4 Inferences about Process Quality

- This chapter contains a review of basic statistical tests of hypotheses and linear regression. We will not cover this in this course.

5 Methods and Philosophy of Statistical Process Control

- We will review the "seven tools of quality" which are very basic statistical process control (SPC) problem-solving tools. We will cover the Shewhart control chart including sample size, sampling interval and rational subgroup selection, placement of control limits, interpretation of control chart signals and patterns, and the average run length in greater detail.

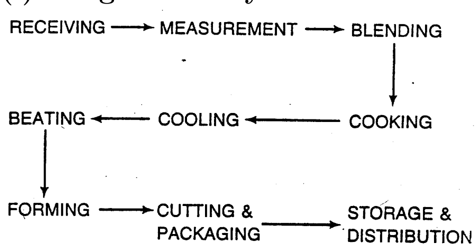
(I)	Flow charts	(IV)	Histograms	(VI)	Pareto charts
(II)	Cause-and-effect diagrams	(V)	Check sheets	(VII)	Scatter plots
(III)	Control charts				

5.1 I. Flow Charts

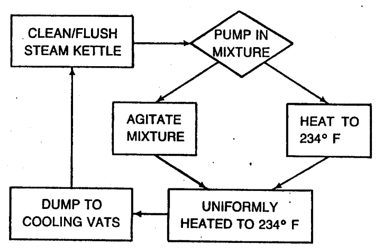
A **flow chart** is a chronological sequence of process steps. That is, it maps the flow of a process. Simple flow charts use directional arrows to indicate the flow direction. More sophisticated flow charts use special symbols to represent different types of flow (e.g., inspection, transportation, delay, storage, etc.).

Fudge Industry Example: The following figures show (i) a simple flow chart of the process, (ii) a detailed flow chart of the cooking stage in the process, and (iii) a pictorial flow chart of the process.

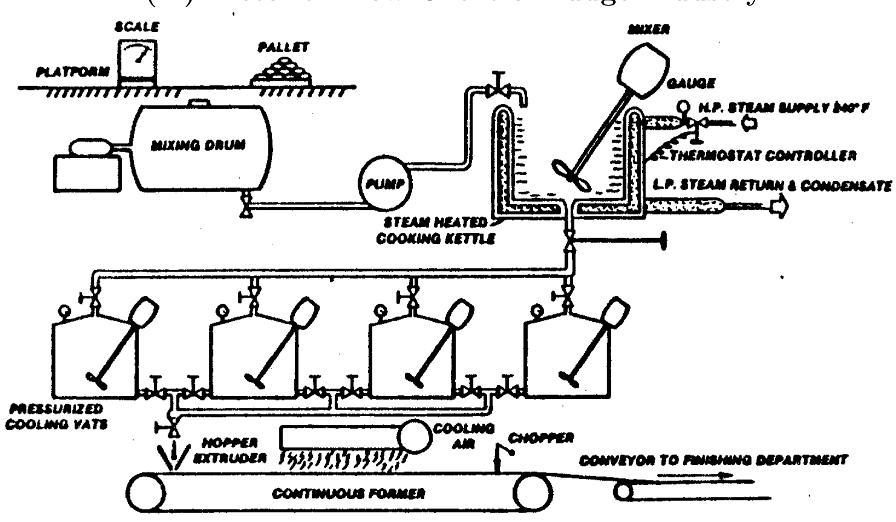
(i) Fudge Industry Flow Chart



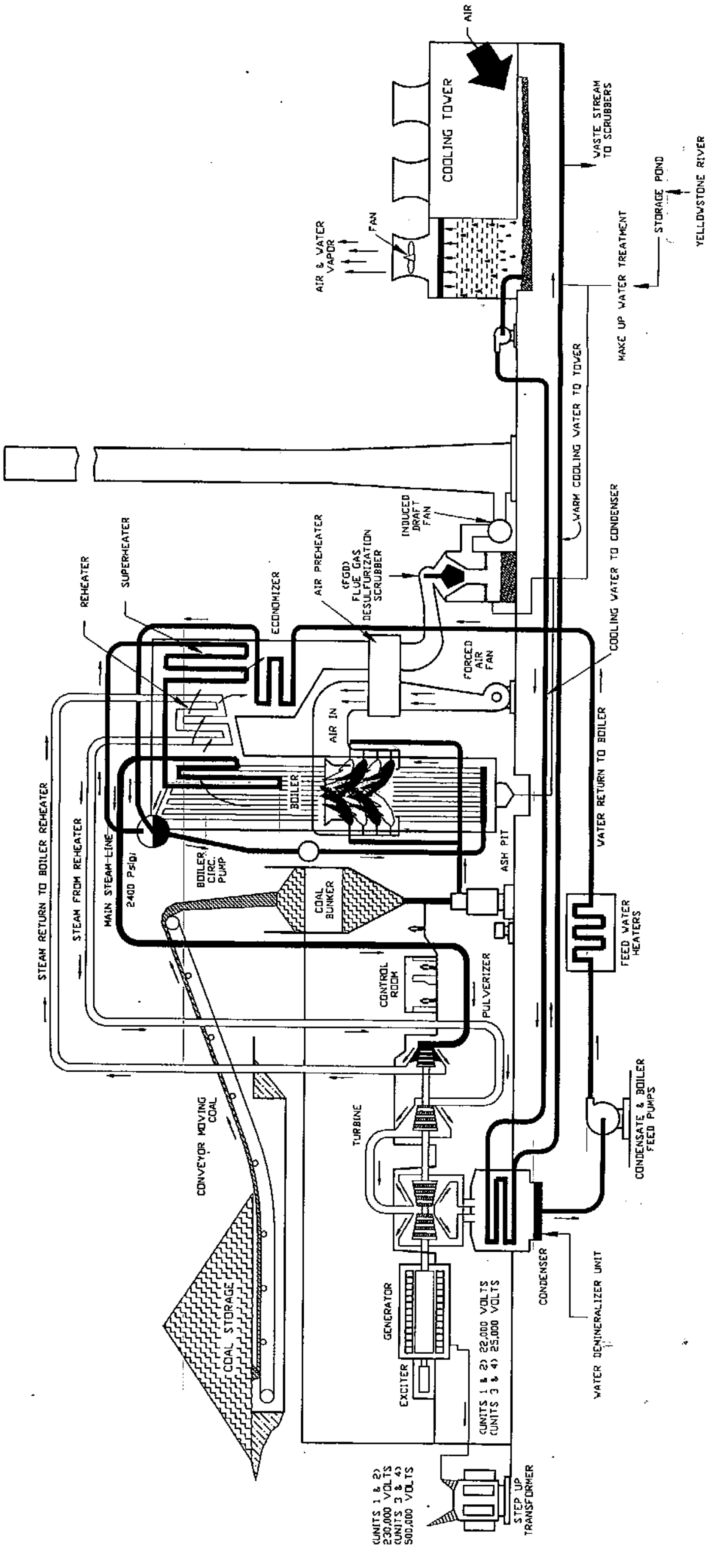
(ii) Cooking Stage Flow Chart



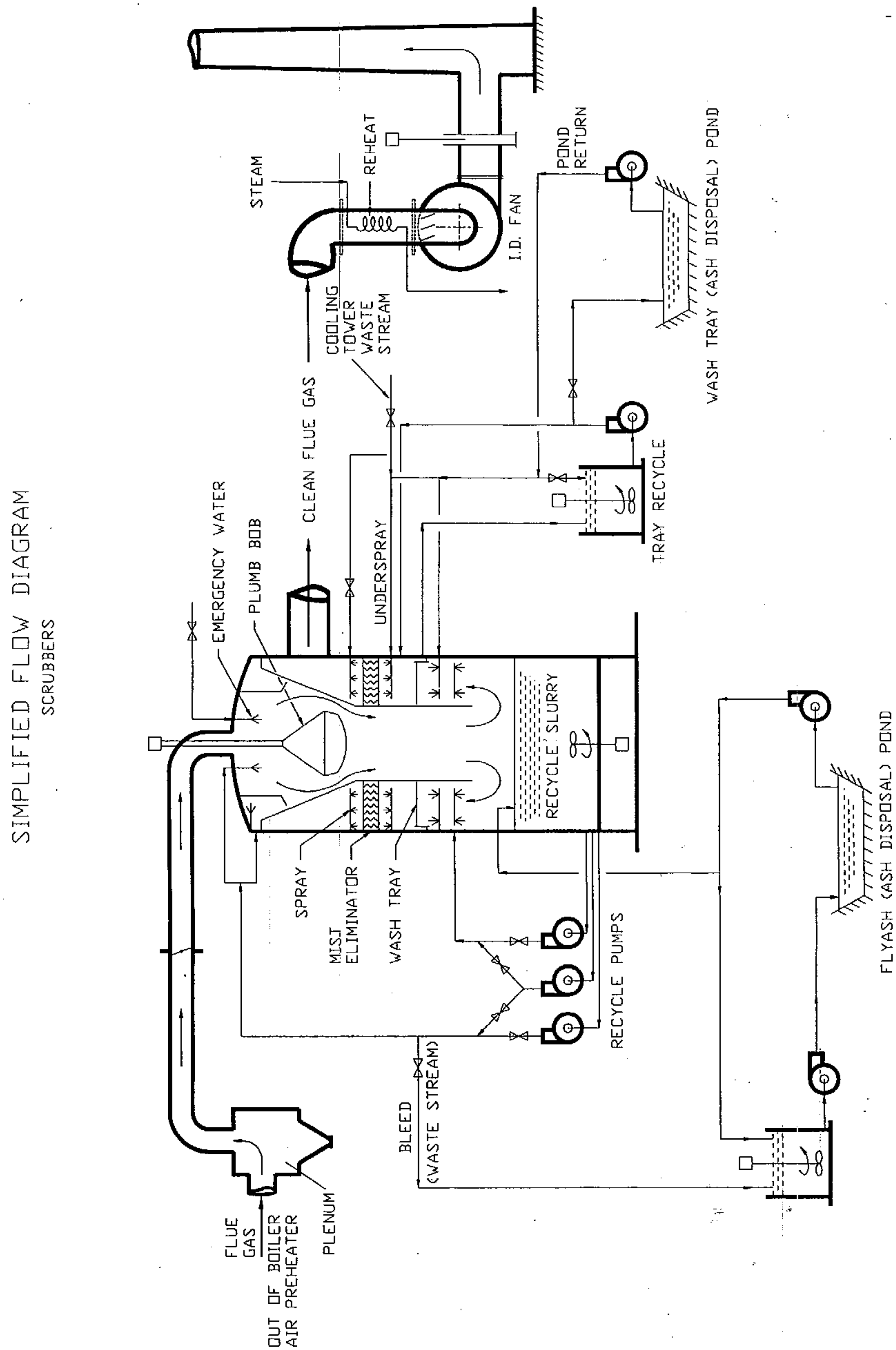
(iii) Pictorial Flow Chart of Fudge Industry



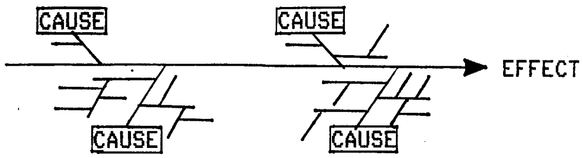
Flow Chart for Coal-Based Electricity Production: *Colstrip Project Division, Generating Units 1-4, Tour Information Booklet*



Flow Chart for Coal-Based Electricity Production Scrubbers



5.2 II. Cause and Effect Diagrams



Represents the relationship between some "effect" and all the discernible possible "causes".

