ENGINEERING COUNCIL

CERTIFICATE LEVEL

THERMODYNAMIC, FLUID AND PROCESS ENGINEERING C106

TUTORIAL 7 – HYDROSTATICS

Elements of this tutorial may be skipped if you are already familiar with the subject matter.

On completion of this tutorial you should be able to do the following.

- Define the main ***fundamental properties*** of liquids.
- Explain ***Archimedes's Principle***
- Calculate the pressure due to the depth of a liquid.
- Calculate the total force on a vertical surface.
- Define and calculate the position of the centre of pressure for various shapes.
- Calculate the turning moments produced on vertically immersed surfaces.

Before you start you should make sure that you fully understand first and second moments of area. If you are not familiar with this, you should do that tutorial before proceeding. Let's start this tutorial by studying the fundamental properties of liquids.

1. SOME FUNDAMENTAL STUDIES

1.1 IDEAL LIQUIDS

An ideal liquid is defined as follows.

It is ***INVISCID***. This means that molecules require no force to separate them. The topic is covered in detail in chapter 3.

It is ***INCOMPRESSIBLE***. This means that it would require an infinite force to reduce the volume of the liquid.

1.2 REAL LIQUIDS

VISCOSITY

Real liquids have ***VISCOSITY***. This means that the molecules tend to stick to each other and to any surface with which they come into contact. This produces fluid friction and energy loss when the liquid flows over a surface. Viscosity defines how easily a liquid flows. The lower the viscosity, the easier it flows.

BULK MODULUS

Real liquids are compressible and this is governed by the ***BULK MODULUS K***. This is defined as follows.

$$K = V\Delta p/\Delta V$$

Δp is the increase in pressure, ΔV is the reduction in volume and V is the original volume.

DENSITY Density ρ relates the mass and volume such that $\rho = m/V$ kg/ m^3

PRESSURE

Pressure is the result of compacting the molecules of a fluid into a smaller space than it would otherwise occupy. Pressure is the force per unit area acting on a surface. The unit of pressure is the N/m^2 and this is called a ***PASCAL***. The Pascal is a small unit of pressure so higher multiples are common.

1 kPa = 10^3 N/m^2
1 MPa = 10^6 N/m^2
Another common unit of pressure is the ***bar*** but this is not an SI unit.
1 bar = 10^5 N/m^2
1 mb = 100 N/m^2

GAUGE AND ABSOLUTE PRESSURE

Most pressure gauges are designed only to measure and indicate the pressure of a fluid above that of the surrounding atmosphere and indicate zero when connected to the atmosphere. These are called ***gauge pressures*** and are normally used. Sometimes it is necessary to add the atmospheric pressure onto the gauge reading in order to find the true or ***absolute pressure***.

Absolute pressure = gauge pressure + atmospheric pressure.

Standard atmospheric pressure is 1.013 bar.

2. HYDROSTATIC FORCES

2.1 HYDROSTATIC PRESSURE

2.1.1 PRESSURE INSIDE PIPES AND VESSELS

Pressure results when a liquid is compacted into a volume. The pressure inside vessels and pipes produce stresses and strains as it tries to stretch the material. An example of this is a pipe with flanged joints. The pressure in the pipe tries to separate the flanges. The force is the product of the pressure and the bore area.

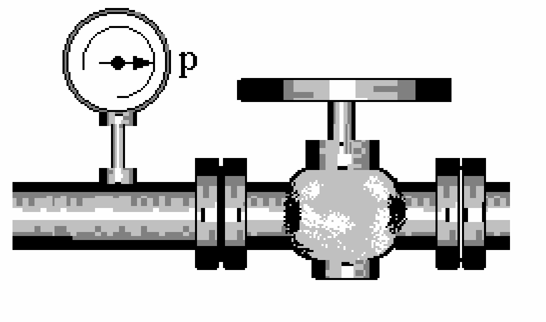


Fig.1

WORKED EXAMPLE No. 1

Calculate the force trying to separate the flanges of a valve (Fig.1) when the pressure is 2 MPa and the pipe bore is 50 mm.

SOLUTION

Force = pressure x bore area
Bore area = $\pi D^2/4 = \pi \times 0.05^2/4 = 1.963 \times 10^{-3}$ m^2
Pressure = 2×10^6 Pa
Force = $2 \times 10^6 \times 1.963 \times 10^{-3} = 3.927 \times 10^3$ N or ***3.927 kN***

2.1.2 PRESSURE DUE TO THE WEIGHT OF A LIQUID

Consider a tank full of liquid as shown. The liquid has a total weight W and this bears down on the bottom and produces a pressure p. Pascal showed that the pressure in a liquid always acts normal (at 90^o) to the surface of contact so the pressure pushes down onto the bottom of the tank. He also showed that the pressure at a given point acts equally in all directions so the pressure also pushes up on the liquid above it and sideways against the walls.

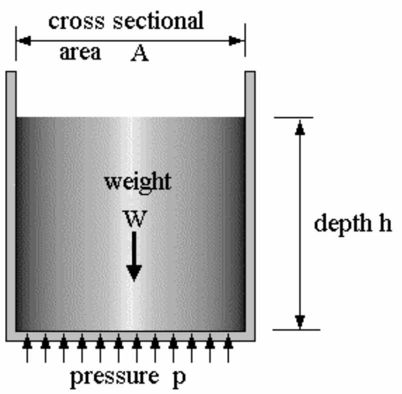


Fig. 2

The volume of the liquid is $V = A\,h$ m^3
The mass of liquid is hence $m = \rho V = \rho A h$ kg
The weight is obtained by multiplying by the gravitational constant g.
$W = mg = \rho A h g$ Newton
The pressure on the bottom is the weight per unit area $p = W/A$ N/m^2
It follows that the pressure at a depth h in a liquid is given by the following equation.

$$p = \rho g h$$

The unit of pressure is the N/m^2 and this is called a ***PASCAL***. The Pascal is a small unit of pressure so higher multiples are common.

WORKED EXAMPLE 2

Calculate the pressure and force on an inspection hatch 0.75 m diameter located on the bottom of a tank when it is filled with oil of density 875 kg/m^3 to a depth of 7 m.

SOLUTION

The pressure on the bottom of the tank is found as follows. $p = \rho g h$
$\rho =$ 875 kg/m^3
g = 9.81 m/s2
h = 7 m
p = 875 x 9.81 x 7 = 60086 N/m^2 or ***60.086 kPa***
The force is the product of pressure and area.
$A = \pi D^2/4 = \pi \times 0.75^2/4 = 0.442$ m^2
F = p A = 60.086 x 10^3 x 0.442 = 26.55 x 10^3 N or ***26.55 Kn***

2.1.3 PRESSURE HEAD

When h is made the subject of the formula, it is called the pressure head. $\boldsymbol{h = p/\rho g}$

Pressure is often measured by using a column of liquid. Consider a pipe carrying liquid at pressure p. If a small vertical pipe is attached to it, the liquid will rise to a height h and at this height, the pressure at the foot of the column is equal to the pressure in the pipe.

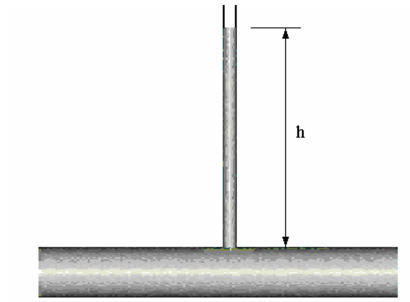
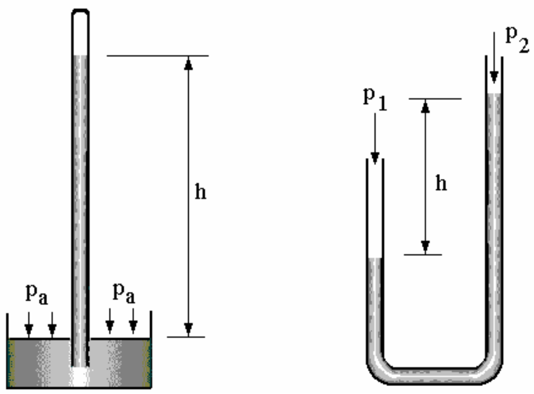


Fig.3

This principle is used in barometers to measure atmospheric pressure and manometers to measure gas pressure.

In the manometer, the weight of the gas is negligible so the height h represents the difference in the pressures p_1 and p_2.

$$p_1 - p_2 = \rho g h$$



Barometer Manometer
Fig.4

In the case of the barometer, the column is closed at the top so that $p_2 = 0$ and $p_1 = p_a$. The height h represents the atmospheric pressure. Mercury is used as the liquid because it does not evaporate easily at the near total vacuum on the top of the column.

$$P_a = \rho g h$$

WORKED EXAMPLE No.3

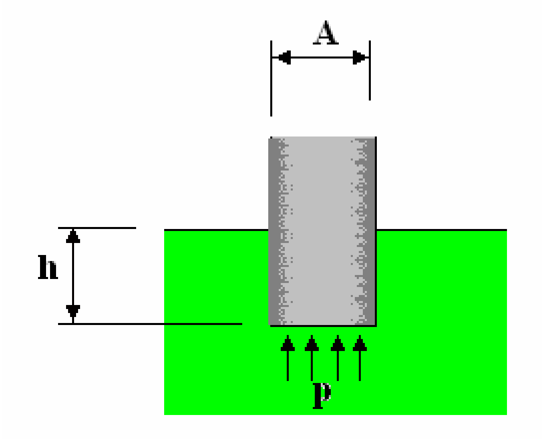
A manometer (fig.4) is used to measure the pressure of gas in a container. One side is connected to the container and the other side is open to the atmosphere. The manometer contains oil of density 750 kg/m^3 and the head is 50 mm. Calculate the gauge pressure of the gas in the container.

SOLUTION

$p_1 - p_2 = \rho g h = 750 \times 9.81 \times 0.05 = 367.9$ Pa

Since p_2 is atmospheric pressure, this is the gauge pressure. ***p_2 = 367.9 Pa (gauge)***

3. ARCHIMEDES' PRINCIPLE



Consider a cylinder floating in a liquid as shown. The pressure on the bottom is $p = \rho g h$

The force pushing upwards is $F = pA = \rho ghA$ and this must be equal to the weight of the cylinder.

hA is the volume of the liquid that is displaced by the cylinder.

ρghA is the weight of the liquid displaced by the cylinder.

It follows that a floating body displaces its own weight of liquid. This is Archimedes' principle.

Since g is a constant it also follows that it follows that a floating body displaces its own mass of liquid.

WORKED EXAMPLE No. 4

A ship is made from steel. It rests in a dry dock as shown. The dry dock is 80 m long and 40 m wide.
When seawater is allowed into the dry dock, it is found that the ship just starts to float when the level reaches 10 m from the bottom. The volume of water that was allowed in was estimated to be 20000 m^3. Calculate:

The mass of the ship.
The volume of steel used to make the ship.
The pressure on the bottom of the ship.

The density of sea water is 1036 kg/m^3
The density of steel is 7830 kg/m^3

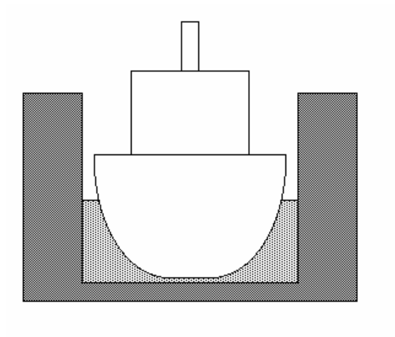


Fig.5

SOLUTION

Volume of water without the ship = 80 x 40 x 10 = 32000 m^3

Volume of water with ship = 20000 m^3

Volume displaced = 12 000 m^3

Mass of water displaced = 12 000 x 1036 = 1243200 kg
Mass of the ship is hence 1243200 kg
This is all steel so the volume of steel is V = Mass/density = 1243200 /7830 = 158.77 m^3

The pressure on the bottom = ρ g h = 1036 x 9.81 x 10 = 101.63 kPa

SELF ASSESSMENT EXERCISE No.1

1. A mercury barometer gives a pressure head of 758 mm. The density is 13 600 kg/m3. Calculate the atmospheric pressure in bar. (1.0113 bar)

2. A manometer (fig.4) is used to measure the pressure of gas in a container. One side is connected to the container and the other side is open to the atmosphere. The manometer contains water of density 1000 kg/m^3 and the head is 250 mm. Calculate the gauge pressure of the gas in the container. (2.452.5 kPa)

3. Calculate the pressure and force on a horizontal submarine hatch 1.2 m diameter when it is at a depth of 800 m in seawater of density 1030 kg/m^3. (8.083 MPa and 9.142 MN)

4. FORCES ON SUBMERGED SURFACES

4.1 TOTAL FORCE

Consider a vertical area submerged below the surface of liquid as shown.

The area of the elementary strip is $dA = B\,dy$

You should already know that the pressure at depth h in a liquid is given by the equation $p = \rho gh$ where ρ is the density and h the depth.

In this case, we are using y to denote depth so $p = \rho gy$

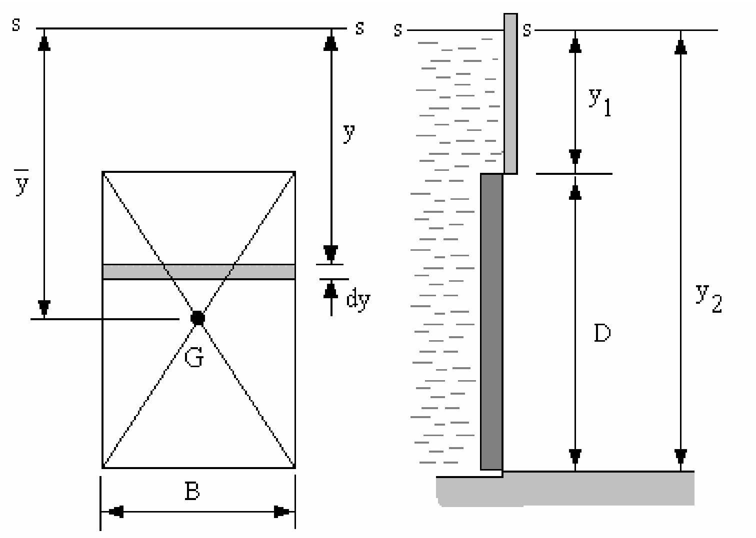


Fig.6

The force on the strip due to this pressure is $dF = p\,dA = \rho B\,gy\,dy$

The total force on the surface due to pressure is denoted R and it is obtained by integrating this expression between the limits of y_1 and y_2.

It follows that $R = \rho gB\left(\dfrac{y_2^2 - y_1^2}{2}\right)$

This may be factorised. $R = \rho gB\dfrac{(y_2 - y_1)(y_2 + y_1)}{2}$

$(y_2 - y_1) = D$ so $B(y_2 - y_1) = BD$ = Area of the surface A

$(y_2 + y_1)/2$ is the distance from the free surface to the centroid $\bar{y}$.

It follows that the total force is given by the expression

$$\boldsymbol{R = \rho g A \bar{y}}$$

The term $A\bar{y}$ is the first moment of area and in general, the total force on a submerged surface is

R = ρg x 1st moment of area about the free surface.

4.2 CENTRE OF PRESSURE

The centre of pressure is the point at which the total force may be assumed to act on a submerged surface. Consider the diagram again. The force on the strip is dF as before. This force produces a turning moment with respect to the free surface s – s. The turning moment due to dF is as follows.

$$dM = y\,dF = \rho g B y^2 dy$$

The total turning moment about the surface due to pressure is obtained by integrating this expression between the limits of y_1 and y_2.

$$M = \int_{y_2}^{y_2} \rho g B y^2 dy = \rho g B \int_{y_2}^{y_2} y^2 dy$$

$$\text{By definition } I_{ss} = B \int_{y_2}^{y_2} y^2 dy$$

Hence $\boldsymbol{M = \rho g I_{SS}}$

This moment must also be given by the total force R multiplied by some distance $\bar{h}$. The position at depth h is called the **CENTRE OF PRESSURE**. $\bar{h}$ is found by equating the moments.

$$M = \bar{h}\,R = \bar{h}\,\rho\,g\,A\,\bar{y} = \rho\,g\,I_{ss}$$

$$\bar{h} = \frac{\rho\,g\,I_{ss}}{\rho\,g\,A\,\bar{y}} = \frac{I_{ss}}{A\,\bar{y}}$$

$$\bar{h} = \frac{2^{nd} \text{ moment of area}}{1^{st} \text{ moment of area}} \text{ about s - s}$$

In order to be competent in this work, you should be familiar with the parallel axis theorem (covered in part 1) because you will need it to solve 2^{nd} moments of area about the free surface. The rule is as follows.

$$\mathbf{I_{SS} = I_{gg} + A\,\bar{y}^2}$$

$\mathbf{I_{SS}}$ is the 2^{nd} moment about the free surface and $\mathbf{I_{gg}}$ is the 2^{nd} moment about the centroid.

You should be familiar with the following standard formulae for 2^{nd} moments about the centroid.

