Innovative design and control of robotic manipulator for chemically aggressive environments

Martin Švejda, Martin Goubej
Department of cybernetics
University of West Bohemia
Pilsen, Czech Republic
Email: msvejda@kky.zcu.cz, mgoubej@kky.zcu.cz

***Abstract*—The paper deals with kineto-static analysis of the AGgressive Environment roBOT (AGEBOT). Serio-parallel kinematic architecture of AGEBOT plays important role in aggressive environment applications such as manipulation in industrial degreasing and paint removing lines in the presence of high pressure and temperature, acid or lye. Standard 4DoF serial manipulator with PRRR joints is equipped with special 3DoF parallel spherical wrist which makes possible to separate vulnerable components (motors, sensors) from the rest of the end-effector by a water-proof barrier. Inverse and direct kinematics are discussed. The algorithm for velocity, acceleration and static forces (gravity compensation) is derived. In the case of known inverse and direct position dependencies proposed algorithm can be generalized for an arbitrary serial or parallel manipulator.**

***Keywords*—serio-parallel manipulator, kinetostatic analysis, gravity compensation**

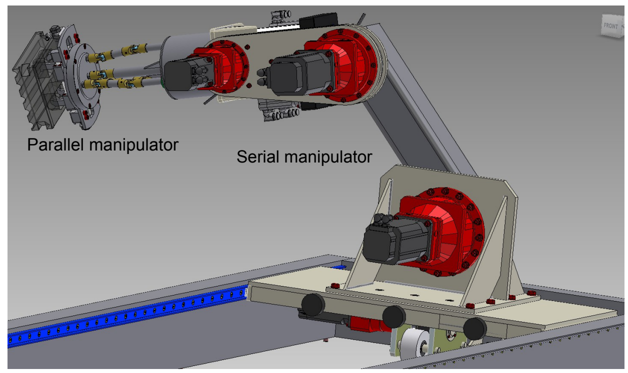

Figure 1. AGEBOT (from 3D CAD vizualization)

I. Introduction

AGEBOT is designed as a special robot architecture which consist of two main parts. Serial manipulator (SM) and parallel manipulator (PM) [1]. SM ensures basic positioning of the end effector of AGEBOT including the translations in x, y, z axes and orientation of the longitudinal axis of PM. This motion is used for handling of parts which are to be exposed of cleaning process in cleaning chambers. PM plays an important role in precise positioning of handled parts inside cleaning chambers because PM allows full orientating about x, y, z axes. Mechanical design of PM is chosen in such a way to be possible separate vulnerable components (motors, sensors, etc.) from an aggressive environment inside the cleaning chamber. Overall view of the AGEBOT is shown in Figure 1.

Position, velocity and acceleration of the actuators of AGEBOT are denoted as joint coordinates $\boldsymbol{\Theta}$ (and their corresponding time derivatives) and position, velocity and acceleration of the end effector as generalized coordinates $\boldsymbol{X}$. The position dependencies between joint and generalized coordinates are referred as direct geometric model (DGM) and inverse geometric model (IGM) and velocity and accelerations dependencies as direct instantaneous geometric model (DIGM) and inverse instantaneous geometric model (IIGM).

A. Serial part of AGEBOT

SM consists of serial kinematic chain **PRRR** where all joints are actuated. The end effector of SM ensures 3 Dof (Degrees of Freedom), three translations and one rotation. The kinematic scheme is shown in Figure 2 and joint $\boldsymbol{\Theta}_S$ and generalized $\boldsymbol{X}_S$ coordinates are set as:

$$\boldsymbol{\Theta}_S = \begin{bmatrix} d_1 & \theta_1 & \theta_2 & \theta_3 \end{bmatrix}^T \tag{1}$$

$$\boldsymbol{X}_S = \begin{bmatrix} \boldsymbol{O}_4^{0_S} \\ \phi \end{bmatrix} = \begin{bmatrix} x & y & z & \phi \end{bmatrix}^T \tag{2}$$

where $\boldsymbol{O}_4^{0_S}$ are the end effector coordinates with respect to coordinate system (CS) F_{0S}. The lengths of individual links are called the kinematic parameters of SM:

$$\boldsymbol{\xi}_S = \begin{bmatrix} L_1 & L_2 & L_3 & L_4 \end{bmatrix}^T$$

The dynamic parameters of SM are supposed to be link's masses $\boldsymbol{M}_S$, origins $\boldsymbol{CG}_S$ (with respect to Link's CSs) and inertia matrices $\boldsymbol{I}_S$ (with respect to Link's origins), m_e is a payload mass:

$$\boldsymbol{M}_S = \begin{bmatrix} m_1 & m_2 & m_3 & m_4 & m_e \end{bmatrix}^T$$

$$\boldsymbol{CG}_S = \begin{bmatrix} \boldsymbol{cg}_1 & \boldsymbol{cg}_2 & \boldsymbol{cg}_3 & \boldsymbol{cg}_4 \end{bmatrix}^T$$
$$\boldsymbol{I}_S = \begin{bmatrix} \boldsymbol{I}_{S_1} & \boldsymbol{I}_{S_2} & \boldsymbol{I}_{S_3} & \boldsymbol{I}_{S_4} \end{bmatrix}$$

CSs of SM are established according to Denavit-Hartenberg notaion (D-H) [2]. Therefore, the homogeneous transformation matrices $\boldsymbol{T}_i^{i-1} \in \Re^{4\times4}$ which describe position and orientation of CS F_i with respect to CS F_{i-1} can be found. Note that CS's correspond to the links of the manipulator. Position $\boldsymbol{r}_{i,j}^{j}$ (the