- EFFECT
  - An effect is a result related to some property of interest obtained from a process.
  - Examples: quality, productivity, cost.
- CAUSE
  - The cause is the direct or indirect influence on the effect.
  - Production process examples: production method, manpower, material, machinery, measurement, and environment.
  - Service process examples: policies and procedures.
- Reasons for using a cause-and-effect diagram:
  - To devise a strategy to control and/or reduce the variability of one or more characteristics.
  - To improve the effect by changing it to a new desirable level.

These diagrams are also called **Ishikawa diagrams** and **fishbone diagrams**.

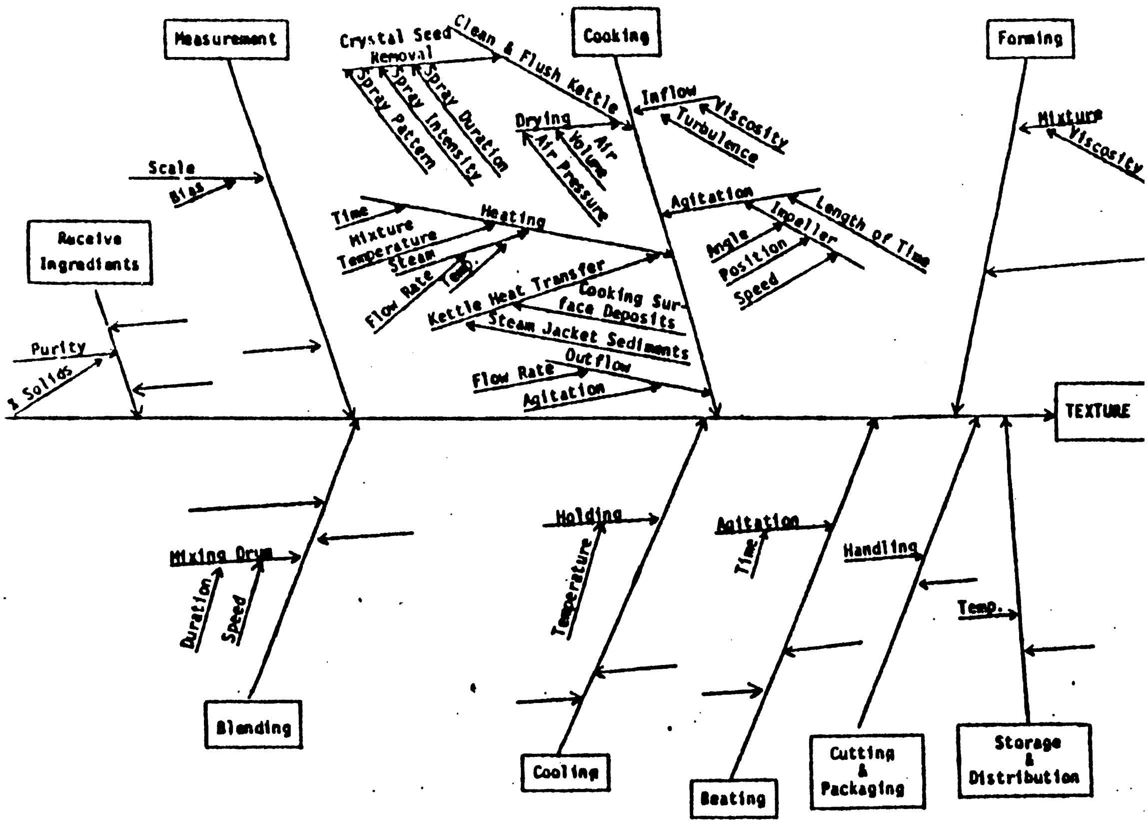
Three Major Types of Cause-and-Effect Diagrams

1. **Variation Type**
   - Break down the components of variance from major to detailed factors.
2. **Production Process Type**
   - Main diagram line follows the process as it relates to the effect studied.
3. **Cause Enumeration Type**
   - List all possible causes regardless of order, logic, etc. (Brainstorming)
   - Organize the list into families that relate to each other.
   - Study interrelationships.

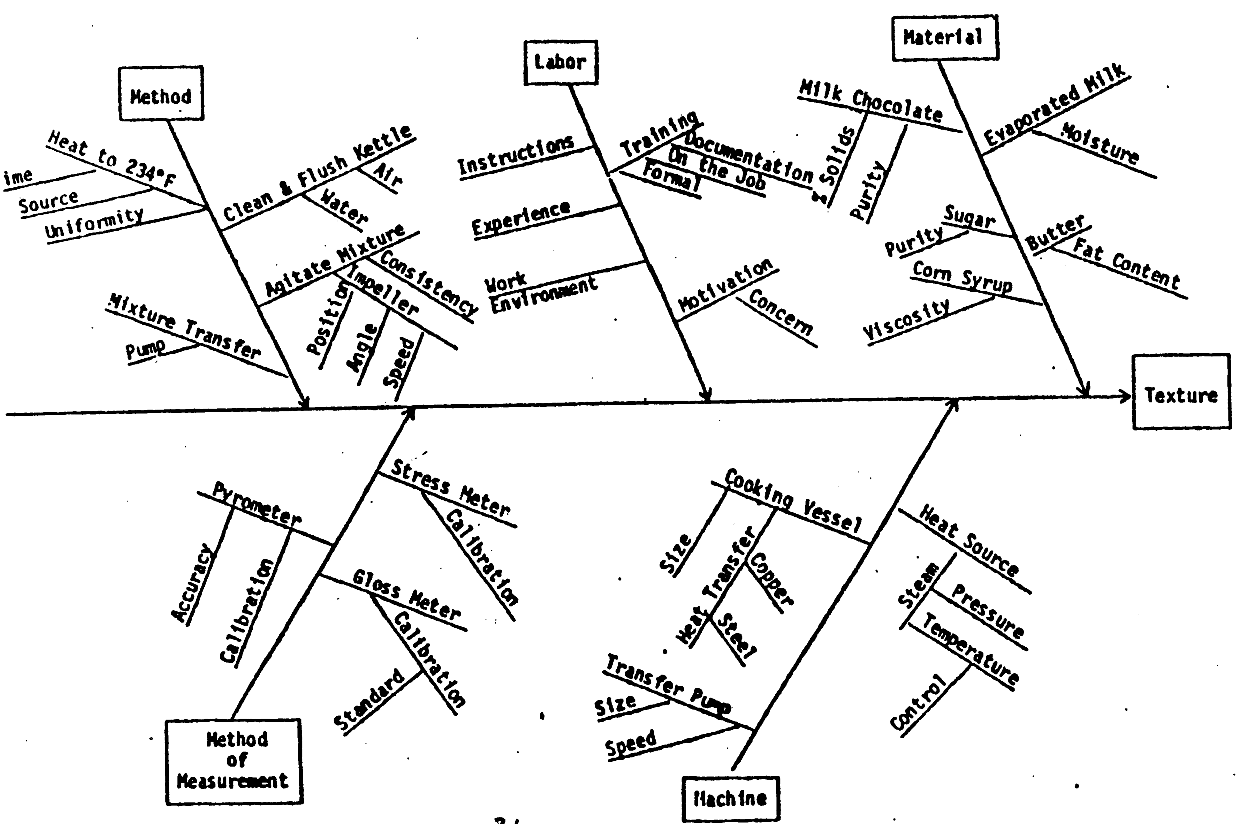
Example: The following diagrams correspond to the fudge industry example discussed earlier.

- The top diagram is for the entire fudge production process.
- The bottom diagram is restricted and expanded for the cooking part of the process.

Cause-and-Effect Diagram of the Entire Fudge Industry Process



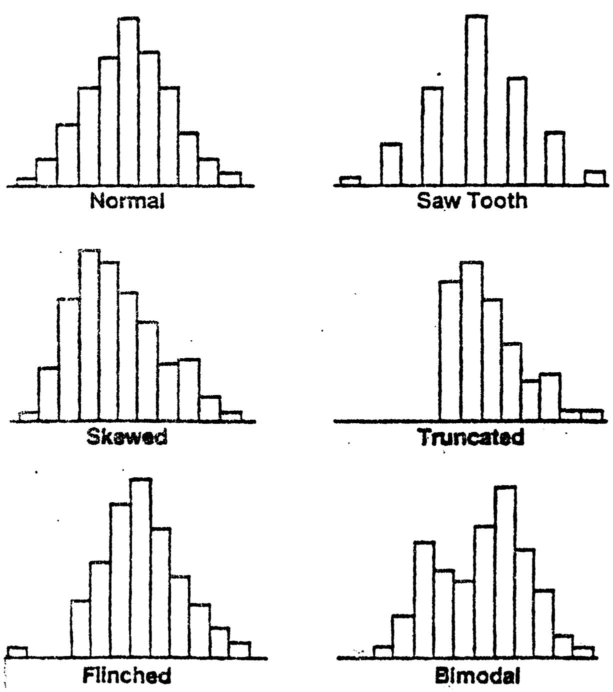
Cause-and-Effect Diagram of the Fudge Industry Cooking Process Only

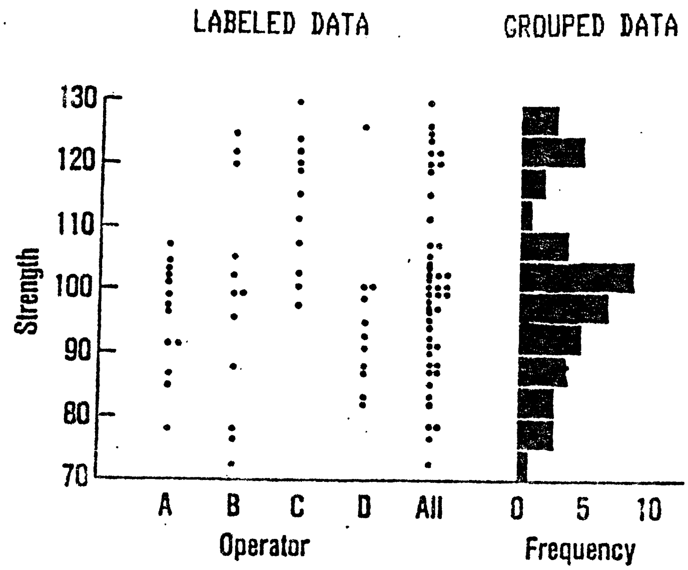


5.3 IV: Histograms

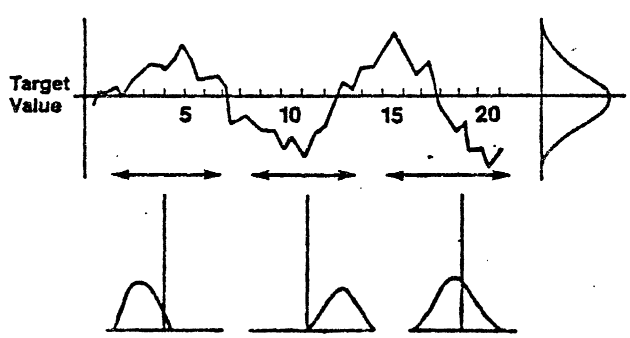
- Histograms display the variability and potentially unusual patterns in a set of measurements taken from a process.
- Histograms from process data are often stratified by
  - Different products, procedures, or quality characteristics of the products or procedures.
  - Different shifts, groups, or teams of personnel.
  - Different areas, machines, or individual workers.
  - Different time periods, such as days or weeks.
  - Different measurement instruments.
- The goal of stratification is to obtain information to narrow the search for large root causes of variability.