Rectangle $I_{gg} = BD^3/12$

Rectangle about its edge $I = BD^3/3$

Circle $I_{gg} = \pi D^4/64$

WORKED EXAMPLE No.5

Show that the centre of pressure on a vertical retaining wall is at 2/3 of the depth. Go on to show that the turning moment produced about the bottom of the wall is given by the expression $\rho gh^3/6$ for a unit width.

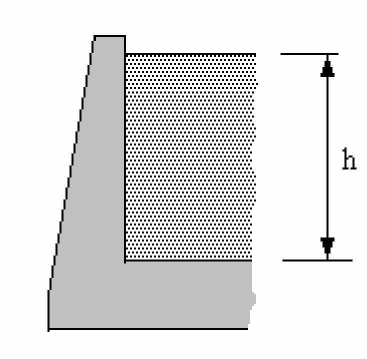


Fig. 7

SOLUTION

For a given width B, the area is a rectangle with the free surface at the top edge.

$$\bar{y} = \frac{h}{2} \qquad A = bh$$

$$1^{st} \text{ moment of area about the top edge is } A\bar{y} = B\frac{h^2}{2}$$

$$2^{nd} \text{ moment of area about the top edge is } B\frac{h^3}{3}$$

$$\bar{h} = \frac{2^{nd} \text{ moment}}{1^{st} \text{ moment}} = \frac{B\frac{h^3}{3}}{B\frac{h^2}{2}}$$

$$\bar{h} = \frac{2h}{3}$$

It follows that the centre of pressure is h/3 from the bottom.

The total force is $R = \rho gAy = \rho gBh^2/2$ and for a unit width this is $\rho gh^2/2$

The moment bout the bottom is $R \times h/3 = (\rho gh^2/2) \times h/3$ = $\boldsymbol{\rho gh^3/6}$

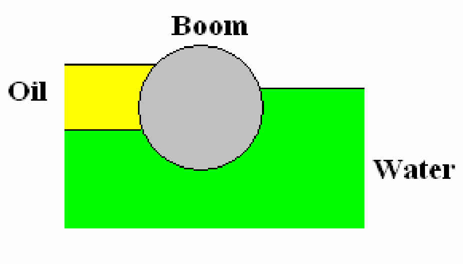
SELF ASSESSMENT EXERCISE No.2

1. A vertical retaining wall contains water to a depth of 20 metres. Calculate the turning moment about the bottom for a unit width. Take the density as 1000 kg/m^3.
   (13.08 MNm)

2. A vertical wall separates seawater on one side from fresh water on the other side. The seawater is 3.5 m deep and has a density of 1030 kg/m^3. The fresh water is 2 m deep and has a density of 1000 kg/m^3. Calculate the turning moment produced about the bottom for a unit width.
   (59.12 kNm)

3. The diagram shows an oil catchment boom.

   Sketch the pressure distribution on the boom. The depth of oil is 0.3 m on top of the water. Determine the horizontal force acting on the boom per metre length.
   The density of water is 1000 kg/m^3 and for oil 890 kg/m^3.



WORKED EXAMPLE No. 6

A concrete wall retains water and has a hatch blocking off an outflow tunnel as shown. Find the total force on the hatch and the position of the centre of pressure. Calculate the total moment about the bottom edge of the hatch. The water density is1000 kg/m^3.

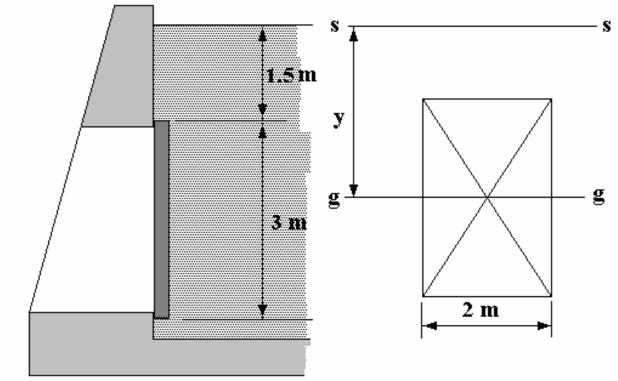


Fig. 8

SOLUTION

Total force = R = $\rho g A \bar{y}$

For the rectangle shown $\bar{y}$ = (1.5 + 3/2) = 3 m. A = 2 x 3 = 6 m^2.

R = 1000 x 9.81 x 6 x 3 = 176580 N or 176.58 kN

$\bar{h}$ = 2nd mom. of Area/ 1st mom. of Area

1^{st} moment of Area = A$\bar{y}$ = 6 x 3 = 18 m^3.

2^{nd} mom of area = $\mathbf{I}_{SS}$ = (BD^3/12) + A$\bar{y}^2$= (2 x 3^3/12) + (6 x 3^2)

$\mathbf{I}_{SS}$ = 4.5 + 54 = 58.5 m^4.

$\bar{h}$ = 58.5/18 = 3.25 m

The distance from the bottom edge is x = 4.5 – 3.25 = 1.25 m

Moment about the bottom edge is = Rx = 176.58 x 1.25 = ***220.725 kNm.***

WORKED EXAMPLE No. 7

Find the force required at the top of the circular hatch shown in order to keep it closed against the water pressure outside. The density of the water is 1030 kg/m^3.

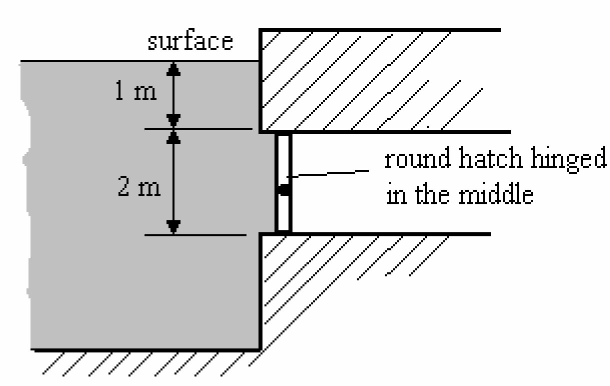


Fig. 9

$\bar{y}$ = 2 m from surface to middle of hatch.

Total Force = R = ρ g A $\bar{y}$ = 1030 x 9.81 x (π x 2^2/4) x 2 = 63487 N or 63.487 kN

Centre of Pressure $\bar{h}$ = 2nd moment/1st moment

2nd moment of area.
$\mathbf{I}_{ss} = \mathbf{I}_{gg} + A\bar{y}^2 = (\pi \times 2^4/64) + (\pi \times 2^2/4) \times 2^2$
$\mathbf{I}_{ss}$ =13.3518 m^4.

1st moment of area
A$\bar{y}$ = (π x 2^2/4) x 2 = 6.283 m^3.
Centre of pressure.
$\bar{h}$ = 13.3518/6.283 = 2.125 m

This is the depth at which, the total force may be assumed to act. Take moments about the hinge.

F = force at top.

R = force at centre of pressure which is 0.125 m below the hinge.

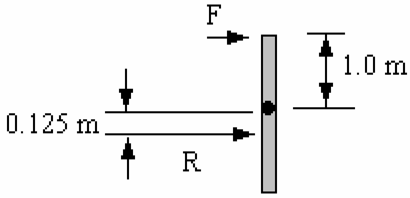


Fig. 10

For equilibrium F x 1 = 63.487 x 0.125

F = 7.936 kN

WORKED EXAMPLE No. 8

The diagram shows a hinged circular vertical hatch diameter D that flips open when the water level outside reaches a critical depth h. Show that for this to happen the hinge must be located at a position x from the bottom given by the formula $x = \frac{D}{2}\left\{\frac{8h - 5D}{8h - 4D}\right\}$

Given that the hatch is 0.6 m diameter, calculate the position of the hinge such that the hatch flips open when the depth reaches 4 metres.

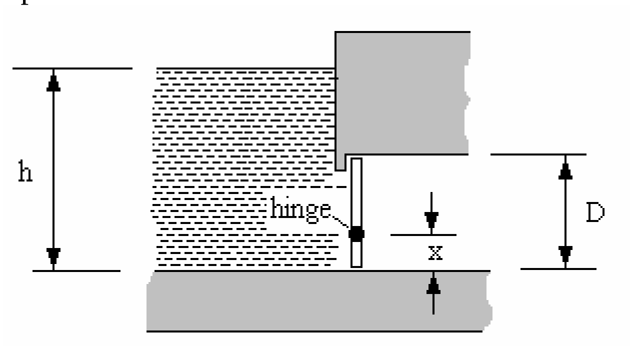


Fig.11

SOLUTION

The hatch will flip open as soon as the centre of pressure rises above the hinge creating a clockwise turning moment. When the centre of pressure is below the hinge, the turning moment is anticlockwise and the hatch is prevented from turning in that direction. We must make the centre of pressure at position x.

$$\bar{y} = h - \frac{D}{2}$$

$$\bar{h} = h - x$$

$$\bar{h} = \frac{\text{second moment of area}}{\text{first moment of area}} \text{ about the surface}$$

$$\bar{h} = \frac{I_{gg} + A\bar{y}^2}{A\bar{y}} = \frac{\frac{\pi D^4}{64} + \frac{\pi D^2}{4}\bar{y}^2}{\frac{\pi D^2}{4}\bar{y}} = \frac{D^2}{16\bar{y}} + \bar{y}$$

Equate for $\bar{h}$

$$\frac{D^2}{16\bar{y}} + \bar{y} = h - x$$

$$x = h - \frac{D^2}{16\bar{y}} - \bar{y} = h - \frac{D^2}{16\left(h - \frac{D}{2}\right)} - \left(h - \frac{D}{2}\right)$$

$$x = -\frac{D^2}{(16h - 8D)} + \frac{D}{2} = \frac{D}{2} - \frac{D^2}{(16h - 8D)}$$

$$x = \frac{D}{2}\left\{1 - \frac{D}{8h - 4D}\right\} = \frac{D}{2}\left\{\frac{8h - 4D - D}{8h - 4D}\right\} = \frac{D}{2}\left\{\frac{8h - 5D}{8h - 4D}\right\}$$

Putting D = 0.6 and h = 4 we get ***x = 0.5 m***

SELF ASSESSMENT EXERCISE No.3

1. A circular hatch is vertical and hinged at the bottom. It is 2 m diameter and the top edge is 2m below the free surface. Find the total force, the position of the centre of pressure and the force required at the top to keep it closed. The density of the water is 1000 kg/m^3.
(92.469 kN, 3.08 m,42.5 kN)

2. A large tank of sea water has a door in the side 1 m square. The top of the door is 5 m below the free surface. The door is hinged on the bottom edge. Calculate the force required at the top to keep it closed. The density of the sea water is 1036 kg/m^3.
(27.11 N)

3. A culvert in the side of a reservoir is closed by a vertical rectangular gate 2m wide and 1 m deep as shown in fig. 11. The gate is hinged about a horizontal axis which passes through the centre of the gate. The free surface of water in the reservoir is 2.5 m above the axis of the hinge. The density of water is 1000 kg/m^3.

Assuming that the hinges are frictionless and that the culvert is open to atmosphere, determine

(i) the force acting on the gate when closed due to the pressure of water. (55.897 kN)

(ii) the moment to be applied about the hinge axis to open the gate. (1635 Nm)

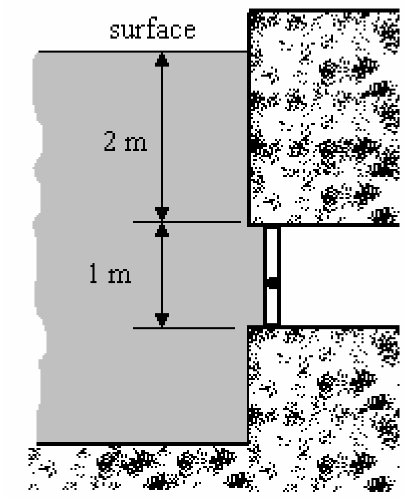


Fig.12

4. The diagram shows a rectangular vertical hatch breadth B and depth D. The hatch flips open when the water level outside reaches a critical depth h. Show that for this to happen the hinge must be located at a position x from the bottom given by the formula

$$x = \frac{D}{2}\left\{\frac{6h - 4D}{6h - 3D}\right\}$$

Given that the hatch is 1 m deep, calculate the position of the hinge such that the hatch flips open when the depth reaches 3 metres. (0.466 m)

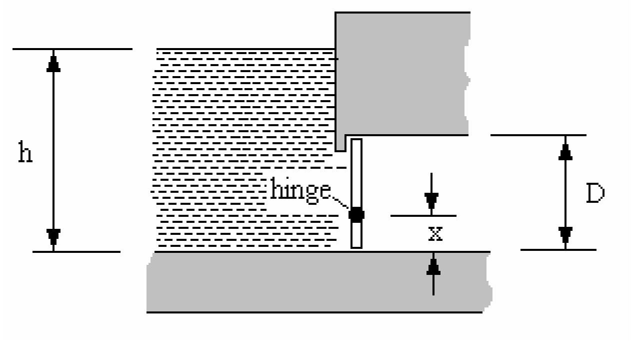


Fig.13

5. Fig.13 shows an L shaped spill gate that operates by pivoting about hinge O when the water level in the channel rises to a certain height H above O. A counterweight W attached to the gate provides closure of the gate at low water levels. With the channel empty the force at sill S is 1.635 kN. The distance l is 0.5m and the gate is 2 m wide.

Determine the magnitude of H.

(i) when the gate begins to open due to the hydrostatic load. (1 m)
(ii) when the force acting on the sill becomes a maximum. What is the magnitude of this force. (0.5 m)
Assume the effects of friction are negligible.

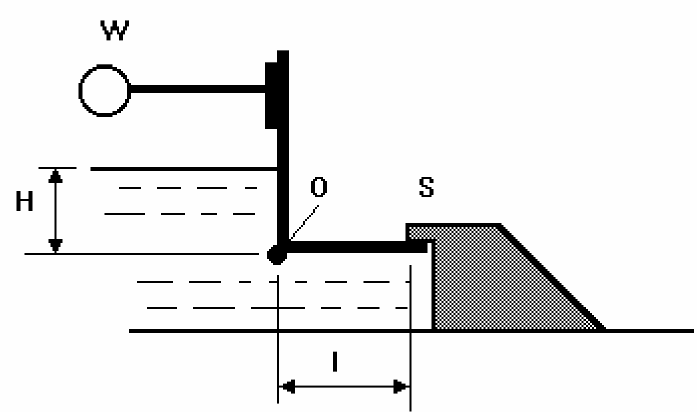


Fig.14

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THERMODYNAMIC, FLUID AND PROCESS ENGINEERING C106

TUTORIAL 5 –THE VISCOUS NATURE OF FLUIDS

On completion of this tutorial you should be able to do the following.

- Define viscosity and its units.
- Define a Newtonian fluid.
- Describe a range of Viscometers

Let's start by examining the meaning of viscosity.

1. VISCOSITY

1.1 BASIC THEORY

Molecules of fluids exert forces of attraction on each other. In liquids this is strong enough to keep the mass together but not strong enough to keep it rigid. In gases these forces are very weak and cannot hold the mass together.

When a fluid flows over a surface, the layer next to the surface may become attached to it (it wets the surface). The layers of fluid above the surface are moving so there must be shearing taking place between the layers of the fluid.

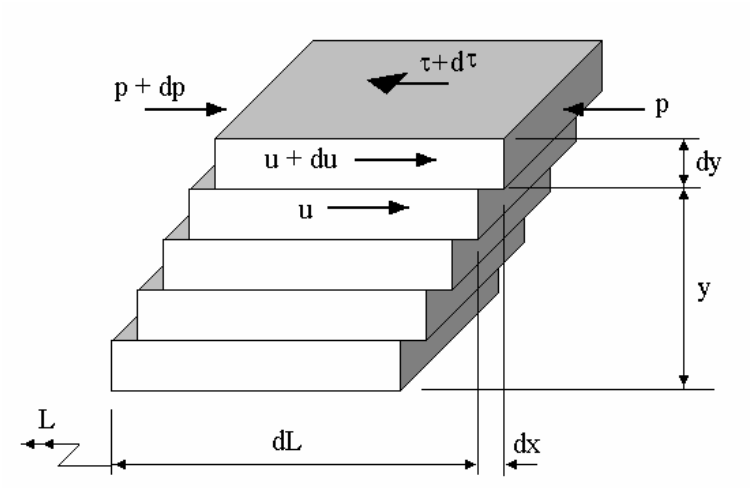


Fig.2.1

Let us suppose that the fluid is flowing over a flat surface in laminated layers from left to right as shown in figure 2.1.

y is the distance above the solid surface (no slip surface)
L is an arbitrary distance from a point upstream.
dy is the thickness of each layer.
dL is the length of the layer.
dx is the distance moved by each layer relative to the one below in a corresponding time dt.
u is the velocity of any layer.
du is the increase in velocity between two adjacent layers.

Each layer moves a distance dx in time dt relative to the layer below it. The ratio dx/dt must be the change in velocity between layers so du = dx/dt.