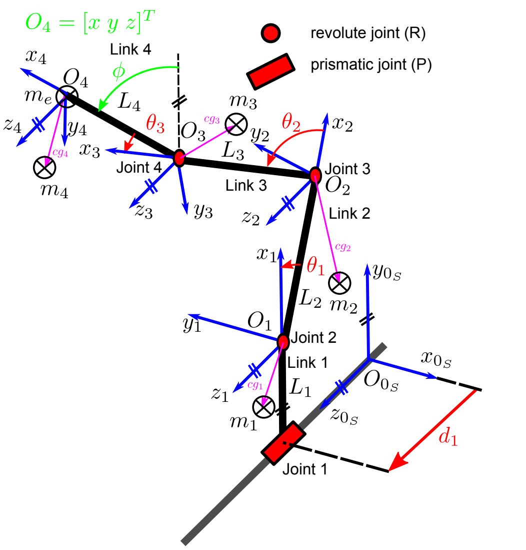

Figure 2. Serial part of AGEBOT

distance between the origins of F_i and F_j) and orientation $\boldsymbol{R}_i^j$ (rotation matrix) of given CS F_i with respect to CS F_j can be obtained directly from a sequence of multiplications of the homogeneous matrices and they are functions of joint coordinates $\boldsymbol{\Theta}$[1]:

$$\boldsymbol{T}_i^j = \left[\begin{array}{c|c} \boldsymbol{R}_i^j & \boldsymbol{r}_{i,j}^i \\ \hline \boldsymbol{0}_{1\times3} & 1 \end{array}\right] = \prod_{m=j}^{i-1} \boldsymbol{T}_{m+1}^m(\boldsymbol{\Theta}[\boldsymbol{m}+\boldsymbol{1}]) \tag{3}$$

The computation mentioned above results in a closed solution of DGM and IGM, for more details see [3][2].

DGM:

$$\boldsymbol{\Theta}_S = \begin{bmatrix} L_4 s_{\theta_{1,2,3}} - L_3 s_{\theta_{1,2}} - L_2 s_{\theta_1} \\ L_4 c_{\theta_{1,2,3}} + L_3 c_{\theta_{1,2}} + L_2 c_{\theta_1} \\ d_1 \end{bmatrix}, \tag{4}$$

IGM:

$$d_1 = z \tag{5}$$

$$w_x = y - L_1 - L_4 c_\phi$$
$$w_y = -x - L_4 s_\phi$$

$$\cos\theta_2 = \frac{w_x^2 + w_y^2 - L_2^2 - L_3^2}{2L_2L_3}$$
$$\sin\theta_2 = \pm\sqrt{1-\cos^2\theta_2}$$
$$\theta_2 = \operatorname{atan2}(\sin\theta_2, \cos\theta_2) \tag{6}$$

$$\sin\theta_1 = \frac{-L_3\sin\theta_2 w_x + (L_2 + L_3\cos\theta_2)w_y}{w_x^2 + w_y^2}$$
$$\cos\theta_1 = \frac{(L_2 + L_3\cos\theta_2)w_x + L_3\sin\theta_2 w_y}{w_x^2 + w_y^2}$$
$$\theta_1 = \operatorname{atan2}(\sin\theta_1, \cos\theta_1) \tag{7}$$

[1] $\boldsymbol{A}[a : b, c : d]$ is sub-matrix of $\boldsymbol{A}$ which consists of $a \dots b$ rows and $c \dots d$ columns.

[2] $s_{\theta_{1,2,3}} = \sin(\theta_1 + \theta_2 + \theta_3)$

$$\theta_3 = \phi - \theta_1 - \theta_2 \tag{8}$$

B. Parallel part of AGEBOT

PM consists of 3 independent kinematic chains **PUS** where only **P** joints are actuated through joint coordinates $\boldsymbol{\Theta}_P$. A passive kinematic chain **S** reduces DoF of the end effector to 3 orientation DoF (XYZ Euler angles corresponding to generalized coordinates $\boldsymbol{X}_P$). The kinematic scheme is shown in Figure 3. The kinematic and dynamic parameters are given in the same manner as for SM and they are identical for each kinematic chain.

$$\boldsymbol{\Theta}_P = \begin{bmatrix} l_{11} & l_{21} & l_{31} \end{bmatrix}^T, \ \boldsymbol{X}_P = \begin{bmatrix} \alpha & \beta & \gamma \end{bmatrix}^T \tag{9}$$

$$\boldsymbol{\xi}_P = \begin{bmatrix} a_1 & a_2 & l & v \end{bmatrix}^T \tag{10}$$

where a_1, a_2 is a side length of the base and end effector triangles, $l = l_{12} = l_{22} = l_{32}$ and v is the manipulator height.

$$\boldsymbol{M}_P = \begin{bmatrix} m_1 & m_3 & m_e \end{bmatrix}^T$$

$$\boldsymbol{CG}_P = \begin{bmatrix} \boldsymbol{cg}_1 & \boldsymbol{cg}_3 & \boldsymbol{cg}_e \end{bmatrix}^T, \ \boldsymbol{I}_P = [\boldsymbol{I}_{P_1} \quad \boldsymbol{I}_{P_3} \quad \boldsymbol{I}_{P_e}]$$

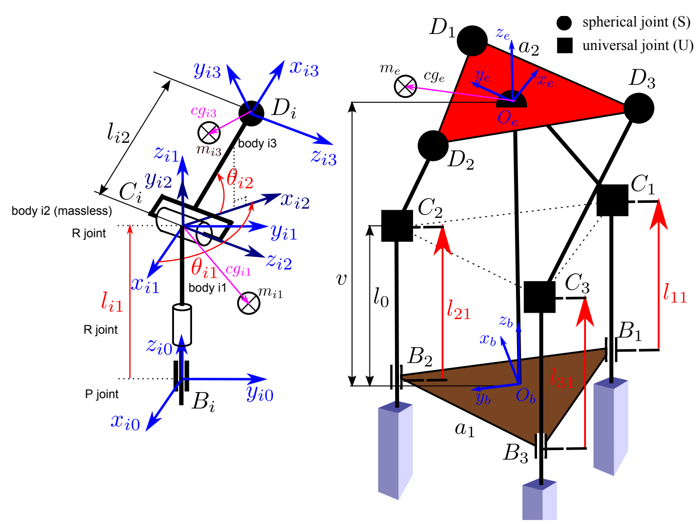

Figure 3. Parallel part of AGEBOT and **PUS** kinematic chain

IGM can be solved in closed form as:

$$l_{i1} = \overrightarrow{\boldsymbol{B}_i\boldsymbol{D}_i}^{i0}[3] \pm \sqrt{l^2 - \overrightarrow{\boldsymbol{B}_i\boldsymbol{D}_i}^{i0}[1]^2 - \overrightarrow{\boldsymbol{B}_i\boldsymbol{D}_i}^{i0}[2]^2} \tag{11}$$

where $\overrightarrow{\boldsymbol{B}_i\boldsymbol{D}_i}^{i0} = \boldsymbol{O}_e^{i0} + \boldsymbol{R}_e^{i0} \cdot \boldsymbol{D}_i^e$, $\boldsymbol{R}_e^{i0} = \boldsymbol{R}_e^b$ is the rotation matrix depending on $\boldsymbol{X}_P$ and $\boldsymbol{O}_e^{i0}$ and $\boldsymbol{D}_i^e$ are constant vectors given by the kinematic parameters with respect to the base and end effector CS respectively.

