0.407 kg/s)

   iii. The inter-stage pressure of the turbines. (2.67 bar)

   iv. The thermal efficiency of the cycle. (30%)

2. A gas turbine draws in air from atmosphere at 1 bar and 300 K and compresses it to 6 bar. The air is heated to 1300 K at constant pressure and then expanded through two stages in series back to 1 bar. The high pressure turbine is connected to the compressor and produces just enough power to drive it.

   $\gamma = 1.4$ and R= 0.287 kJ/kg K for both the turbine and compressor. Neglect the increase in mass due to the addition of fuel for burning.

   Calculate the following for a mass flow of 1 kg/s.

   i. The inter-stage pressure. (3.33 bar)

   ii. The net power output. (322 kW)

   iii. The thermal efficiency of the cycle. (40%)

1.4 EXHAUST GAS HEAT EXCHANGER

The exhaust gas from a turbine is hotter than the air leaving the compressor. If heat is passed to the air from the exhaust gas, then less fuel is needed in the combustion chamber to raise the air to the operating temperature. This requires an exhaust heat exchanger. Fig.5 shows the layout required.

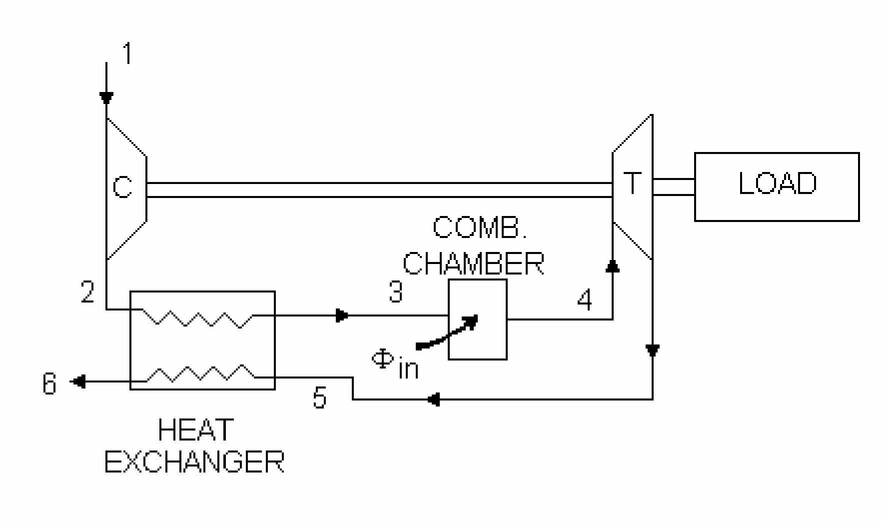

Fig.5

In order to solve problems associated with this cycle, it is necessary to determine the temperature prior to the combustion chamber (T_3).

A perfect heat exchanger would heat up the air so that T_3 is the same as T_5. It would also cool down the exhaust gas so that T_6 becomes T_2. In reality this is not possible so the concept of *THERMAL RATIO* is used. This is defined as the ratio of the enthalpy given to the air to the maximum possible enthalpy lost by the exhaust gas. The enthalpy lost by the exhaust gas is

$$\Delta H = m_g c_{pg}(T_5 - T_6)$$

This would be a maximum if the gas is cooled down such that $T_6 = T_2$. Of course in reality this does not occur and the maximum is not achieved and the gas turbine does not perform as well as predicted by this idealisation.

$$\Delta H \text{ (maximum)} = m_g c_{pg}(T_5 - T_2)$$

The enthalpy gained by the air is

$$\Delta H \text{ (air)} = m_a c_{pa}(T_3 - T_2)$$

Hence the thermal ratio is

$$\text{T.R.} = m_a c_{pa}(T_3 - T_2)/m_g c_{pg}(T_5 - T_2)$$

The suffix a refers to the air and g to the exhaust gas. Since the mass of fuel added in the combustion chamber is small compared to the air flow we often neglect the difference in mass and the equation becomes

$$\mathbf{T.R.} = c_{pa}(T_3 - T_2)/c_{pg}(T_5 - T_2)$$

WORKED EXAMPLE No.2

A gas turbine uses a pressure ratio of 7.5/1. The inlet temperature and pressure are respectively 10°C and 105 kPa. The temperature after heating in the combustion chamber is 1300 °C.
The specific heat capacity c_p for air is 1.005 kJ/kg K and for the exhaust gas is 1.15 kJ/kg K. The adiabatic index is 1.4 for air and 1.33 for the gas. Assume isentropic compression and expansion. The mass flow rate is 1kg/s.