When any material is deformed sideways by a (shear) force acting in the same direction, a shear stress τ is produced between the layers and a corresponding shear strain γ is produced. Shear strain is defined as follows.

$$\gamma = \frac{\text{sideways deformation}}{\text{height of the layer being deformed}} = \frac{dx}{dy}$$

The rate of shear strain is defined as follows.

$$\dot{\gamma} = \frac{\text{shear strain}}{\text{time taken}} = \frac{\gamma}{dt} = \frac{dx}{dt\,dy} = \frac{du}{dy}$$

It is found that fluids such as water, oil and air, behave in such a manner that the shear stress between layers is directly proportional to the rate of shear strain.

$\tau = \text{constant x } \dot{\gamma}$

Fluids that obey this law are called ***NEWTONIAN FLUIDS***.

It is the constant in this formula that we know as the dynamic viscosity of the fluid.

$$\textbf{\textit{DYNAMIC VISCOSITY }}\boldsymbol{\mu} = \frac{\text{shear stress}}{\text{rate of shear}} = \frac{\tau}{\dot{\gamma}} = \tau\frac{dy}{du}$$

FORCE BALANCE and VELOCITY DISTRIBUTION

A shear stress τ exists between each layer and this increases by dτ over each layer. The pressure difference between the downstream end and the upstream end is dp.

The pressure change is needed to overcome the shear stress. The total force on a layer must be zero so balancing forces on one layer (assumed 1 m wide) we get the following.

$$dp\,dy + d\tau\,dL = 0$$
$$\frac{d\tau}{dy} = -\frac{dp}{dL}$$

It is normally assumed that the pressure declines uniformly with distance downstream so the ***pressure gradient*** $\frac{dp}{dL}$ is assumed constant. The minus sign indicates that the pressure falls with distance. Integrating between the no slip surface (y = 0) and any height y we get

$$-\frac{dp}{dL} = \frac{d\tau}{dy} = \frac{d\left(\mu\frac{du}{dy}\right)}{dy}$$
$$-\frac{dp}{dL} = \mu\frac{d^2u}{dy^2}\text{.......... ....(2.1)}$$

Integrating twice to solve u we get the following.

$$-y\frac{dp}{dL} = \mu\frac{du}{dy} + A$$
$$-\frac{y^2}{2}\frac{dp}{dL} = \mu u + Ay + B$$

A and B are constants of integration that should be solved based on the known conditions (boundary conditions). For the flat surface considered in figure 2.1 one boundary condition is that u = 0 when y = 0 (the no slip surface). Substitution reveals the following.

0 = 0 +0 +B hence B = 0

At some height δ above the surface, the velocity will reach the mainstream velocity u_o. This gives us the second boundary condition $u = u_o$ when $y = \delta$. Substituting we find the following.

$$-\frac{\delta^2}{2}\frac{dp}{dL} = \mu u_o + A\delta$$

$$A = -\frac{\delta}{2}\frac{dp}{dL} - \frac{\mu u_o}{\delta} \quad \text{hence}$$

$$-\frac{y^2}{2}\frac{dp}{dL} = \mu u + \left(-\frac{\delta}{2}\frac{dp}{dL} - \frac{\mu u_o}{\delta}\right)y$$

$$u = y\left(\frac{\delta}{2\mu}\frac{dp}{dL} + \frac{u_o}{\delta}\right)$$

Plotting u against y gives figure 2.2.

BOUNDARY LAYER.

The velocity grows from zero at the surface to a maximum at height δ. In theory, the value of δ is infinity but in practice it is taken as the height needed to obtain 99% of the mainstream velocity. This layer is called the boundary layer and δ is the boundary layer thickness. It is a very important concept and is discussed more fully in chapter 3. The inverse gradient of the boundary layer is du/dy and this is the rate of shear strain γ.

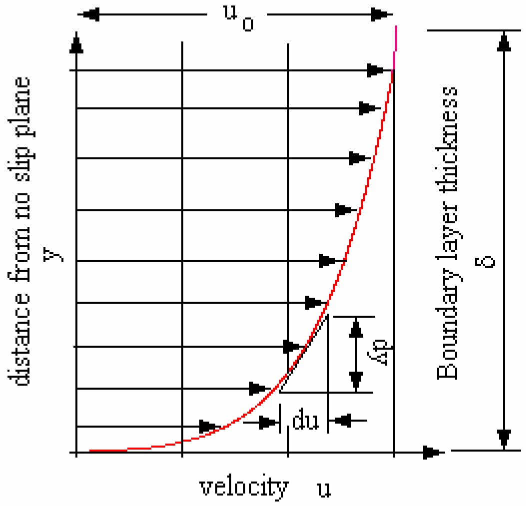


Fig.2.2

1.2. UNITS of VISCOSITY

1.2.1 DYNAMIC VISCOSITY μ

The units of dynamic viscosity μ are N s/m^2. It is normal in the international system (SI) to give a name to a compound unit. The old metric unit was a dyne.s/cm^2 and this was called a POISE after Poiseuille. It follows that the SI unit is related to the Poise such that 10 Poise = 1 Ns/m^2
This is not an acceptable multiple. Since, however, 1 CentiPoise (1cP) is 0.001 N s/m^2 then the cP is the accepted SI unit.

1cP = 0.001 N s/m^2.

The symbol η is also commonly used for dynamic viscosity. There are other ways of expressing viscosity and this is covered next.

1.2.2 KINEMATIC VISCOSITY ν

This is defined as follows. $\nu = \frac{\text{dynamic viscosity}}{\text{density}} = \frac{\mu}{\rho}$

The basic units are m^2/s. The old metric unit was the cm^2/s and this was called the STOKE after the British scientist. It follows that 1 Stoke (St) = 0.0001 m^2/s and this is not an acceptable SI multiple. The centiStoke (cSt) ,however, is 0.000001 m^2/s and this is an acceptable multiple.

$$1cSt = 0.000001\ m^2/s = 1\ mm^2/s$$

1.2.3 OTHER UNITS

Other units of viscosity have come about because of the way viscosity is measured. For example REDWOOD SECONDS comes from the name of the Redwood viscometer. Other units are Engler Degrees, SAE numbers and so on. Conversion charts and formulae are available to convert them into useable engineering or SI units.

1.2.4 VISCOMETERS

The measurement of viscosity is a large and complicated subject. The principles rely on the resistance to flow or the resistance to motion through a fluid. Many of these are covered in British Standards 188. The following is a brief description of some types.

U TUBE VISCOMETER

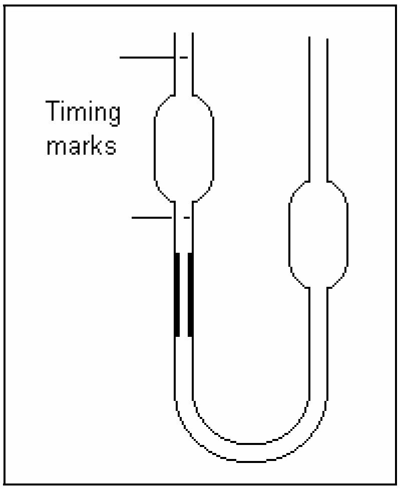


Fig.2.3

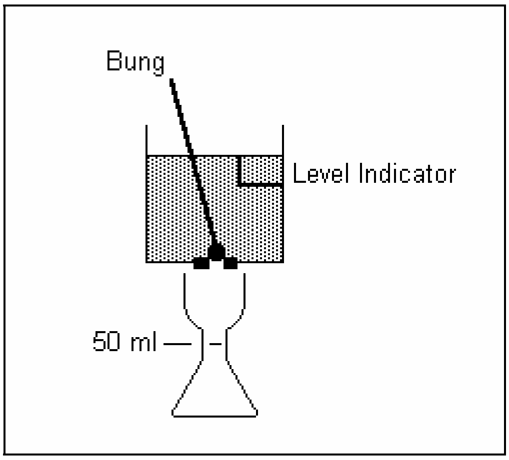
The fluid is drawn up into a reservoir and allowed to run through a capillary tube to another reservoir in the other limb of the U tube.

The time taken for the level to fall between the marks is converted into cSt by multiplying the time by the viscometer constant.

$$\nu = ct$$

The constant c should be accurately obtained by calibrating the viscometer against a master viscometer from a standards laboratory.

REDWOOD VISCOMETER



This works on the principle of allowing the fluid to run through an orifice of very accurate size in an agate block.

50 ml of fluid are allowed to empty from the level indicator into a measuring flask. The time taken is the viscosity in Redwood seconds. There are two sizes giving Redwood No.1 or No.2 seconds. These units are converted into engineering units with tables.

Fig.2.4

FALLING SPHERE VISCOMETER

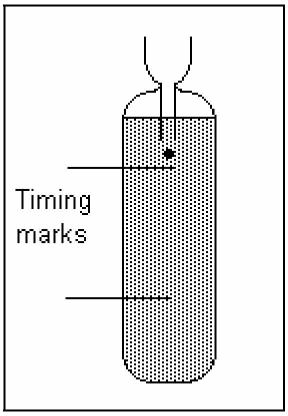


Fig.2.5

This viscometer is covered in BS188 and is based on measuring the time for a small sphere to fall in a viscous fluid from one level to another. The buoyant weight of the sphere is balanced by the fluid resistance and the sphere falls with a constant velocity. The theory is based on Stoke's Law and is only valid for very slow velocities. The theory is covered later in the section on laminar flow where it is shown that the terminal velocity (u) of the sphere is related to the dynamic viscosity (μ) and the density of the fluid and sphere (ρ_f and ρ_S) by the formula

$$\mu = F\ gd^2(\rho_S - \rho_f)/18u$$

F is a correction factor called the Faxen correction factor, which takes into account a reduction in the velocity due to the effect of the fluid being constrained to flow between the wall of the tube and the sphere.

ROTATIONAL TYPES

There are many types of viscometers, which use the principle that it requires a torque to rotate or oscillate a disc or cylinder in a fluid. The torque is related to the viscosity. Modern instruments consist of a small electric motor, which spins a disc or cylinder in the fluid. The torsion of the connecting shaft is measured and processed into a digital readout of the viscosity in engineering units.

You should now find out more details about viscometers by reading BS188, suitable textbooks or literature from oil companies.

SELF ASSESSMENT EXERCISE No. 2.1

1. Describe the principle of operation of the following types of viscometers.

a. Redwood Viscometers.

b. British Standard 188 glass U tube viscometer.

c. British Standard 188 Falling Sphere Viscometer.

d. Any form of Rotational Viscometer

ENGINEERING COUNCIL

CERTIFICATE LEVEL

THERMODYNAMIC, FLUID AND PROCESS ENGINEERING C106

TUTORIAL 9 – FLOW THROUGH PIPES

On completion of this outcome you should be able to do the following.

- Derive Bernoulli's equation for liquids.
- Define and explain laminar and turbulent flow.
- Find the pressure losses in piped systems due to fluid friction.
- Find the minor frictional losses in piped systems.

Let's start by revising basics. The flow of a fluid in a pipe depends upon two fundamental laws, the conservation of mass and energy.

PIPE FLOW

The solution of pipe flow problems requires the applications of two principles, the law of conservation of mass (continuity equation) and the law of conservation of energy (Bernoulli's equation)

CONSERVATION OF MASS

When a fluid flows at a constant rate in a pipe or duct, the mass flow rate must be the same at all points along the length. Consider a liquid being pumped into a tank as shown (fig.3.1).

The mass flow rate at any section is $m = \rho A u_m$

ρ = density (kg/m^3)
u_m = mean velocity (m/s)
A = Cross Sectional Area (m^2)

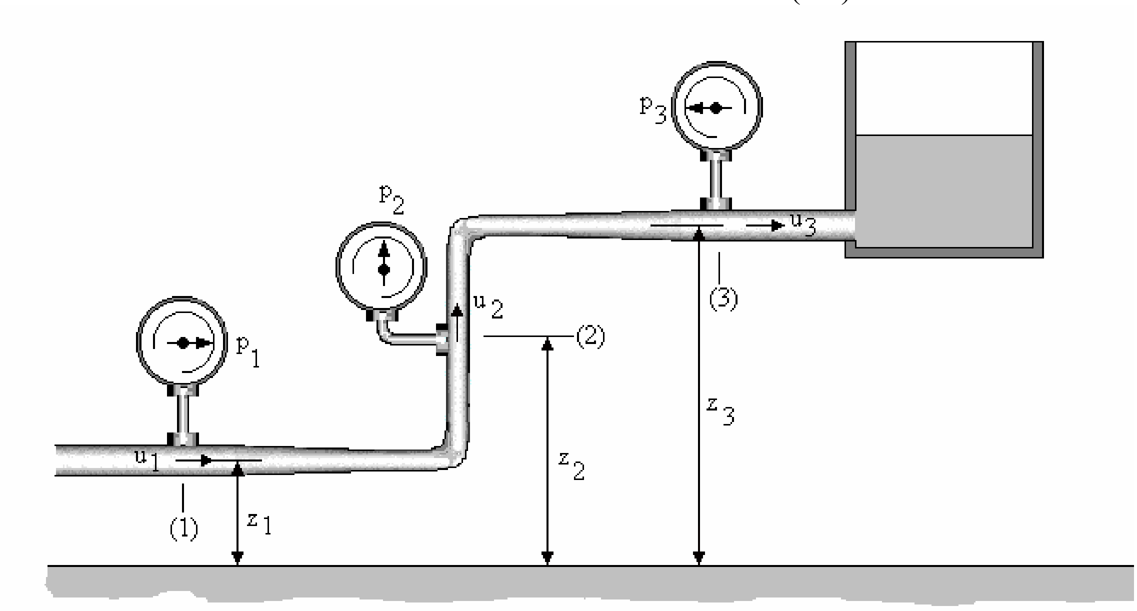


Fig.1

For the system shown the mass flow rate at (1), (2) and (3) must be the same so

$$\rho_1 A_1 u_1 = \rho_2 A_2 u_2 = \rho_3 A_3 u_3$$

In the case of liquids the density is equal and cancels so

$$A_1 u_1 = A_2 u_2 = A_3 u_3 = Q$$

CONSERVATION OF ENERGY

ENERGY FORMS

FLOW ENERGY

This is the energy a fluid possesses by virtue of its pressure.

The formula is ***F.E. = pQ Joules***

p is the pressure (Pascals)
Q is volume rate (m^3)

POTENTIAL OR GRAVITATIONAL ENERGY

This is the energy a fluid possesses by virtue of its altitude relative to a datum level.

The formula is ***P.E. = mgz Joules***

m is mass (kg)
z is altitude (m)

KINETIC ENERGY

This is the energy a fluid possesses by virtue of its velocity.

The formula is ***K.E.*** $= \frac{1}{2} m u_m^2$ ***Joules***

u_m is mean velocity (m/s)

INTERNAL ENERGY

This is the energy a fluid possesses by virtue of its temperature. It is usually expressed relative to 0^oC.

The formula is $U = mc\theta$

c is the specific heat capacity (J/kg ^oC)
θ is the temperature in ^oC

In the following work, internal energy is not considered in the energy balance.

SPECIFIC ENERGY

Specific energy is the energy per kg so the three energy forms as specific energy are as follows.

F.E./m = pQ/m = p/ρ Joules/kg
P.E/m. = gz Joules/kg
K.E./m $= \frac{1}{2} u^2$ ***Joules/kg***

ENERGY HEAD

If the energy terms are divided by the weight mg, the result is energy per Newton. Examining the units closely we have J/N = N m/N = metres.

It is normal to refer to the energy in this form as the energy head. The three energy terms expressed this way are as follows.

F.E./mg $= p/\rho g = h$
P.E./mg $= z$
K.E./mg $= u^2/2g$

The flow energy term is called the pressure head and this follows since earlier it was shown p/ρg = h. This is the height that the liquid would rise to in a vertical pipe connected to the system.

The potential energy term is the actual altitude relative to a datum.

The term $u^2/2g$ is called the kinetic head and this is the pressure head that would result if the velocity is converted into pressure.

BERNOULLI'S EQUATION

Bernoulli's equation is based on the conservation of energy. If no energy is added to the system as work or heat then the total energy of the fluid is conserved. Remember that internal (thermal energy) has not been included.

The total energy E_T at (1) and (2) on the diagram (fig.3.1) must be equal so :

$$E_T = p_1Q_1 + mgz_1 + m\frac{u_1^2}{2} = p_2Q_2 + mgz_2 + m\frac{u_2^2}{2}$$

Dividing by mass gives the specific energy form

$$\frac{E_T}{m} = \frac{p_1}{\rho_1} + gz_1 + \frac{u_1^2}{2} = \frac{p_2}{\rho_2} + gz_2 + \frac{u_2^2}{2}$$

Dividing by g gives the energy terms per unit weight

$$\frac{E_T}{mg} = \frac{p_1}{g\rho_1} + z_1 + \frac{u_1^2}{2g} = \frac{p_2}{g\rho_2} + z_2 + \frac{u_2^2}{2g}$$

Since p/ρg = pressure head h then the total head is given by the following.

$$h_T = h_1 + z_1 + \frac{u_1^2}{2g} = h_2 + z_2 + \frac{u_2^2}{2g}$$

This is the head form of the equation in which each term is an energy head in metres. z is the potential or gravitational head and $u^2/2g$ is the kinetic or velocity head.