Detecting Problems in Variability





Caution
Histograms show data distribution over a time period but do not show change with time



Typical patterns of variation

The bell-shaped distribution: a symmetrical shape with a peak in the middle of the range of data. This is the normal, natural distribution of data from a process. Deviations from this bell shape might indicate the presence of complicating factors or outside influences. While deviations from a bell shape should be investigated, such deviations are not necessarily bad. As we will see, some non-bell distributions are to be expected in certain cases.

The double-peaked distribution: a distinct valley in the middle of the range of the data with peaks on either side. This pattern is usually a combination of two bell-shaped distributions and suggests that two distinct processes are at work.

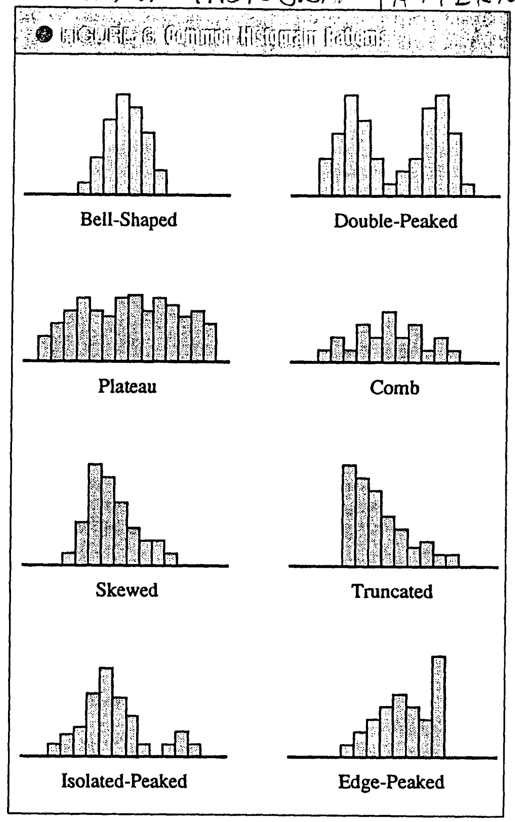
There is more than one possible interpretation for this pattern. Trying various stratification schemes to isolate the distinct processes or conditions is one method of further analysis.

The plateau distribution: a flat top with no distinct peak and slight tails on either side. This pattern is likely to be the result of many different bell-shaped distributions with centers spread evenly throughout the range of data.

Diagram the flow and observe the operation to identify the many different processes at work. An extreme case occurs in organizations that have no defined processes or training—everyone does the job his or her own way. The wide variability in process leads to the wide variability observed in the data. Defining and implementing standard procedures will reduce this variability.

The comb distribution: high and low values alternating in a regular fashion. This pattern typically indicates measurement error, errors in the way the data were grouped to construct the histogram, or a systematic bias in the way the data were rounded off. A less likely alternative is that this is a type of plateau distribution.

COMMON HISTOGRAM PATTERNS



The skewed distribution: an asymmetrical shape in which the peak is off-center in the range of data and the distribution tails off sharply on one side and gently on the other. The illustration in Figure 3 is called a positively skewed distribution because the long tail extends rightward, toward increasing values. A negatively skewed distribution would have a long tail extending leftward, toward decreasing values.

The skewed pattern typically occurs when a practical limit, or a specification limit, exists on one side and is relatively close to the nominal value. In these cases, there simply are not as many values available on one side as there are on the other. Practical limits occur frequently when the data consist of time measurements or counts of things.

For example, tasks that take a very short time can never be completed in zero or less time. Those occasions when the task takes a little longer than average to complete create a positively skewed tail on this distribution of task time.

The number of weaving defects per 100 yards of fabric can never be less than zero. If the process averages about 0.7 defects per 100 yards, then sporadic occurrences of three or four defects per 100 yards will result in a positively skewed distribution.

Such skewed distributions are not inherently bad, but a team should question the impact of the values in the long tail. Could they cause customer dissatisfaction (e.g., long waiting times)? Could they lead to higher costs (e.g., overfilling containers)? Could the extreme values cause problems in downstream operations? If the long tail has a negative impact on quality, the team should investigate and determine the causes for those values.

The truncated distribution: an asymmetrical shape in which the peak is at or near the edge of the range of the data, and the distribution ends very abruptly on one side and tails off gently on the other. The illustration in Figure 3 shows truncation on the left side with a positively skewed tail. Of course, one might also encounter truncation on the right side with a negatively skewed tail. Truncated distributions are often smooth, bell-shaped distributions with a part of the distribution removed, or truncated, by some external force such as screening, 100% inspection, or a review process. Note that these truncation efforts are an added cost and are, therefore, good candidates for removal.

The isolated peaked distribution: a small, separate group of data in addition to the larger distribution. Like the double-peaked distribution, this pattern is a combination and suggests that two distinct processes are at work. But the small size of the second peak indicates an abnormality, something that doesn't happen often or regularly.

Look closely at the conditions surrounding the data in the small peak to see if you can isolate a particular time, machine, input source, procedure, operator, etc. Such small isolated peaks in conjunction with a truncated distribution might result from the lack of complete effectiveness in screening out defective items. It is also possible that the small peak represents errors in measurements or in transcribing the data. Re-check measurements and calculations.

The edge-peaked distribution: a large peak is appended to an otherwise smooth distribution. This shape occurs when the extended tail of the smooth distribution has been cut off and lumped into a single category at the edge of the range of the data. This shape very frequently indicates inaccurate recording of the data (e.g., values outside the "acceptable" range are reported as being just inside the range).

5.4 V: Check Sheets

Why Collect Data?

- It is not practical or wise to rely only on experience, intuition, or professional judgement as the basis for analysis or decision-making.
- Properly and accurately collected data will reflect the facts and should be used as a basis for taking action.

Questions to Ask

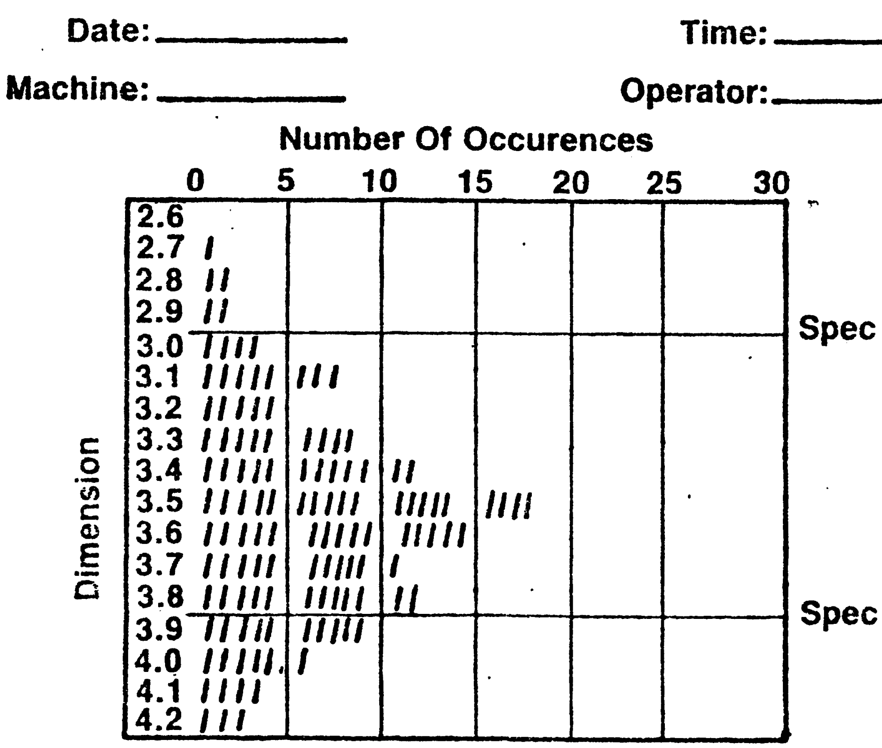
- What is the purpose of your data collection?
- What is the appropriate data to address the purpose?
- How are you going to analyze the data?
- What are your resources for collecting data?

Once these questions are addressed then a simple data recording form known as a **check sheet** can be developed so that proper and accurate data will be collected.

Typical Check Sheet Types

(1) Dispersion of continuous data.
(2) Check by cause.
(3) Defect location.
(4) Cause and effect.
(5) Confirmation

(1) Dispersion of Continuous Data Check Sheet: Dimension Specifications Check



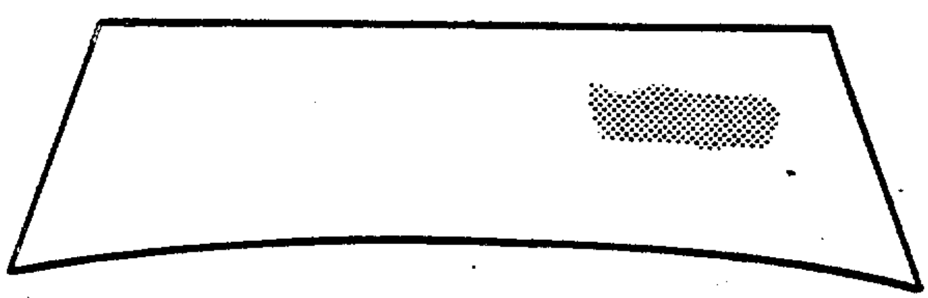
(2) Check-by-Cause Check Sheet: Errors in Expense Reports

Time Period	Division	
Types of Errors	**Checks**	**Subtotal**
Hotel Receipt Missing		
Car Rental Receipt Missing		
Other Receipt Missing		
Wrong Allocation		
Incomplete Allocation		
Not Signed by Employee		
Not Authorized Properly		
No Insurance Statement		
Incorrect Arithmetic		
Others		
Total Reports	Grand Total of Errors	

(3) Defect Location Check Sheet: Defects on Windshields

Date/Time:
Type:

Operator:
Machine:



Mark defect location by shading the corresponding area

Remarks:

(4) Cause-and-Effect Check Sheet: Defects in Molded Parts

PART TYPE: WEEK OF:

MACHINES	OPERATOR	MON TUE WED THU FRI
I	A	
	B	
II	A	
	B	

○ surface scratch
x blowhole
● improper shape
▲ defective finish
◘ all others

(4) Cause-and-Effect Check Sheet: Preventive Maintenance for a Plastic Extruder

MACHINE NO.:
MANUFACTURER:
SERIAL NO.:
LOCATION:

YEAR:
MANAGER:
SUPERVISOR:
FOREMAN:

PART INSPECTED	CHECK POINT	J	F	M	A	M	J	J	A	S	O	N	D
DRIVE													
GEAR BOX	NOISE, VIBR												
BEARING	TEMP, VIBR												
TRANSMISSION	OPERATION												
PULLEY, BELT	DAMAGE, TENSION												
ELECTRICAL													
MOTOR	TEMP, LOOSE BELT												
CONTROL	OPERATION, ERROR												
WIRING	AGING												
ILLUMINATION	DIRTY, AGING												
HEATER	BREAK												
CONNECTION	INSULATION												
DIE													
FLANGE	BOLT, LEAKAGE												
FITTING	LEAKAGE												
HEATING PANEL	WIRING, BOLT												
DIE LIP	FLAW												
ADJUSTING BOLT	OUT OF ROOM												
OTHERS													
PRESSURE GAUGE	ERROR, LEAKAGE												
BLOWER	NOISE, VIBRATION												
COOLING WATER	FLOW, LEAK, TEMP												
O: OK	X: NG												

Other Check Sheet Examples

Figure 2. A Check Sheet

SOLDER BATH TEMPERATURE

- Read temperature to nearest degree off meter number 5. Date: 6-7-88
- Plot the temperature and time on the grid using a dot (•). Connect the dots with a line. Line #: 13
- Reading should be taken on the hour (± 5 minutes). Inspector: GINNY SMITH
- Use the "Notes" section to record anything unusual.