DGM is much more difficult to solve and it can be proven that there does not exist closed form solution and up to eight different positions of the end effector can be found for given joint coordinates. For more details see [3], [4]. On the other hand there are some efficient numerical methods for dealing with this problem.

II. Kinetostatic analysis

Kinetostatic analysis is very useful in a process of manipulators design and it can be decomposed into two parts. Kinematic analysis which is given by the DIGM and IIGM and static analysis which makes possible to determine force dependencies between joint forces/moments and generalized forces/moments if a manipulator is supposed to be stationary. These relations have to be taken into account especially in an early design phase of manipulators when mechanical components (motor, gearboxes, links, etc.) should be specified.

A. Kinematic analysis for serial chain

Generally, DIGM and IIGM for serial kinematic chain with n joints (**P** or **R**), joint coordinates $\boldsymbol{\Theta}$ and generalized coordinates $\boldsymbol{X}$ can be given directly by a time derivative of DGM and IGM respectively. But despite of all possibilities of symbolic computation software like Maple, Mathematica, etc. the resulting terms are too complicated for implementation to control systems. Fortunately, we can use a geometrical methodology which makes possible to establish DIGM/IIGM with the help of an algebraic computation depending on the elements of the homogeneous transformation matrices (3).

Let DIGM is formulated as:

$$\dot{\boldsymbol{X}} = \boldsymbol{J}(\boldsymbol{\Theta}) \cdot \dot{\boldsymbol{\Theta}} \text{ (velocities)} \tag{12}$$

$$\ddot{\boldsymbol{X}} = \dot{\boldsymbol{J}}(\boldsymbol{\Theta}) \cdot \dot{\boldsymbol{\Theta}} + \boldsymbol{J}(\boldsymbol{\Theta}) \cdot \ddot{\boldsymbol{\Theta}} \text{ (accelerations)} \tag{13}$$

And IIGM is formulated as:

$$\dot{\boldsymbol{\Theta}} = \boldsymbol{J}^{-1}(\boldsymbol{\Theta}) \cdot \dot{\boldsymbol{X}} \text{ (velocities)} \tag{14}$$

$$\ddot{\boldsymbol{\Theta}} = \boldsymbol{J}^{-1}(\boldsymbol{\Theta}. \left(\ddot{\boldsymbol{X}} - \dot{\boldsymbol{J}}(\boldsymbol{\Theta}) \cdot \dot{\boldsymbol{\Theta}}\right) \text{ accelerations} \tag{15}$$

Where generalized coordinates $\boldsymbol{X} = \begin{bmatrix} \boldsymbol{O}_n^0 & \boldsymbol{\omega}_n^0 \end{bmatrix}^T$ consists of the translation and rotation part of the last link's CS F_n with respect to the base CS F_0 and $\boldsymbol{J}$ is a kinematic jacobian which is given as follows[3]:

$$\underbrace{\begin{bmatrix} \dot{\boldsymbol{O}}_i^0 \\ \boldsymbol{\omega}_i^0 \end{bmatrix}}_{\boldsymbol{X}} = \underbrace{\begin{bmatrix} \boldsymbol{j}_1^p & \cdots & \boldsymbol{j}_j^p & \cdots & \boldsymbol{j}_i^p \\ \boldsymbol{j}_1^o & \cdots & \boldsymbol{j}_j^o & \cdots & \boldsymbol{j}_i^o \end{bmatrix}}_{\boldsymbol{J}_i^0(\boldsymbol{\Theta})} \cdot \underbrace{\begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_i \end{bmatrix}}_{\dot{\boldsymbol{\Theta}}} \quad i = n \tag{16}$$

Where:

$$\begin{bmatrix} \boldsymbol{j}_j^p \\ \boldsymbol{j}_j^o \end{bmatrix} = \begin{bmatrix} \boldsymbol{z}_{j-1}^0 \\ 0 \end{bmatrix} \quad \text{(for } \mathbf{P} \text{ joint)}$$

$$\begin{bmatrix} \boldsymbol{j}_j^p \\ \boldsymbol{j}_j^o \end{bmatrix} = \begin{bmatrix} \boldsymbol{z}_{j-1}^0 \times \boldsymbol{r}_{j-1,i}^0 \\ \boldsymbol{z}_{j-1}^0 \end{bmatrix} \quad \text{(for } \mathbf{R} \text{ joint)}$$

And $\boldsymbol{z}_j^0$ a $\boldsymbol{r}_{j,i}^0$ is given by the sequence of matrices (3) as:

$$\boldsymbol{z}_j^0 = \boldsymbol{T}_j^0[1:3,3] \quad \text{and} \quad \boldsymbol{r}_{j,i}^0 = \boldsymbol{T}_i^0[1:3,4] - \boldsymbol{T}_j^0[1:3,4]$$

[3]Note, that generally the kinematic jacobian $\boldsymbol{J}$ is matrix of type $[6 \times n]$ but it can be reduced appropriately to $[n \times n]$ matrix for a non-redundant manipulator. It means that for SM the kinematic jacobian is $[4 \times 4]$ matrix because only the rotation about z axis is possible. Hence, the inversion $\boldsymbol{J}^{-1}$ can be performed if the manipulator is supposed to be out of a singularity.