Calculate the air standard efficiency if no heat exchanger is used and compare it to the thermal efficiency when an exhaust heat exchanger with a thermal ratio of 0.8 is used.

SOLUTION

Referring to the numbers used on fig.6 the solution is as follows.

Air standard efficiency = $1 - r_p^{(1-1/\gamma)} = 1 - 7.5^{-0.286}$ = **0.438 0r 43.8%**

Solution with heat exchanger

$T_2 = T_1 r_p^{(1-1/\gamma)} = 283\ (7.5)^{0.286} = 503.6$ K
$T_5 = T_4 / r_p^{(1-1/\gamma)} = 1573/(7.5)^{0.248} = 954.1$ K

Use the thermal ratio to find T_3.

$0.8 = 1.005(T_3 - T_2)/1.15(T_5 - T_2)$
$0.8 = 1.005(T_3 - 503.6)/1.15(954.1 - 503.6)$
$T_3 = 916$ K

In order find the thermal efficiency, it is best to solve the energy transfers.

$P(in) = mc_{pa}(T_2 - T_1) = 1 \times 1.005\ (503.6 - 283) = 221.7$ kW

$P(out) = mc_{pg}(T_4 - T_5) = 1 \times 1.15\ (1573 - 954.1) = 711.7$ kW

P(net) = P(out) - P(in) = 490 kW

Φ(in)combustion chamber) = $mc_{pg}(T_4 - T_3)$
$\Phi(in) = 1.15(1573 - 916) = 755.5$ kW

$\eta_{th} = P(net)/\Phi(in) = 490/755.5$ = **0.65 or 65%**

SELF ASSESSMENT EXERCISE No.2

1. A gas turbine uses a pressure ratio of 7/1. The inlet temperature and pressure are respectively 10^{o}C and 100 kPa. The temperature after heating in the combustion chamber is 1000 ^{o}C. The specific heat capacity C_p is 1.005 kJ/kg K and the adiabatic index is 1.4 for air and gas. Assume isentropic compression and expansion. The mass flow rate is 0.7 kg/s.

   Calculate the net power output and the thermal efficiency when an exhaust heat exchanger with a thermal ratio of 0.8 is used.
   (234 kW and 57%)

2. A gas turbine draws in air from the atmosphere at 1.02 bar and 300 K. The air is compressed to 6.4 bar isentropically. The air entering the turbine is at 1500 K and it expands isentropically to 1.02 bar. Assume the specific heat Cp is 1.005 kJ/kg K and γ is 1.4 for both the turbine and compressor. Ignore the addition of mass in the burner. Calculate the following.

   i. The air standard efficiency. (40.8%)
   ii. The efficiency when an exhaust heat exchanger with a thermal ratio of 0.75 is added. (70.7%)

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OUTCOME 4
STEAM AND GAS TURBINE POWER PLANT

TUTORIAL No. 10 – ISENTROPIC EFFICIENCY

Steam and gas turbine

Principles of operation: impulse and reaction turbines; condensing; pass-out and back pressure steam turbines; single and double shaft gas turbines; regeneration and re-heat in gas turbines; combined heat and power plants

Circuit and property diagrams: circuit diagrams to show boiler/heat exchanger; superheater; turbine; condenser; condenser cooling water circuit; hot well; economiser/feedwater heater; condensate extraction and boiler feed pumps; temperature entropy diagram of Rankine cycle

Performance characteristics: Carnot, Rankine and actual cycle efficiencies; turbine isentropic efficiency; power output; use of property tables and enthalpy-entropy diagram for steam

On completion of this tutorial you should be able to do the following.

