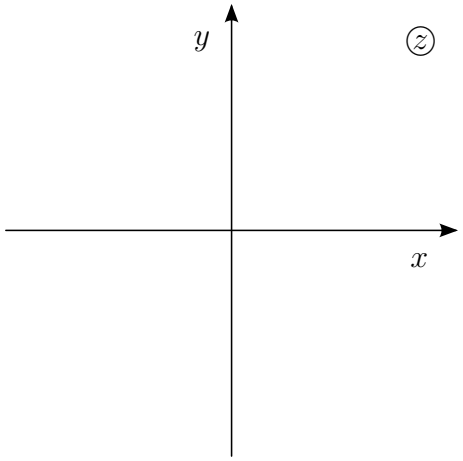
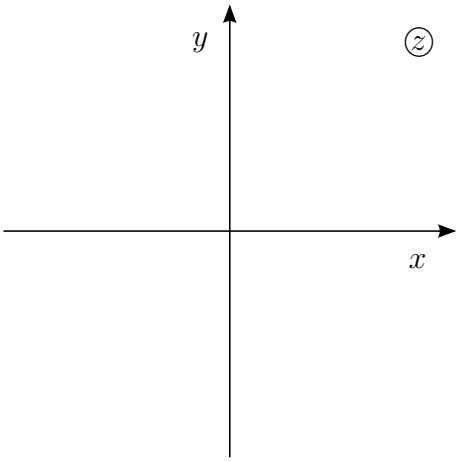
Jméno a PŘÍJMENÍ: .....

Příklad 1. (křivky)
Znázorněte množinu $\langle\varphi\rangle = \{\varphi(t) : \ t \in D(\varphi)\}$:

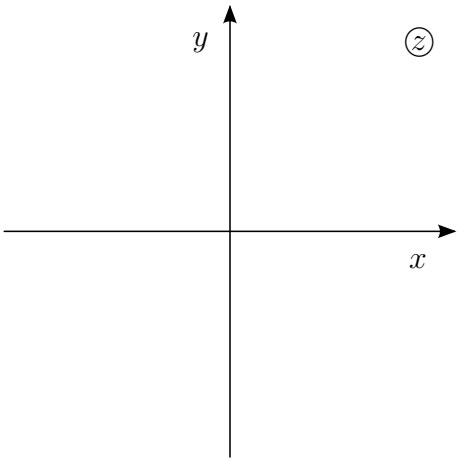
$\varphi(t) = 1 - \mathrm{i}\,t, \quad D(\varphi) = \langle 0, 2\rangle,$



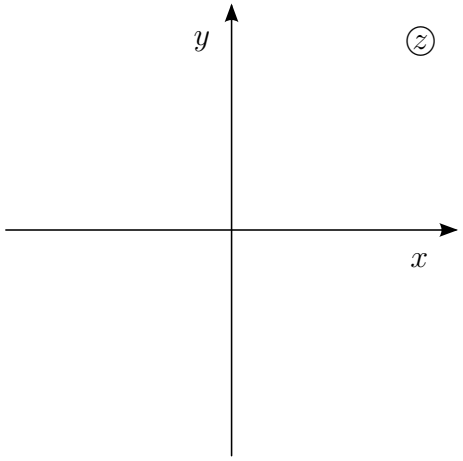
$\varphi(t) = t - \mathrm{i}\,t^2, \quad D(\varphi) = \langle -1, 2\rangle,$



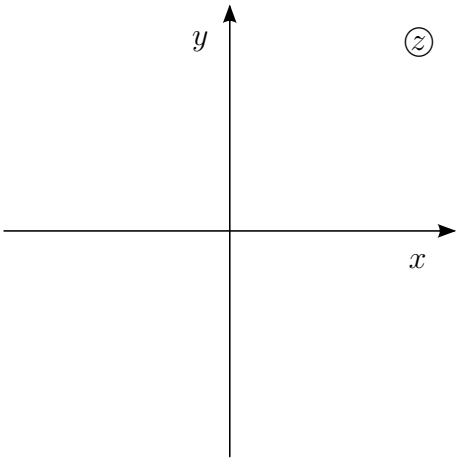
$\varphi(t) = 1 + \mathrm{e}^{-\mathrm{i}\,t}, \quad D(\varphi) = \langle 0, 2\pi\rangle,$



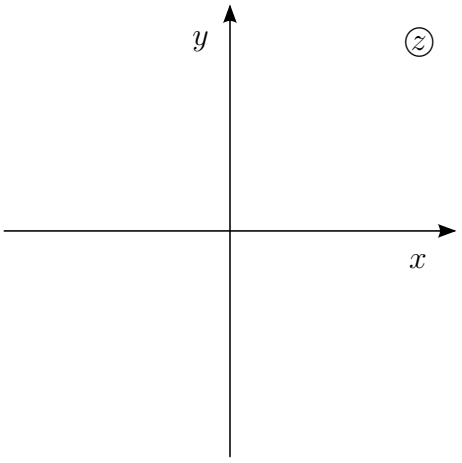
$\varphi(t) = \mathrm{e}^{2\,\mathrm{i}\,t} - 1, \quad D(\varphi) = \langle 0, 2\pi\rangle,$



$$\varphi(t) = \begin{cases} \mathrm{e}^{\mathrm{i}\,\pi t}, & t \in \langle 0, 1\rangle, \\ t - 2, & t \in \langle 1, 3\rangle, \end{cases}$$