For liquids the density is the same at both points so multiplying by ρg gives the pressure form. The total pressure is as follows.

$$p_T = p_1 + \rho gz_1 + \frac{\rho u_1^2}{2} = p_2 + \rho gz_2 + \frac{\rho u_2^2}{2}$$

In real systems there is friction in the pipe and elsewhere. This produces heat that is absorbed by the liquid causing a rise in the internal energy and hence the temperature. In fact the temperature rise will be very small except in extreme cases because it takes a lot of energy to raise the temperature. If the pipe is long, the energy might be lost as heat transfer to the surroundings. Since the equations did not include internal energy, the balance is lost and we need to add an extra term to the right side of the equation to maintain the balance. This term is either the head lost to friction h_L or the pressure loss p_L.

$$h_1 + z_1 + \frac{u_1^2}{2g} = h_2 + z_2 + \frac{u_2^2}{2g} + h_L$$

The pressure form of the equation is as follows.

$$p_1 + \rho gz_1 + \frac{\rho u_1^2}{2} = p_2 + \rho gz_2 + \frac{\rho u_2^2}{2} + p_L$$

The total energy of the fluid (excluding internal energy) is no longer constant.

Note that if one of the points is a ***free surface*** the pressure is normally atmospheric but if gauge pressures are used, the pressure and pressure head becomes zero. Also, if the surface area is large (say a large tank), the velocity of the surface is small and when squared becomes negligible so the kinetic energy term is neglected (made zero).

WORKED EXAMPLE No. 1

The diagram shows a pump delivering water through as pipe 30 mm bore to a tank. Find the pressure at point (1) when the flow rate is 1.4 dm^3/s. The density of water is 1000 kg/m^3. The loss of pressure due to friction is 50 kPa.

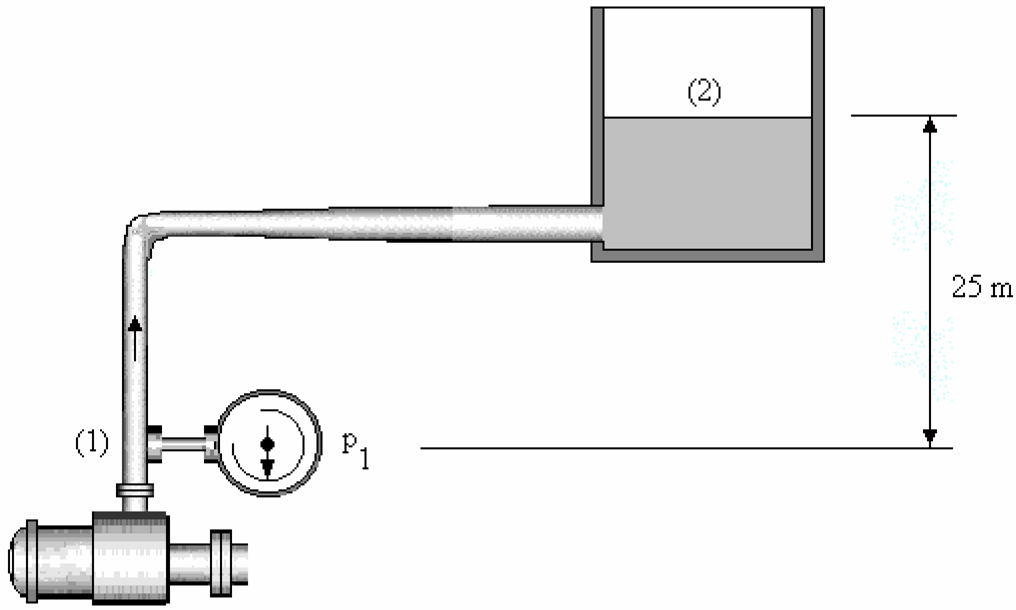


Fig. 2

SOLUTION

Area of bore $A = \pi \times 0.03^2/4 = 706.8 \times 10^{-6}\ m^2$.
Flow rate $Q = 1.4\ dm^3/s = 0.0014\ m^3/s$
Mean velocity in pipe = Q/A = 1.98 m/s
Apply Bernoulli between point (1) and the surface of the tank.

$$p_1 + \rho g z_1 + \frac{\rho u_1^2}{2} = p_2 + \rho g z_2 + \frac{\rho u_2^2}{2} + p_L$$

Make the low level the datum level and $z_1 = 0$ and $z_2 = 25$.

The pressure on the surface is zero gauge pressure. $P_L = 50\ 000$ Pa

The velocity at (1) is 1.98 m/s and at the surface it is zero.

$$p_1 + 0 + \frac{1000 x 1.98^2}{2} = 0 + 1000 x 9.9125 + 0 + 50000$$
$$p_1 = 293.29 kPa \text{ gauge pressure}$$

WORKED EXAMPLE 2

The diagram shows a tank that is drained by a horizontal pipe. Calculate the pressure head at point (2) when the valve is partly closed so that the flow rate is reduced to 20 dm^3/s. The pressure loss is equal to 2 m head.

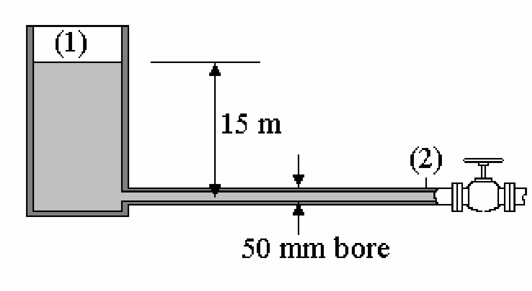


Fig. 3

SOLUTION

Since point (1) is a free surface, $h_1 = 0$ and u_1 is assumed negligible.

The datum level is point (2) so $z_1 = 15$ and $z_2 = 0$. Q = 0.02 m3/s

$A_2 = \pi d^2/4 = \pi \times (0.05^2)/4 = 1.963 \times 10^{-3}\ m^2$.

$u_2 = Q/A = 0.02/1.963 \times 10^{-3} = 10.18$ m/s

Bernoulli's equation in head form is as follows.

$$h_1 + z_1 + \frac{u_1^2}{2g} = h_2 + z_2 + \frac{u_2^2}{2g} + h_L$$

$$0 + 15 + 0 = h_2 + 0 + \frac{10.18^2}{2 \times 9.81} + 2 \qquad h_2 = 7.72\text{m}$$

WORKED EXAMPLE 3

The diagram shows a horizontal nozzle discharging into the atmosphere. The inlet has a bore area of 600 mm^2 and the exit has a bore area of 200 mm^2. Calculate the flow rate when the inlet pressure is 400 Pa. Assume there is no energy loss.

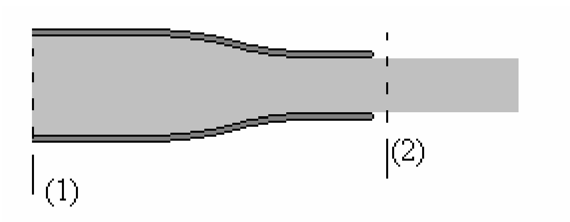


Fig. 4

SOLUTION

Apply Bernoulli between (1) and (2)

$$p_1 + \rho g z_1 + \frac{\rho u_1^2}{2} = p_2 + \rho g z_2 + \frac{\rho u_2^2}{2} + p_L$$

Using gauge pressure, p2 = 0 and being horizontal the potential terms cancel. The loss term is zero so the equation simplifies to the following.

$$p_1 + \frac{\rho u_1^2}{2} = \frac{\rho u_2^2}{2}$$

From the continuity equation we have

$$u_1 = \frac{Q}{A_1} = \frac{Q}{600 \text{ x } 10^{-6}} = 1666.7Q$$

$$u_2 = \frac{Q}{A_2} = \frac{Q}{200 \text{ x } 10^{-6}} = 5\,000\text{ Q}$$

Putting this into Bernoulli's equation we have the following.

$$400 + 1000 \text{ x } \frac{(1666.7Q)^2}{2} = 1000 \text{ x } \frac{(5000Q)^2}{2}$$

$$400 + 1.389 \text{ x} 10^9 Q^2 = 12.5 \text{ x } 10^9 Q^2$$

$$400 = 11.11 \text{ x} 10^9 Q^2$$

$$Q^2 = \frac{400}{11.11 \text{ x} 10^9} = 36 \text{ x} 10^{-9}$$

$$Q = 189.7 \text{ x } 10^{-6} \text{ m}^3/\text{s or } 189.7 \text{ cm}^3/\text{s}$$

HYDRAULIC GRADIENT

Consider a tank draining into another tank at a lower level as shown. There are small vertical tubes at points along the length to indicate the pressure head (h). Relative to a datum, the total energy head is $h_T = h + z + u^2/2g$ and this is shown as line A.

The hydraulic grade line is the line joining the free surfaces in the tubes and represents the sum of h and z only. This is shown as line B and it is always below the line of h_T by the velocity head $u^2/2g$. Note that at exit from the pipe, the velocity head is not recovered but lost as friction as the emerging jet collides with the static liquid. The free surface of the tank does not rise.

The only reason why the hydraulic grade line is not horizontal is because there is a frictional loss h_f. The actual gradient of the line at any point is the rate of change with length $i = \delta h_f/\delta L$

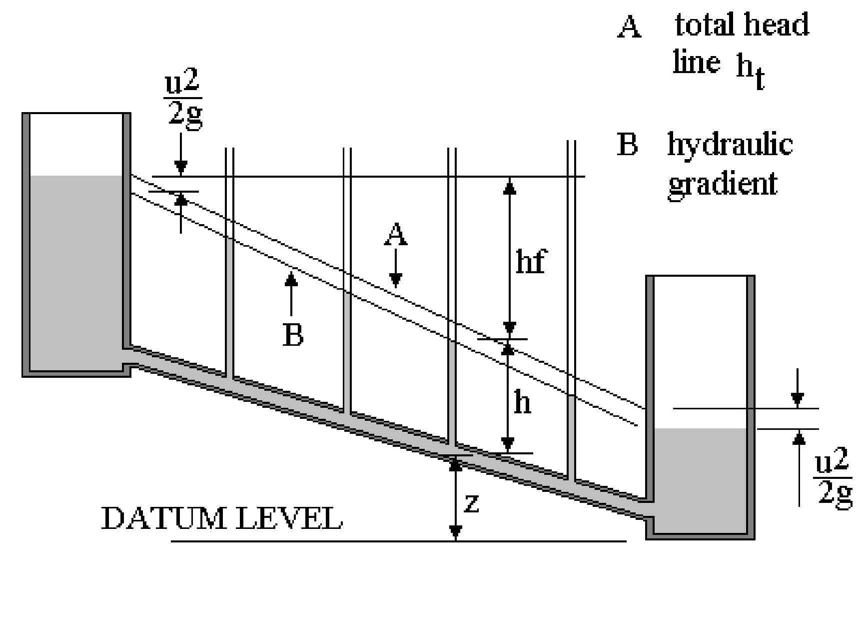


Fig. 5

SELF ASSESSMENT EXERCISE No.1

1. A pipe 100 mm bore diameter carries oil of density 900 kg/m^3 at a rate of 4 kg/s. The pipe reduces to 60 mm bore diameter and rises 120 m in altitude. The pressure at this point is atmospheric (zero gauge). Assuming no frictional losses, determine:

   i. The volume/s (4.44 dm^3/s)
   ii. The velocity at each section (0.566 m/s and 1.57 m/s)
   iii. The pressure at the lower end. (1.06 MPa)

2. A pipe 120 mm bore diameter carries water with a head of 3 m. The pipe descends 12 m in altitude and reduces to 80 mm bore diameter. The pressure head at this point is 13 m. The density is 1000 kg/m^3. Assuming no losses, determine

   i. The velocity in the small pipe (7 m/s)
   ii. The volume flow rate. (35 dm^3/s)

3. A horizontal nozzle reduces from 100 mm bore diameter at inlet to 50 mm at exit. It carries liquid of density 1000 kg/m^3 at a rate of 0.05 m^3/s. The pressure at the wide end is 500 kPa (gauge). Calculate the pressure at the narrow end neglecting friction. (196 kPa)

4. A pipe carries oil of density 800 kg/m^3. At a given point (1) the pipe has a bore area of 0.005 m^2 and the oil flows with a mean velocity of 4 m/s with a gauge pressure of 800 kPa. Point (2) is further along the pipe and there the bore area is 0.002 m^2 and the level is 50 m above point (1). Calculate the pressure at this point (2). Neglect friction. (374 kPa)

5. A horizontal nozzle has an inlet velocity u_1 and an outlet velocity u_2 and discharges into the atmosphere. Show that the velocity at exit is given by the following formulae.

   $$u_2 = \{2\Delta p/\rho + u_1^2\}^{½}$$

   and

   $$u_2 = \{2g\Delta h + u_1^2\}^{½}$$

LAMINAR and TURBULENT FLOW

The following work only applies to Newtonian fluids

LAMINAR FLOW

A ***stream line*** is an imaginary line with no flow normal to it, only along it. When the flow is laminar, the streamlines are parallel and for flow between two parallel surfaces we may consider the flow as made up of parallel laminar layers. In a pipe these laminar layers are cylindrical and may be called ***stream tubes***. In laminar flow, no mixing occurs between adjacent layers and it occurs at low average velocities.

TURBULENT FLOW

The shearing process causes energy loss and heating of the fluid. This increases with mean velocity. When a certain critical velocity is exceeded, the streamlines break up and mixing of the fluid occurs. The diagram illustrates Reynolds coloured ribbon experiment. Coloured dye is injected into a horizontal flow. When the flow is laminar the dye passes along without mixing with the water. When the speed of the flow is increased turbulence sets in and the dye mixes with the surrounding water. One explanation of this transition is that it is necessary to change the pressure loss into other forms of energy such as angular kinetic energy as indicated by small eddies in the flow.

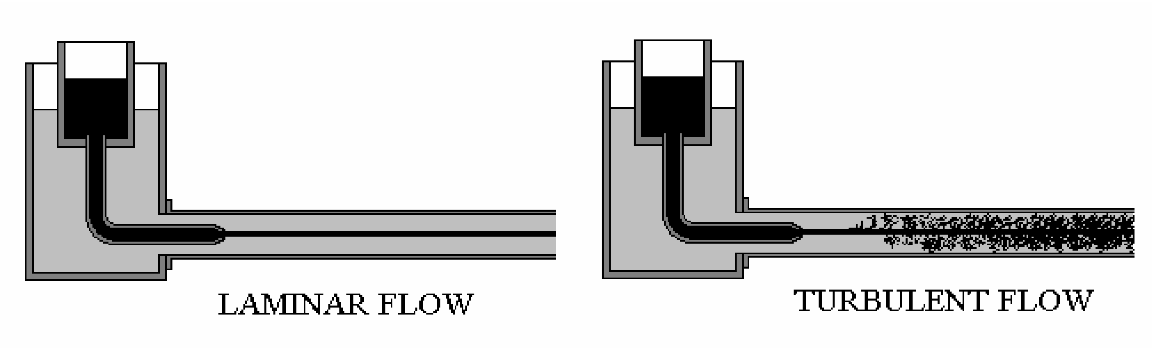


Fig. 6

LAMINAR AND TURBULENT BOUNDARY LAYERS

In chapter 2 it was explained that a ***boundary layer*** is the layer in which the velocity grows from zero at the wall (no slip surface) to 99% of the maximum and the thickness of the layer is denoted δ. When the flow within the boundary layer becomes turbulent, the shape of the boundary layers waivers and when diagrams are drawn of turbulent boundary layers, the mean shape is usually shown. Comparing a laminar and turbulent boundary layer reveals that the turbulent layer is thinner than the laminar layer.

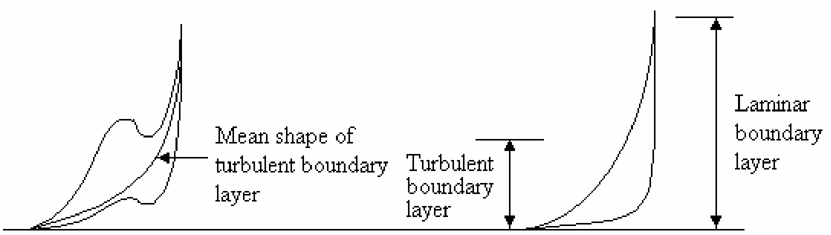


Fig. 7

CRITICAL VELOCITY - REYNOLDS NUMBER

When a fluid flows in a pipe at a volumetric flow rate Q m^3/s the average velocity is defined $u_m = \frac{Q}{A}$ A is the cross sectional area.

The Reynolds number is defined as $R_e = \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu}$

If you check the units of R_e you will see that there are none and that it is a dimensionless number. You will learn more about such numbers in section ….?.

Reynolds discovered that it was possible to predict the velocity or flow rate at which the transition from laminar to turbulent flow occurred for any Newtonian fluid in any pipe. He also discovered that the critical velocity at which it changed back again was different. He found that when the flow was gradually increased, the change from laminar to turbulent always occurred at a Reynolds number of 2500 and when the flow was gradually reduced it changed back again at a Reynolds number of 2000. Normally, 2000 is taken as the critical value.

WORKED EXAMPLE No. 4

Oil of density 860 kg/m^3 has a kinematic viscosity of 40 cSt. Calculate the critical velocity when it flows in a pipe 50 mm bore diameter.

