A CONTRIBUTION TO THE STATE MODEL OF ELECTRIC NETWORK WITH EXCESS CAPACITORS

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1. MOTIVATION OF THE PROBLEM

From the theory of linear electric networks it is well known (see e.g. [1], [2]) that the behaviour of networks for which at least one proper tree exists, is fairly "reasonable", that is, the formulation and solution of the state variable model presents no substantial difficulties, the state quantities (i.e. the currents in the inductors and the voltages on the capacitors) depend continuously on time and always have – as well as the powers of all the elements of the network – finite values. However, the situation is more complicated with networks for which no proper tree exists, so that some of its inductors or capacitors are excess ones. For such networks, at the moment when the parameter of a certain active or passive element of the network changes by a jump, some state quantities may have discontinuities while some co-state quantities (i.e. voltages on the inductors and currents in the capacitors) may have the character of a "distribution function"; the power need not be finite for all elements in the newtork. We can simply avoid these complications by modelling physically real systems only as models with proper trees, which from the viewpoint of the physical interpretation of both voltage and current responses apparently represent better models. However, taking into account the fact that every network is just a more or less adequate model of the physical reality, we can accept the above mentioned physically unjustifiable properties of the voltage and current response of the network as approximations of the corresponding response in the physical system. Moreover, the requirement of existence of a proper tree for the network in question sometimes causes greater complexity of the network and may lead to networks different from the usual ones, which are currently used in applications.

The construction of the state variable model of an electric network has been described in detail in a number of publications even in the case that the given network admits no proper tree, so that the network involves excess capacitors and inductors (see e.g. [1], [3]). Nonetheless, it is our opinion that there is still one open question left, namely, how to determine – for networks with excess elements – the initial conditions necessary for the integration of the state variable equations. This problem was suggested in the book [4] and in the paper [5]. In the present paper we will show that in networks with excess capacitors we can find sub-networks containing only capacitors and voltage sources – we shall call them voltage-excess blocks, briefly ve-blocks. The state variables will have discontinuities at $t = 0$ only in these ve-blocks, while they will be continuous outside them. This will enable us to find the vector of the initial conditions from the right. If this initial condition is known, then the integration of the corresponding state variable equation of the network presents no problem any more.

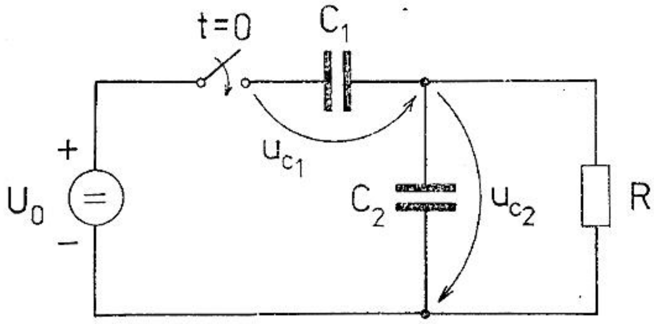

Fig. 1.

On the duality basis we could formulate an analogous procedure for networks with excess inductors.

Our main idea can be illustrated by a simple example: the network in Fig. 1 evidently has no proper tree; one of his capacitors is an excess one. The state variables are voltages $u_{C1}(t)$, $u_{C2}(t)$, which evidently have discontinuities at $t = 0$, the moment when the switch is connected: $u_{C1}(0_-) \neq u_{C1}(0_+)$, $u_{C2}(0_-) \neq u_{C2}(0_+)$. At the moment $t = 0_-$ the both voltages have given values. However, in order to be able to solve the state equations of the network for $t > 0$ we need the initial condition

$$\boldsymbol{x}_0(0_+) = {}^t[u_{C1}(0_+);\ u_{C2}(0_+)],$$

which is not apriori known.

2. EXCESS OF THE TREE AND A NORMAL TREE OF A NETWORK

Consider a network, each of whose branches contains exactly one element. It may be either a passive element R, L, C or a voltage source $u_0(t)$, or possibly a current source $i_0(t)$. Taking into account the element of a given branch we speak about C-branches, u_0-branches atc.

If a network $\mathcal{N}$ has a proper tree $\mathcal{T}_p$, then all the state variables of the network are known to be continuous [1], [2], since

– every loop of the network $\mathcal{N}$ is incident with at least one link; according to Kirchhoff's voltage law, the sum of discontinuities of the voltage sources of branches incident with this loop equals the discontinuity of the voltage of the link considered, and

– every cut set of the network $\mathcal{N}$ is incident with at least one branch of the proper tree; according to Kirchhoff's current law, the sum of discontinuities of the current sources of branches incident with this cut set equals the discontinuity of the current in this tree branch.

However, we are not going to deal with networks which have proper trees but solely with networks with excess elements, that is, with such networks for which no proper tree exists. In our considerations we shall use the following characteristics of a tree:

Definition 1. Let $\mathcal{T}$ be a tree of a network $\mathcal{N}$. Then

a) the number of all C-branches and u_0-branches of the network $\mathcal{N}$ which are not incident with the tree $\mathcal{T}$, will be denoted by $E_u(\mathcal{T})$ and called the *voltage excess* of the tree $\mathcal{T}$;

b) the number of all L-branches and i_0-branches of the network $\mathcal{N}$, which are incident with the tree $\mathcal{T}$, will be denoted by $E_i(\mathcal{T})$ and called the *current excess* of the tree $\mathcal{T}$, and

c) the number

$$E(\mathcal{T}) = E_u(\mathcal{T}) + E_i(\mathcal{T})$$

will be called the *excess* of the tree $\mathcal{T}$.

We introduce the notion of a normal tree, which will be of crucial significance in what follows:

Definition 2. A tree $\mathcal{T}_n$ of a network $\mathcal{N}$, whose excess assumes the minimal value, is called a *normal tree* of the network $\mathcal{N}$.