• Questions? Contact Larry Fine x2222

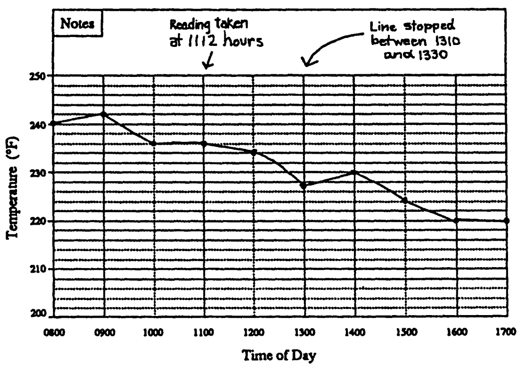


Figure 3. A Data Sheet

SOLDER BATH TEMPERATURE

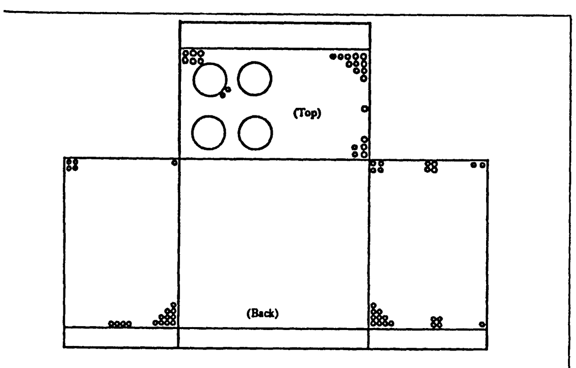
- Read temperature to nearest degree off meter number 5. Date: 6-7-88
- Record the temperature in the table below. Line #: 13
- Reading should be taken on the hour (±5 minutes).
- Use the "Notes" section to record anything unusual. Inspector: GINNY SMITH

• Questions? Contact Larry Fine x2222

Time of Day	Temperature (°F)	Time of Day	Temperature (°F)
0800	240	1300	227
0900	242	1400	230
1000	236	1500	224
1100	236	1600	220
1200	234	1700	220

Notes: -1100 hours reading taken at 1112
-The line was stopped between 1310 and 1330

Figure 5. Location Plot of Chipped Enamel on Range



From J.M. Juran, "Quality Improvement," *Quality Control Handbook*, third edition (New York: McGraw-Hill Book Co., 1979), p. 16-30.

Figure 4. Check Sheet for TV Component Failures

COMPONENTS REPLACED BY LAB
Enter a mark for each component replaced. Mark like the following: I II III IIII HHT
TIME PERIOD: *22 Feb to 27 Feb. 1988*
REPAIR TECHNICIAN: **Bob**

TV SET MODEL 1013

Integrated circuits	HHT
Capacitors	HHT HHT HHT HHT HHT II
Resistors	II
Transformers	IIII
Commands	
CRT	I

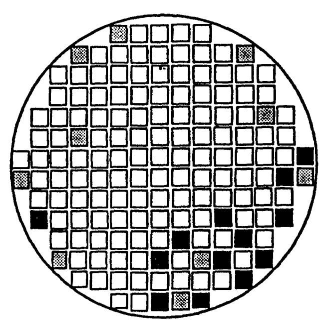
TV SET MODEL 1017

Integrated circuits	III
Capacitors	HHT HHT HHT HHT HHT II
Resistors	I
Transformers	II
Commands	HHT HHT HHT III
CRT	I

TV SET MODEL 1019

Integrated circuits	I
Capacitors	HHT HHT HHT HHT III
Resistors	I
Transformers	II
Commands	
CRT	I

Figure 6. Location Plot of Chip Rejects



■ PARAMETRIC REJECTS
▩ FUNCTIONAL REJECTS

5.5 VI: Pareto Charts

- A pareto chart is a tool for prioritizing importance of opportunities.
- Named for V.F.D. Pareto (1848-1923), an economist who studied the distribution of individual income in Italy. He found that a large part of the wealth was held by few people.

Finding Opportunities

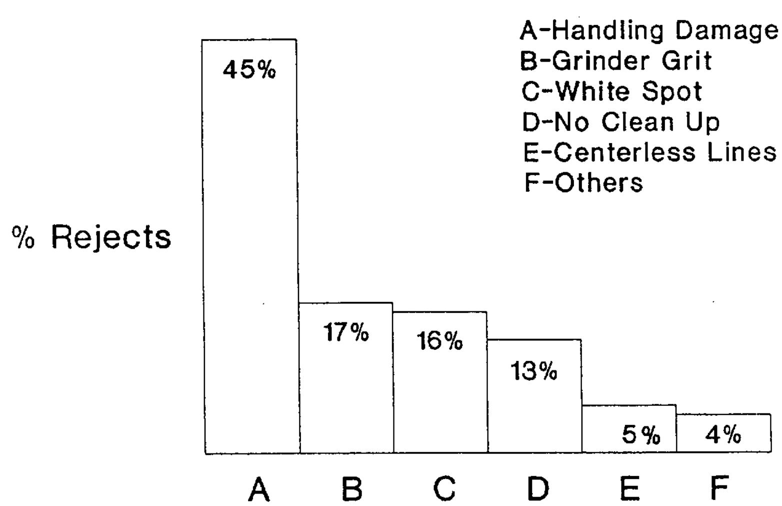
- "The Vital Few" Philosophy
  - 80% of the losses are due to 20% of the causes
  - 80% of the sales are due to 20% of the customers
- First Step in Improvement
  - Identify the causes and customers to form a basis for continual improvement

Making a Pareto Chart

1. Decide categories/classification items and make a check list.
2. Determine the period to collect data.
3. Calculate the percentage of occurrence for each category.
4. Plot the bars in a bar chart in order of decreasing percentages.
5. Plot the cumulative percentage at the horizontal value at right side of each bar. (This step is not always included.)
6. Title the chart, label axes, etc. so that it is easily understood.

Typical Pareto Charts

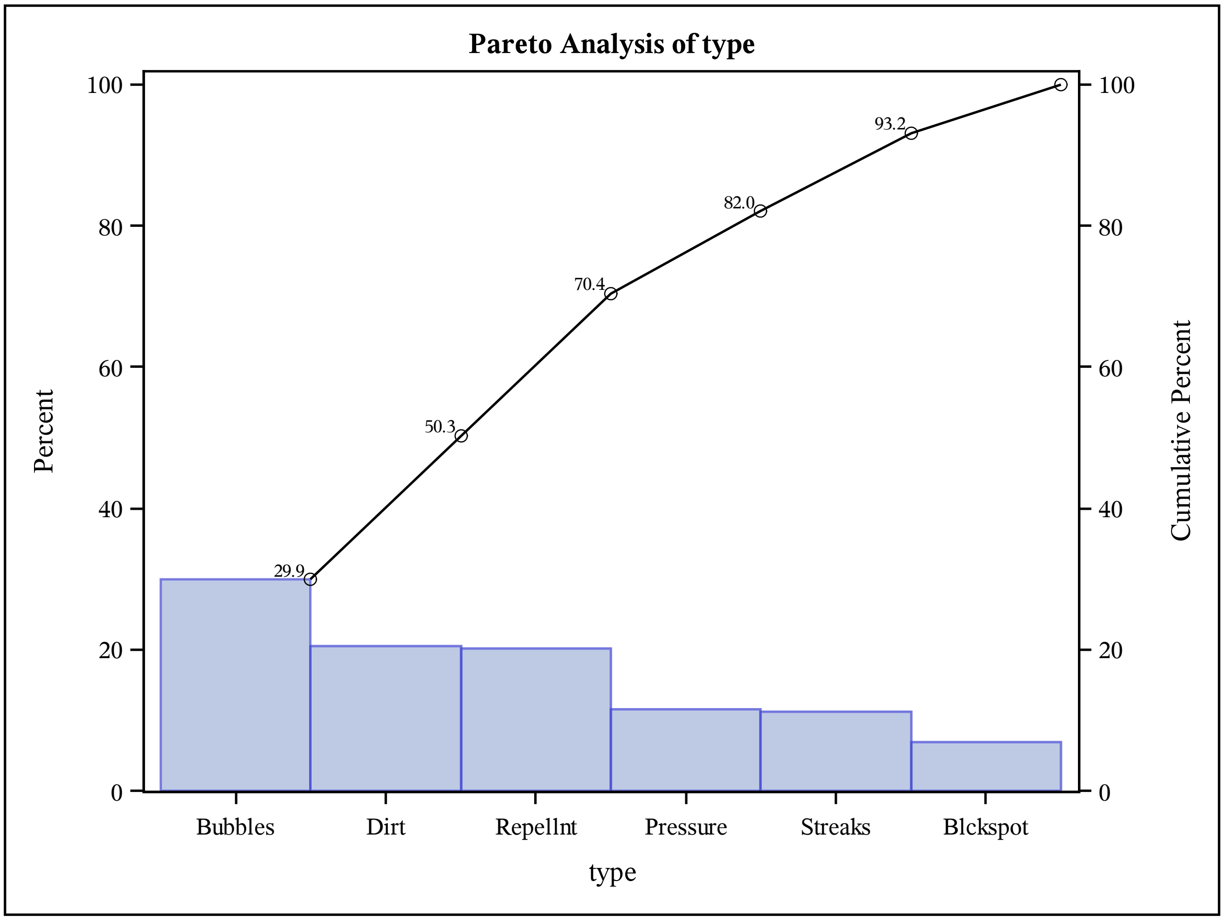
- Product types by defect.
- Inventory values by item.
- Product quality problems by type.
- Sales volume by salesperson.
- Product defects by cause.
- Product quality problems by customer.
- Complaints by customer.



Example: Pareto chart for the types of defects in a lamination process.

Defect Type	Square Feet	Percent	Cumulative %
Bubbles	446,000	29.89 %	29.89 %
Dirt	305,000	20.44 %	50.33 %
Repellents	300,000	20.11 %	70.44 %
Pressure	173,000	11.59 %	82.03 %
Streaks	166,000	11.13 %	93.16 %
Blackspots	102,000	6.84 %	100 %
Total	1,492,000		

Pareto Chart for Loss in Square Feet



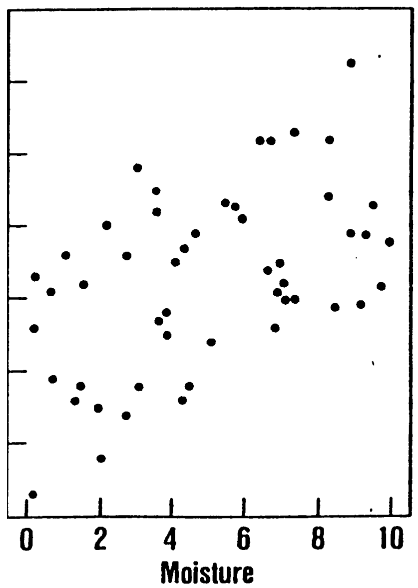
SAS Code to make a Pareto chart.