SOLUTION

$$R_e = \frac{u_m D}{\nu}$$

$$u_m = \frac{R_e \nu}{D} = \frac{2000 x 40 x 10^{-6}}{0.05} = 1.6 \text{ m/s}$$

DERIVATION OF POISEUILLE'S EQUATION for LAMINAR FLOW

Poiseuille did the original derivation shown below which relates pressure loss in a pipe to the velocity and viscosity for LAMINAR FLOW. His equation is the basis for measurement of viscosity hence his name has been used for the unit of viscosity. Consider a pipe with laminar flow in it. Consider a stream tube of length ΔL at radius r and thickness dr.

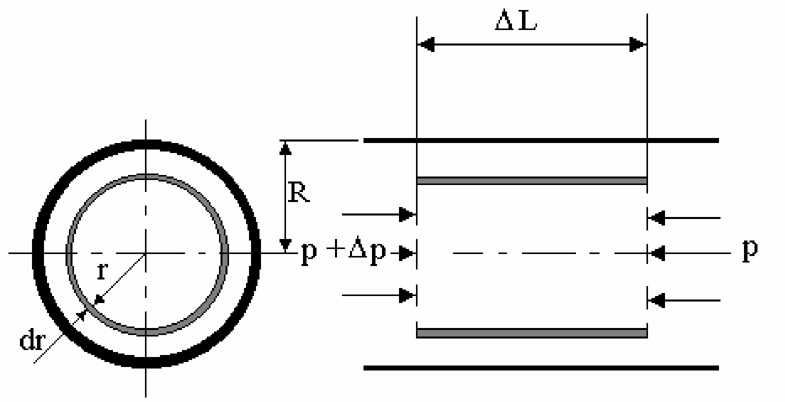


Fig. 8

y is the distance from the pipe wall. $y = R - r \qquad dy = -dr \qquad \frac{du}{dy} = -\frac{du}{dr}$

The shear stress on the outside of the stream tube is τ. The force (F_s) acting from right to left is due to the shear stress and is found by multiplying τ by the surface area.

$Fs = \tau \times 2\pi r\ \Delta L$

For a Newtonian fluid , $\tau = \mu \frac{du}{dy} = -\mu \frac{du}{dr}$. Substituting for τ we get the following.

$$F_s = -2\pi r \Delta L \mu \frac{du}{dr}$$

The pressure difference between the left end and the right end of the section is Δp. The force due to this (F_p) is Δp x circular area of radius r.

$F_p = \Delta p \times \pi r^2$

Equating forces we have $-2\pi r \mu \Delta L \frac{du}{dr} = \Delta p\ \pi\ {}^2$

$$du = -\frac{\Delta p}{2\mu\mu\Delta L} rdr$$

In order to obtain the velocity of the streamline at any radius r we must integrate between the limits u = 0 when r = R and u = u when r = r.

$$\int_0^u du = -\frac{\Delta p}{2\mu\Delta L}\int_R^r rdr$$

$$u = -\frac{\Delta p}{4\mu\Delta L}\left(r^2 - R^2\right)$$

$$u = \frac{\Delta p}{4\mu L}\left(R^2 - r^2\right)$$

This is the equation of a Parabola so if the equation is plotted to show the boundary layer, it is seen to extend from zero at the edge to a maximum at the middle.

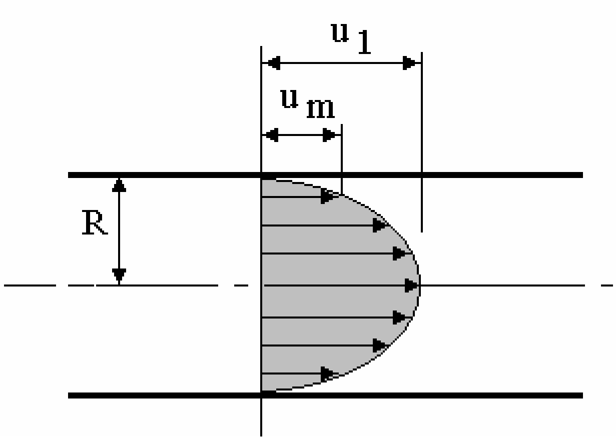


Fig. 9

For maximum velocity put r = 0 and we get $\qquad u_1 = \frac{\Delta p R^2}{4\mu\Delta L}$

The average height of a parabola is half the maximum value so the average velocity is

$$u_m = \frac{\Delta p R^2}{8\mu\Delta L}$$

Often we wish to calculate the pressure drop in terms of diameter D. Substitute R=D/2 and rearrange.

$$\Delta p = \frac{32\,\mu\,\Delta L\,u_m}{D^2}$$

The volume flow rate is average velocity x cross sectional area.

$$Q = \frac{\pi R^2 \Delta p R^2}{8\,\mu\Delta L} = \frac{\pi R^4 \Delta p}{8\,\mu\Delta L} = \frac{\pi D^4 \Delta p}{128\,\mu\Delta L}$$

This is often changed to give the pressure drop as a friction head.

The friction head for a length L is found from $h_f = \Delta p/\rho g$ $\qquad h_f = \frac{32\,\mu L u_m}{\rho g D^2}$

This is Poiseuille's equation that applies only to laminar flow.

WORKED EXAMPLE No. 5

A capillary tube is 30 mm long and 1 mm bore. The head required to produce a flow rate of 8 mm^3/s is 30 mm. The fluid density is 800 kg/m^3.

Calculate the dynamic and kinematic viscosity of the oil.

SOLUTION

Rearranging Poiseuille's equation we get

$$\mu = \frac{h_f \rho g D^2}{32 L u_m} \qquad A = \frac{\pi d^2}{4} = \frac{\pi \text{ x } 1^2}{4} = 0.785\text{ mm}^2$$

$$u_m = \frac{Q}{A} = \frac{8}{0.785} = 10.18\text{ mm/s} \qquad \mu = \frac{0.03 \text{ x } 800 \text{ x } 9.81 \text{ x } 0.001^2}{32 \text{ x } 0.03 \text{ x } 0.01018} = 0.0241\text{ N s/m or } 24.1\text{ cP}$$

$$\nu = \frac{\mu}{\rho} = \frac{0.0241}{800} = 30.11 \text{ x } 10^{-6}\text{ m}^2\text{/s or } 30.11\text{ cSt}$$

WORKED EXAMPLE No. 6

Oil flows in a pipe 100 mm bore with a Reynolds number of 250. The dynamic viscosity is 0.018 Ns/m^2. The density is 900 kg/m^3.
Determine the pressure drop per metre length, the average velocity and the radius at which it occurs.

SOLUTION

$Re = \rho u_m D/\mu$. Hence $u_m = Re\,\mu/\rho D u_m$ = (250 x 0.018)/(900 x 0.1) = 0.05 m/s

$\Delta p = 32\mu L\, u_m/D^2$ = 32 x 0.018 x 1 x 0.05/0.1^2
Δp= 2.88 Pascals.

$u = \{\Delta p/4L\mu\}(R^2 - r^2)$ which is made equal to the average velocity 0.05 m/s

$0.05 = (2.88/4 \text{ x } 1 \text{ x } 0.018)(0.05^2 - r^2)$ r = 0.035 m or 35.3 mm.

SELF ASSESSMENT EXERCISE No. 2

1. Oil flows in a pipe 80 mm bore diameter with a mean velocity of 0.4 m/s. The density is 890 kg/m^3 and the viscosity is 0.075 Ns/m^2.

   Show that the flow is laminar and hence deduce the pressure loss per metre length. (150 Pa)

2 Calculate the maximum velocity of water that can flow in laminar form in a pipe 20 m long and 60 mm bore. Determine the pressure loss in this condition. The density is 1000 kg/m^3 and the dynamic viscosity is 0.001 N s/m^2. (0.0333 m/s and 5.92 Pa)

3 Oil flow in a pipe 100 mm bore diameter with a Reynolds Number of 500. The density is 800 kg/m^3. The dynamic viscosity μ = 0.08 Ns/m^2.

Calculate the velocity of a streamline at a radius of 40 mm. (0.36 m/s)

4a When a viscous fluid is subjected to an applied pressure it flows through a narrow horizontal passage as shown below. By considering the forces acting on the fluid element and assuming steady fully developed laminar flow, show that the velocity distribution is given by

$$-\frac{dp}{dx} = \mu\frac{d^2u}{dy^2}$$

b. Using the above equation show that for flow between two flat parallel horizontal surfaces distance t apart the velocity at any point is given by the following formula.

u = (1/2μ)(dp/dx)(y^2 - ty)

c. Carry on the derivation and show that the volume flow rate through a gap of height 't' and width 'B' is given by $Q = -B\frac{dp}{dx}\frac{t^3}{12\mu}$.

d. Show that the mean velocity 'u_m' through the gap is given by $u_m = -\frac{1}{12\mu}\frac{dp}{dx}t^2$

5 The volumetric flow rate of glycerine between two flat parallel horizontal surfaces 1 mm apart and 10 cm wide is 2 cm^3/s. Determine the following.

i. the applied pressure gradient dp/dx. (240 kPa per metre)

ii. the maximum velocity. (0.06 m/s)

For glycerine assume that μ= 1.0 Ns/m^2 and the density is 1260 kg/m^3.

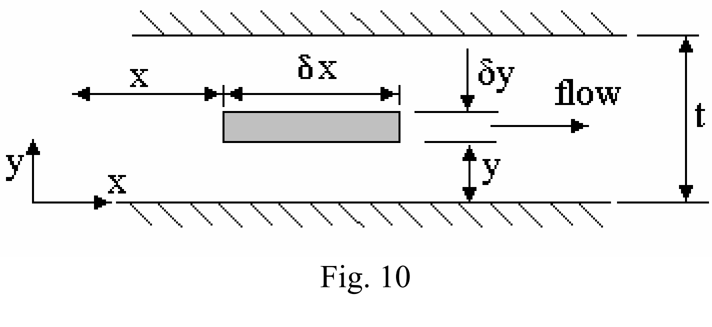


Fig. 10

FRICTION COEFFICIENT

The friction coefficient is a convenient idea that can be used to calculate the pressure drop in a pipe. It is defined as follows.

$$C_f = \frac{\text{Wall Shear Stress}}{\text{Dynamic Pressure}}$$

DYNAMIC PRESSURE

Consider a fluid flowing with mean velocity u_m. If the kinetic energy of the fluid is converted into flow or fluid energy, the pressure would increase. The pressure rise due to this conversion is called the dynamic pressure.

KE = ½ mu_m^2

Flow Energy = p Q   Q is the volume flow rate and $\rho = m/Q$

Equating ½ $mu_m^2 = p\,Q$ $p = mu^2/2Q = ½\,\rho\,u_m^2$

WALL SHEAR STRESS τ_o

The wall shear stress is the shear stress in the layer of fluid next to the wall of the pipe.

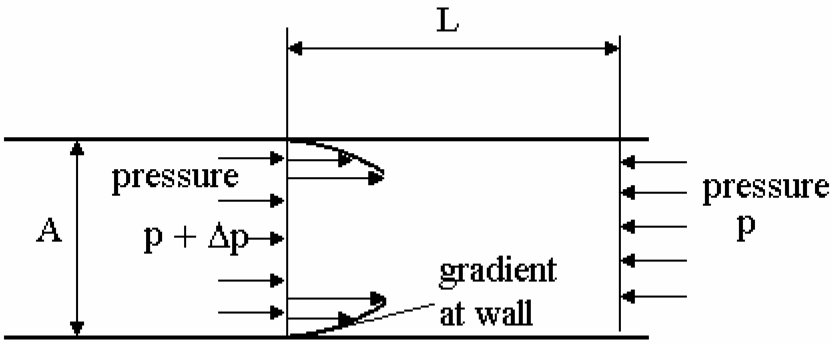


Fig. 11

The shear stress in the layer next to the wall is $\tau_o = \mu\left(\frac{du}{dy}\right)_{wall}$

The shear force resisting flow is $F_s = \tau_o \pi L D$

The resulting pressure drop produces a force of $F_p = \frac{\Delta p\,\pi\,D^2}{4}$

Equating forces gives $\tau_o = \frac{D\,\Delta p}{4L}$

FRICTION COEFFICIENT for LAMINAR FLOW

$$C_f = \frac{\text{Wall Shear Stress}}{\text{Dynamic Pressure}} = \frac{2D\Delta p}{4L\rho u_m^2}$$

From Poiseuille's equation $\Delta p = \frac{32\mu L u_m}{D^2}$ Hence $C_f = \left(\frac{2D}{4L\rho u_m^2}\right)\left(\frac{32\mu L u}{D^2}\right) = \frac{16\mu}{\rho u_m^2 D} = \frac{16}{R_e}$

DARCY FORMULA

This formula is mainly used for calculating the pressure loss in a pipe due to turbulent flow but it can be used for laminar flow also.

Turbulent flow in pipes occurs when the Reynolds Number exceeds 2500 but this is not a clear point so 3000 is used to be sure. In order to calculate the frictional losses we use the concept of friction coefficient symbol C_f. This was defined as follows.

$$C_f = \frac{\text{Wall Shear Stress}}{\text{Dynamic Pressure}} = \frac{2D\Delta p}{4L\rho u_m^2}$$

Rearranging equation to make Δp the subject

$$\Delta p = \frac{4C_f L\rho u_m^2}{2D}$$

This is often expressed as a friction head h_f

$$h_f = \frac{\Delta p}{\rho g} = \frac{4C_f L u_m^2}{2gD}$$

This is the Darcy formula. In the case of laminar flow, Darcy's and Poiseuille's equations must give the same result so equating them gives

$$\frac{4C_f L u_m^2}{2gD} = \frac{32\mu L u_m}{\rho g D^2}$$

$$C_f = \frac{16\mu}{\rho u_m D} = \frac{16}{R_e}$$

This is the same result as before for laminar flow.

FLUID RESISTANCE

The above equations may be expressed in terms of flow rate Q by substituting u = Q/A

$$h_f = \frac{4C_f L u_m^2}{2gD} = \frac{4C_f L Q^2}{2gDA^2}$$

Substituting A $=\pi D^2/4$ we get the following.

$$h_f = \frac{32C_f L Q^2}{g\pi^2 D^5} = RQ^2$$

R is the fluid resistance or restriction. $R = \frac{32C_f L^2}{g\pi^2 D^5}$

If we want pressure loss instead of head loss the equations are as follows.

$$p_f = \rho g h_f = \frac{32\rho C_f L Q^2}{\pi^2 D^5} = RQ^2$$

R is the fluid resistance or restriction. $R = \frac{32\rho C_f L^2}{\pi^2 D^5}$

It should be noted that R contains the friction coefficient and this is a variable with velocity and surface roughness so R should be used with care.

MOODY DIAGRAM AND RELATIVE SURFACE ROUGHNESS

In general the friction head is some function of u_m such that $h_f = \phi u_m{}^n$. Clearly for laminar flow, n =1 but for turbulent flow n is between 1 and 2 and its precise value depends upon the roughness of the pipe surface. Surface roughness promotes turbulence and the effect is shown in the following work.

Relative surface roughness is defined as $\varepsilon = k/D$ where k is the mean surface roughness and D the bore diameter.

An American Engineer called Moody conducted exhaustive experiments and came up with the Moody Chart. The chart is a plot of C_f vertically against R_e horizontally for various values of ε. In order to use this chart you must know two of the three co-ordinates in order to pick out the point on the chart and hence pick out the unknown third co-ordinate. For smooth pipes, (the bottom curve on the diagram), various formulae have been derived such as those by Blasius and Lee.