It is evident from this definition that the values $E_u(\mathcal{T}_n)$, $E_i(\mathcal{T}_n)$ and hence also $E(\mathcal{T}_n)$ are independent of the particular choice of the normal tree $\mathcal{T}_n$. Thus they characterize not only any normal tree $\mathcal{T}_n$ of the network $\mathcal{N}$ but the whole network $\mathcal{N}$ as well, and we can use for them the notation $E_u(\mathcal{N})$, $E_i(\mathcal{N})$ and $E(\mathcal{N})$.

It is also evident that a proper tree $\mathcal{T}_p$ (for definition see e.g. [1], [2]) is at the same time a normal tree with $E(\mathcal{T}_p) = 0$. While in a given network $\mathcal{N}$ a proper tree need not exist, a normal tree always exists. Neither a normal tree nor a proper one are uniquely determined; nevertheless, if there are more of them, their excesses coincide.

The following result gives the basic topological characteristic of a normal tree:

Theorem 1. A tree of a network $\mathcal{N}$ is a normal tree $\mathcal{T}_n$, if and only if the following conditions are fulfilled:

1. By adding to a normal tree $\mathcal{T}_n$ either a u_0-branch or a C-branch which are not incident with the tree $\mathcal{T}_n$, we obtain a loop (according to [2], it is unique), which contains only u_0-branches and C-branches.

2. If we delete from a normal tree $\mathcal{T}_n$ an i_0-branch or an L-branch, then by means of a Jordan curve intersecting only this branch of the tree $\mathcal{T}_n$ we can obtain a cut set (it is again unique, according to [2]), which contains only i_0-branches and L-branches.

Proof.

a) *Necessity*: if the obtained loop is incident with either an i_0-branch or an L-branch or an R-branch, we modify the tree $\mathcal{T}_n$ by including the u_0-branch or C-branch into $\mathcal{T}_n$ and deleting the i_0-branch or the L-branch or the R-branch. In this way the excess of the network decreases. Analogously, if the cut set is incident with a u_0-branch or a C-branch or an R-branch, we include it into the tree and delete the i_0-branch or the L-branch, which previously belonged to the cut set. This again decreases the excess, which contradicts the minimality.

b) *Sufficiency*: from the conditions it follows that the network $\mathcal{N}$ contains exactly $E_u(\mathcal{T}_n)$ independent loops incident only with the u_0- and C-branches, and exactly $E_i(\mathcal{T}_n)$ independent cut sets incident only with the i_0- and L-branches. Since in each tree of the network $\mathcal{N}$ at least one branch from each loop of the network must be missing, while the tree must contain at least one branch each cut set, its excess is necessarily greater than or equal to $E(\mathcal{T}_n)$, Q.E.D.

The algorithm suggested in the first part of the proof of Theorem 1 makes it possible to find a normal tree of a given network $\mathcal{N}$ and to evaluate its excess; the structure of the "excess parts" of the network $\mathcal{N}$ is seen from the second part of the proof.

3. NETWORKS WITH EXCESS CAPACITORS

In this section we shall restrict our considerations to networks with excess capacitors, that is, to networks $\mathcal{N}$ with $E_i(\mathcal{N}) = 0$ and $E_u(\mathcal{N}) > 0$. In the network $\mathcal{N}$ we indicate a sub-network containing only u_0-branches and C-branches and denote it by $\mathcal{N}_u$. Theorem 1 implies that the sub-network $\mathcal{N}_u$ contains exactly $E_u(\mathcal{N})$ links. For our further considerations, the sub-networks obtained from the sub-network $\mathcal{N}_u$ by deleting the branches which are incident with no loop, are of crucial importance. They are introduced by

Definition 3. The blocks (i.e. the maximal 2-connected parts) of the network $\mathcal{N}_u$ are called the *voltage excess blocks* of the network $\mathcal{N}$ briefly *ve-blocks*.

Let us recall that a network is called 2-connected, if each of its node-cut set contains at least two nodes, or equivalently, if it is connected and remains so after deleting an arbitrary one of its nodes. For example, the networks in Fig. 2ab are not 2-connected while that in Fig. 2c is.