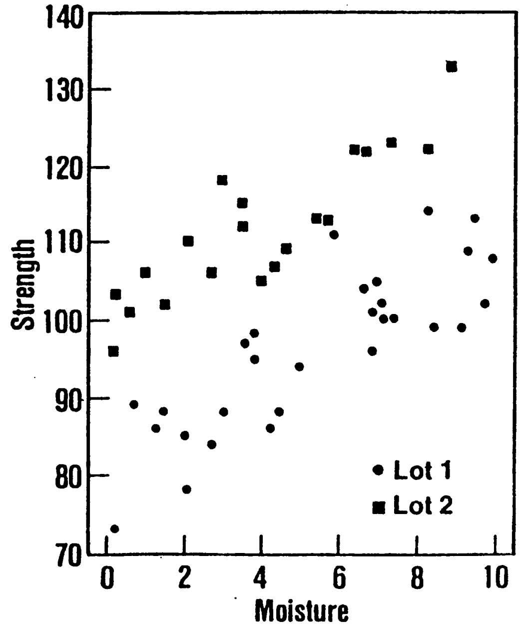
```
DM 'LOG;CLEAR;OUT;CLEAR;';
* ODS PRINTER PDF file='C:\COURSES\ST528\SAS\paretoex.pdf';
OPTIONS NODATE NONUMBER;

TITLE 'Pareto Chart for Loss in Square Feet';

DATA loss; INPUT type \$ sqr_feet @@;
lines;
Bubbles  446000   Dirt    305000   Repellnt 300000
Pressure 173000   Streaks 166000   Blckspot 102000
;
PROC PARETO DATA = loss;
   VBAR type / FREQ = sqr_feet MARKERS CMPCTLABEL;
RUN;
```

5.6 VII: Scatter Plots

- In scatter plots, the goal is to identify and quantify relationships among continuous variables.
- When there is also a categorical variable of interest, a symbolic scatter plot (in which plotted points are labeled using different symbols) is another useful scatter plot.

Product Strength vs. Moisture





5.7 III: Control Charts

- A process that is operating when only random chance causes of variation (random noise) are present is said to be **in statistical control**.
- We refer to sources of variation that are not attributable to chance as **assignable causes**. Three common assignable causes arise from improperly adjusted machines, operator errors, and defective raw materials.
- A process that is operating in the presence of assignable causes is said to be in a state that is **out-of-control**.

Quality control procedures are

- Used to monitor process characteristics to ensure that process specifications are met.
- Designed to indicate the point in time when a process begins to produce units which do not meet the process specifications, that is, when the process has shifted to an out-of-control state.
  - If the process is in-control, the output should vary randomly about the parameter associated with the process characteristic of interest. If well-designed, the procedure would require a minimum number of runs to detect that this shift occurs.
  - Once the shift has been detected, an assignable cause needs to be found quickly so the process can be adjusted and returned to the in-control state. Detecting the shift quickly helps reduce the number of substandard units produced, thereby, reducing production costs.

We will review several basic control chart procedures used for monitoring a process that:

- Provide graphical and computational rules for making process adjustments.
- Are simple to use.
- Are based on sound statistical principles.

In this course, three procedures will be reviewed: (i) Shewhart control charts, (ii) cumulative sum (CUSUM) charts, and (iii) exponentially weighted moving average (EWMA) charts.

Use of Control Charts

- We assume that processes will not operate in a state of statistical control forever. That is, something will eventually happen to take the process out of control.
- Often production processes will operate in an in-control state producing acceptable product for relatively long periods of time. Eventually, a shift will occur resulting in an out-of-control state such that a larger proportion of the process output does not conform to specifications.
- Only when control charts are routinely used can assignable causes be identified. If the causal effects can be significantly reduced or eliminated then variability is reduced. Reduced variability improves the process.
- Warning: Control charts will only *detect* assignable causes. It is only through the actions of those in charge that can *eliminate* the assignable causes.
- Additionally, from a control chart for an in-control process, we can estimate certain parameters (e.g. mean, standard deviation, fraction nonconforming).

Control charts are a popular quality tool because they

- Are a proven technique for improving productivity through reduction in scrap and rework.
- Are effective in defect prevention by helping keep the process in control.
- Prevent unnecessary process adjustments by separating noise from abnormal variation.
- Provide diagnostic information through pattern detection.
- Provide information about process capability (i.e., the stability of process parameters).

5.8 Shewhart Control Charts

- There are two general types of Shewhart control charts:
  1. If the decision scheme is to classify each unit as conforming or nonconforming to the specifications of certain quality characteristics, then the **attributes control chart** would be the proper choice.
  2. If a specific numerical measurement is to be used to judge the control status of a process, then the **variables control chart** should be used.
- Both types of control charts follow the same general form:

- An investigator is concerned with some quality characteristic θ. For example, θ corresponds to a population proportion nonconforming (attribute data) or corresponds to the mean of a continuous random variable (variable data).
- Control charts display the realized values of $\widehat{\theta}$, an estimator of θ, for each successive sample drawn.
  * The sample numbers are plotted along the horizontal axis.
  * The values of $\hat{\theta}$ are plotted along the vertical axis.
  * Two horizontal lines, called **control limits**, are drawn on the control chart equidistant from a centerline.
  * Often, two other horizontal lines, called **warning limits**, will be included on the control chart.

<u>Control Limits</u>

- Let $\hat{\theta}$ be an estimator of θ based on a random sample of n independent units drawn from an in-control process.
- Let $\mu_{\hat{\theta}}$ and $\sigma_{\hat{\theta}}$ be the mean and standard deviation of the distribution of $\hat{\theta}$.
- The following set of formulas (proposed by Dr. Walter Shewhart) are used to construct control limits for the process characteristic of interest:

$$\begin{array}{l} \text{Upper Control Limit(UCL)} = \mu_{\hat{\theta}} + k_1\sigma_{\hat{\theta}} \\ \text{Centerline} = \mu_{\hat{\theta}} \\ \text{Lower Control Limit(LCL)} = \mu_{\hat{\theta}} - k_1\sigma_{\hat{\theta}}, \end{array} \tag{1}$$

  where k_1 is the number of standard deviations a particular value of $\hat{\theta}$ is allowed to vary from $\mu_{\hat{\theta}}$ without signalling an out-of-control process.

- Control limits provide upper and lower bounds for values of $\hat{\theta}$ that are acceptable to the producer.

<u>Warning Limits</u>

- The warning limits lie between the centerline and the control limits and are determined by the following formulas:

$$\begin{array}{l} \text{Upper Warning Limit(UWL)} = \mu_{\hat{\theta}} + k_2\sigma_{\hat{\theta}} \\ \text{Centerline} = \mu_{\hat{\theta}} \\ \text{Lower Warning Limit(LWL)} = \mu_{\hat{\theta}} - k_2\sigma_{\hat{\theta}}, \end{array} \tag{2}$$

  where k_2 is the number of standard deviations a particular value of $\hat{\theta}$ is allowed to vary from $\mu_{\hat{\theta}}$ without giving a warning signal.

- Figure 1 illustrates the appearance of a control chart for $\hat{\theta}$ with $k_1\sigma_{\hat{\theta}}$ control limits and $k_2\sigma_{\hat{\theta}}$ warning limits.

Once the control chart is set up, the values of $\hat{\theta}$ for each successive sample are plotted.

- From formulas (1) and (2) it can be seen that the design of a control chart is dependent upon the values of k_1, k_2, n, and the sampling frequency.

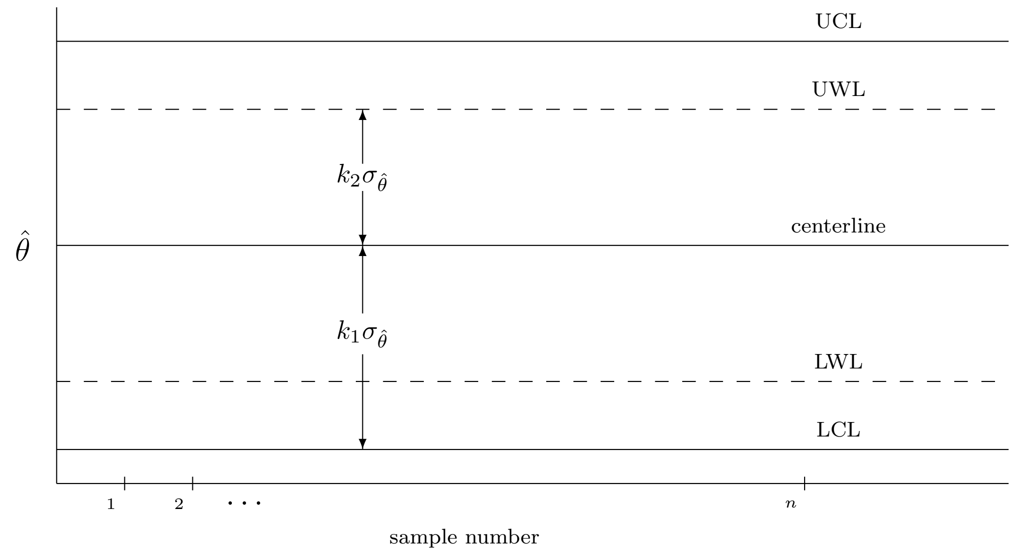


Figure 1: Shewhart Control Chart

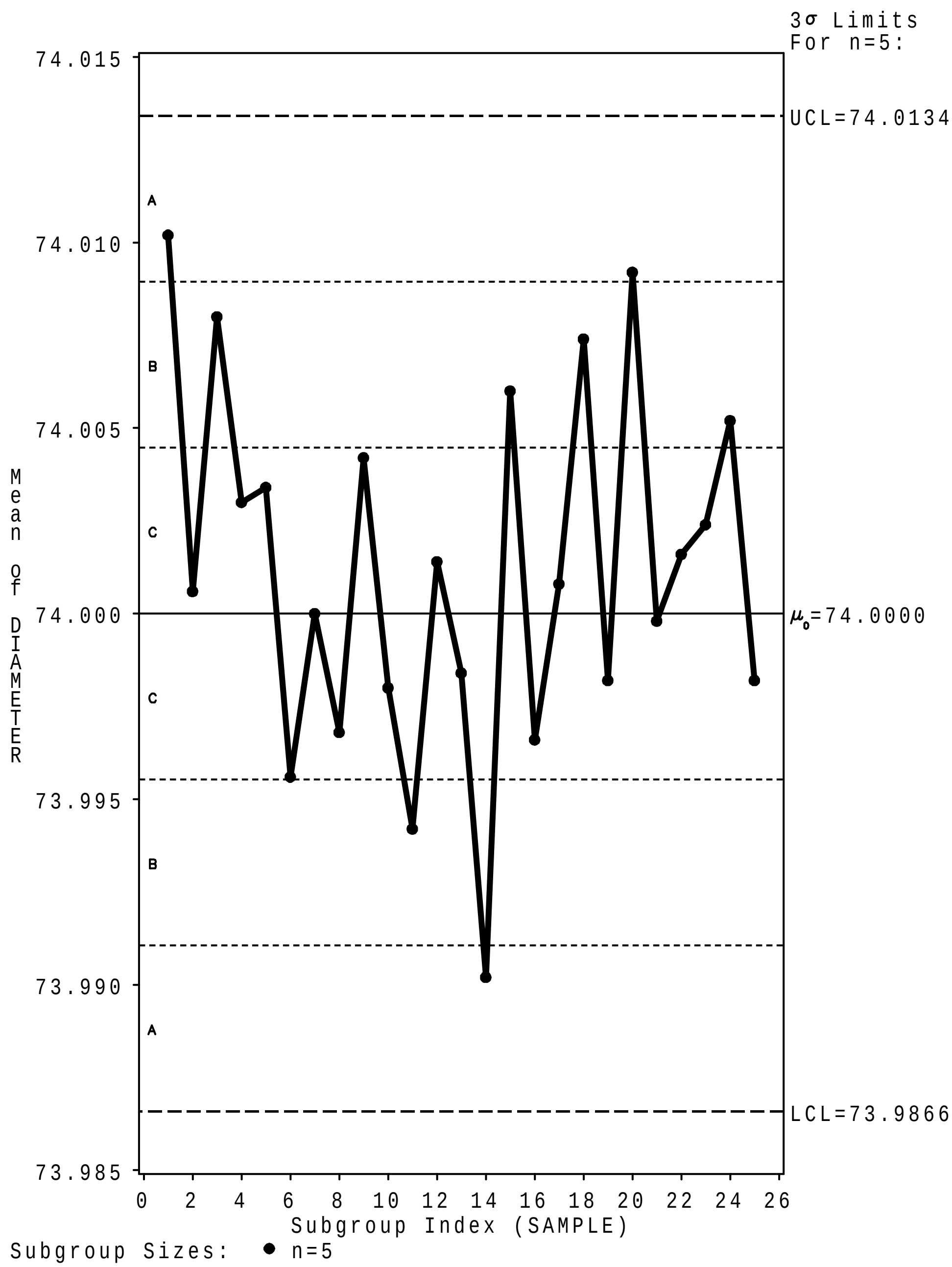
- The values of k_1 and k_2 are based on cumulative probabilities of the distribution of $\hat{\theta}$.
- Computing k_1 and k_2 from the exact distribution of $\hat{\theta}$ can often be difficult, because the exact distribution of the characteristic may not be known.
- It is common in practice to choose $k_1 = 3$ and $k_2 = 2$. These are suggested values of k_1 and k_2 which have been shown to work well in practice.
- Certain considerations, such as losses due to producing substandard products, may require the use of smaller values of k_1 and k_2.