- Explain the effect of friction on steam and gas expansions in turbines.
- Solve steam cycle problems taking into account friction.
- Solve gas turbine cycle problems taking into account friction.

1. ISENTROPIC EFFICIENCY

1.1 THE EFFECT OF FRICTION

When a fluid is expanded or compressed with fluid friction occurring, a degree of irreversibility is present. The result is the generation of internal heat equivalent to a heat transfer. **This always results in an increase in entropy**.

Figure 1 shows expansion and compression processes on a T-s diagram. In the case of vapour, the line crosses the saturation curve. In the case of gas, the process takes place well away from the saturation curve and indeed the saturation curve would not normally be shown for gas processes. Note that in every case, the ideal process is from (1) to (2') but the real process is from (1) to (2).

Friction does the following.

- Increases the entropy.
- Increases the enthalpy.
- The true process path on property diagrams is always to the right of the ideal process.
- When the final point (2) is in the gas (superheat) region, the result is a hotter temperature.
- When the final point (2) is in the wet region, the result is a dryer vapour.

Gas and vapour processes should be described by sketching them on an appropriate property diagram and these effects of friction clearly shown.

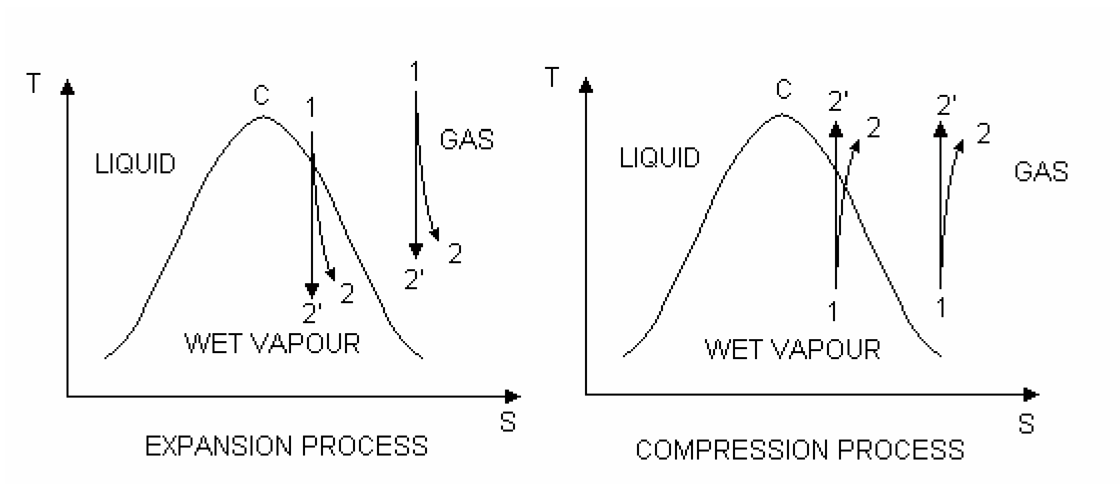

Figure 1

The same points are also apparent on the h-s diagram. Figure 2 shows a vapour expansion from (1) to (2) with the ideal being from (1) to (2'). Note how it ends up dryer at the same pressure with an increase in entropy. Vapour is not normally compressed so this is not shown.