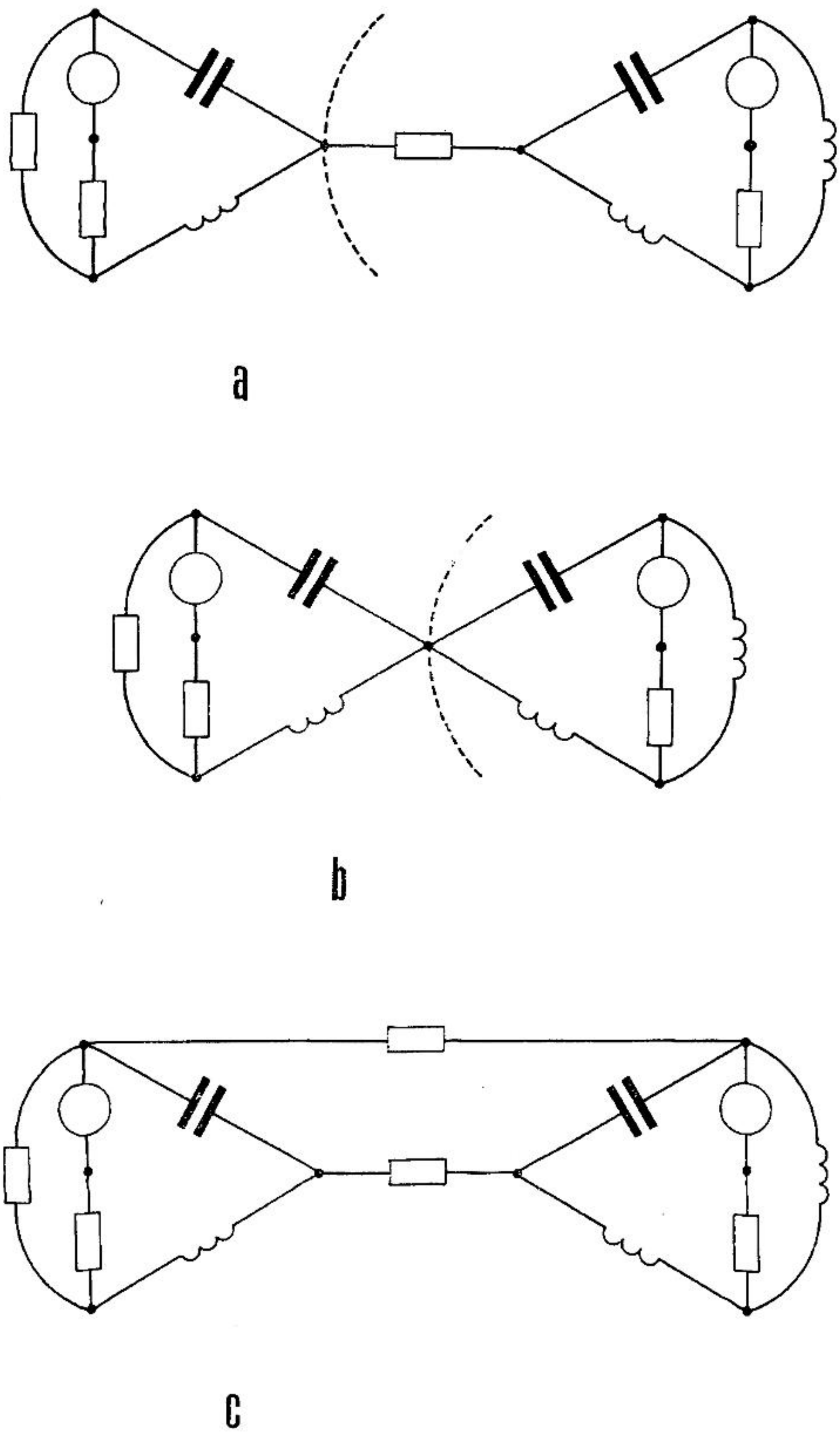

Fig. 2.

Fig. 3 illustrates the notions just introduced: a network $\mathcal{N}$ (Fig. 3a), its subnetwork $\mathcal{N}_u$ (Fig. 3b) and the three corresponding ve-blocks (Fig. 3c).

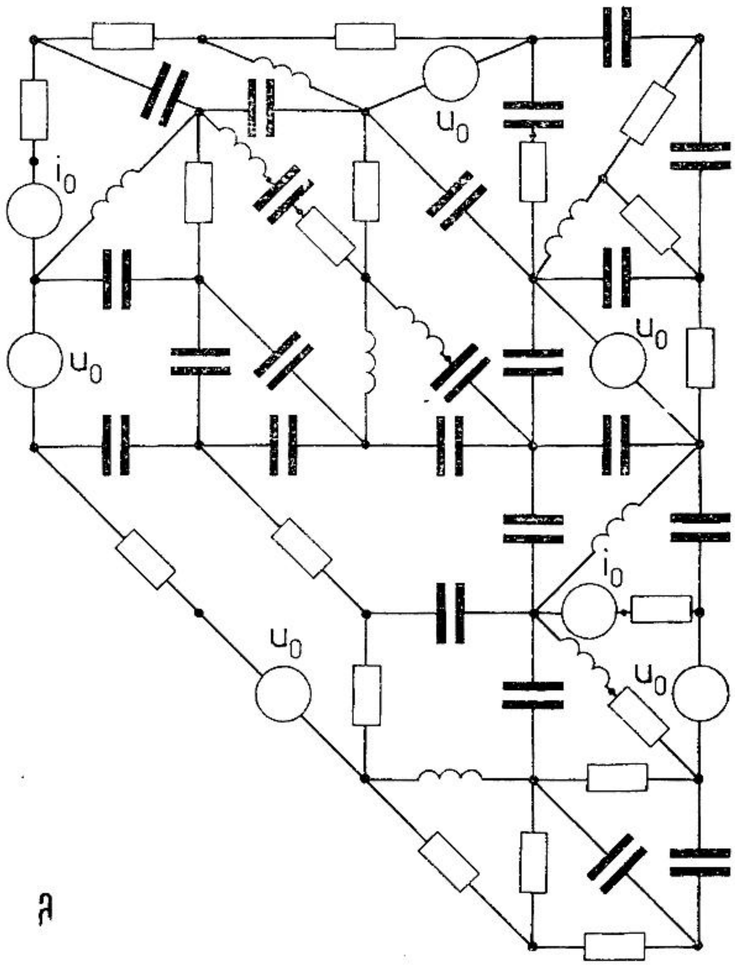

Fig. 3a.

The reason of introducing the ve-blocks is the fact that the phenomena connected with the non-continuity of the state variables can be studied in each ve-block separately, that is, independently of the rest of the network $\mathcal{N}$. This is asserted by

Theorem 2. Let a network $\mathcal{N}$ with excess capacitors be given, i.e. $E_i(\mathcal{N}) = 0$, $E_u(\mathcal{N}) > 0$. Let each loop of $\mathcal{N}$ contain at least one passive branch. Then all the state variables are continuous on the branches which are not incident with the ve-blocks.

Before passing to the proof of Theorem 2, let us illustrate the idea of the proof by an example of a network with a single ve-block with a single loop (i.e. $E_u(\mathcal{N}) = 1$, $E_i(\mathcal{N}) = 0$), and then prove an auxiliary proposition.