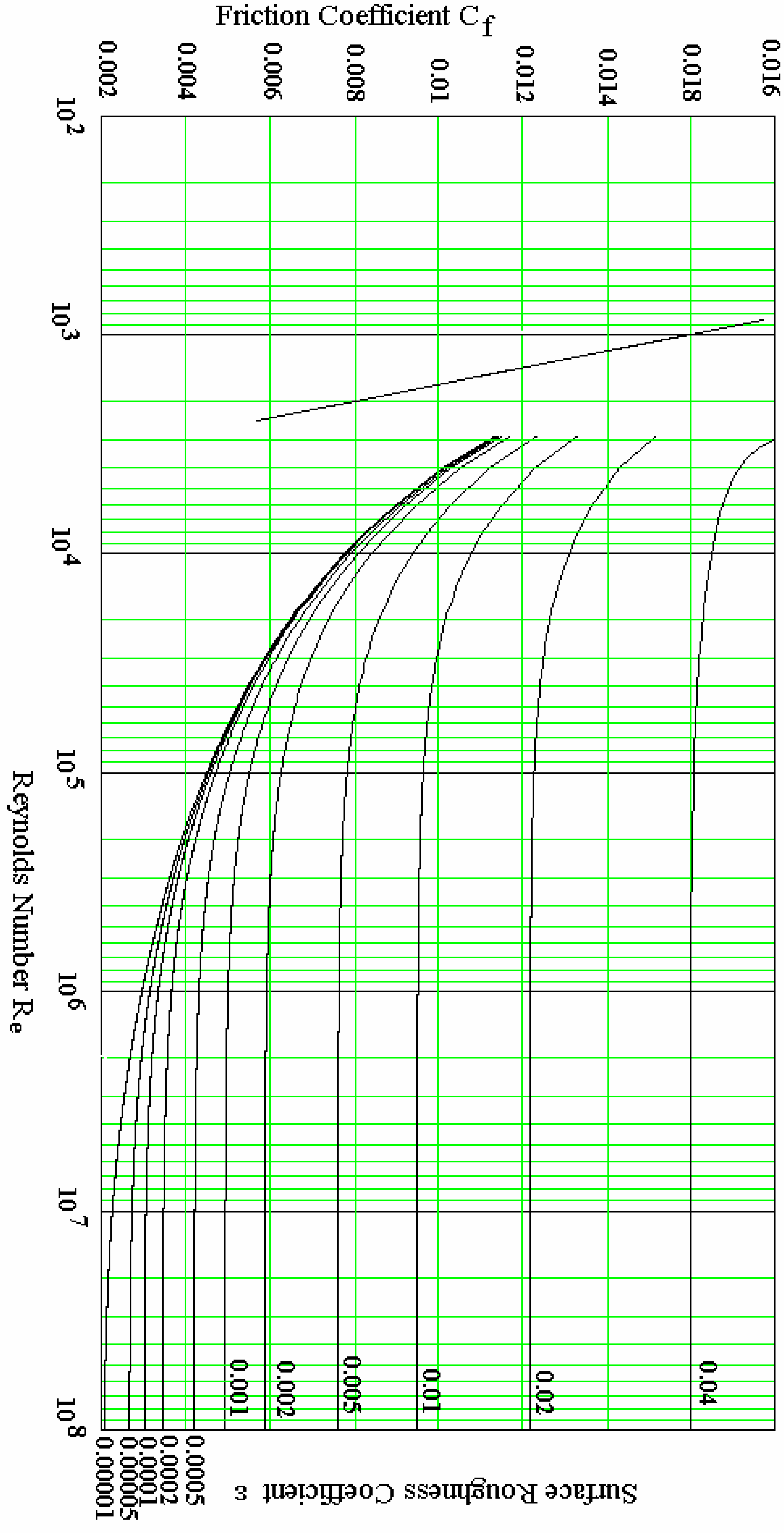
BLASIUS $C_f = 0.0791\ R_e{}^{0.25}$

LEE $C_f = 0.0018 + 0.152\ R_e{}^{0.35}$.

The Moody diagram shows that the friction coefficient reduces with Reynolds number but at a certain point, it becomes constant. When this point is reached, the flow is said to be fully developed turbulent flow. This point occurs at lower Reynolds numbers for rough pipes.

A formula that gives an approximate answer for any surface roughness is that given by Haaland.

$$\frac{1}{\sqrt{C_f}} = -3.6\log_{10}\left\{\frac{6.9}{R_e} + \left(\frac{\varepsilon}{3.71}\right)^{1.11}\right\}$$



Friction Coefficient C_f
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.018
0.016
10^2
10^3
10^4
10^5
10^6
10^7
10^8
Reynolds Number R_e
0.001
0.002
0.005
0.01
0.02
0.04
0.0005
0.0002
0.0001
0.00005
0.00001
Surface Roughness Coefficient ε

Fig. 12 CHART

WORKED EXAMPLE No.7

Determine the friction coefficient for a pipe 100 mm bore with a mean surface roughness of 0.06 mm when a fluid flows through it with a Reynolds number of 20 000.

SOLUTION

The mean surface roughness ε = k/d = 0.06/100 = 0.0006
Locate the line for ε = k/d = 0.0006.
Trace the line until it meets the vertical line at Re = 20 000. Read of the value of C_f horizontally on the left. Answer C_f = 0.0067

Check using the formula from Haaland.

$$\frac{1}{\sqrt{C_f}} = -3.6\log_{10}\left\{\frac{6.9}{R_e} + \left(\frac{\varepsilon}{3.71}\right)^{1.11}\right\}$$

$$\frac{1}{\sqrt{C_f}} = -3.6\log_{10}\left\{\frac{6.9}{20000} + \left(\frac{0.0006}{3.71}\right)^{1.11}\right\}$$

$$\frac{1}{\sqrt{C_f}} = -3.6\log_{10}\left\{\frac{6.9}{20000} + \left(\frac{0.0006}{3.71}\right)^{1.11}\right\}$$

$$\frac{1}{\sqrt{C_f}} = 12.206$$

$$C_f = 0.0067$$

WORKED EXAMPLE No. 8

Oil flows in a pipe 80 mm bore with a mean velocity of 4 m/s. The mean surface roughness is 0.02 mm and the length is 60 m. The dynamic viscosity is 0.005 N s/m^2 and the density is 900 kg/m^3. Determine the pressure loss.

SOLUTION

Re = ρud/μ = (900 x 4 x 0.08)/0.005 = 57600

ε= k/d = 0.02/80 = 0.00025

From the chart C_f = 0.0052

$h_f = 4C_fLu2/2dg$ = (4 x 0.0052 x 60 x 4^2)/(2 x 9.81 x 0.08) = 12.72 m

Δp = $\rho g h_f$ = 900 x 9.81 x 12.72 = 112.32 kPa.

SELF ASSESSMENT EXERCISE No.3

1. A pipe is 25 km long and 80 mm bore diameter. The mean surface roughness is 0.03 mm. It caries oil of density 825 kg/m^3 at a rate of 10 kg/s. The dynamic viscosity is 0.025 N s/m^2.

   Determine the friction coefficient using the Moody Chart and calculate the friction head. (Ans. 3075 m.)

2. Water flows in a pipe at 0.015 m^3/s. The pipe is 50 mm bore diameter. The pressure drop is 13 420 Pa per metre length. The density is 1000 kg/m^3 and the dynamic viscosity is 0.001 N s/m^2.

   Determine the following.
   i. The wall shear stress (167.75 Pa)
   ii. The dynamic pressures (29180 Pa).
   iii. The friction coefficient (0.00575)
   iv. The mean surface roughness (0.0875 mm)

3. Explain briefly what is meant by fully developed laminar flow. The velocity u at any radius r in fully developed laminar flow through a straight horizontal pipe of internal radius r_o is given by

$$\mathbf{u = (1/4\mu)(r_o^2 - r^2)dp/dx}$$

   dp/dx is the pressure gradient in the direction of flow and μ is the dynamic viscosity. The wall skin friction coefficient is defined as $C_f = 2\tau_o/(\rho u_m^2)$.

   Show that $C_f = 16/R_e$ where $R_e = \rho u_m D/\mu$ an ρ is the density, u_m is the mean velocity and τ_o is the wall shear stress.

4. Oil with viscosity 2 x 10^{-2} Ns/m^2 and density 850 kg/m^3 is pumped along a straight horizontal pipe with a flow rate of 5 dm^3/s. The static pressure difference between two tapping points 10 m apart is 80 N/m^2. Assuming laminar flow determine the following.

   i. The pipe diameter.
   ii. The Reynolds number.

   Comment on the validity of the assumption that the flow is laminar.

MINOR LOSSES

Minor losses occur in the following circumstances.

i. Exit from a pipe into a tank.
ii. Entry to a pipe from a tank.
iii. Sudden enlargement in a pipe.
iv. Sudden contraction in a pipe.
v. Bends in a pipe.
vi. Any other source of restriction such as pipe fittings and valves.

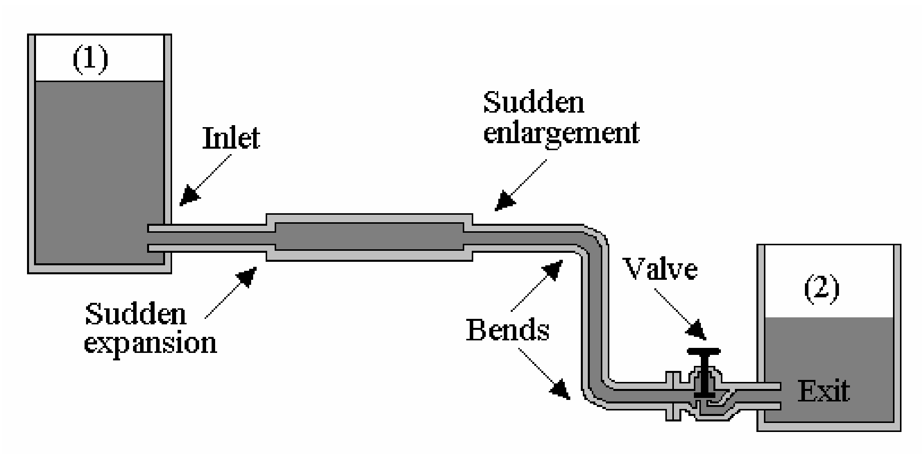


Fig. 13

In general, minor losses are neglected when the pipe friction is large in comparison but for short pipe systems with bends, fittings and changes in section, the minor losses is the dominant factor.

In general, the minor losses are expressed as a fraction of the kinetic head or dynamic pressure in the smaller pipe.

Minor head loss = k $u^2/2g$ Minor pressure loss = ½ $k\rho u^2$

Values of k can be derived for standard cases but for items like elbows and valves in a pipeline, it is determined by experimental methods.

Minor losses can also be expressed in terms of fluid resistance R as follows.

$$h_L = k\frac{u^2}{2} = k\frac{Q^2}{2A^2} = k\frac{8Q^2}{\pi^2 D^4} = RQ^2 \text{ Hence } R = \frac{8k}{\pi^2 D^4}$$

$$p_L = k\frac{8\rho g Q^2}{\pi^2 D^4} = RQ^2 \text{ hence } R = \frac{8k\rho g}{\pi^2 D^4}$$

Before you go on to look at the derivations, you must first learn about the coefficients of contraction and velocity.

COEFFICIENT OF CONTRACTION Cc

The fluid approaches the entrance from all directions and the radial velocity causes the jet to contract just inside the pipe. The jet then spreads out to fill the pipe. The point where the jet is smallest is called the *VENA CONTRACTA*.

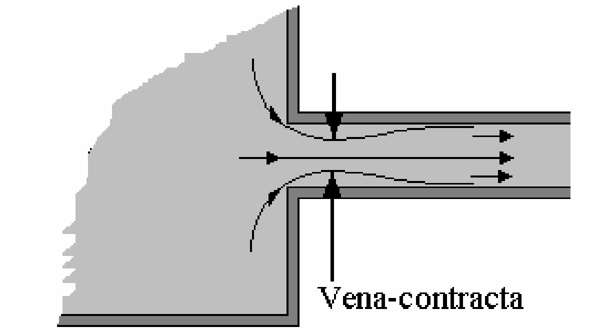


Fig. 14

The coefficient of contraction C_c is defined as $C_c = A_j/A_o$

A_j is the cross sectional area of the jet and A_o is the c.s.a. of the pipe. For a round pipe this becomes $C_c = d_j^2/d_o^2$.

COEFFICIENT OF VELOCITY C_v

The coefficient of velocity is defined as **C_v = actual velocity/theoretical velocity**

In this instance it refers to the velocity at the vena-contracta but as you will see later on, it applies to other situations also.

EXIT FROM A PIPE INTO A TANK.

The liquid emerges from the pipe and collides with stationary liquid causing it to swirl about before finally coming to rest. All the kinetic energy is dissipated by friction. It follows that all the kinetic head is lost so k = 1.0

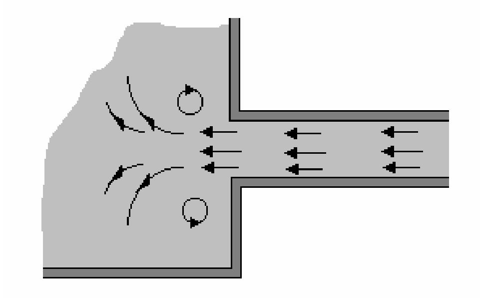


Fig. 15

ENTRY TO A PIPE FROM A TANK

The value of k varies from 0.78 to 0.04 depending on the shape of the inlet. A good rounded inlet has a low value but the case shown is the worst.

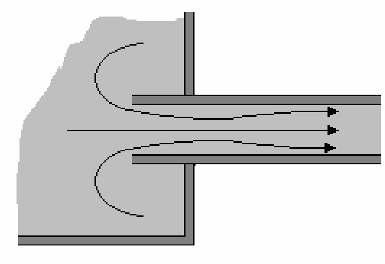


Fig.16

SUDDEN ENLARGEMENT

This is similar to a pipe discharging into a tank but this time it does not collide with static fluid but with slower moving fluid in the large pipe. The resulting loss coefficient is given by the following expression.

$$k = \left\{1 - \left(\frac{d_1}{d_2}\right)^2\right\}^2$$

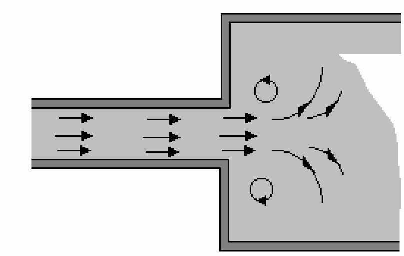


Fig. 17

SUDDEN CONTRACTION

This is similar to the entry to a pipe from a tank. The best case gives k = 0 and the worse case is for a sharp corner which gives k = 0.5.

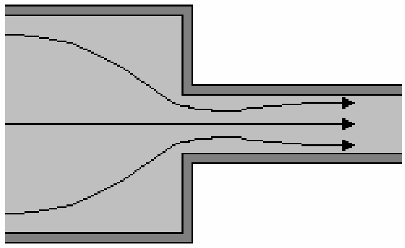


Fig. 18

BENDS AND FITTINGS

The k value for bends depends upon the radius of the bend and the diameter of the pipe. The k value for bends and the other cases is on various data sheets. For fittings, the manufacturer usually gives the k value. Often instead of a k value, the loss is expressed as an equivalent length of straight pipe that is to be added to L in the Darcy formula.

WORKED EXAMPLE No. 9

A tank of water empties by gravity through a horizontal pipe into another tank. There is a sudden enlargement in the pipe as shown. At a certain time, the difference in levels is 3 m. Each pipe is 2 m long and has a friction coefficient C_f = 0.005. The inlet loss constant is K = 0.3.

Calculate the volume flow rate at this point.

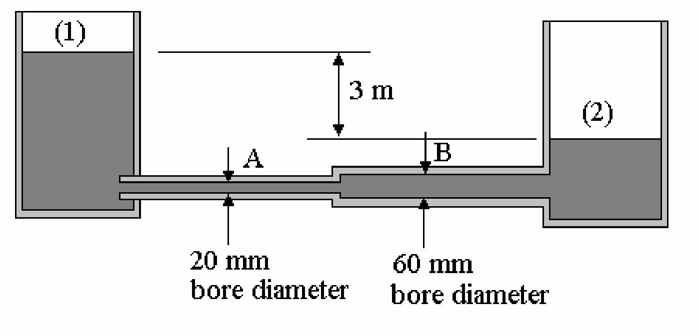


Fig. 19

SOLUTION

There are five different sources of pressure loss in the system and these may be expressed in terms of the fluid resistance as follows.

The head loss is made up of five different parts. It is usual to express each as a fraction of the kinetic head as follows.

Resistance pipe A $$R_1 = \frac{32C_f L}{gD_A{}^5\pi^2} = \frac{32 \text{ x } 0.005 \text{ x } 2}{g \text{ x } 0.02^5\pi^2} = 1.0328 \text{ x } 10^6 \text{ s}^2\text{m}^{-5}$$

Resistance in pipe B $$R_2 = \frac{32C_f L}{gD_B^5\pi^2} = \frac{32 \text{ x } 0.005 \text{ x } 2}{g \text{ x } 0.06^5\pi^2} = 4.250\text{x}10^3 \text{ s}^2\text{m}^{-5}$$

Loss at entry K=0.3 $$R_3 = \frac{8K}{g\pi^2 D_A{}^4} = \frac{8 \text{ x } 0.3}{g\,\pi^2 \text{x } 0.02^4} = 158 \text{ s}^2\text{m}^{-5}$$

Loss at sudden enlargement. $$k = \left\{1-\left(\frac{d_A}{d_B}\right)^2\right\}^2 = \left\{1-\left(\frac{20}{60}\right)^2\right\}^2 = 0.79$$

$$R_4 = \frac{8K}{g\pi^2 D_A{}^4} = \frac{8\text{x}0.79}{g\pi^2 \text{x } 0.02^4} = 407.7 \text{ s}^2\text{m}^{-5}$$

Loss at exit K=1 $$R_5 = \frac{8K}{g\pi^2 D_B{}^4} = \frac{8\text{x}1}{g\pi^2 \text{x } 0.06^4} = 63710 \text{ s}^2\text{m}^{-5}$$

$$h_L = R_1Q^2 + R_2Q^2 + R_3Q^2 + R_4Q^2 + R_5Q^2$$

Total losses. $$h_L = (R_1 + R_2 + R_2 + R_4 + R_5)Q^2$$

$$h_L = 1.101 \text{ x } 10^6 Q^2$$

BERNOULLI'S EQUATION

Apply Bernoulli between the free surfaces (1) and (2)

$$h_1 + z_1 + \frac{u_1^2}{2g} = h_2 + z_2 + \frac{u_2^2}{2g} + h_L$$

On the free surface the velocities are small and about equal and the pressures are both atmospheric so the equation reduces to the following.

