Příklad 1. Najděte všechna řešení Cauchyových počátečních úloh

1) $\begin{cases} ty'(t) + y(t) = 0, \quad t > 4, \\ y(4) = -3, \end{cases}$

2) $\begin{cases} y'(t)(1+t^2) = 2ty, \quad t > 1, \\ y(1) = 3. \end{cases}$

Příklad 2. Najděte obecná řešení diferenciálních rovnic prvního řádu

1) $ty'(t) + y(t) = y^2(t)$,

2) $ty'(t) + y(t) = t \sin t$.

Příklad 3. Užitím substituce $u(t) = \frac{y(t)}{t}$ řešte diferenciální rovnice

1) $t^2 y'(t) = ty(t) + y^2(t)$,

2) $y^2(t) - 2ty(t) + t^2 y'(t) = 0$.

Příklad 4. V rovině xy je zadán systém křivek s parametrem C. Tento systém křivek načrtněte. Dále určete a načrtněte systém kolmých křivek (sestavte diferenciální rovnice pro oba dva systémy křivek a najděte obecné řešení diferenciální rovnice, která popisuje systém kolmých křivek).

1) $y = Cx, \quad C \in \mathbb{R}$,

2) $xy = C, \quad C \in \mathbb{R}$,

3) $x^2 + (y - C)^2 = C^2, \quad C \in \mathbb{R} - \{0\}$.

Příklad 5. (dobrodružství kriminalistiky) Ráno byl ve sklepě nalezen mrtvý muž. V 8:00 byla teplota jeho těla 30 °C. V 11:00 klesla teplota oběti na 28 °C. Teplota vzduchu ve sklepě se po celý den drží na hodnotě 11 °C.

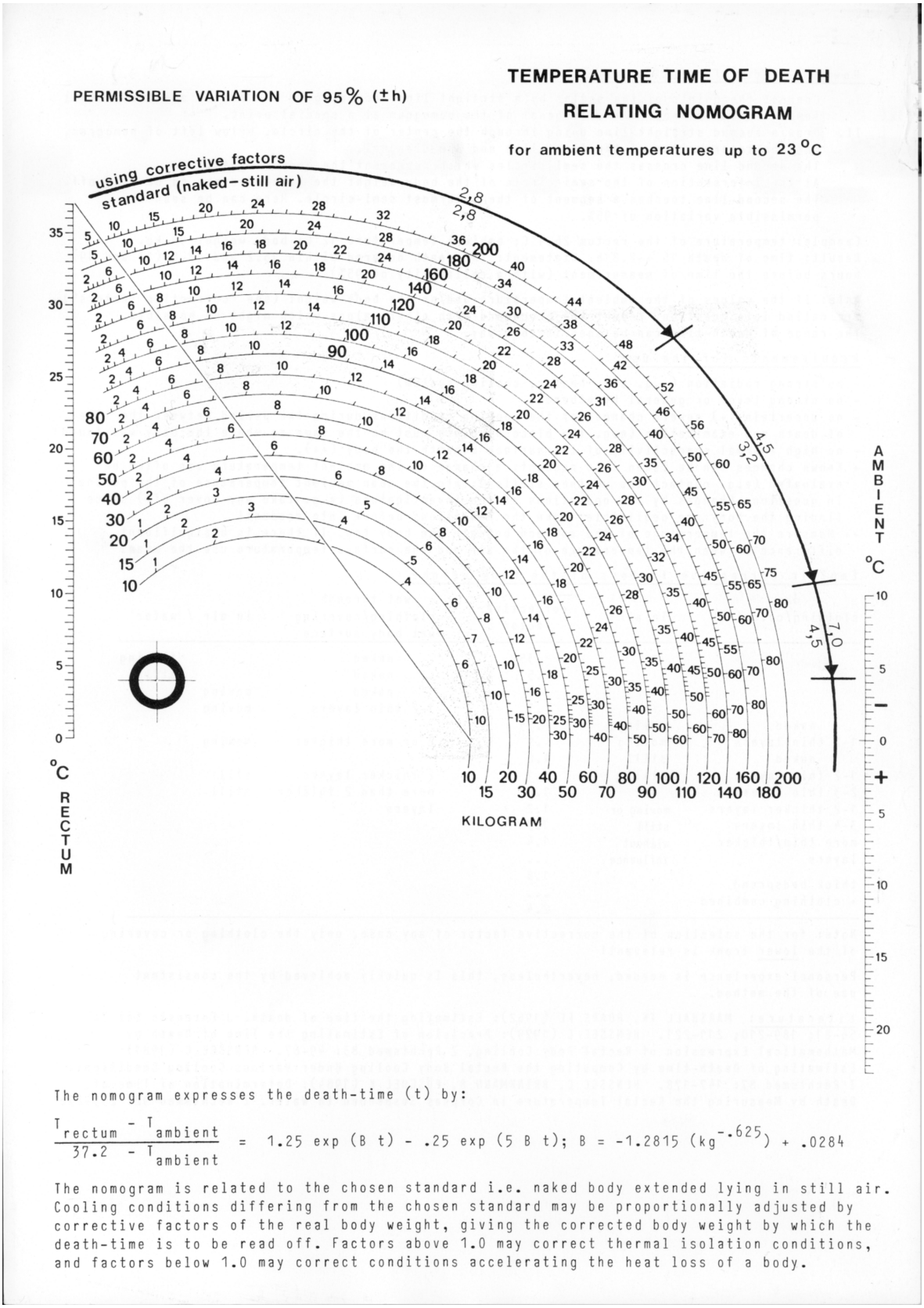
Matematický model chladnutí lidského těla post mortem má tvar počáteční úlohy