Example 1. Let us investigate the network from Fig. 4. It contains a single ve-block which is formed by the branches with capacitors C_2, C_3, C_4, C_5 and voltage source

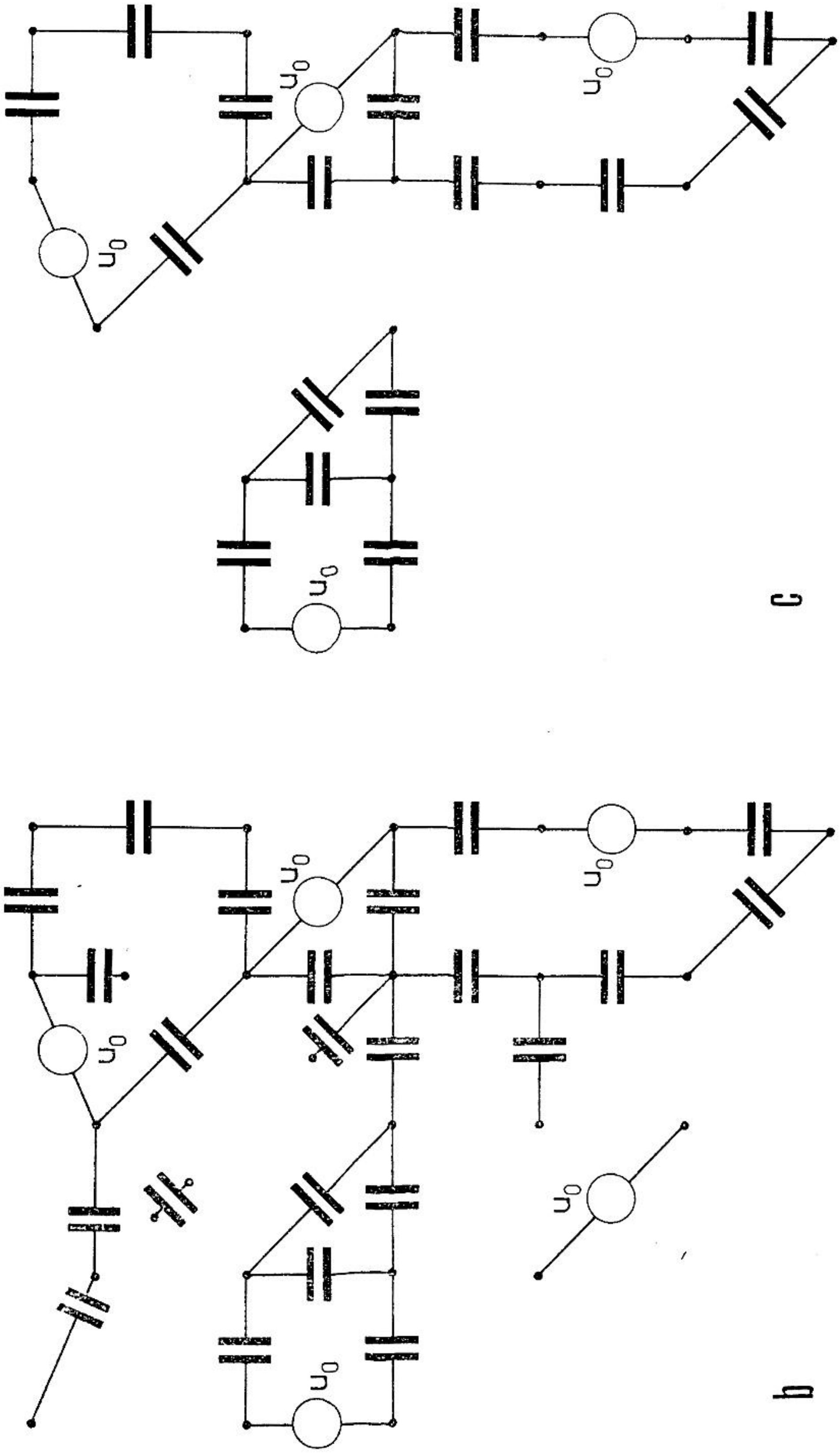

Fig. 3bc.

$u_{01}(t)$. The method of loop currents yields

$$(1)\quad \frac{1}{C_2}\int (i_1 - i_2) + \frac{1}{C_3}\int (i_1 - i_6) + \frac{1}{C_4}\int (i_1 - i_5) + \frac{1}{C_5}\int (i_1 - i_4) = u_{01}(t),$$

$$R_1 i_2 + L_1 \frac{di_2}{dt} + \frac{1}{C_2}\int (i_2 - i_1) + \frac{1}{C_1}\int (i_2 - i_3) = -u_{01}(t),$$

$$R_2 i_3 + \frac{1}{C_1}\int (i_3 - i_2) + R_3(i_3 - i_4) = 0,$$

$$R_3(i_4 - i_3) + \frac{1}{C_5}\int (i_4 - i_1) + R_4(i_4 - i_5) = u_{02}(t),$$

$$R_5 i_5 + R_4(i_5 - i_4) + \frac{1}{C_4}\int (i_5 - i_1) + L_2 \frac{d}{dt}(i_5 - i_6) = 0,$$

$$R_6 i_6 + L_2 \frac{d}{dt}(i_6 - i_5) + \frac{1}{C_3}\int (i_6 - i_1) = 0,$$

when we write $\int (i_1 - i_2)$ etc. instead of $\int_0^t (i_1(\tau) - i_2(\tau))\, d\tau$ for the sake of brevity.