Certainly, it is necessary to determine a time derivative of the kinematic jacobian $\boldsymbol{J}$ if acceleration dependencies are taken into account. Similarly it can be done as:

$$\dot{\boldsymbol{J}}_i^0(\boldsymbol{\Theta}, \dot{\boldsymbol{\Theta}}) = \begin{bmatrix} \dot{\boldsymbol{j}}_1^p & \cdots & \dot{\boldsymbol{j}}_j^p & \cdots & \dot{\boldsymbol{j}}_i^p \\ \dot{\boldsymbol{j}}_1^o & \cdots & \dot{\boldsymbol{j}}_j^o & \cdots & \dot{\boldsymbol{j}}_i^o \end{bmatrix}, \quad i = n \tag{17}$$

Where:

$$\begin{bmatrix} \dot{\boldsymbol{j}}_j^p \\ \dot{\boldsymbol{j}}_j^o \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{z}}_{j-1}^{0s} \\ 0 \end{bmatrix} \quad \text{(for } \mathbf{P} \text{ joint)}$$

$$\begin{bmatrix} \dot{\boldsymbol{j}}_j^p \\ \dot{\boldsymbol{j}}_j^o \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{z}}_{j-1}^0 \times \boldsymbol{r}_{j-1,i}^0 + \boldsymbol{z}_{j-1}^0 \times \dot{\boldsymbol{r}}_{j-1,i}^0 \\ \dot{\boldsymbol{z}}_{j-1}^0 \end{bmatrix} \quad \text{(for } \mathbf{R} \text{ joint)}$$

And:

$$\dot{\boldsymbol{z}}_i^0 = \boldsymbol{\omega}_i^0 \times \boldsymbol{z}_i^0 \quad \text{a} \quad \dot{\boldsymbol{r}}_{j,i}^0 = \dot{\boldsymbol{O}}_i^0 - \dot{\boldsymbol{O}}_j^0$$

Where $\dot{\boldsymbol{\omega}}_i^0$, $\dot{\boldsymbol{O}}_i^0$ is computed from (16).

DIKM and IIKM for SM is established according to this algorithm.

B. Static analysis for serial chain

If the kinematic jacobian is known, a relationship between joint forces/moments $\boldsymbol{\tau}$ and generalized forces and moments $\boldsymbol{F}$ can be derived by virtue of a virtual work principle for the manipulator in a static equilibrium [5], [6]:

$$\boldsymbol{\tau} = \boldsymbol{J}_n^T \cdot \boldsymbol{F} \tag{18}$$

If we consider that the position of the origin of the ith link is given with respect to CS F_{i-1} as:

$$\boldsymbol{T}_{i\,cg}^{i-1} = \left[\begin{array}{c|c} \boldsymbol{R}_i^{i-1} & \boldsymbol{r}_{i-1,i}^{i-1} + \boldsymbol{R}_i^{i-1} \cdot \boldsymbol{cg}_i \\ \hline \boldsymbol{0}_{1\times 3} & 1 \end{array}\right] \tag{19}$$

Where $\boldsymbol{R}_i^{i-1} = \boldsymbol{T}_i^{i-1}[1:3,1:3]$ and $\boldsymbol{r}_{i-1,i}^{i-1} = \boldsymbol{T}_i^{i-1}[1:3,4]$ and $\boldsymbol{cg}_i$ is the coordinates of the ith link's origin with respect to link's CS F_i.

Then it is possible to express overall static kinematic forces (**P** joint) and/or moments (**R** joints) as a sum of individual contributions of the gravity forces acting on the link's and end effector's origins in the sense of equation (18).

Because the SM is considered as a serial kinematic chain and the first **P** joint is not influenced by gravity, the static gravity compensation $\boldsymbol{\tau}_S$ for remaining joints can be expressed as:

$$\underbrace{\begin{bmatrix} M_2 \\ M_3 \\ M_4 \end{bmatrix}}_{\boldsymbol{\tau}_S} = \begin{bmatrix} \left(\boldsymbol{J}_{2\,cg}^1\right)^T & \left(\boldsymbol{J}_{3\,cg}^1\right)^T & & \\ \boldsymbol{0}_{1\times 3} & & \left(\boldsymbol{J}_{4\,cg}^1\right)^T & \left(\boldsymbol{J}_4^1\right)^T \\ \boldsymbol{0}_{1\times 3} & \boldsymbol{0}_{1\times 3} & & \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{F}_{g2}^1 \\ \boldsymbol{F}_{g3}^1 \\ \boldsymbol{F}_{g4}^1 \\ \boldsymbol{F}_{ge}^1 \end{bmatrix} \tag{20}$$

Where $\boldsymbol{J}_4^1$ is the kinematic jacobian computed analogously as in (16) with respect to CS F_1, it means from the homogeneous matrix sequence $\boldsymbol{T}_2^1$, $\boldsymbol{T}_3^1$, $\boldsymbol{T}_4^1$. The kinematic jacobians $\boldsymbol{J}_{i\,cg}^1$ are computed in a similar way only with difference as follows: The homogeneous transformation matrices sequence:

- For $\boldsymbol{J}_{2\,cg}^1$: $\boldsymbol{T}_{2\,cg}^1$

- For $\boldsymbol{J}^1_{3\,cg}$: $\boldsymbol{T}^1_2$, $\boldsymbol{T}^1_{3\,cg} = \boldsymbol{T}^1_2 \cdot \boldsymbol{T}^2_{3\,cg}$
- For $\boldsymbol{J}^1_{4\,cg}$: $\boldsymbol{T}^1_2$, $\boldsymbol{T}^1_3$, $\boldsymbol{T}^1_{4\,cg} = \boldsymbol{T}^1_2 \cdot \boldsymbol{T}^2_3 \cdot \boldsymbol{T}^3_{4\,cg}$

Where $\boldsymbol{T}^{i-1}_{i\,cg}$ are given in (19).

