Př: Je dána uspořádaná trojice $\mathcal{A}(\mathcal{B}, V, \varphi)$, kde $\mathcal{B} = \mathbb{R}^3$, $V = \mathbb{R}^3$ a
$\forall X, Y \in \mathcal{B} \quad \varphi(X, Y) = (x_1 - y_2, x_2 - y_3, x_3 - y_1) = (u_1, u_2, u_3) = \vec{u} \in V$.
Rozhodněte, zda struktura $\mathcal{A}$ je afinní prostor.

Řešení: Ověříme podmínky (i) a (ii) z D.1.1

(i) $\forall X, Y, Z \in \mathcal{B} \quad \varphi(X, Y) + \varphi(Y, Z) = \varphi(X, Z)$???

$$\varphi(X, Y) + \varphi(Y, Z) = (x_1 - y_2, x_2 - y_3, x_3 - y_1) + (y_1 - z_2, y_2 - z_3, y_3 - z_1) =$$
$$= (x_1 - y_2 + y_1 - z_2,\ x_2 - y_3 + y_2 - z_3,\ x_3 - y_1 + y_3 - z_1)$$
$$\varphi(X, Z) = (x_1 - z_2, x_2 - z_3, x_3 - z_1)$$

$$\varphi(X, Y) + \varphi(Y, Z) \neq \varphi(X, Z)$$
$$\Downarrow$$
(i) neplatí

(ii) není nutné ověřovat

NEJDE O AFINNÍ PROSTOR

Př: Je dána uspořádaná trojice $\mathcal{A}(\mathcal{B}, V, \varphi)$, kde $\mathcal{B} = \{[x_1, x_2] \in \mathbb{R}^2, x_2 > 0\}$,
$V = \mathbb{R}^2$ a
$\forall X, Y \in \mathcal{B} \quad \varphi(X, Y) = \left(x_1 - y_1 + \log \frac{x_2}{y_2}, x_1 - y_1 - \log \frac{x_2}{y_2}\right) = (u_1, u_2) = \vec{u} \in V$.
Rozhodněte, zda struktura $\mathcal{A}$ je afinní prostor.

Řešení: Ověříme podmínky (i) a (ii) z D.1.1

(i) $\forall X, Y, Z \in \mathcal{B} \quad \varphi(X, Y) + \varphi(Y, Z) = \varphi(X, Z)$???

$$\varphi(X,Y)+\varphi(Y,Z) = \left(x_1-y_1+\log\frac{x_2}{y_2},\ x_1-y_1-\log\frac{x_2}{y_2}\right) + \left(y_1-z_1+\log\frac{y_2}{z_2},\ y_1-z_1-\log\frac{y_2}{z_2}\right) =$$

$$= \left(x_1\cancel{-y_1}\cancel{+y_1}-z_1+\log\frac{x_2}{y_2}+\log\frac{y_2}{z_2},\ x_1\cancel{-y_1}\cancel{+y_1}-z_1-\log\frac{x_2}{y_2}-\log\frac{y_2}{z_2}\right) =$$

$$= \left(x_1-z_1+\log\left(\frac{x_2}{\cancel{y_2}}\cdot\frac{\cancel{y_2}}{z_2}\right),\ x_1-z_1-\log\left(\frac{x_2}{\cancel{y_2}}\cdot\frac{\cancel{y_2}}{z_2}\right)\right) =$$

$$= (x_1-z_1) + \log\frac{x_2}{z_2},\ x_1-z_1-\log\frac{x_2}{z_2}) = \varphi(X,Z)$$

(i) platí

(ii) $\forall X\in\mathcal{B}\quad \forall\vec{u}\in V\quad \exists!\, Y\in\mathcal{B}\quad \varphi(X,Y)=\vec{u}$

$$\varphi(X,Y) = \left(x_1-y_1+\log\frac{x_2}{y_2},\ x_1-y_1-\log\frac{x_2}{y_2}\right) = (u_1,u_2) = \vec{u}\in V$$

$$x_1-y_1+\log\frac{x_2}{y_2} = u_1$$

$$x_1-y_1-\log\frac{x_2}{y_2} = u_2$$

známe x_1, x_2 a u_1, u_2 a hledáme jednoznačně určené y_1, y_2

R1+R2 / R1−R2

$$2(x_1-y_1) = u_1+u_2$$

$$x_1-y_1 = \frac{u_1+u_2}{2}$$

$$y_1 = x_1 - \frac{u_1+u_2}{2}$$

$$2\log\frac{x_2}{y_2} = u_1-u_2$$

$$\log\frac{x_2}{y_2} = \frac{u_1-u_2}{2}$$

$$\frac{x_2}{y_2} = 10^{\frac{u_1-u_2}{2}}$$

$$y_2 = x_2\cdot 10^{\frac{u_2-u_1}{2}}$$

(ii) platí

(i) a (ii) jsou splněny $\Rightarrow$ $\mathcal{A}$ je afinní prostor

Př.: Jsou dány dvě soustavy souřadnic φ a φ' dané repéry $\langle P; \vec{u}, \vec{v} \rangle$ a $\langle P'; \vec{u}', \vec{v}' \rangle$, přičemž platí

$$P' = P + 2\vec{u} - \vec{v}$$
$$\vec{u}' = \vec{u} - 3\vec{v}$$
$$\vec{v}' = -\vec{u} + \vec{v}$$

Znáte-li $\{A\}_{\varphi} = [-1,3]$ a $\{D\}_{\varphi'} = [0,3]$, určete $\{A\}_{\varphi'}$ a $\{D\}_{\varphi}$.