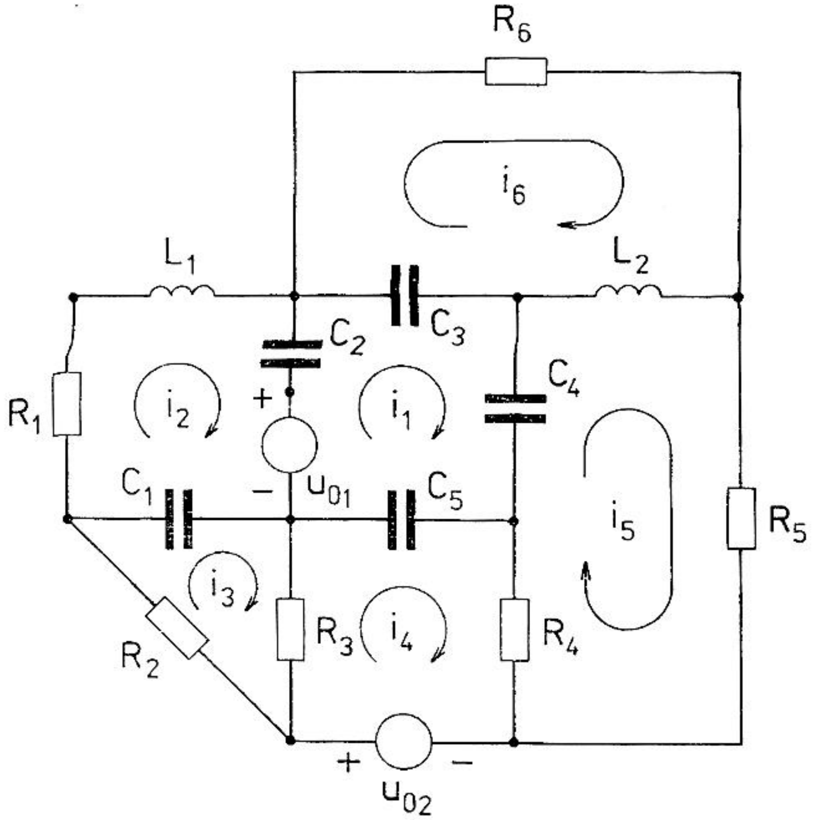

Fig. 4.

Since the network in Fig. 4 has at least one passive branch in each of its loop, the equations (1) have exactly one solution (which is well known – see e.g. [6]). Let us denote it by $\bar{i}_1(t), \ldots, \bar{i}_6(t)$. Denoting

(2)
$$\tilde{u}_1(t) = \frac{1}{C_2}\int_0^t (\bar{i}_1(\tau) - \bar{i}_2(\tau))\,\mathrm{d}\tau\,,$$

$$\tilde{u}_2(t) = \frac{1}{C_4}\int_0^t (\bar{i}_1(\tau) - \bar{i}_5(\tau))\,\mathrm{d}\tau\,,$$

$$\tilde{u}_3(t) = \frac{1}{C_5}\int_0^t (\bar{i}_1(\tau) - i_4(\tau))\,\mathrm{d}\tau\,,$$

and

$$\tilde{u}_4(t) = \frac{1}{C_3}\int_0^t (\bar{i}_1(\tau) - \bar{i}_6(\tau))\,\mathrm{d}\tau - R(\bar{i}_1(t) - \bar{i}_6(t))\,,$$

where $R > 0$, we obtain after substituting (2) into the equations (1):

(3)
$$R(i_1 - i_6) = u_{01} - \tilde{u}_1 - \tilde{u}_2 - \tilde{u}_3 - \tilde{u}_4\,,$$

$$R_1 i_2 + L_1\frac{\mathrm{d}i_2}{\mathrm{d}t} + \frac{1}{C_1}\int (i_2 - i_3) = -u_{01} - \tilde{u}_1\,,$$

$$R_2 i_3 + \frac{1}{C_1}\int (i_3 - i_2) + R_3(i_3 - i_4) = 0\,,$$

$$R_3(i_4 - i_3) + R_4(i_4 - i_5) = u_{02} + \tilde{u}_3\,,$$

$$R_5 i_5 + R_4(i_5 - i_4) + L_2\frac{\mathrm{d}}{\mathrm{d}t}(i_5 - i_6) = \tilde{u}_2\,,$$

$$R_6 i_6 + L_2\frac{\mathrm{d}}{\mathrm{d}t}(i_6 - i_5) + R(i_6 - i_1) = \tilde{u}_4\,.$$

From the system of equations (3) it is clear that the network from Fig. 4 can be replaced by an equivalent one (in the sense that the voltages as well as currents of the corresponding branches are equal), which is shown in Fig. 5. Comparing the both networks we see that the possibly discontinuous state variables in the network from Fig. 4 have been replaced by voltage sources (with posibly discontinuous dependence on the time variable). While there is no proper tree for the network in Fig. 4, there is one for the network in Fig. 5. Consequently, the state quantities for the network from Fig. 5 are continuous, hence the same is true for the corresponding state quantities of the network from Fig. 4. Let us note that in both networks the only mutually corresponding state variables are those which do not belong to the ve-block of the network from Fig. 4.

Before passing to the proof of Theorem 2 let us establish an auxiliary result.

Lemma 1. If the assumptions of the Theorem 2 are fulfilled, then there exists a normal tree $\mathcal{T}_n$ in the network $\mathcal{N}$, such that all u_0-branches of the network $\mathcal{N}$ are incident with the tree $\mathcal{T}_n$.