The gravity vectors with respect to CS F_1 (the vector are taken negatively because of compensation purposes):

$$\boldsymbol{F}^1_{gi} = \begin{bmatrix} 9.81 \cdot m_i & 0 & 0 \end{bmatrix}^T \tag{21}$$

C. Kinematic analysis for parallel manipulator

PM manipulator can be decomposed into three independent serial kinematic chains $\boldsymbol{B}_i\boldsymbol{C}_i\boldsymbol{D}_i$ (of type **PRR**) with joint coordinates $\boldsymbol{\Theta}_i = \begin{bmatrix} l_{i1} & \theta_{i1} & \theta_{i2} \end{bmatrix}^T$ and the generalized coordinates are supposed to be position coordinates of the end effector connecting points $\boldsymbol{X}_i = \boldsymbol{D}_i$. The transformation of CSs of the ith kinematic chain is given by the homogeneous transformation matrices $\boldsymbol{T}^{i(k-1)}_{ik}(\boldsymbol{\Theta_i}[k])$. The passive joints coordinates θ_{i1}, θ_{i2} can be computed from known active joint coordinates l_{i1} in (11) because the vector $\overrightarrow{\boldsymbol{C}_i\boldsymbol{D}_i}^{i0} = \overrightarrow{\boldsymbol{B}_i\boldsymbol{D}_i}^{i0} - \begin{bmatrix} 0 & 0 & l_{i1} \end{bmatrix}^T = \begin{bmatrix} cdi_x & cdi_y & cdi_z \end{bmatrix}^T$ is known:

$$s_{i2} = cd_{iz}, \quad c_{i2} = \pm\sqrt{cd^2_{ix} + cd^2_{iy}} \;\Rightarrow\; \theta_{i2} = \mathrm{atan2}(s_{i2}, c_{i2})$$

$$s_{i1} = \frac{cd_{iy}}{c_{i2}}, \quad c_{i1} = \frac{cd_{ix}}{c_{i2}} \;\Rightarrow\; \theta_{i1} = \mathrm{atan2}(s_{i1}, c_{i1}) \tag{22}$$

IIGM is established separately for each serial kinematic chain because velocities $\dot{\boldsymbol{X}}_i$ and accelerations $\ddot{\boldsymbol{X}}_i$ of the generalized coordinates of each kinematic chain are known for given generalized coordinates of PM as follows:

$$\begin{aligned} \dot{\boldsymbol{X}}_i &= \dot{\boldsymbol{D}}^{i0}_i = \boldsymbol{\omega}^{i0}_e \times \overrightarrow{\boldsymbol{O}_e\boldsymbol{D}_i}^{i0} = \boldsymbol{\omega}^{i0}_e \times \boldsymbol{R}^{i0}_e \boldsymbol{D}^e_i \\ \ddot{\boldsymbol{X}}_i &= \ddot{\boldsymbol{D}}^{i0}_i = \dot{\boldsymbol{\omega}}^{i0}_e \times \boldsymbol{R}^{i0}_e \boldsymbol{D}^e_i + \boldsymbol{\omega}^{i0}_e \times \dot{\boldsymbol{R}}^{i0}_e \boldsymbol{D}^e_i \end{aligned} \tag{23}$$

where

- Euler kinematic equation for XYZ rotations:

$$\boldsymbol{\omega}^{i0}_e = \boldsymbol{\omega}^b_e = \begin{bmatrix} 1 & 0 & s_\beta \\ 0 & c_\alpha & -s_\alpha c_\beta \\ 0 & s_\alpha & c_\alpha c_\beta \end{bmatrix} \cdot \dot{\boldsymbol{X}}_P$$

- A time derivative of the rotation matrix:

$$\dot{\boldsymbol{R}}^{i0}_e = \dot{\boldsymbol{R}}^b_e = \boldsymbol{S}(\boldsymbol{\omega}^b_e)\cdot\boldsymbol{R}^b_e, \quad \boldsymbol{S}(\boldsymbol{\omega}^b_e) = \begin{bmatrix} 0 & -\boldsymbol{\omega}^b_e[3] & \boldsymbol{\omega}^b_e[2] \\ \boldsymbol{\omega}^b_e[3] & 0 & -\boldsymbol{\omega}^b_e[1] \\ -\boldsymbol{\omega}^b_e[2] & \boldsymbol{\omega}^b_e[1] & 0 \end{bmatrix}$$

- A time derivative of $\dot{\boldsymbol{\omega}}^{i0}_e$:

$$\dot{\boldsymbol{\omega}}^{i0}_e = \dot{\boldsymbol{\omega}}^b_e = \begin{bmatrix} c_\beta\dot{\beta}\dot{\gamma} \\ -s_\beta\dot{\alpha}\dot{\beta} - c_\alpha c_\beta\dot{\alpha}\dot{\gamma} + s_\alpha s_\beta\dot{\beta}\dot{\gamma} \\ c_\alpha\dot{\alpha}\dot{\beta} - s_\alpha c_\beta\dot{\alpha}\dot{\gamma} - c_\alpha s_\beta\dot{\beta}\dot{\gamma} \end{bmatrix} + \begin{bmatrix} 1 & 0 & s_\beta \\ 0 & c_\alpha & -s_\alpha c_\beta \\ 0 & s_\alpha & c_\alpha c_\beta \end{bmatrix} \ddot{\boldsymbol{X}}_P$$

The kinematic jacobians $\boldsymbol{J}^{i0}_{i3}$ and their time derivative $\dot{\boldsymbol{J}}^{i0}_{i3}$ for the serial kinematic chains $\boldsymbol{B}_i\boldsymbol{C}_i\boldsymbol{D}_i$ can be determined in the same way as in the equations (16), (17). Therefore, IIGM for serial kinematic chains results in:

$$\dot{\boldsymbol{\Theta}}_i = \begin{bmatrix} \dot{l}_{i1} \\ \dot{\theta}_{i1} \\ \dot{\theta}_{i2} \end{bmatrix} = (\boldsymbol{J}^{i0}_{i3})^{-1} \cdot \dot{\boldsymbol{X}}_i \tag{24}$$

$$\ddot{\boldsymbol{\Theta}}_i = \begin{bmatrix} \ddot{l}_{i1} \\ \ddot{\theta}_{i1} \\ \ddot{\theta}_{i2} \end{bmatrix} = (\boldsymbol{J}^{i0}_{i3})^{-1} \left(\ddot{\boldsymbol{X}}_i - \dot{\boldsymbol{J}}^{i0}_{i3} \cdot \dot{\boldsymbol{\Theta}}_i \right) \tag{25}$$

It is clear that IIGM for PM is easily given by choosing the first coordinates from the equations (24), (25)[4].