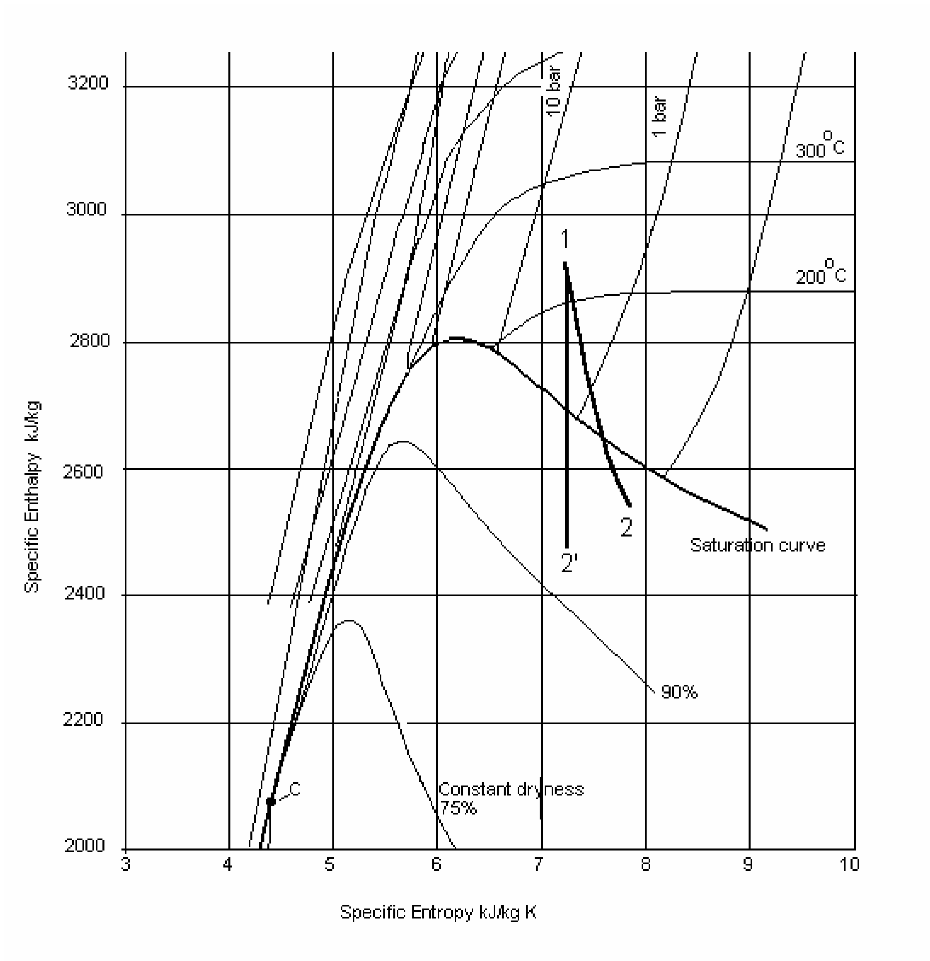

Figure 2

1.2 ISENTROPIC EFFICIENCY

An ideal reversible adiabatic process would be constant entropy as shown on the diagrams from (1) to (2').

When friction is present, the process is (1) to (2).

The ideal change in enthalpy is $\Delta h \text{ (ideal)} = h_{2'} - h_1$

The actual change is $\Delta h\text{(actual)} = h_2 - h_1$

The isentropic efficiency is defined as follows.

EXPANSION

$$\eta_{is} = \frac{\Delta h \text{ (actual)}}{\Delta h \text{ (ideal)}} = \frac{h_2 - h_1}{h_{2'} - h_1}$$

COMPRESSION

$$\eta_{is} = \frac{\Delta h \text{ (ideal)}}{\Delta h \text{ (actual)}} = \frac{h_{2'} - h_1}{h_2 - h_1}$$

For gas only $h = c_p T$

EXPANSION

$$\eta_{is} = \frac{T_2 - T_1}{T_{2'} - T_1}$$

COMPRESSION

$$\eta_{is} = \frac{T_{2'} - T_1}{T_2 - T_1}$$

Note that for an expansion negative changes are obtained on the top and bottom lines that cancel.

If the work transfer rate is only due to the change in enthalpy we may also define isentropic efficiency as follows.

$$\eta_{is} = \frac{\text{Actual Power Output}}{\text{Ideal Power Output}} \quad \text{for a turbine}$$

$$\eta_{is} = \frac{\text{Ideal Power Input}}{\text{Actual Power Input}} \quad \text{for a compressor}$$

WORKED EXAMPLE No.1

A turbine expands steam adiabatically from 70 bar and 500°C to 0.1 bar with an isentropic efficiency of 0.9. The power output is 35 MW. Determine the steam flow rate.