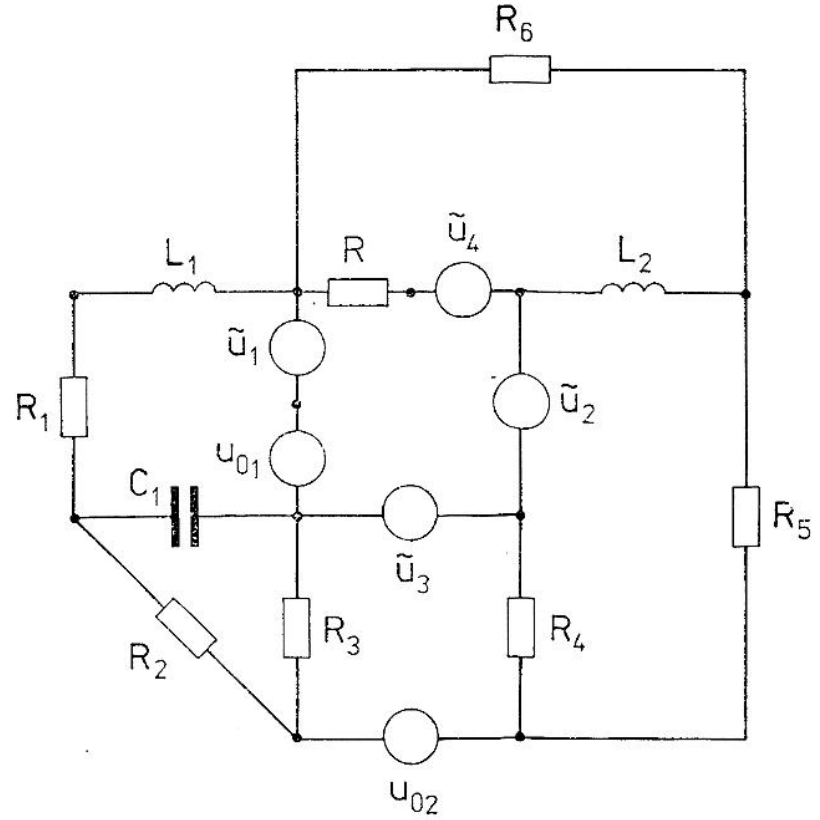

Fig. 5.

Proof. Let us choose a normal tree $\mathcal{T}_n^0$ in the network $\mathcal{N}$. In the corresponding system of links $\mathcal{N} - \mathcal{T}_n$ there are exactly $E_u(\mathcal{N})$ u_0-branches and C-branches. Let k of them $(k \leqq E_u(\mathcal{N}))$ be u_0-branches, say $b_1^n, \ldots, b_k^n$. Adding the branch b_1^n to the tree $\mathcal{T}_n^0$ we obtain a network $\mathcal{N}_1$ containing exactly one loop $\mathcal{L}_1$. By assumption this loop is incident with at least one passive branch. In virtue of the fact that the branches of the loop $\mathcal{L}_1$ are either u_0-branches or C-branches, the branch incident with the loop $\mathcal{L}_1$ necessarily is a C-branch, say b_1^C. By deleting this branch b_1^C from the network $\mathcal{N}_1$ we obtain a tree $\mathcal{T}_n^1$. It is easily seen that $\mathcal{T}_n^1$ again is a normal tree of the network $\mathcal{N}$.

By adding the branch b_2^n to the network $\mathcal{T}_n^1$ we obtain a network $\mathcal{N}_2$, which contains a loop $\mathcal{L}_2$. Deleting another C-branch from the loop $\mathcal{L}_2$, say the branch b_2^C, we obtain a normal tree $\mathcal{T}_n^2$. After k such steps we obtain a normal tree $\mathcal{T}_n^k$, such that the corresponding system of links $\mathcal{N} - \mathcal{T}_n^k$ contains no u_0-branches. The proof is then completed by putting $\mathcal{T}_n = \mathcal{T}_n^k$.

Now we proceed to

Proof of Theorem 2. In the network $\mathcal{N}$ let us choose a normal tree $\mathcal{T}_n$ with the property from 1. Let us denote the branches of the corresponding system of links $\mathscr{N} - \mathscr{T}_n$ by $b_1, \ldots, b_l$ (l is the cyclomatic number of the graph of the network). By adding the i-th branch b_i to the tree $\mathscr{T}_n$ we obtain a single loop $\mathscr{L}_i$ ($i = 1, \ldots, l$). As is well known (see e.g. [2]), the family of loops $\{\mathscr{L}_i\}_{i=1}^l$ forms the complete system of independent loops of the network $\mathscr{N}$. Given an arbitrary orientation of the branches of the tree $\mathscr{N}$ and the independent loops $\mathscr{L}_i$, the method of loop currents can be used to formulate the system of equations, which represent the mathematical model of the network $\mathscr{N}$. Under the assumptions of Theorem 2 (see e.g. [6]) this system of equations has exactly one solution $\bar{i}_1(t), \ldots, \bar{i}_l(t)$. Now it is possible to find the current $i_b(t)$ in each branch b of the network $\mathscr{N}$.

Starting from the network $\mathscr{N}$ we use the following construction to obtain a network $\overline{\mathscr{N}}$. Let b be a branch of the network $\mathscr{N}$. If the branch b

(i) is incident with no ve-block of the network $\mathscr{N}$, then it will belong to the network $\overline{\mathscr{N}}$ as well;

(ii) is incident with a ve-block, then

– it will belong to $\overline{\mathscr{N}}$ as well provided it is a u_0-branch (and hence necessarily belongs to the tree $\mathscr{T}_n$),

– provided it is a C-branch (with a capacitor whose capacity is C_b) and this C-branch

(α) is incident with the tree $\mathscr{T}_n$, we replace it by a u_0-branch, the value of the voltage source being

$$u_{0b}(t) = \frac{1}{C_b}\int_0^t \bar{i}_b(\tau)\,\mathrm{d}\tau\,, \tag{4}$$

(β) is not incident with a tree $\mathscr{T}_n$, we choose a real number $R_b > 0$ and replace the branch b by a pair of branches coupled in series, one of them being an R-branch (containing a resistor with the resistance R_b) and the other a u_0-branch with a voltage source with the value

$$u_{0b}(t) = \frac{1}{C_b}\int_0^t \bar{i}_b(\tau)\,\mathrm{d}\tau - R_b\,\bar{i}_b(t)\,. \tag{5}$$

It is seen from this construction that the network $\overline{\mathscr{N}}$ has a proper tree, since all the excess branches of the tree $\mathscr{T}_n$ have been replaced by an R-branch and a u_0-branch connected in series. Consequently, the state quantities of the network $\overline{\mathscr{N}}$ are continuous. In virtue of the relations (4) and (5) the systems of equations describing the networks $\mathscr{N}$ and $\overline{\mathscr{N}}$ have the same solutions. The continuity of the state variables of the network $\overline{\mathscr{N}}$ implies the continuity of the state variables in that part of the network $\mathscr{N}$ which has not charged by the construction, that is, in all the branches which do not belong to the ve-blocks.