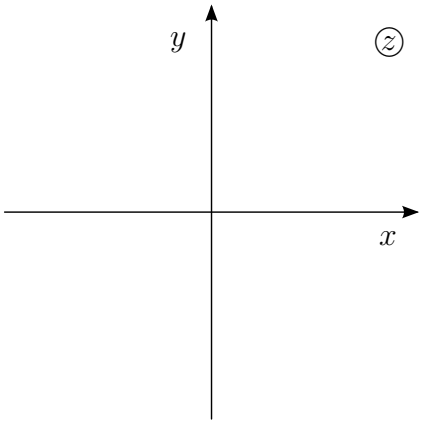


$$\varphi(t) = \begin{cases} \mathrm{e}^{\mathrm{i}\,t}, & t \in \langle -\frac{\pi}{2}, \pi\rangle, \\ \frac{3t}{\pi} - 4, & t \in \langle \pi, 2\pi\rangle. \end{cases}$$

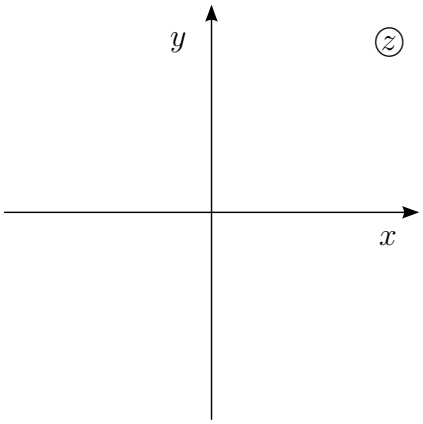


Příklad 2. (parametrizace množiny)
Načrtněte a parametrizujte množinu Ω (tzn. najděte křivku $\varphi:\ \mathbb{R}\to\mathbb{C}$ takovou, že $\langle\varphi\rangle=\Omega$):

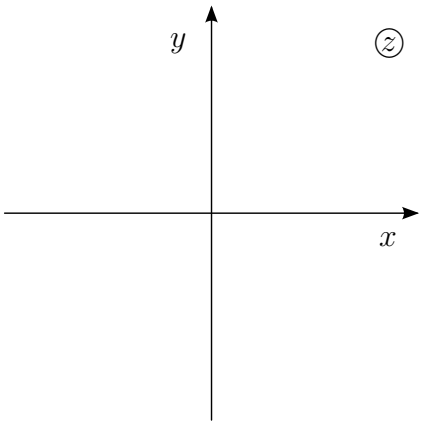
1. $\Omega=\{z\in\mathbb{C}:\ |z-2+3\,\mathrm{i}\,|=2\}$,



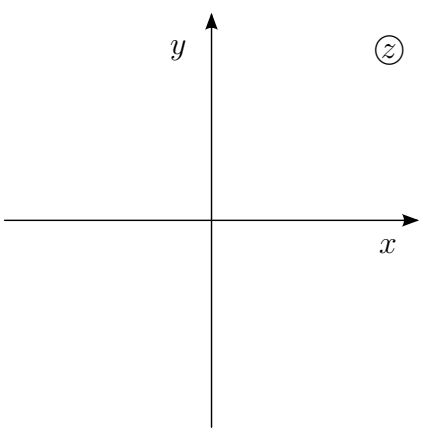
2. Ω je úsečka s krajními body $a,b\in\mathbb{C}$, $a\neq b$,



3. $\Omega=\{z\in\mathbb{C}:\ \mathrm{Re}\,z=2\,\mathrm{Im}\,z\}$,

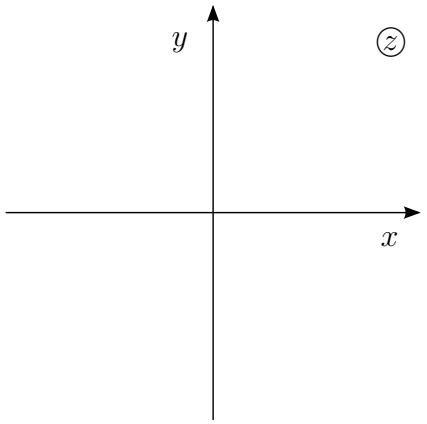


4. $\Omega=\left\{z\in\mathbb{C}:\ \mathrm{Re}\left(\frac{1}{z}\right)=2\right\}$,