D. Static analysis for parallel manipulator

A relation between the static joint forces $\boldsymbol{\tau}_P$ and moments $\boldsymbol{M}^b_e$ actuating the end effector of the PM is given in the sense of equation (18) through the kinematic jacobian $\boldsymbol{J}^b_e$ of the PM (the relation between joint velocities $\dot{\boldsymbol{\Theta}}_P$ and angular velocity of the end effector $\boldsymbol{\omega}^b_e$). The inverse of $\boldsymbol{J}^b_e$ can be derived from the equation (24) and the velocities of the connecting points $\dot{\boldsymbol{X}}_i$ can be expressed from (23) as a function of the angular velocity $\boldsymbol{\omega}^e_b$ as follows:

$$\dot{\boldsymbol{\Theta}}_i = \begin{bmatrix} \dot{l}_{i1} \\ \dot{\theta}_{i1} \\ \dot{\theta}_{i2} \end{bmatrix} = (\boldsymbol{J}^{i0}_{i3})^{-1} \cdot \dot{\boldsymbol{X}}_i = -(\boldsymbol{J}^{i0}_{i3})^{-1} \cdot \boldsymbol{S}(\boldsymbol{R}^{i0}_e \boldsymbol{D}^e_i) \cdot \boldsymbol{\omega}^{i0}_e$$

$$\dot{\boldsymbol{Q}}_P = \begin{bmatrix} \dot{l}_{11} \\ \dot{l}_{21} \\ \dot{l}_{31} \end{bmatrix} = -\underbrace{\begin{bmatrix} \left((\boldsymbol{J}^{10}_{13})^{-1} \cdot \boldsymbol{S}(\boldsymbol{R}^b_e \boldsymbol{D}^e_1)\right)[1,:] \\ \left((\boldsymbol{J}^{20}_{23})^{-1} \cdot \boldsymbol{S}(\boldsymbol{R}^b_e \boldsymbol{D}^e_2)\right)[1,:] \\ \left((\boldsymbol{J}^{30}_{33})^{-1} \cdot \boldsymbol{S}(\boldsymbol{R}^b_e \boldsymbol{D}^e_3)\right)[1,:] \end{bmatrix}}_{(\boldsymbol{J}^b_e)^{-1}} \cdot \boldsymbol{\omega}^b_e \tag{26}$$

Finding static forces for parallel manipulators in order to compensate an influence of gravity is more difficult because of dependencies among kinematic chains through the end effector. Therefore, it is not possible to compute static force compensation of the active joints of each kinematic chain separately (the gravity influence on the links) and add it to the static force which is given by the gravity actuating the end effector. Hence, the main idea for computation of static forces for PM is as follows:

1) Analogously to (20) it is possible to compute static forces and moments $\boldsymbol{\tau}_i$ of each kinematic chains. These forces and moments are caused by the mass of the chain's links.

$$\boldsymbol{\tau}_i = \begin{bmatrix} F_{i1} \\ M_{i1} \\ M_{i2} \end{bmatrix} = \begin{bmatrix} (\boldsymbol{J}^{i0}_{i1\,cg})^T & \\ \boldsymbol{0}_{1\times3} & (\boldsymbol{J}^{i0}_{i3\,cg})^T \\ \boldsymbol{0}_{1\times3} & \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{F}^{i0}_{gi1} \\ \boldsymbol{F}^{i0}_{gi3} \end{bmatrix} \tag{27}$$

where $(\boldsymbol{J}^{i0}_{i1\,cg})^T$ is the kinematic jacobian of the ith kinematic chain with respect to CS F_{i0} given by the transformation matrix $\boldsymbol{T}^{i0}_{i1\,cg}$, $(\boldsymbol{J}^{i0}_{i3\,cg})^T$ is the kinematic jacobian of the ith kinematic chain with respect to CS F_{i0} given by the sequence of the transformation matrices $\boldsymbol{T}^{i0}_{i1}$, $\boldsymbol{T}^{i1}_{i2}$, $\boldsymbol{T}^{i2}_{i3\,cg}$, see (16), the transformation matrices $\boldsymbol{T}^{i(k-1)}_{ik\,cg}$ are given similarly as in (19):

$$\boldsymbol{T}^{i(k-1)}_{ik\,cg} = \left[\begin{array}{c|c} \boldsymbol{R}^{i(k-1)}_{ik} & \boldsymbol{r}^{i(k-1)}_{i(k-1),ik} + \boldsymbol{R}^{i(k-1)}_{ik} \cdot \boldsymbol{cg}_{ik} \\ \hline \boldsymbol{0}_{1\times3} & 1 \end{array}\right] \tag{28}$$

[4]DIGM can be established by the inversion of IIGM and it can be shown that it leads to solving linear equations system.

where $\boldsymbol{R}_{ik}^{i(k-1)} = \boldsymbol{T}_{ik}^{i(k-1)}[1:3,1:3]$ and $\boldsymbol{r}_{i(k-1),ik}^{i(k-1)} = \boldsymbol{O}_{ik}^{i(k-1)} = \boldsymbol{T}_{ik}^{i(k-1)}[1:3,4]$ are given by the transformation matrices of the serial chains $\boldsymbol{T}_{ik}^{i(k-1)}(\boldsymbol{\Theta_i}[k])$. $\boldsymbol{cg}_{ik}$ are origins and m_{ik} are masses of ith kinematic chain. The gravity forces are supposed to be:

$$\boldsymbol{F}_{gik}^{ik} = \begin{bmatrix} 0 & 0 & 9.81 \cdot m_{ik} \end{bmatrix}^T \tag{29}$$

2) The joint forces and moments $\boldsymbol{\tau}_i$ of each kinematic chain can be recomputed to forces $\boldsymbol{F}_{\boldsymbol{D}_i}$ actuating the end effector in the connecting points $\boldsymbol{D}_i$:

$$\boldsymbol{F}_{\boldsymbol{D}_i} = \left((\boldsymbol{J}_{i3}^{i0})^T\right)^{-1} \cdot \boldsymbol{\tau}_i \tag{30}$$