4. DISCUSSION OF DISCONTINUITIES OF THE STATE VARIABLES IN THE VE-BLOCKS

We have found out that the state variables may have discontinuities only in the ve-blocks and that when investigating these discontinuities, we may solve each ve-block separately, independently of the rest of the network. Hence the problem of finding the limit from the right of the initial values for a network with excess capacitors reduces to finding the limit from the right of the initial values in a network consisting solely of u_0-branches and C-branches, briefly a UC-network.

The UC-networks satisfy relations analogous to the Kirchhoff's laws [1], [2]:

a) the charge preserving law holds for any node B_j which is incident only with C-branches:

$$\sum_k a_{jk} Q_{k-} = \sum_k a_{jk} Q_k \,, \tag{6}$$

where $Q_{k-} > 0$ is the value of the charge of the electrode of the capacitor at the end of the k-th C-branch at the time moment $t = 0_-$ while $Q_k > 0$ is the value of charge of the same electrode at $t = 0_+$; the coefficient $a_{jk} = 1$ provided the node B_j is incident with the k-th C-branch and the corresponding electrode of the capacitor has a positive charge, $a_{jk} = -1$ provided the node B_j is incident with the k-th C-branch and the corresponding electrode of the capacitor has a negative charge, and $a_{jk} = 0$ provided the node B_j is not incident with the k-th C-branch.

b) For an arbitrary loop $\mathscr{L}_j$, the second Maxwell equation in the integral form yields

$$\sum_k b_{jk} u_{Ck} = \sum_k b_{jk} \frac{Q_k}{C_k} = \sum_k b_{jk} u_{0k} \,, \tag{7}$$

where $u_{Ck} > 0$ is the voltage on the capacitor with a capacity C_k, $Q_k > 0$ is the value of charge of its positive electrode and u_{0k} is the voltage of the k-th u_0-branch; the coefficient $b_{jk} = 1$ provided the k-th branch is incident with the loop $\mathscr{L}_j$ and their orientations coincide, $b_{jk} = -1$ provided the k-th branch is incident with the loop $\mathscr{L}_j$ and they have different orientations, and $b_{jk} = 0$ provided the k-th branch is not incident with the loop $\mathscr{L}_j$.

Let a UC-network containing p C-branches be given. For the moments $t = 0_-$ and $t = 0_+$ let us apply the relation (6) to the independent nodes and for $t = 0_+$ the relation (7) to the independent loops. Thus we obtain a system of p linearly independent algebraic equations for the charges $Q_1, \ldots, Q_p$. By solving this system of equations we determine these charges, and hence also the voltages at $t = 0_+$ for the branches with capacitors whose capacities are $C_1, \ldots, C_p$:

$$u_k = \frac{Q_k}{C_k} \quad (k = 1, \ldots, p) \,.$$

Example 2. For the UC-network in Fig. 6, B_1 is an independent node and $\mathscr{L}_1$, $\mathscr{L}_2$ are independent loops. By applying the relations (6), (7) we obtain the system of linear equations

$$\begin{aligned} -\quad Q_1 + \quad Q_2 - \quad Q_3 &= \quad 0\,, \\ -\,C_3Q_1 \qquad\qquad + C_1Q_3 &= -C_1C_3u_{01}\,, \\ -\,C_3Q_2 - C_2Q_3 &= -C_2C_3u_{02} \end{aligned}$$

and hence the initial values for $t = 0_+$:

$$u_{Ck} = \frac{Q_k}{C_k} \quad (k = 1, 2, 3)\,.$$

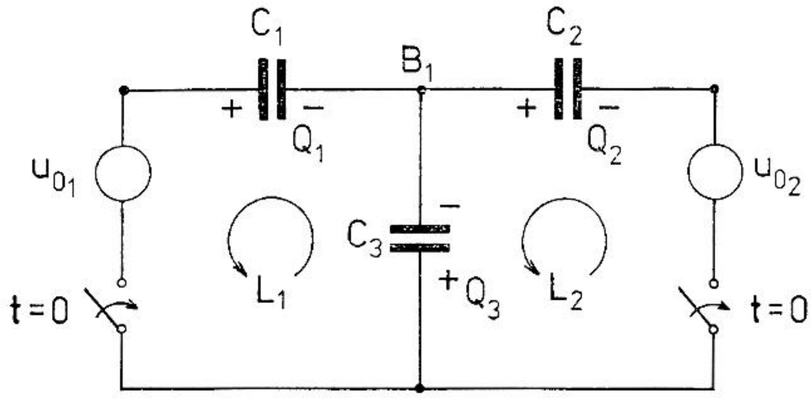

Fig. 6.

5. MATHEMATICAL MODEL OF A NETWORK WITH EXCESS CAPACITORS

It remains to construct a mathematical model of a network with excess capacitors, and then to solve it for $t > 0$. We have just described a way of finding the voltages on all capacitors at the time $t = 0_+$. The mathematical model can be obtained by only slightly modifying the well known method applicable to the case of networks with proper trees (see e.g. [2]): We introduce the state variables (i.e. the voltages on the C-branches and the currents in the L-branches) as well as the auxiliary variables (i.e. the currents or voltages on the R-branches) in the usual way. By means of the normal tree we obtain the system of independent loops and independent cut sets (see e.g. [1], [3]) and by applying the Kirchhoff's current law and Kirchhoff's voltage law we obtain the equations of the network. Here the voltages on the excess C-branches are not introduced as independent state variables but expressed in terms of the voltages on the C-branches of the proper tree.