Řešení:

$$P' = P + 2\vec{u} - \vec{v} \Rightarrow \{P'\}_{\varphi} = \mathbb{b} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\left.\begin{aligned} \vec{u}' &= \vec{u} - 3\vec{v} \\ \vec{v}' &= -\vec{u} + \vec{v} \end{aligned}\right\} \Rightarrow \left.\begin{aligned} \{\vec{u}'\}_{\varphi} &= (1,-3)^T \\ \{\vec{v}'\}_{\varphi} &= (-1,1)^T \end{aligned}\right\} \text{tj. } \mathbb{A} = \begin{pmatrix} 1 & -1 \\ -3 & 1 \end{pmatrix}$$

$$\mathbb{x} = \mathbb{A} \cdot \mathbb{x}' + \mathbb{b}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -3 & 1 \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}; \text{ tj. } \quad \boxed{\begin{aligned} x &= x' - y' + 2 \\ y &= -3x' + y' - 1 \end{aligned}}$$

$$\{A\}_{\varphi} = [-1,3]; \text{ tj.: } \left.\begin{aligned} -1 &= x' - y' + 2 \\ 3 &= -3x' + y' - 1 \end{aligned}\right\} \Rightarrow \begin{aligned} x' &= -\tfrac{1}{2} \\ y' &= \tfrac{5}{2} \end{aligned}$$

$$\{A\}_{\varphi'} = \left[-\tfrac{1}{2}; \tfrac{5}{2}\right]$$

$$\{D\}_{\varphi'} = [0,3]; \text{ tj.: } \left.\begin{aligned} x &= 0 - 3 + 2 \\ y &= -3 \cdot 0 + 3 - 1 \end{aligned}\right\} \Rightarrow \begin{aligned} x &= -1 \\ y &= 2 \end{aligned}$$

$$\{D\}_{\varphi} = [-1,2]$$

Př.: V A_2 jsou dány dvě soustavy souřadnic $\mathcal{S} = \langle [1,0]^T; (1,1)^T, (0,1)^T \rangle$
a $\mathcal{S}' = \langle [1,-1]^T; (2,0)^T, (1,2)^T \rangle$.
Najděte souřadnice bodu A v soustavě souřadnic $\mathcal{S}'$, jestliže
$\{A\}_{\mathcal{S}} = [-2,1]^T$.

Řešení:

$$\mathcal{S}: \quad X = [1,0] + x_1 \cdot (1,1) + x_2 \cdot (0,1)$$

$$\mathcal{S}': \quad X = [1,-1] + x_1' \cdot (2,0) + x_2' \cdot (1,2)$$

tj. pro bod A

$$\{A\}_{\mathcal{S}} = [\underset{a_1}{-2}, \underset{a_2}{1}] \Rightarrow \quad A = [1,0] + \overset{a_1}{(-2)} \cdot (1,1) + \overset{a_2}{1} \cdot (0,1) = \underline{\underline{[-1,-1]}}$$

$$\{A\}_{\mathcal{S}'} = [a_1', a_2'] \Rightarrow \quad [-1,-1] = [1,-1] + a_1' \cdot (2,0) + a_2' \cdot (1,2)$$

$$(-2,0) = a_1' \cdot (2,0) + a_2' \cdot (1,2)$$

$$\left.\begin{aligned} -2 &= 2a_1' + a_2' \\ 0 &= \quad\; 2a_2' \end{aligned}\right\} \quad \begin{aligned} a_1' &= -1 \\ a_2' &= 0 \end{aligned}$$

$$A = [-1,-1]$$

$$\{A\}_{\mathcal{S}} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\boxed{\{A\}_{\mathcal{S}'} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}}$$

Jiné řešení:

$$\varphi = \langle [1,0]^T; (1,1)^T, (0,1)^T \rangle = \langle P; \vec{u}, \vec{v} \rangle$$

$$\varphi' = \langle [1,-1]^T; (2,0)^T, (1,2)^T \rangle = \langle P'; \vec{u}', \vec{v}' \rangle$$

Najdeme transformační rovnice $\varphi \to \varphi'$

$$P' = P + b_1 \cdot \vec{u} + b_2 \cdot \vec{v}$$

$$[1,-1]^T = [1,0]^T + b_1 \cdot (1,1)^T + b_2 \cdot (0,1)^T$$

$$b_1 = 0 \qquad \boxed{\mathbb{b} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}}$$

$$b_2 = -1$$

$$\vec{u}' = a_{11} \cdot \vec{u} + a_{21} \cdot \vec{v} \qquad \vec{v}' = a_{12} \cdot \vec{u} + a_{22} \cdot \vec{v}$$

$$(2,0)^T = a_{11} \cdot (1,1)^T + a_{21} \cdot (0,1)^T \qquad (1,2)^T = a_{12} \cdot (1,1)^T + a_{22} \cdot (1,2)^T$$

$$a_{11} = 2 \qquad a_{12} = 1$$

$$a_{21} = -2 \qquad a_{22} = 1$$

$$\boxed{A = \begin{pmatrix} 2 & 1 \\ -2 & 1 \end{pmatrix}}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -2 & 1 \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\{A\}_\varphi = [-2, 1]^T \qquad \left.\begin{aligned} -2 &= 2x' + y' \\ 1 &= -2x' + y' - 1 \end{aligned}\right\} \qquad \begin{aligned} x' &= -1 \\ y' &= 0 \end{aligned}$$

$$\{A\}_{\varphi'} = [-1, 0]^T$$