$$\begin{cases} y'(t) = -a(y(t) - b), \quad t > t_0, \\ y(t_0) = y_0, \end{cases} \tag{1}$$

kde $y(t)$ je teplota těla v čase t, a je koeficient chladnutí těla a b je teplota prostředí.

1. Určete řešení $y = y(t)$ počáteční úlohy (1).
2. Vypočtěte $\lim\limits_{t \to +\infty} y(t)$ a získaný výsledek interpretujte.
3. Předpokládejte, že teplota těla před usmrcením byla 37 °C. Pomocí modelu (1) stanovte čas mordu (zaokrouhleno na minuty).

Henssgeho nomogram



How to read off the time of death

I. Connect the points of the scales by a straight line according to the rectal and the ambient temperature. It crosses the diagonal of the nomogram at a special point.

II. Draw a second straight line going through the center of the circle, below left of nomogram, and the intersection of the first line and the diagonal.
The second line crosses the semi-circles which represent the body weights.
At the intersection of the semi-circle of the body weight the time of death ca be read off.
The second line touches a segment of the outermost semi-circle. Here can be seen the permissible variation of 95%.

Example: temperature of the rectum 26.4°C; ambient temperature 12°C; body weight 90 kg.
Result: time of death 16 +-2.8 h. Statement: the death occured within 13.2 and 18.8 (13 and 19) hours before the time of measurement (with a reliability of 95%).

Note: if the values of the ambient temperature and/or the body weight (see "corrective factors") are called into question, repeat the procedure with other values which might be possible.
The range of death-time can be seen in this way.

Requirements for the use

- no strong radiation (e.g. sun, heater, cooling system)
- no strong fever or general hypothermia
- no uncertain (+) severe changes of the cooling conditions during the period between the time of death and examination (e.g. the place of death must be the same as where the body was found)
- no high thermal conductivity of the surface beneath the body (++).

\+ Known changes can be taken into account: a change of the ambient temperature can often be evaluated (e.g. contact the weather station); use the mean ambient temperature of the period in question. Changes by the operations of the investigators (e.g. take any cover off) since finding the body are negligible: take the conditions before into account!

++ Measure the temperature of the surface beneath the body too. If there is a significant difference between the temperature of the air and the surface temperature use the mean.

Empiric corrective factors of the body weight

dry clothing/covering	in air	corrective factor	wet through clothing/covering wet body surface	in air / water
		.35	naked	flowing
		.5	naked	still
		.7	naked	moving
		.7	1-2 thin layers	moving
naked	moving	.75		
1-2 thin layers	moving	.9	2 or more thicker	moving
naked	still	1.0		
1-2 thin layers	still	1.1	2 thicker layers	still
2-3 thin layers		1.2	more than 2 thicker	still
1-2 thicker layers	moving or	1.2	layers	
3-4 thin layers	still	1.3		
more thin/thicker	without	1.4		
layers	influence	...		
		1.8		
thick bedspread		...		
+ clothing combined		2.4		

Note: for the selection of the corrective factor of any case, only the clothing or covering of the lower trunk is relevant!

Personel experience is needed, nevertheless, this is quickly achieved by the consistent use of the method.

Literature: MARSHALL TK, HOARE FE (1962): Estimating the time of death. J Forensic Sci 7: 56-81; 189-210; 211-221. HENSSGE C (1979): Precision of Estimating the Time of Death by Mathematical Expression of Rectal Body Cooling. Z Rechtsmed 83: 49-67. HENSSGE C (1981): Estimating of Death-time by Computing the Rectal Body Cooling Under Various Cooling Conditions. Z Rechtsmed 87: 147-178. HENSSGE C, BRINKMANN B, PÜSCHEL K (1984): Determination of Time of Death by Measuring the Rectal Temperature in Corpses Suspended in Water. Z Rechtsmed 92: 255-276