3) The static forces $\boldsymbol{F}_{\boldsymbol{D}_i}$ induce the moment of the end effector as (the influence of the gravity actuating the kinematic chains):

$$\boldsymbol{M}_{e\,cg\,\text{chains}}^b = \sum_{i=1}^{3} \boldsymbol{R}_e^b \cdot \boldsymbol{D}_i^e \times \boldsymbol{F}_{\boldsymbol{D}_i} \tag{31}$$

4) The moment actuating the end effector because of an gravity is given as:

$$\boldsymbol{M}_{e\,cg}^b = \boldsymbol{R}_e^b \cdot \boldsymbol{cg}_e \times \boldsymbol{F}_{ge}^b \tag{32}$$

where $\boldsymbol{cg}_e$ is the origin with respect to CS F_e and the gravity force is:

$$\boldsymbol{F}_{ge}^b = \begin{bmatrix} 0 & 0 & 9.81 \cdot m_e \end{bmatrix}^T$$

The resulting static moment of the end effector which has to be compensated is expressed as:

$$\boldsymbol{M}_e^b = \boldsymbol{M}_{e\,cg\,\text{chains}}^b + \boldsymbol{M}_{e\,cg}^b \tag{33}$$

5) The static joint forces $\boldsymbol{\tau}_P$ corresponding to the static moment $\boldsymbol{M}_e^b$ of the end effector can be expressed in the similar way as in (18) because the inverse jacobian is known, see (26):

$$\boldsymbol{\tau}_P = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = (\boldsymbol{J}_e^b)^T \cdot \boldsymbol{M}_e^b \tag{34}$$

III. Simulation results

The simulation model of AGEBOT was created in the toolbox SimMechanics in Matlab [7]. Therefore, it makes possible to start simulation in so-called *inverse dynamic mode* which returns required forces/moments of the joints for their given positions, velocities and accelerations. Required values of the joint coordinates and their derivatives are computed from required motion of the manipulator's end effector (given in generalized coordinates) through IGM and IIGM. So it is possible to compare overall required forces/moments of the actuators with required *static* forces/moments of these ones. This analysis plays an important role in the control design concerning feedforward compensation where the choice of suitable model of manipulator - complete inverse dynamic model or inverse static model (only a gravity compensation) - has to be taken into account in order to reduce a computation load.

A. Serial manipulator

Parameters:

$$\boldsymbol{\xi}_S = \begin{bmatrix} 0.26 & 0.67 & 0.44 & 0.84 \end{bmatrix}^T$$
$$\boldsymbol{M}_S = \begin{bmatrix} 282 & 106 & 52 & 84 & 15 \end{bmatrix}^T$$
$$\boldsymbol{CG}_S = \begin{bmatrix} -0.13 & -0.19414 & -0.13227e & -0.51987 \\ 0 & 0.000097 & 0.000010 & 0.003837 \\ 0 & 0.092685 & 0.057912 & 0.016341 \end{bmatrix}$$

$$\boldsymbol{I}_{S_2} = \begin{bmatrix} 3.80 & 0.002 & -2.67 \\ 0.002 & 11.5 & 0.004 \\ -2.67 & 0.004 & 8.06 \end{bmatrix}, \ \boldsymbol{I}_{S_3} = \begin{bmatrix} 0.63 & 0.001 & -0.53 \\ 0.001 & 2.11 & -0.007 \\ -0.53 & -0.007 & 1.77 \end{bmatrix},$$
$$\boldsymbol{I}_{S_4} = \begin{bmatrix} 0.99 & 0.16 & 0.39 \\ 0.16 & 10.5 & 0.0 \\ 0.39 & 0.0 & 10.4 \end{bmatrix}$$

Required trajectory of the end effector $\boldsymbol{X}_S$ was chosen as a line motion among the points $\boldsymbol{A}$-$\boldsymbol{F}$ with bang-bang profile of an acceleration with limited values of acceleration $(1\,[-])$ and velocity $(1\,[-])$[5].

B. Parallel manipulator

Parameters:

$$\boldsymbol{\xi}_P = \begin{bmatrix} 0.1298 & 0.10108 & 0.1465 & 0.278 \end{bmatrix}^T$$
$$\boldsymbol{M}_P = \begin{bmatrix} 1.5 & 2 & 5 \end{bmatrix}^T, \ \boldsymbol{CG}_P = \begin{bmatrix} 0 & -0.07325 & 0 \\ 0 & 0 & 0 \\ 0.08 & 0 & 0 \end{bmatrix}$$
$$\boldsymbol{I}_{P_1} = \boldsymbol{I}_{P_3} = \boldsymbol{I}_{P_e} \begin{bmatrix} 3.80 & 0.002 & -2.67 \\ 0.002 & 11.5 & 0.00400 \\ -2.67 & 0.004 & 8.06 \end{bmatrix}$$

Required trajectory of the end effector $\boldsymbol{X}_P$ was chosen as the motion with constant γ coordinate and α, β to be changed in such a way that the z axis of CS F_e is aligned step by step with three given direction vectors. The angular acceleration is again considered to be bang bang profile with limited values of acceleration $(1\ \frac{rad}{s^2})$ and velocity $(1\ \frac{rad}{s})$.

IV. Conclusion

The paper deals with kineto-static analysis of the special serio-parallel manipulator AGEBOT. It was shown that there are some efficient methods for finding velocity and acceleration dependencies between the joint and generalized coordinates (DIGM, IIGM) for serial manipulators as well as for parallel manipulators where the end effector is connected to the serial kinematic chains via **S** joints (**R** joints in planar cases). These geometrical methods are based on the known relations between the position of the joint and generalized coordinates (DGM, IGM) and there is no necessity of the symbolic derivation. The relationship between the static joints and generalized forces/moments can be derived in a similar way. Therefore, there is no problem to compute gravity compensation forces/moments which are necessary for maintaining the robot in the given position (without motion).