$z_1 - z_2 = h_L = 3$ $3 = 1.101 \text{ x } 10^6 Q^2$ $Q^2 = 2.724 \text{ x } 10^{-6}$ $Q = 1.65 \text{ x } 10^{-3} \text{ m}^3/\text{s}$

MOMENTUM and PRESSURE FORCES

Changes in velocities mean changes in momentum and Newton's second law tells us that this is accompanied by a force such that

Force = rate of change of momentum.

Pressure changes in the fluid must also be considered as these also produce a force. Translated into a form that helps us solve the force produced on devices such as those considered here, we use the equation $F = \Delta(pA) + m\,\Delta u$.

When dealing with devices that produce a change in direction, such as pipe bends, this has to be considered more carefully and this is covered in chapter 4. In the case of sudden changes in section, we may apply the formula

$$F = (p_1A_1 + mu_1) - (p_2A_2 + mu_2)$$

point 1 is upstream and point 2 is downstream.

WORKED EXAMPLE No. 10

A pipe carrying water experiences a sudden reduction in area as shown. The area at point (1) is 0.002 m^2 and at point (2) it is 0.001 m^2. The pressure at point (2) is 500 kPa and the velocity is 8 m/s. The loss coefficient K is 0.4. The density of water is 1000 kg/m^3. Calculate the following.

i. The mass flow rate.
ii. The pressure at point (1)
iii. The force acting on the section.

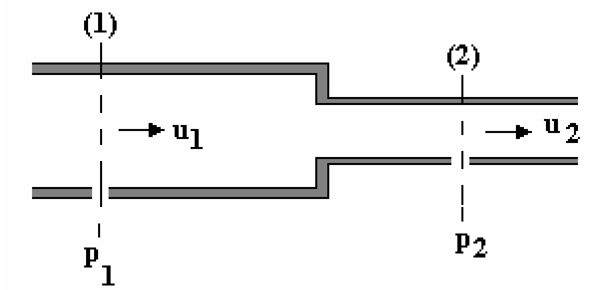


Fig. 20

SOLUTION

$u_1 = u_2A_2/A_1 = (8 \times 0.001)/0.002 = 4$ m/s

$m = \rho A_1u_1 = 1000 \times 0.002 \times 4 = 8$ kg/s. $\qquad Q = A_1u_1 = 0.002 \times 4 = 0.008\ m^3/s$

Pressure loss at contraction = $\frac{1}{2}\rho k u_1^2 = \frac{1}{2} \times 1000 \times 0.4 \times 4^2 = 3200$ Pa

Apply Bernoulli between (1) and (2)

$$p_1 + \frac{\rho u_1^2}{2} = p_2 + \frac{\rho u_2^2}{2} + p_L$$

$$p_1 + \frac{1000 \times 4^2}{2} = 500 \times 10^3 + \frac{1000 \times 8^2}{2} + 3200 \qquad p_1 = 527.2\ \text{kPa}$$

$F = (p_1A_1 + mu_1) - (p_2A_2 + mu_2)$

$F = [(527.2 \times 10^3 \times 0.002) + (8 \times 4)] - [500 \times 10^3 \times 0.001) + (8 \times 8)]$

$F = 1054.4 + 32 - 500 - 64 = 522.4$ N

SELF ASSESSMENT EXERCISE No. 4

1. A pipe carries oil at a mean velocity of 6 m/s. The pipe is 5 km long and 1.5 m diameter. The surface roughness is 0.8 mm. The density is 890 kg/m^3 and the dynamic viscosity is 0.014 N s/m^2. Determine the friction coefficient from the Moody chart and go on to calculate the friction head h_f.
(Ans. C_f = 0.0045 h_f = 110.1 m)

2. The diagram shows a tank draining into another lower tank through a pipe. Note the velocity and pressure is both zero on the surface on a large tank. Calculate the flow rate using the data given on the diagram. (Ans. 7.16 dm^3/s)

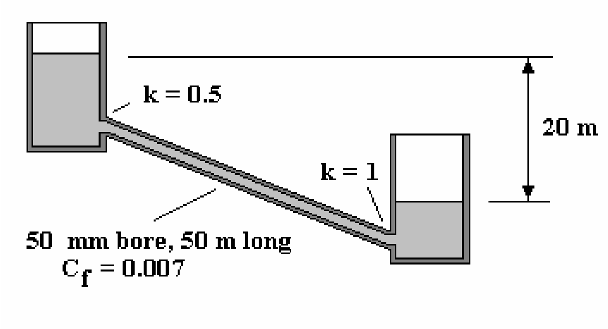


Fig. 21

3. Water flows through the sudden pipe expansion shown below at a flow rate of 3 dm^3/s. Upstream of the expansion the pipe diameter is 25 mm and downstream the diameter is 40 mm. There are pressure tappings at section (1), about half a diameter upstream, and at section (2), about 5 diameters downstream. At section (1) the gauge pressure is 0.3 bar.

Evaluate the following.
(i) The gauge pressure at section (2) (0.387 bar)
(ii) The total force exerted by the fluid on the expansion. (-23 N)

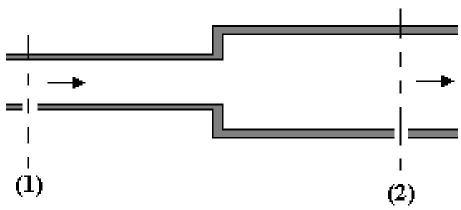


Fig. 22

4. A domestic water supply consists of a large tank with a loss free-inlet to a 10 mm diameter pipe of length 20 m, that contains 9 right angles bends. The pipe discharges to atmosphere 8.0 m below the free surface level of the water in the tank.

Evaluate the flow rate of water assuming that there is a loss of 0.75 velocity heads in each bend and that friction in the pipe is given by the Blasius equation C_f=0.079(Re)$^{-0.25}$ (0.118 dm^3/s).

The dynamic viscosity is 0.89 x 10^{-3} and the density is 997 kg/m^3.

ENGINEERING COUNCIL

CERTIFICATE LEVEL

THERMODYNAMIC, FLUID AND PROCESS ENGINEERING C106

TUTORIAL 5 - DRAG

When you have completed this tutorial you should be able to explain how fluids exert a drag force on a body. The tutorial covers more than enough for the Syllabus and the student should concentrate on the descriptive parts and do the calculations only if they wish to have a deeper understanding of the topic.

1. DRAG

When a fluid flows around the outside of a body, it produces a force that tends to drag the body in the direction of the flow. The drag acting on a moving object such as a ship or an aeroplane must be overcome by the propulsion system. Drag takes two forms, skin friction drag and form drag.

1.1 SKIN FRICTION DRAG

Skin friction drag is due to the viscous shearing that takes place between the surface and the layer of fluid immediately above it. This occurs on surfaces of objects that are long in the direction of flow compared to their height. Such bodies are called ***STREAMLINED***. When a fluid flows over a solid surface, the layer next to the surface may become attached to it (it wets the surface). This is called the 'no slip condition'. The layers of fluid above the surface are moving so there must be shearing taking place between the layers of the fluid. The shear stress acting between the wall and the first moving layer next to it is called ***the wall shear stress*** and denoted τ_w.

The result is that the velocity of the fluid grows from zero at the surface to a maximum u_o at some distance δ above it. This layer is called the ***BOUNDARY LAYER*** and δ is the boundary layer thickness. Fig. 1 Shows how the velocity "u" varies with height "y" for a typical boundary layer.

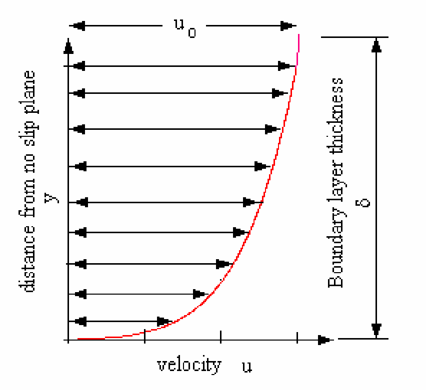


Fig. 1

In a pipe, this is the only form of drag and it results in a pressure and energy lost along the length. A thin flat plate is an example of a streamlined object. Consider a stream of fluid flowing with a uniform velocity u_o. When the stream is interrupted by the plate (fig. 2), the boundary layer forms on both sides. The diagram shows what happens on one side only.

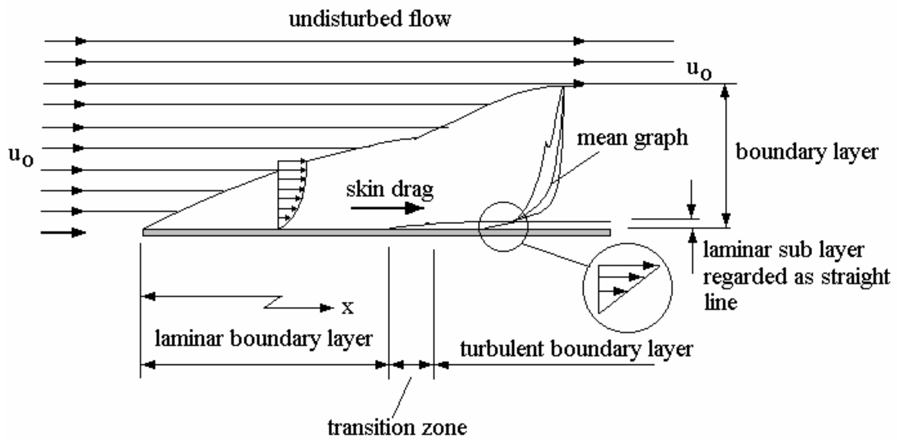


Fig. 2

The boundary layer thickness δ grows with distance from the leading edge. At some distance from the leading edge, it reaches a constant thickness. It is then called a ***FULLY DEVELOPED BOUNDARY LAYER***.

The Reynolds number for these cases is defined as: $(R_e)_x = \dfrac{\rho u_o x}{\mu}$

x is the distance from the leading edge. At low Reynolds numbers, the boundary layer may be laminar throughout the entire thickness. At higher Reynolds numbers, it is turbulent. This means that at some distance from the leading edge the flow within the boundary layer becomes turbulent. A turbulent boundary layer is very unsteady and the streamlines do not remain parallel. The boundary layer shape represents an average of the velocity at any height. There is a region between the laminar and turbulent section where transition takes place

The turbulent boundary layer exists on top of a thin laminar layer called the ***LAMINAR SUB LAYER***. The velocity gradient within this layer is linear as shown. A deeper analysis would reveal that for long surfaces, the boundary layer is turbulent over most of the length. Many equations have been developed to describe the shape of the laminar and turbulent boundary layers and these may be used to estimate the skin friction drag.

Note that for this ideal example, it is assumed that the velocity is the undisturbed velocity u_o everywhere outside the boundary layer and that there is no acceleration and hence no change in the static pressure acting on the surface. There is hence no drag force due to pressure changes.

CALCULATING SKIN DRAG

The skin drag is due to the wall shear stress τ_w and this acts on the surface area (wetted area).
The drag force is hence: **R = τ_w x wetted area.** The dynamic pressure is the pressure resulting from the conversion of the kinetic energy of the stream into pressure and is defined by the expression $\dfrac{\rho u_o^2}{2}$.The drag coefficient is defined as

$$C_{Df} = \frac{\text{Drag force}}{\text{dynamic pressure} \quad \text{x wetted area}}$$

$$C_{Df} = \frac{2R}{\rho u_0^2 \text{ x wetted area}} = \frac{2\tau_w}{\rho u_0^2}$$

Note that this is the same definition for the pipe friction coefficient C_f and it is in fact the same thing. It is used in the Darcy formula to calculate the pressure lost in pipes due to friction. For a smooth surface, it can be shown that $C_{Df} = 0.074\ (R_e)_x^{-1/5}$

$(Re)_l$ is the Reynolds number based on the length. $(R_e)_x = \dfrac{\rho u_o L}{\mu}$

WORKED EXAMPLE No.1

Calculate the drag force on each side of a thin smooth plate 2 m long and 1 m wide with the length parallel to a flow of fluid moving at 30 m/s. The density of the fluid is 800 kg/m^3 and the dynamic viscosity is 8 cP.

SOLUTION

$$(R_e)_x = \frac{\rho u_o L}{\mu} = \frac{800 \text{ x } 30 \text{ x } 2}{0.008} = 6 \text{ x } 10^6$$

$$C_{Df} = 0.074 \text{ x } (6 \text{ x} 10^6)^{-\frac{1}{5}} = 0.00326$$

$$\text{Dynamic pressure} = \frac{\rho u_0^2}{2} = \frac{800 \text{ x } 30^2}{2} = 360 \text{ kPa}$$

$$\tau_w = C_{Df} \text{ x dynamic pressure} = 0.00326 \text{ x } 360 \text{ x } 10^3 = 1173.6 \text{ Pa}$$

$$R = \tau_w \text{ x Wetted Area} = 1173.6 \text{ x } 2 \text{ x } 1 = 2347.2 \text{ N}$$

On a small area the drag is dR = τ_w dA. If the body is not a thin plate and has an area inclined at an angle θ to the flow direction, the drag force in the direction of flow is τ_w dA cosθ.

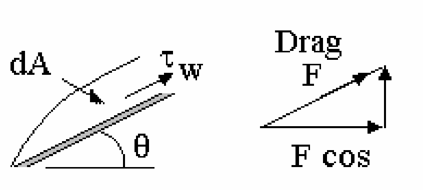


Fig.3

The drag force acting on the entire surface area is found by integrating over the entire area.

$$R = \oint \tau_w \cos\theta \, dA$$

Solving this equation requires more advanced studies concerning the boundary layer and students should refer to the classic textbooks on this subject.

SELF ASSESSMENT EXERCISE No.1

1. A smooth thin plate 5 m long and 1 m wide is placed in an air stream moving at 3 m/s with its length parallel with the flow. Calculate the drag force on each side of the plate. The density of the air is 1.2 kg/m^3 and the kinematic viscosity is 1.6 x 10^{-5} m^2/s. (0.128 N)

2. A pipe bore diameter D and length L has fully developed laminar flow throughout the entire length with a centre line velocity u_o. Given that the drag coefficient is given as C_{Df} = 16/Re where $Re = \frac{\rho u_o D}{\mu}$, show that the drag force on the inside of the pipe is given as $R = 8\pi\mu u_o L$ and hence the pressure loss in the pipe due to skin friction is $p_L = 32\mu \, u_o L/D^2$

1.2 **FORM DRAG and WAKES**

Form or pressure drag applies to bodies that are tall in comparison to the length in the direction of flow. Such bodies are called ***BLUFF BODIES***.

Consider the case below that could for example, be the pier of a bridge in a river. The water speeds up around the leading edges and the boundary layer quickly breaks away from the surface. Water is sucked in from behind the pier in the opposite direction. The total effect is to produce eddy currents or whirl pools that are shed in the wake. There is a build up of positive pressure on the front and a negative pressure at the back. The pressure force resulting is the form drag. When the breakaway or separation point is at the front corner, the drag is almost entirely due to this effect but if the separation point moves along the side towards the back, then a boundary layer forms and skin friction drag is also produced. In reality, the drag is always a combination of skin friction and form drag. The degree of each depends upon the shape of the body.

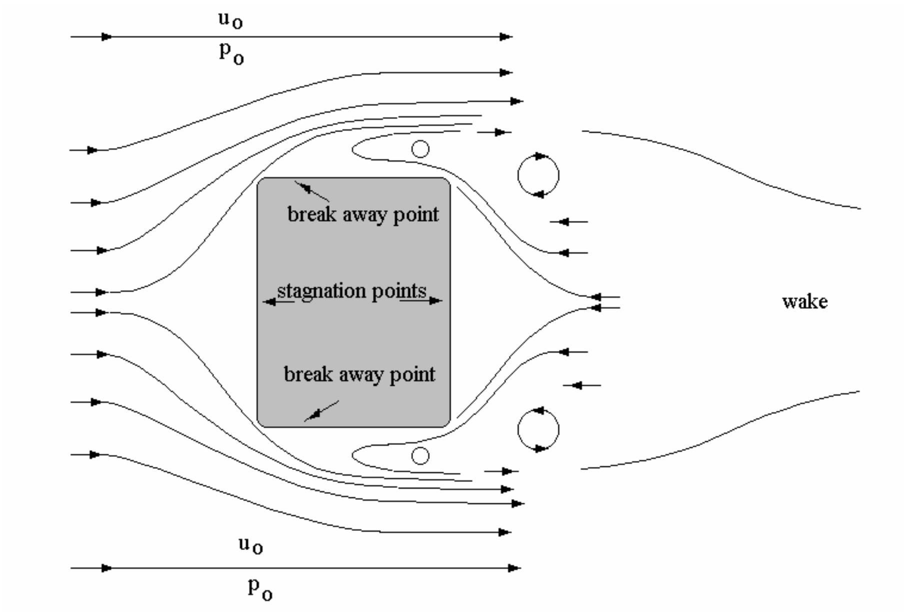


Fig. 4

The next diagram typifies what happens when fluid flows around a bluff object. The fluid speeds up around the front edge. Remember that the closer the streamlines, the faster the velocity. The line representing the maximum velocity is shown but also remember that this is the maximum at any point and this maximum value also increases as the stream lines get closer together.