5.8.1 Control Charts and Hypothesis Testing

- Control charts are used to test the null hypothesis, H_0, that the process is in control versus the alternative, H_a, that the process is out of control.
- **Type I error** (α): the risk indicating an assignable cause when no assignable cause is present (or, indicating an out-of-control condition when the process is in-control).
  - In a Shewhart control chart this is the risk of a point falling *outside* the control limits when no assignable cause is present.
- **Type II error** (β): the risk of failing to find an assignable cause when an assignable cause is present (or, failing to detect an out-of-control condition when the process is out-of-control).
  - In a Shewhart control chart this is the risk of a point falling *inside* the control limits when the process is in an out-of-control condition.
- Various rules have been suggested to decide whether or not to reject H_0.
  - If any of these rules are satisfied, a search for an assignable cause should be conducted.
  - If an assignable cause is found, the production process should be adjusted.
  - After the adjustment has been made, continue with process control testing.

- A control chart may indicate an out-of-control condition when one or more points fall beyond the control limits or when the points exhibit some nonrandom pattern of behavior.
- A **run** is a sequence of consecutive points of the same type.
  - Example: If the sequence of points are all increasing or all decreasing then this type of run is called a **run up** or a **run down**.
- In practice, a subset of these rules can be chosen for implementation. If one or more of the implemented rules is satisfied, reject H_0.
- Data for the following example is based on 25 samples of 5 different piston ring diameter measurements. The aim is 74 mm. It is assumed that $\sigma = .01$.

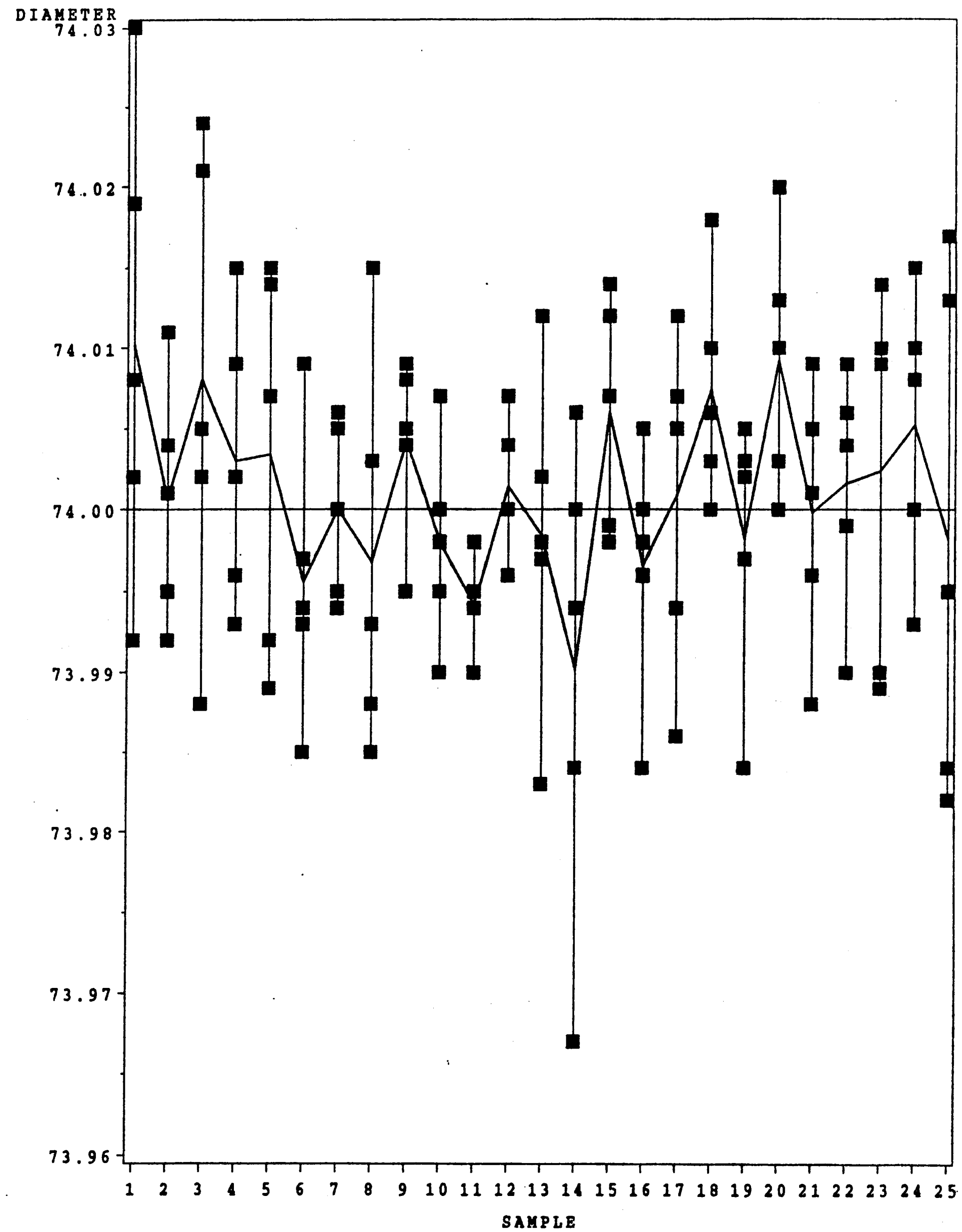
SHEWHART CHART for Mean Piston–Ring Diameters across Samples