The simulation of the serial and parallel parts of AGEBOT was performed in SimMechanics toolbox with inverse dynamic

[5]Line motion has mixtured units due to the generalized coordinates consist of $[m]$ and $[rad]$.

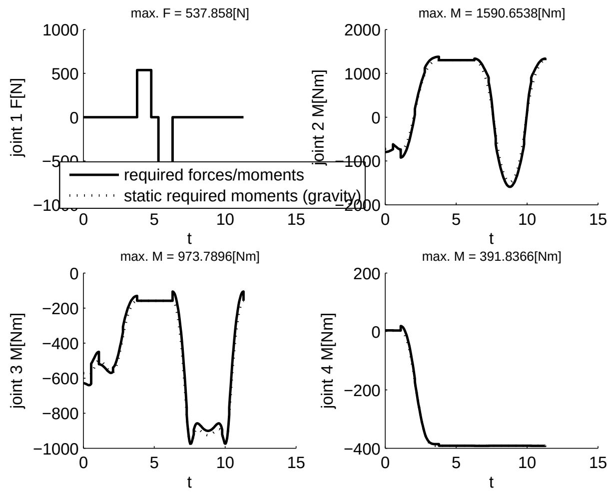

Figure 4. Required force and moments of the joints of SM. Overall (static and dynamic) force/moments from the SimMechanics model are depicted by a solid line and static moments from algorithm mentioned above are depicted by a dash line. Note, that the first **P** joint is not influenced by a gravity.

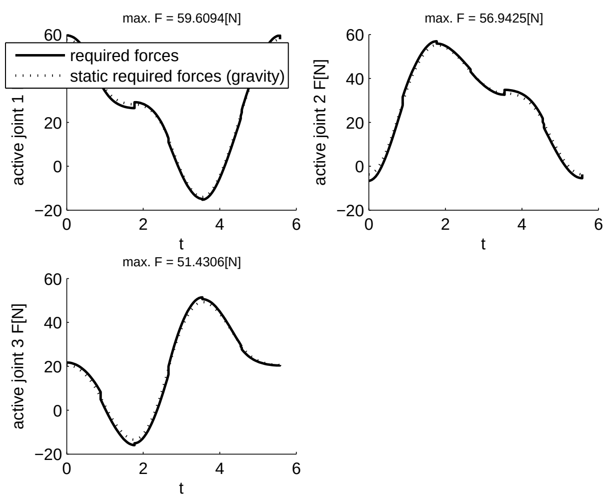

Figure 5. Required forces of the joints of PM. Overall (static and dynamic) forces from the SimMechanics model are depicted by a solid line and static forces from algorithm mentioned above are depicted by a dash line.

mode and the results are collected in Figure 4, Figure 5. It is clear that (for required trajectory of the end effector) the dynamic effects of joint forces/moments can be neglected in comparison with the static compensation forces/moments which are caused by a gravity. Therefore, only static feed-forward compensation can be aim of the interest during the control design of AGEBOT.

The main design drawback arises from using standard construction components (gear boxes, links, motors) which are not a priori determined for manipulator manufacturing. Especially for serial manipulators small values of the payload-mass ratio results in large static actuator moments compensating the impact of the gravity forces. Note that serial part of AGEBOT has the moving links with the total mass about 240 kg (including PM) and considered mass of payload is only 5 kg. The possibility of improving payload-mass ratio is to substitute $joint$ 2 and $joint$ 3 of SM with a special 2SCARA planar parallel manipulator, see Figure 6. The simulation model of this innovative design of AGEBOT was derived (based on methodology mentioned above) and there are supposed to be some advantages in comparison with the standard construction: higher rigidity, lower mass and possibility to use cheaper gears (two of three gears are mounted on the rigid base).

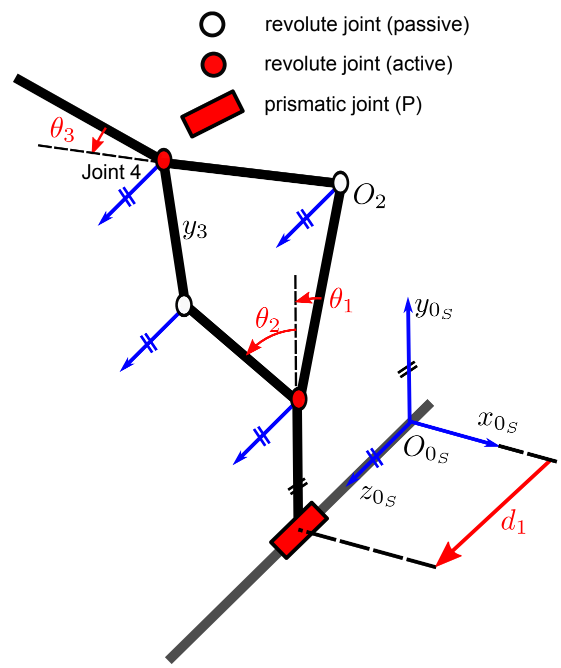

Figure 6. Alternative archtecture of AGEBOT

Acknowledgment

The paper was supported by grant FRTI1/174 from the Ministry of Industry and Trade of Czech Republic.

References

[1] M. Goubej and M. Švejda, "Research and design of modular robotic manipulator for chemical aggressive environment," in *Carpathian Control Conference (ICCC), 2011 12th International*, may 2011, pp. 374 –378.

[2] J. Denavit and R. S. Hartenberg, "A kinematic notation for lower-pair mechanisms based on matrices," *J. Appl. Mechanics, June 1955*, vol. 22, pp. 215–221, 1955.

[3] M. Švejda, "Inverse kinematics and statics of agebot manipulator," Department of Cybernetics, UWB in Pilsen, Tech. Rep., 2011.

[4] M. Švejda, "Kinematics of robot's architectures," Work for PhD exam, Department of Cybernetics, UWB in Pilsen, 2011.

[5] B. S. L. Sciavicco, *Modelling and Control of Robot Manipulators*, 2nd ed. Springer, 2000.

[6] E. D. W. Khalil, *Modeling, Identification and Control of Robots*. Butterworth-Heinemann, 2004.

[7] T. MathWorks, *SimMechanics User's Guide, www.mathworks.com*.