Example 3. Let us give a sketch of how to formulate the equations for the network from Fig. 7a. The corresponding proper tree with the state variables (u_{C1},

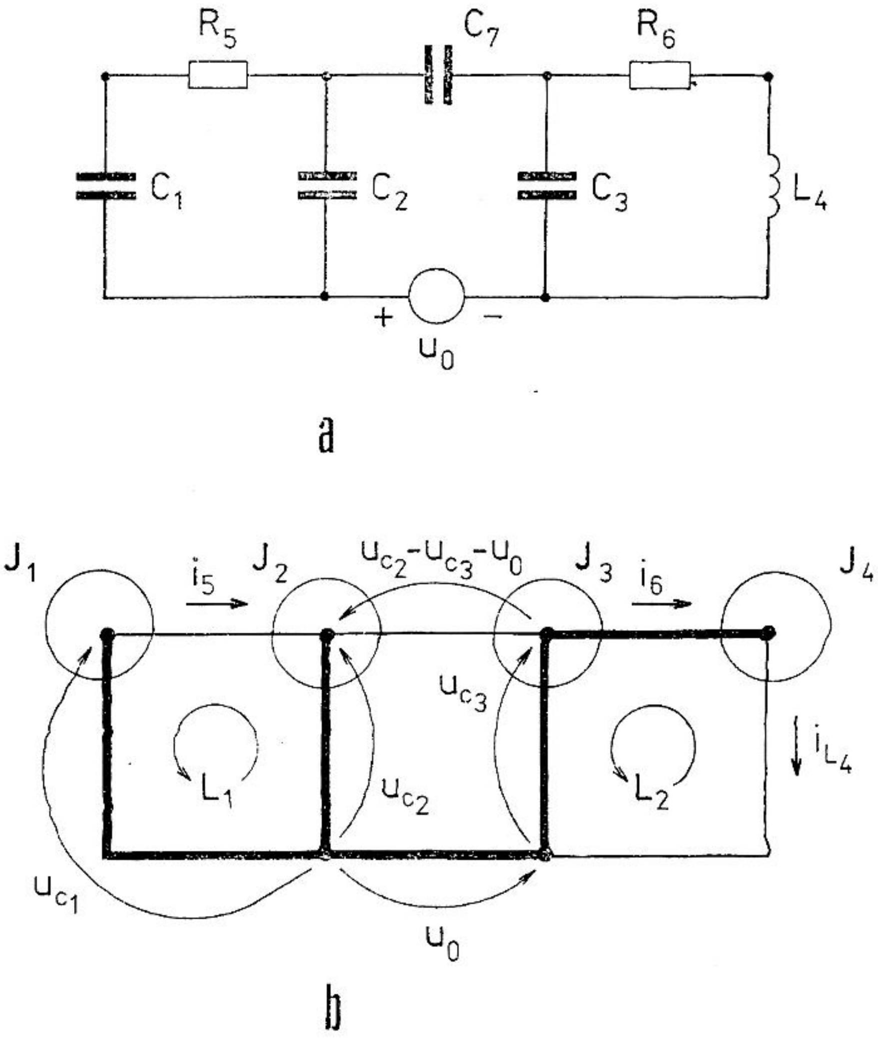

Fig. 7.

u_{C2}, u_{C3}, i_{L4}) and the auxiliary variables (i_5, i_6) is given in Fig. 7b, where two loops ($\mathscr{L}_1$, $\mathscr{L}_2$) and four cut sets ($\mathscr{J}_1$, $\mathscr{J}_2$, $\mathscr{J}_3$, $\mathscr{J}_4$) are indicated. Hence we obtain:

$$\mathscr{L}_1: - u_{C1} + u_{C2} - R_5 i_5 = 0 ,$$

$$\mathscr{L}_2: - u_{C3} - L_4 \frac{\mathrm{d}i_{L4}}{\mathrm{d}t} - R_6 i_6 = 0 ,$$

$$\mathscr{J}_1: \quad - C_1 \frac{\mathrm{d}u_{C1}}{\mathrm{d}t} + i_5 = 0 ,$$

$$\mathscr{J}_2: - i_5 - C_2 \frac{\mathrm{d}u_{C2}}{\mathrm{d}t} - C_7 \frac{\mathrm{d}}{\mathrm{d}t} (u_{C2} - u_{C3} - u_0) = 0 ,$$

$$\mathscr{J}_3: C_7 \frac{\mathrm{d}}{\mathrm{d}t}(u_{C2} - u_{C3} - u_0) - C_3 \frac{\mathrm{d}u_{C3}}{\mathrm{d}t} + i_6 = 0\,,$$

$$\mathscr{J}_4: \qquad - i_6 + i_{L4} \qquad = 0\,.$$

Eliminating the auxiliary variables and re-arranging this system we obtain the usual state-variable model of the network.

REFERENCES

[1] Balabanian, N., and Bickart, A.: Electrical Network Theory. New York, J. Wiley and Sons, 1969.

[2] Mayer, D.: An Introduction to the Theory of Electric Networks. Praha, SNTL, 1978, 1980.

[3] Mayer, D.: The State Variable Method of Electric Network Analysis. Acta Technica ČSAV, **15** (1970), No. 6, pp. 761—789.
Mayer, D.: Formulation State Equations of Electric Networks by a Simple Method. Acta Technica ČSAV, **20** (1975), No. 3, pp. 275—300.

[4] Polivanov, K. M.: Teoretičeskije osnovy elektrotechniky. Tom 1. Moskva, „Energija", 1972.

[5] Artěm'ěv, V. M.: K teorii pěrechodnych processov v linejnych cepjach pri „nekorektno" postavlennych zadačach. Električestvo, 1979, No. 5, pp. 66—68.

[6] Čulík, K., Doležal, V. and Fiedler, M.: Applicable Combinatorial Analysis. (Czech) Praha, SNTL, 1967.

A CONTRIBUTION TO THE STATE MODEL OF ELECTRIC NETWORK WITH EXCESS CAPACITORS

When applying the state variable method to networks with excess elements, the problem of finding of the initial conditions from the right arises. The present paper concerns the theory of networks with excess capacitors. It is shown that certain sub-networks can be selected from the given network — the so called voltage excess blocks, briefly ve-blocks — in which the state variables have discontinuities, while in the remaining part of the network they are continuous. Further, it is shown that in order to find the initial conditions from the right, these ve-blocks can be investigated independently. On the basis of these fact it is possible to formulate a simple algorithm for construction of a mathematical model of a network with excess capacitors.

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