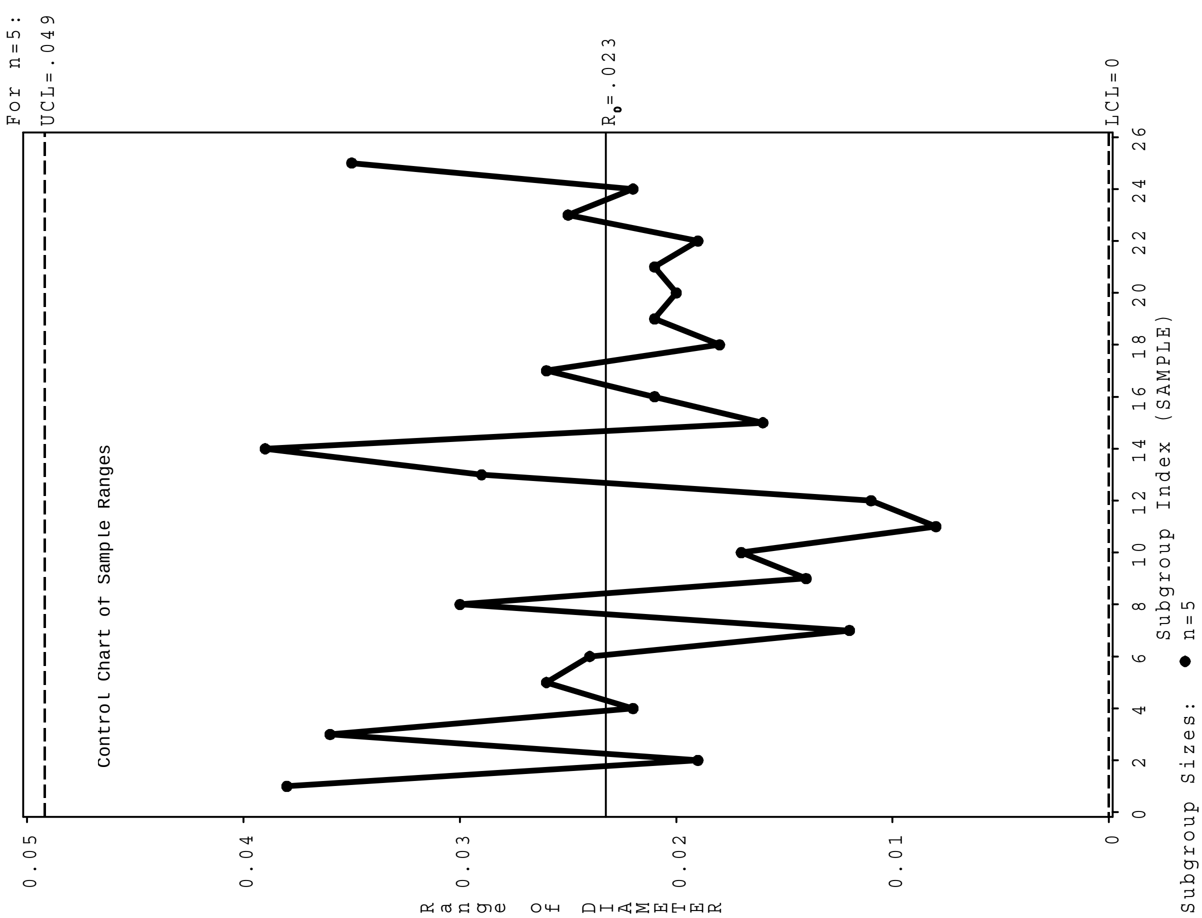


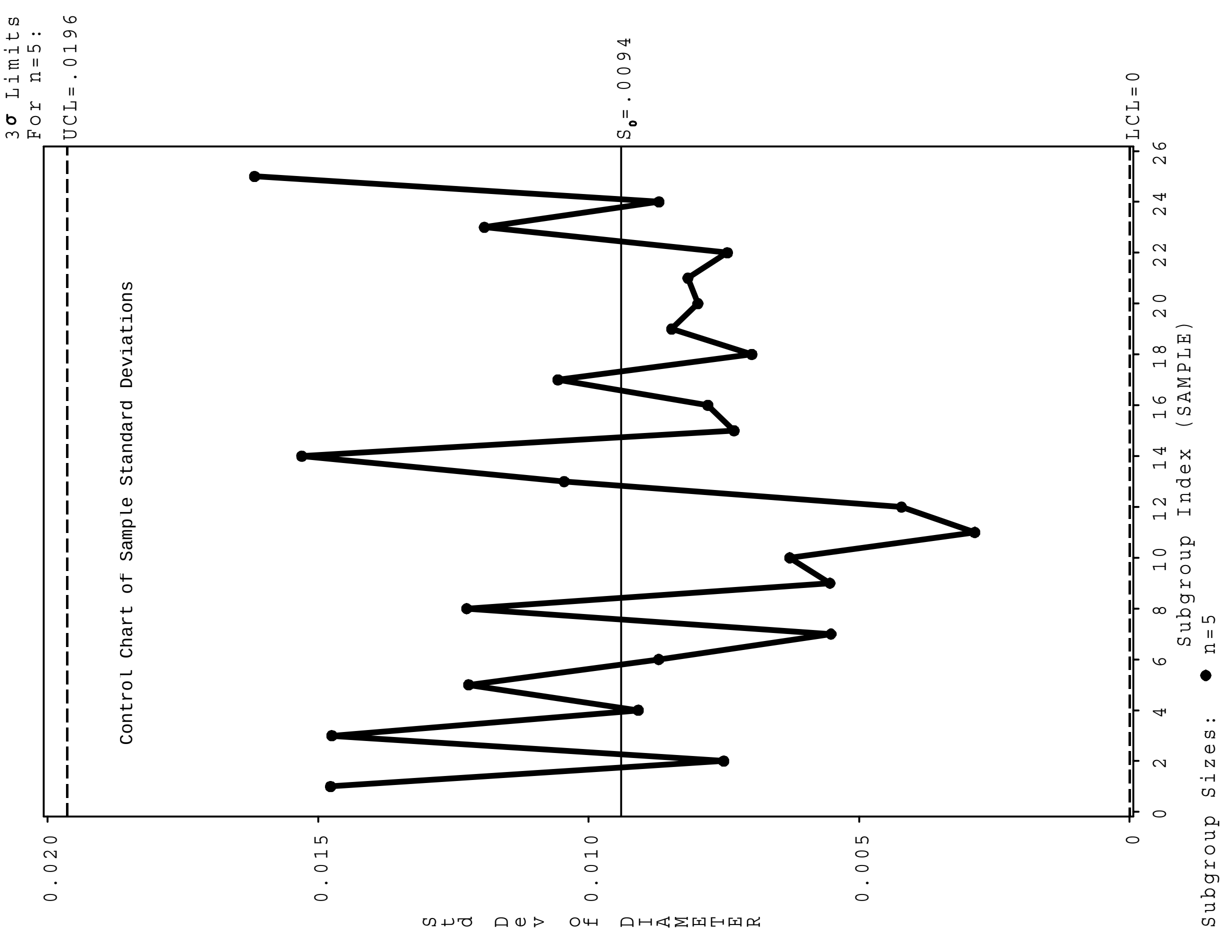
Forged Piston-Ring Inside Diameter (mm)

Sample Number	Observations				
1	74.030	74.002	74.019	73.992	74.008
2	73.995	73.992	74.001	74.011	74.004
3	73.988	74.024	74.021	74.005	74.002
4	74.002	73.996	73.993	74.015	74.009
5	73.992	74.007	74.015	73.989	74.014
6	74.009	73.994	73.997	73.985	73.993
7	73.995	74.006	73.994	74.000	74.005
8	73.985	74.003	73.993	74.015	73.988
9	74.008	73.995	74.009	74.005	74.004
10	73.998	74.000	73.990	74.007	73.995
11	73.994	73.998	73.994	73.995	73.990
12	74.004	74.000	74.007	74.000	73.996
13	73.983	74.002	73.998	73.997	74.012
14	74.006	73.967	73.994	74.000	73.984
15	74.012	74.014	73.998	73.999	74.007
16	74.000	73.984	74.005	73.998	73.996
17	73.994	74.012	73.986	74.005	74.007
18	74.006	74.010	74.018	74.003	74.000
19	73.984	74.002	74.003	74.005	73.997
20	74.000	74.010	74.013	74.020	74.003
21	73.988	74.001	74.009	74.005	73.996
22	74.004	73.999	73.990	74.006	74.009
23	74.010	73.989	73.990	74.009	74.014
24	74.015	74.008	73.993	74.000	74.010
25	73.982	73.984	73.995	74.017	74.013

Piston – Ring Diameters by Sample




Control Chart of Sample Ranges
3σ Limits
For n=5:
UCL=.049
R_0=.023
LCL=0
Range of DIAMETER
Subgroup Index (SAMPLE)
Subgroup Sizes: ● n=5


Control Chart of Sample Standard Deviations
3σ Limits
For n=5:
UCL=.0196
S_0=.0094
LCL=0
Std Dev of DIAMETER
Subgroup Index (SAMPLE)
Subgroup Sizes: ● n=5

5.8.2 Rules for Modified Shewhart Control Charts

1: One point is outside the control limits. Because the design of the control chart depends on the construction of the control limits, this is the primary rule used by many practitioners, even though it is not the most sensitive in detecting an out-of-control process.

2: n_2 consecutive points on the same side of the centerline. n_2 is typically 7, 8 or 9.

3: n_3 consecutive points are steadily increasing or decreasing. n_3 is typically 6, 7, or 8.

4: Fourteen points in a row alternating up and down.

5: Two out of three consecutive points are between the warning limits and the control limits on the same side of the centerline.

6: Four out of five consecutive points plot beyond the $\pm 1\,\sigma$ limits on the same side of the centerline.

7: Fifteen points in a row within the warning limits on either or both sides of the centerline.

8: Eight points in a row on either or both sides of the centerline with no points within the warning limits.

9: From ten consecutive points, any subset of eight points follow a monotone increasing or decreasing pattern. Changing the rule from ten consecutive points to nine consecutive points increases this rule's sensitivity.

10: The second of two consecutive points is at least $4\sigma_{\hat{\theta}}$ above or below the first.

- Rules #1 to #8 are rules that *SAS* can check. By itself, Rule #1 corresponds to the traditional Shewhart control chart. Use of Rule #1 with any additional rules yields a **modified** Shewhart control chart.
- These rules are not completely reliable in detecting an out-of-control process. It is possible that a small shift or a cyclic pattern in the process parameter can go undetected.
- Widening control limits will decrease the Type I error and increase the Type II error.
- Narrowing control limits will increase the Type I error and decrease the Type II error.

5.8.3 Sample Size and Sampling Frequency

- In designing a control chart, the **sample size** and **sampling frequency** must be specified.
- The general problem is the **allocation of sampling effort**. That is, do we take smaller samples at shorter intervals or larger samples at longer intervals.
- A method of evaluating sample size and sampling frequency is through the **average run length (ARL)** of the control chart.
- The ARL is the average number of samples until the first out-of-control signal occurs.
  - For the traditional Shewhart control chart, $ARL = 1/p$ where p is the probability that any point exceeds the control limits.

  - For any modified Shewhart control chart, the ARL depends on combining probabilities associated with each rule. Review the *Technometrics* handout.

- Some people prefer to use the **average time to signal (ATS)**. If samples are taken at fixed intervals of h units of time, then $ATS = ARL \times h$.

- The sample size must be both small enough to ensure that losses due to sampling do not exceed the benefits, and are large enough to give reasonably accurate results.

Rational Subgroups

- The **rational subgroup concept** means that subgroups or samples should be selected so that if assignable causes are present, the chance for detecting differences *between subgroups* will be maximized while the chance for differences due to these assignable causes *within a subgroup* will be minimized.

- When control charts are applied to production processes, the time order of production is a logical basis for forming rational subgroups. Time order is often used for forming subgroups because it allows the detection of assignable causes that occur over time.

- Two general approaches for constructing rational subgroups:

  1. Each sample consists of units that were produced at the same time or were produced relatively close in time.
     - This method is used when the primary purpose is to detect process shifts.
     - It minimizes the chance of variability due to assignable causes *within* a sample and maximizes the chance of variability *between* samples if assignable causes are present.
  2. Each sample consists of units of products that are representative of all units that have been produced since the last sample was taken.
     - Each subgroup is a random sample of all process output over the sampling interval.
     - This method is used when the primary purpose is to make decisions about the quality of all units of product since the last sample.
     - This approach gives a "snapshot" of the process at each point in time when a sample is collected.

- If the process shifts to an out-of-control state and then back to an in-control state between samples, the first method will be ineffective against these types of short-term shifts. In such cases, the second method should be used.

Other Sample Size Considerations

- The cost of sampling: What will the budget allow?

- Destructive vs. nondestructive sampling: Destructive sampling renders the sampling unit unfit for future use.

Exact Results for Shewhart Control Charts With Supplementary Runs Rules

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2. SUPPLEMENTARY RUNS RULES

Consider a random variable X that measures the quality of a product. We observe successively the independent sample means $\bar{X}_1, \bar{X}_2, \ldots$, assuming $\bar{X}_i \sim N(\mu_i, \sigma^2/n)$, $i = 1, 2, \ldots$, where σ^2 is known and remains constant.

Assuming that $\mu_i = \mu$ $(i = 1, 2, \ldots)$, the condition $\mu = \mu_0$ is to be maintained. It is convenient to base decisions on the standardized sample means, $Z_i = (\bar{X}_i - \mu_0)/(\sigma/\sqrt{n})$, $i = 1, 2, \ldots$. The Shewhart control chart is based on the present sample mean with a signal being given at the first stage N such that $|Z_N| > c$, where usually $c = 3$. This procedure has the disadvantage of not being sensitive to small shifts in the mean. One way to improve the chart's sensitivity to small changes in the mean is to add more rules for signaling that the process is out-of-control. One type of rule that is often suggested is the runs rule.

The runs rule, which signals if k of the last m standardized sample means fall in the interval (a, b), $a < b$, will be denoted by $T(k, m, a, b)$. Thus the usual Shewhart $\bar{X}$ chart is denoted by $\{T(1, 1, -\infty, -3), T(1, 1, 3, \infty)\}$, and the $\bar{X}$ chart with supplementary runs rules is given by a larger collection of rules of the form $T(k, m, a, b)$.

Various combinations of the following rules will be considered in this article:

Rule 1: $C_1 = \{T(1, 1, -\infty, -3), T(1, 1, 3, \infty)\}$.
Rule 2: $C_2 = \{T(2, 3, -3, -2), T(2, 3, 2, 3)\}$.
Rule 3: $C_3 = \{T(4, 5, -3, -1), T(4, 5, 1, 3)\}$.
Rule 4: $C_4 = \{T(8, 8, -3, 0), T(8, 8, 0, 3)\}$.
Rule 5: $C_5 = \{T(2, 2, -3, -2), T(2, 2, 2, 3)\}$.
Rule 6: $C_6 = \{T(5, 5, -3, -1), T(5, 5, 1, 3)\}$.
Rule 7: $C_7 = \{T(1, 1, -\infty, -3.09), T(1, 1, 3.09, \infty)\}$.
Rule 8: $C_8 = \{T(2, 3, -3.09, -1.96), T(2, 3, 1.96, 3.09)\}$.
Rule 9: $C_9 = \{T(8, 8, -3.09, 0), T(8, 8, 0, 3.09)\}$.

3. ARL COMPARISONS

The ARL's of several Shewhart control charts with supplementary runs rules are shown in Table 1. The shifts in the mean are measured in units of the process standard deviation, that is, the shift is $d = \sqrt{n}\,|\mu - \mu_0|/\sigma$. Table 1 shows that the supplementary runs rules increase the sensitivity of the Shewhart chart to small shifts in the mean. The supplementary runs rules also reduce the ARL at the target value μ_0, however, and thus result in more false alarms. The in-control ARL for C_{1234} is only 91.75, and the use of shorter runs or additional stopping rules would reduce the ARL to an even smaller value. Any desired ARL at the target value could be obtained, however, by widening the control limits.