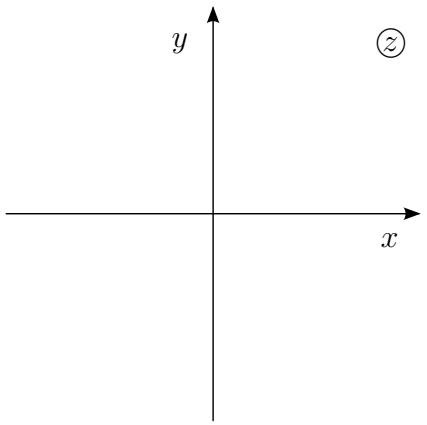


Příklad 3. (křivkový integrál)
Vypočtěte

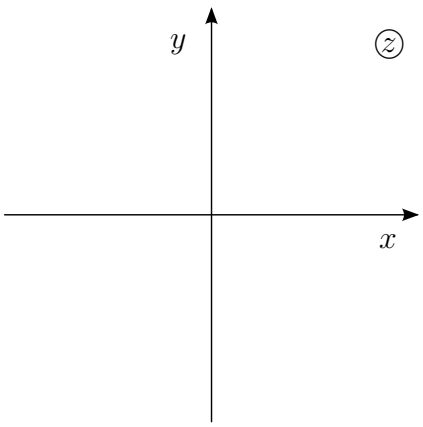
1. $\int\limits_{\varphi} \frac{1}{z}\,\mathrm{d}z$, kde φ je úsečka spojující body $1+\mathrm{i}$ a i,



2. $\oint\limits_{\varphi} \mathrm{Re}(z)\,\mathrm{d}z$, kde φ je kružnice se středem v počátku a poloměrem 5,



3. $\oint\limits_{\varphi} \frac{1}{1+z^2}\,\mathrm{d}z$, kde φ je kružnice se středem v počátku a poloměrem 3,



Příklad 4. (určitý integrál)
Vypočtěte

1. $\int_0^{1+\mathrm{i}} \mathrm{e}^z \,\mathrm{d}z,$

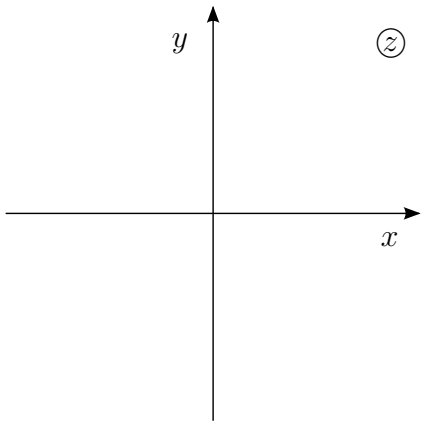
2. $\int_0^{1+\mathrm{i}} z^3 \,\mathrm{d}z,$

3. $\int_0^{\mathrm{i}} z^2 \sin z \,\mathrm{d}z,$

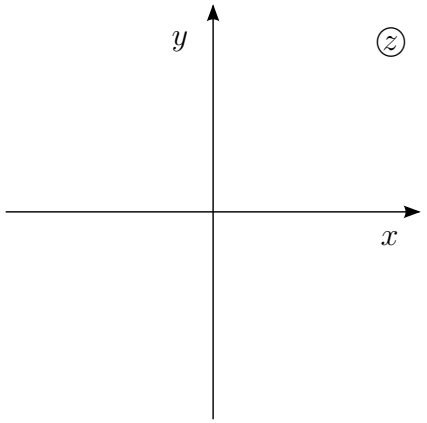
4. $\int_\pi^{\mathrm{i}} \mathrm{e}^z \cos z \,\mathrm{d}z,$

Příklad 5. (křivkový integrál)
Vypočtěte pomocí Cauchyových integrálních vzorců:

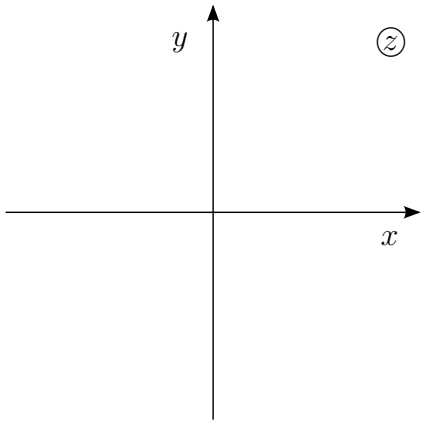
1. $\oint\limits_{\varphi} \dfrac{z^2+\mathrm{i}}{z}\,\mathrm{d}z$, kde $\varphi : |z-2\,\mathrm{i}\,| = 1$,



2. $\oint\limits_{\varphi} \dfrac{\sin z}{z+\mathrm{i}}\,\mathrm{d}z$, kde $\varphi :\ |z+\mathrm{i}\,| = 1$,



3. $\oint\limits_{\varphi} \dfrac{\sin z}{z^2-7z+10}\,\mathrm{d}z$, kde $\varphi :\ |z| = 3$,



4. $\oint\limits_{\varphi} \dfrac{\sin z}{(z-2\,\mathrm{i})^3}\,\mathrm{d}z$, kde $\varphi :\ |z| = 3$,