SOLUTION

The solution is easier with a h-s chart but we will do it with tables only.

h_1 = 3410 kJ/kg at 70 bar and 500°C.

s_1 = 6.796 kJ/kg K at 70 bar and 500°C.

For an ideal expansion from (1) to (2') we calculate the dryness fraction as follows.

$s_1 = s_2 = s_f + x's_{fg}$ at 0.1 bar.

6.796 = 0.649 + x'(7.5)   x' = 0.8196

Note that you can never be certain if the steam will go wet. It may still be superheated after expansion. If x' came out to be larger than unity, then because this is impossible, it must be superheated and you need to deduce its temperature by referring to the superheat tables.

Now we find the ideal enthalpy $h_{2'}$

$h_{2'} = h_f + x'h_{fg}$ at 0.1 bar.

$h_{2'}$ = 192 + 0.8196(2392) = 2152.2 kJ/kg

Now we use the isentropic efficiency to find the actual enthalpy h_2.

$$\eta_{is} = \frac{\Delta h(ideal)}{\Delta h(actual)}$$

$$0.9 = \frac{2152.2 - 3410}{h_2 - 3410}$$

$$h_2 = 2278.3 \text{ kJ/kg}$$

Now we may use the SFEE to find the mass flow rate.

$\dot{\Phi} + P = m(h_2 - h_1)$

$\dot{\Phi} = 0$ since it is an adiabatic process.

P = -35 000 kW (out of system) = m(2278.3-3410)

m = 30.926 kg/s

WORKED EXAMPLE No.2

A turbine expands gas adiabatically from 1 MPa and 600°C to 100 kPa. The isentropic efficiency is 0.92. The mass flow rate is 12 kg/s. Calculate the power output.

c_p = 1.005 kJ/kg K c_v = 0.718 kJ/kg K.

SOLUTION

The process is adiabatic so the ideal temperature $T_{2'}$ is given by

$$T_{2'}=T_1(r_p)^{1-1/\gamma}$$

r_p is the pressure ratio
$r_p = p2/p1 = 0.1$

$$\gamma = c_p/c_v = 1.005/0.718 = 1.4$$

$$T_{2'}=873(0.1)^{1-1/1.4} = 451.9 \text{ K}$$

Now we use the isentropic efficiency to find the actual final temperature.

$$\eta_{is} = (T_2 - T_1)/(T_{2'} - T_1)$$

$$0.92 = (T_2 - 873)/(451.9 - 873)$$

$$T_2 = 485.6 \text{ K}$$

Now we use the SFEE to find the power output.

$$\dot{\Phi} + P = m\, c_p(T_2 - T_1)$$

The process is adiabatic $\Phi = 0$.

P = 12(1.005)(485.6 - 873) = - 4672 kW (out of system)

WORKED EXAMPLE No.3

A simple steam power plant uses the Rankine cycle. The boiler supplies superheated steam to the turbine at 40 bar and 400°C. The condenser operates at 0.2 bar and produces saturated water. The power input to the pump is negligible.

i. Calculate the thermal efficiency of the ideal cycle.

ii. Calculate the thermal efficiency when the turbine has an isentropic efficiency of 89%.

SOLUTION

The solution is easier with a h-s chart.