Table 1. Average Run Lengths for Shewhart Control Charts With Supplementary Runs Rules

Shift	Control charts															
d	C_1	C_7	C_{12}	C_{78}	C_{15}	C_{13}	C_{14}	C_{79}	C_{16}	C_{123}	C_{156}	C_{124}	C_{789}	C_{134}	C_{1456}	C_{1234}
.0	370.40	499.62	225.44	239.75	278.03	166.05	152.73	170.41	349.38	132.89	266.82	122.05	126.17	105.78	133.21	91.75
.2	308.43	412.01	177.56	185.48	222.59	120.70	110.52	120.87	279.53	97.86	208.44	89.14	91.19	76.01	96.37	66.80
.4	200.08	262.19	104.46	106.15	134.17	63.88	59.76	63.80	165.48	52.93	119.47	48.71	49.19	40.95	51.94	36.61
.6	119.67	153.86	57.92	57.80	75.27	33.99	33.64	35.46	89.07	28.70	63.70	27.49	27.57	23.15	29.01	20.90
.8	71.55	90.41	33.12	32.75	42.96	19.78	21.07	22.09	48.40	16.93	34.96	17.14	17.14	14.62	17.94	13.25
1.0	43.89	54.55	20.01	19.70	25.61	12.66	14.58	15.26	27.74	10.95	20.43	11.73	11.71	10.19	12.19	9.22
1.2	27.82	34.03	12.81	12.62	16.06	8.84	10.90	11.42	17.05	7.68	12.83	8.61	8.59	7.66	8.90	6.89
1.4	18.25	21.97	8.69	8.58	10.60	6.62	8.60	9.05	11.28	5.76	8.65	6.63	6.62	6.08	6.84	5.41
1.6	12.38	14.68	6.21	6.16	7.36	5.24	7.03	7.44	7.98	4.54	6.22	5.27	5.27	5.01	5.42	4.41
1.8	8.69	10.15	4.66	4.64	5.36	4.33	5.85	6.24	5.97	3.73	4.71	4.27	4.27	4.24	4.39	3.68
2.0	6.30	7.25	3.65	3.65	4.07	3.68	4.89	5.25	4.67	3.14	3.72	3.50	3.52	3.65	3.61	3.13
2.2	4.72	5.36	2.96	2.98	3.22	3.18	4.08	4.41	3.78	2.70	3.04	2.91	2.94	3.17	3.01	2.70
2.4	3.65	4.08	2.48	2.51	2.64	2.78	3.38	3.67	3.14	2.35	2.55	2.47	2.50	2.77	2.54	2.35
2.6	2.90	3.20	2.13	2.17	2.22	2.43	2.81	3.05	2.64	2.07	2.19	2.13	2.16	2.43	2.19	2.07
2.8	2.38	2.59	1.87	1.91	1.93	2.14	2.35	2.54	2.26	1.85	1.91	1.87	1.91	2.14	1.91	1.85
3.0	2.00	2.15	1.68	1.71	1.70	1.89	1.99	2.14	1.95	1.67	1.70	1.68	1.71	1.89	1.70	1.67

Table 41.1. Definitions of Tests 1 to 4

Test *Index*	Pattern Description
1	One point beyond Zone A (outside the control limits)
2	Nine points in a row in Zone C or beyond on one side of the central line (see Note 1 below)
3	Six points in a row steadily increasing or steadily decreasing (see Note 2 below)
4	Fourteen points in a row alternating up and down

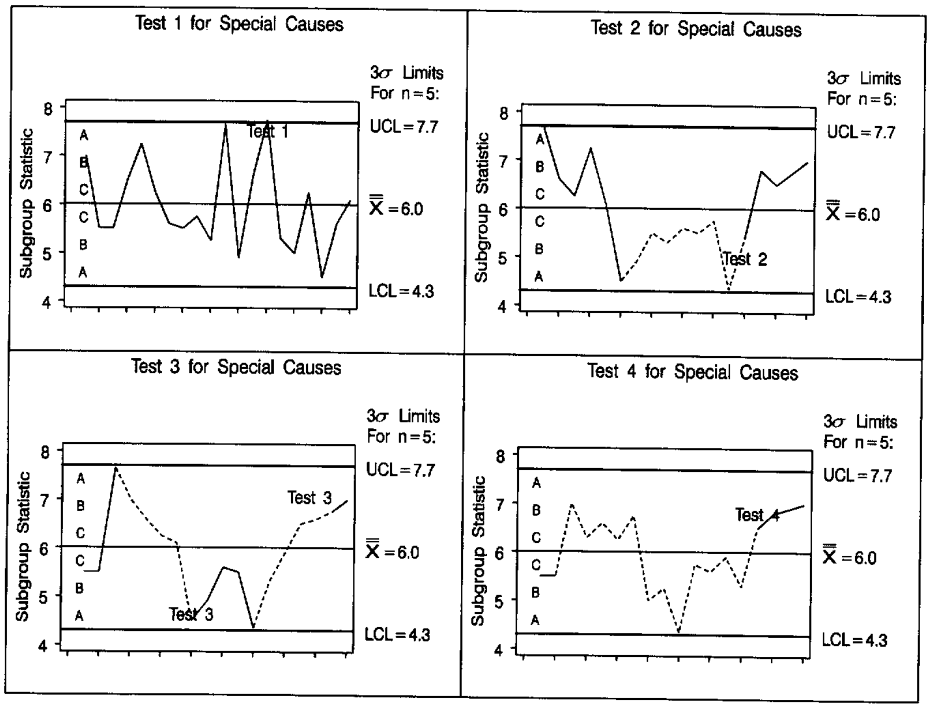

Figure 41.1. Examples of Tests 1 to 4

Notes:

1. The number of points in Test 2 can be specified as 7, 8, 9, 11, 14, or 20 with the TEST2RUN= option.
2. The number of points in Test 3 can be specified as 6, 7, or 8 with the TEST3RUN= option.

Table 41.2. Definitions of Tests 5 to 8

Test *Index*	Pattern Description
5	Two out of three points in a row in Zone A or beyond
6	Four out of five points in a row in Zone B or beyond
7	Fifteen points in a row in Zone C on either or both sides of the central line
8	Eight points in a row on either or both sides of the central line with no points in Zone C

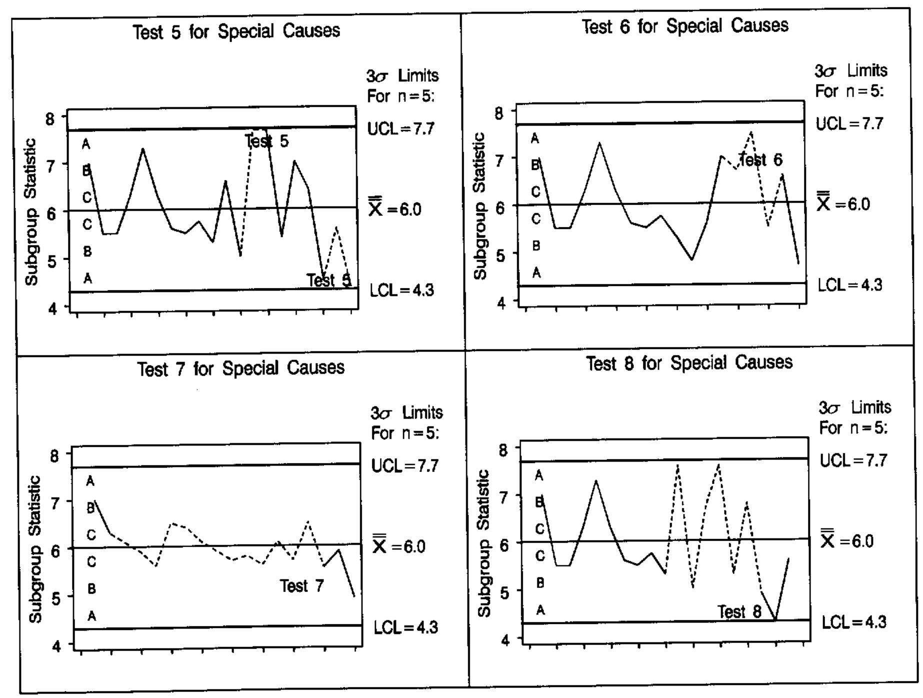

Figure 41.2. Examples of Tests 5 to 8

Interpreting Standard Tests for Special Causes

Nelson (1984, 1985) makes the following comments concerning the interpretation of the tests:

- When a process is in statistical control, the chance of a false signal for each test is less than five in one thousand.
- Test 1 is positive if there is a shift in the process mean, if there is an increase in the process standard deviation, or if there is a "single aberration in the process such as a mistake in calculation, an error in measurement, bad raw material, a breakdown of equipment, and so on" (Nelson 1985).
- Test 2 signals a shift in the process mean. The use of nine points (rather than seven as in Grant and Leavenworth 1988) for the pattern that defines Test 2 makes the chance of a false signal comparable to that of Test 1. (To control the number of points for the pattern in test 2, use the TEST2RUN= option in the chart statement.)
- Test 3 signals a drift in the process mean. Nelson (1985) states that causes can include "tool wear, depletion of chemical baths, deteriorating maintenance, improvement in skill, and so on."
- Test 4 signals "a systematic effect such as produced by two machines, spindles, operators or vendors used alternately" (Nelson 1985).
- Tests 1, 2, 3, and 4 should be applied routinely; the combined chance of a false signal from one or more of these tests is less than one in a hundred. Nelson (1985) describes these tests as "a good set that will react to many commonly occurring special causes."
- In the case of charts for variables, the first four tests should be augmented by Tests 5 and 6 when earlier warning is desired. The chance of a false signal increases to two in a hundred.
- Tests 7 and 8 indicate stratification (observations in a subgroup have multiple sources with different means). Test 7 is positive when the observations in the subgroup always have multiple sources. Test 8 is positive when the subgroups are taken from one source at a time.

Nelson (1985) also comments that "the probabilities quoted for getting false signals should not be considered to be very accurate" since the probabilities are based on assumptions of normality and independence that may not be satisfied. Consequently, he recommends that the tests "should be viewed as simply practical rules for action rather than tests having specific probabilities associated with them." Nelson cautions that "it is possible, though unlikely, for a process to be out of control yet not show any signals from these eight tests."

Modifying Standard Tests for Special Causes

Some textbooks and references present slightly different versions of Tests 2 and 3. You can use the following options to request these modifications:

- TEST2RUN=*run-length* specifies the length of the pattern for Test 2. The form of the test for each *run-length* is given in the following table. The default *run-length* is 9.

Run-length	Number of Points on One Side of Central Line
7	7 in a row
8	8 in a row
9	9 in a row
11	at least 10 out of 11 in a row
14	at least 12 out of 14 in a row
20	at least 16 out of 20 in a row

- TEST3RUN=*run-length* specifies the length of the pattern for Test 3. The *run-length* values allowed are 6, 7, and 8. The default *run-length* is 6.

The Western Electric Company (now AT&T) *Statistical Quality Control Handbook* and Montgomery (1991) discuss a test that is signaled by eight points in a row in Zone C or beyond (on one side of the central line). You can request this test by specifying TESTS=2 and TEST2RUN=8. The *Handbook* also discusses tests corresponding to Tests 1, 5, 6, 7, and 8.

Kume (1985) recommends a number of tests for special causes that can be regarded as modifications of Tests 2 and 3:

- seven points in a row on one side of the central line. Specify TESTS=2 and TEST2RUN=7.
- at least 10 out of 11 points in a row on one side of the central line. Specify TESTS=2 and TEST2RUN=11.
- at least 12 out of 14 points in a row on one side of the central line. Specify TESTS=2 and TEST2RUN=14.
- at least 16 out of 20 points in a row on one side of the central line. Specify TESTS=2 and TEST2RUN=20.
- seven points in a row steadily increasing or decreasing. Specify TESTS=3 and TEST3RUN=7.