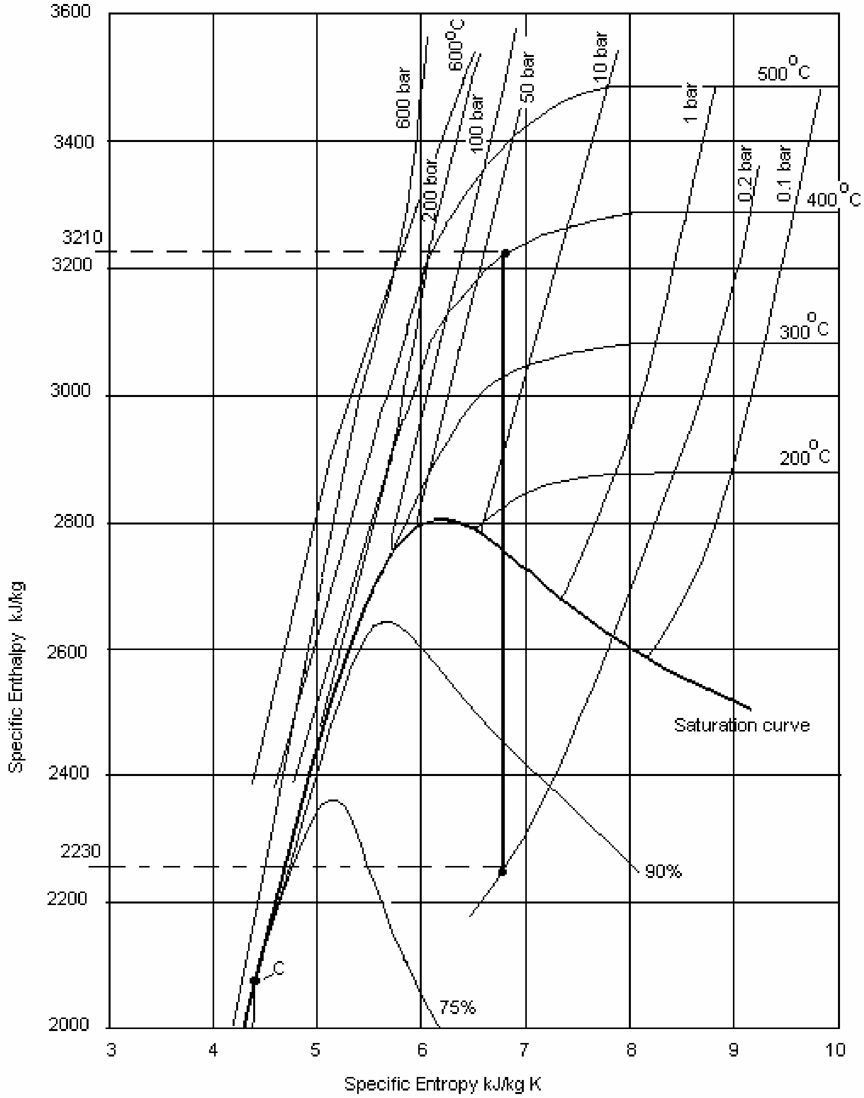

Figure 3

IDEAL CONDITIONS

From the chart h_1= 3210 kJ/kg and h_2 = 2230 kJ/kg/k

The ideal work output = 3210 – 2230 = 980 kJ/kg

When the power input to the pump is ignored, the power out is the net power and the enthalpy at inlet to the boiler is h_f at 0.2 bar

The heat input to the boiler = 3210 – 251 = 2959 kJ/kg

$$\eta_{th} = \frac{980}{2959} = 0.331 \text{ 0r } 33.1\%$$

TAKING ACCOUNT OF ISENTROPIC EFFICIENCY

$$\eta_{is} = \frac{\text{Actual work output}}{\textit{Ideal}\text{ work output}} = \frac{\text{Actual work output}}{980}$$

$$0.89 = \frac{\text{Actual work output}}{980}$$

$$\text{Actual work output} = 980 \text{ x } 0.89 = 872.2 \text{ kJ/kg}$$

$$\eta_{th} = \frac{872.2}{2959} = 0.295 \text{ or } 29.5\%$$

WORKED EXAMPLE No.4

A simple gas turbine uses the Joule cycle. The pressure ratio is 6.5. The air temperature is 300 K at inlet to the compressor and 1373 K at inlet to the turbine. The adiabatic index is 1.4 throughout and the specific heat capacities may be considered constant.

i. Calculate the thermal efficiency of the ideal cycle.

ii. Calculate the thermal efficiency when the turbine and compressor has an isentropic efficiency of 90%.

iii. Sketch the cycle on a T-s diagram showing the effect of friction.

SOLUTION

IDEAL CYCLE

Compressor $T_2 = 300(6.5)^{0.286} = 512.4\text{ K}$

Turbine $T_4 = 1373(6.5)^{-0.286} = 803.8\text{ K}$

$$\eta_{th} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{803.8 - 300}{1373 - 512.4} = 0.415 \text{ or } 41.5\%$$

INCLUDING ISENTROPIC EFFICIENCY

Compressor $\eta_{is} = 0.9 = \dfrac{512.4 - 300}{T_2 - 300}$

$T_2 = 536\text{ K}$

Turbine $\eta_{is} = 0.9 = \dfrac{1373 - T_4}{1373 - 803.8}$

$T_4 = 860.7\text{ K}$

$$\eta_{th} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{860.7 - 300}{1373 - 536} = 0.33 \text{ or } 33\%$$

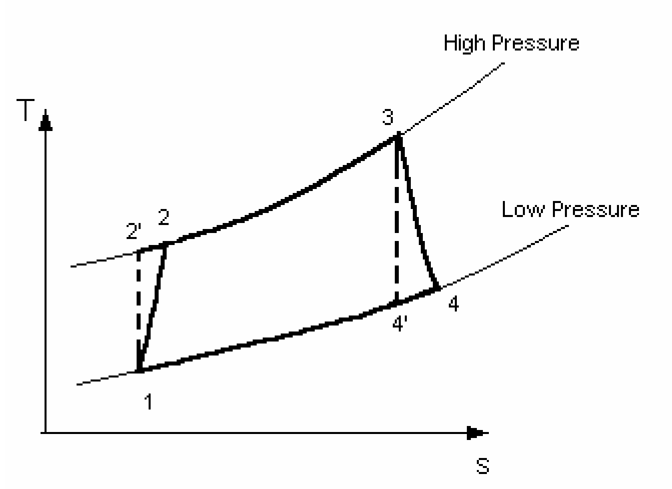

Fig. 4

SELF ASSESSMENT EXERCISE No.1

1. Steam is expanded adiabatically in a turbine from 100 bar and 600°C to 0.09 bar with an isentropic efficiency of 0.88. The mass flow rate is 40 kg/s. Calculate the power output.
(51 mw)

2. A compressor takes in gas at 1 bar and 15°C and compresses it adiabatically to 10 bar with an isentropic efficiency of 0.89. The mass flow rate is 5 kg/s. Calculate the final temperature and the power input. c_p = 1.005 kJ/kg K γ=1.4
(590 K and 1.51 MW)

3. A turbine is supplied with 3 kg/s of hot gas at 10 bar and 920°C. It expands adiabatically to 1 bar with an isentropic efficiency of 0.92. Calculate the final temperature and the power output. c_p = 1.005 kJ/kg K γ=-1.4
(663 K and 1.6 MW)

4. A turbine is supplied with 7 kg/s of hot gas at 9 bar and 850°C that it expands adiabatically to 1 bar with an isentropic efficiency of 0.87. Calculate the final temperature and the power output. c_p = 1.005 kJ/kg K γ=1.4
(667 K and 3.2 MW)

5. A simple steam power plant uses the Rankine cycle. The boiler supplies superheated steam to the turbine at 100 bar and 550°C. The condenser operates at 0.05 bar and produces saturated water. The power input to the pump is negligible.

   i. Calculate the thermal efficiency of the ideal cycle. (42.5%)

   ii. Calculate the thermal efficiency when the turbine has an isentropic efficiency of 85%. (36.1%)

6. A simple gas turbine uses the Joule cycle. The pressure ratio is 7.5. The air temperature is 288 K at inlet to the compressor and 1400 K at inlet to the turbine. The adiabatic index is 1.4 throughout and the specific heat capacities may be considered constant.

   i. Calculate the thermal efficiency of the ideal cycle. (43.8%)

   ii. Calculate the thermal efficiency when the turbine and compressor has an isentropic efficiency of 92%. (36.9%)