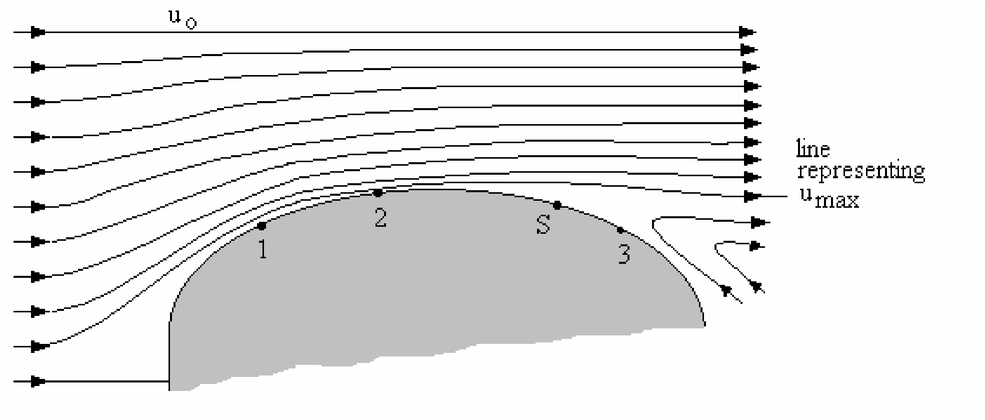


Fig. 5

Two important effects affect the drag.

Outside the boundary layer, the velocity increases up to point 2 so the pressure acting on the surface goes down. The boundary layer thickness δ gets smaller until at point S it is reduced to zero and the flow separates from the surface. At point 3, the pressure is negative. ***This change in pressure is responsible for the form drag.***

Inside the boundary layer, the velocity is reduced from u_{max} to zero and skin friction drag results.

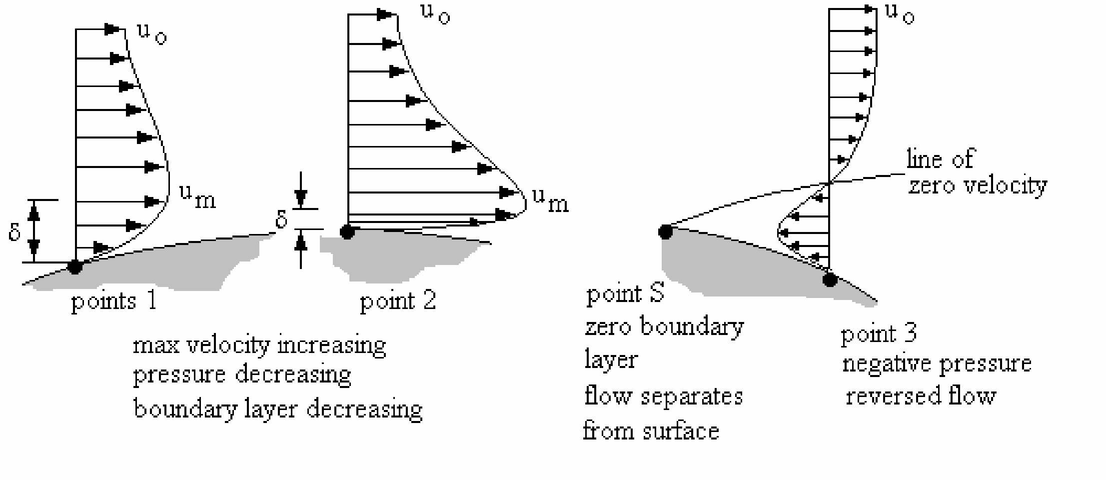


Fig. 6

In problems involving liquids with a free surface, a negative pressure shows up as a drop in level and the pressure build up on the front shows as a rise in level. If the object is totally immersed, the pressure on the front rises and a vacuum is formed at the back. This results in a pressure force opposing movement (form drag). The swirling flow forms ***vortices*** and the wake is an area of great turbulence behind the object that takes some distance to settle down and revert to the normal flow condition.

Here is an outline of the mathematical approach needed to solve the form drag.

Form drag is due to pressure changes only. The drag coefficient due to pressure only is denoted C_{Dp} and defined as

$$C_{Dp} = \frac{\text{Drag force}}{\text{dynamic pressure} \quad \text{x projected area}}$$

$$C_{Dp} = \frac{2R}{\rho u_0^2 \text{ x projected area}}$$

The projected area is the area of the outline of the shape projected at right angles to the flow. The pressure acting at any point on the surface is p. The force exerted by the pressure on a small surface area is p dA. If the surface is inclined at an angle θ to the general direction of flow, the force is p $\cos\theta$ dA. The total force is found by integrating all over the surface.

$$R = \oint p\cos\theta \, dA$$

The pressure distribution over the surface is often expressed in the form of a pressure coefficient defined as follows.

$$C_p = \frac{2(p - p_o)}{\rho u_o^2}$$

p_o is the static pressure of the undisturbed fluid, u_o is the velocity of the undisturbed fluid and $\frac{\rho u_o^2}{2}$ is the dynamic pressure of the stream.

Consider any streamline that is affected by the surface. Applying Bernoulli between an undisturbed point and another point on the surface, we have the following.

$$p_0 + \frac{\rho u_o^2}{2} = p + \frac{\rho u^2}{2} \qquad p - p_o = \frac{\rho}{2}(u_o^2 - u^2)$$

$$C_p = \frac{2(p - p_o)}{\rho u_o^2} = \frac{2\left(\frac{\rho}{2}(u_o^2 - u^2)\right)}{\rho u_o^2} = \frac{(u_o^2 - u^2)}{u_o^2} = 1 - \frac{u^2}{u_o^2}$$

In order to calculate the drag force, further knowledge about the velocity distribution over the object would be needed and students are again recommended to study the classic textbooks on this subject. The equation shows that if $u<u_o$ then the pressure is positive and if $u>u_o$ the pressure is negative.

1.3 TOTAL DRAG

It has been explained that a body usually experiences both skin friction drag and form drag. The total drag is the sum of both. This applies to aeroplanes and ships as well as bluff objects such as cylinders and spheres. The drag force on a body is very hard to predict by purely theoretical methods. Much of the data about drag forces is based on experimental data and the concept of a drag coefficient is widely used.

The ***DRAG COEFFICIENT*** is denoted $\mathbf{C_D}$ and is defined by the following expression.

$$C_D = \frac{\text{Resistance force}}{\text{Dynamic pressure x projected Area}} \qquad C_D = \frac{2R}{\rho u_o^2 \text{ x projected Area}}$$

WORKED EXAMPLE No.2

A cylinder 80 mm diameter and 200 mm long is placed in a stream of fluid moving at 0.5 m/s. The axis of the cylinder is normal to the direction of flow. The density of the fluid is 800 kg/m^3. The drag force is measured and found to be 30 N.

Calculate the drag coefficient.

At a point on the surface, the pressure is measured as 96 Pa above ambient.

Calculate the velocity at this point.

SOLUTION

Projected area = 0.08 x 0.2 = 0.016 m^2 R = 30 N, u_o = 0.5 m/s ρ = 800 kg/m^3

Dynamic pressure = $\rho u^2/2$ = 800 x 0.5^2/2 = 100 Pa

$$C_D = \frac{\text{Resistance force}}{\text{Dynamic pressure x projected Area}} = \frac{30}{100 \text{ x } 0.016} = 18.75$$

$$p - p_o = \frac{\rho}{2}(u_o^2 - u^2) \qquad 96 = \frac{800}{2}(0.5^2 - u^2) \qquad \frac{96 \text{ x } 2}{800} = (0.5^2 - u^2)$$

$$0.24 = 0.25 - u^2 \qquad u^2 = 0.01 \qquad u = 0.1 \text{ m/s}$$

1.4 APPLICATION TO A CYLINDER

The drag coefficient is defined as : $C_D = \frac{2R}{\rho u_o^2 \text{ x projected Area}}$ The projected Area is LD where L is the length and D the diameter. The drag around long cylinders is more predictable than for short cylinders and the following applies to long cylinders. Much research has been carried out into the relationship between drag and Reynolds number. $Re = \frac{\rho u_o d}{\mu}$ and d is the diameter of the cylinder. At very small velocities, (Re <0.5) the fluid sticks to the cylinder all the way round and never separates from the cylinder. This produces a streamline pattern similar to that of an ideal fluid. The drag coefficient is at its highest and is mainly due to skin friction. The pressure distribution shows that the dynamic pressure is achieved at the front stagnation point and vacuum equal to three dynamic pressures exists at the top and bottom where the velocity is at its greatest.

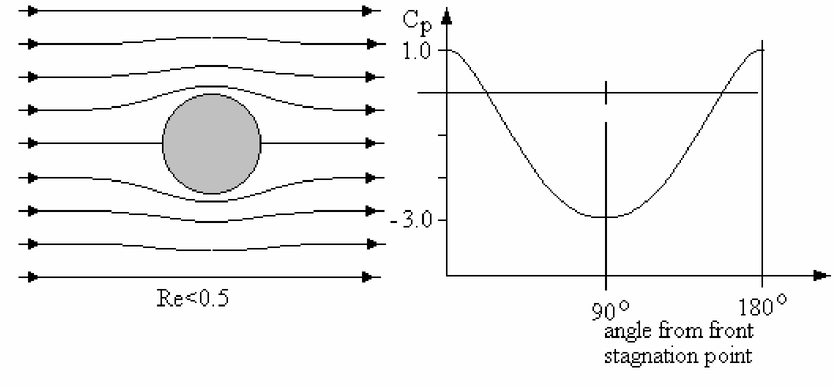


Fig. 7

As the velocity increases the boundary layer breaks away and eddies are formed behind. The drag becomes increasingly due to the pressure build up at the front and pressure drop at the back.

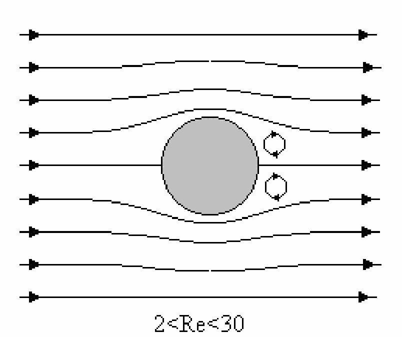


Fig. 8

Further increases in the velocity cause the eddies to elongate and the drag coefficient becomes nearly constant. The pressure distribution shows that ambient pressure exists at the rear of the cylinder.

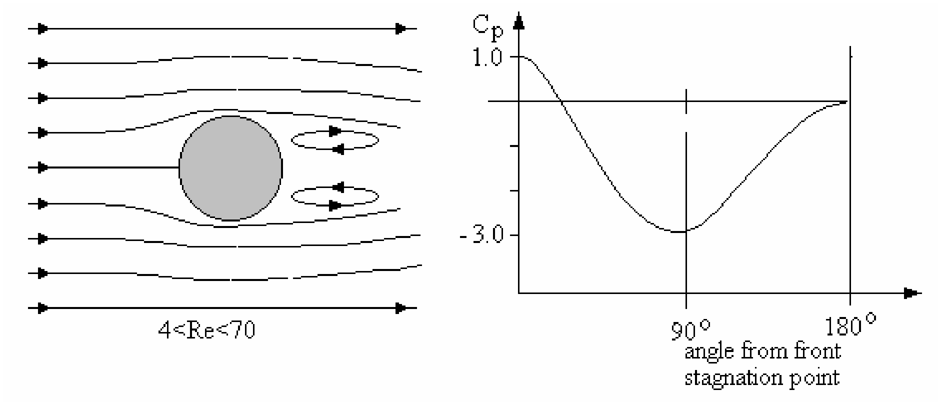


Fig. 9

At a Reynolds number of around 90 the vortices break away alternatively from the top and bottom of the cylinder producing a vortex street in the wake. The pressure distribution shows a vacuum at the rear.

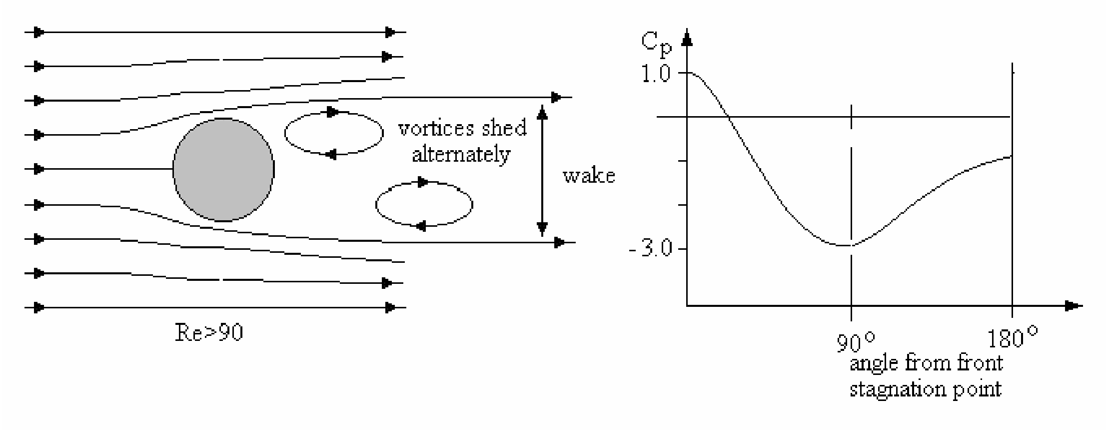


Fig.10

Up to a Reynolds number of about 2 x 10^5, the drag coefficient is constant with a value of approximately 1. The drag is now almost entirely due to pressure. Up to this velocity, the boundary layer has remained laminar but at higher velocities, flow within the boundary layer becomes turbulent. The point of separation moves back producing a narrow wake and a pronounced drop in the drag coefficient.

When the wake contains vortices shed alternately from the top and bottom, they produce alternating forces on the structure. If the structure resonates with the frequency of the vortex shedding, it may oscillate and produce catastrophic damage. This is a problem with tall chimneys and suspension bridges. The vortex shedding may produce audible sound.

Fig. 12 shows an approximate relationship between C_D and R_e for a cylinder and a sphere.

SELF ASSESSMENT EXERCISE No.2

1. Calculate the drag force for a cylindrical chimney 0.9 m diameter and 50 m tall in a wind blowing at 30 m/s given that the drag coefficient is 0.8. The density of the air is 1.2 kg/m^3. (19.44 N)

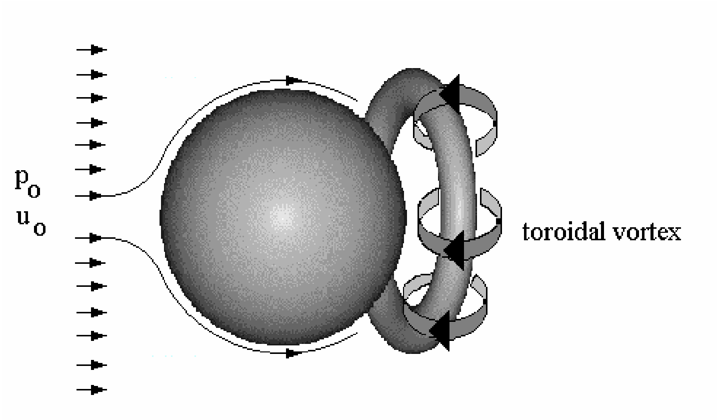
2. Using the graph (fig.12) to find the drag coefficient, determine the drag force per metre length acting on an overhead power line 30 mm diameter when the wind blows at 8 m/s. The density of air may be taken as 1.25 kg/m^3 and the kinematic viscosity as 1.5 x 10^{-5} m^2/s. (1.8 N).

1.5 APPLICATION TO SPHERES

The relationship between drag and Reynolds number is roughly the same as for a cylinder but it is more predictable. The Reynolds number is $Re = \frac{\rho u_o d}{\mu}$ where d is the diameter of the sphere. The projected area of a sphere of diameter d is ¼ πd^2. In this case, the expression for the drag coefficient is as follows. $C_D = \frac{8R}{\rho u^2 \text{ x } \pi x^2}$.

At very small Reynolds numbers (less than 0.2) the flow stays attached to the sphere all the way around and this is called Stokes flow. The drag is at its highest in this region.

As the velocity increases, the boundary layer separates at the rear stagnation point and moves



forward. A toroidal vortex is formed. For 0.2<Re<500 the flow is called Allen flow.

Fig.11

The breakaway or separation point reaches a stable position approximately 80^o from the front stagnation point and this happens when R_e is about 1000. For 500<R_e the flow is called Newton flow. The drag coefficient remains constant at about 0.4. Depending on various factors, when R_e reaches 10^5 or larger, the boundary layer becomes totally turbulent and the separation point moves back again forming a smaller wake and a sudden drop in the drag coefficient to about 0.3. An empirical formula that covers the range $0.2 < R_e < 10^5$ is as follows.

$$C_D = \frac{24}{R_e} + \frac{6}{1+\sqrt{R_e}} + 0.4$$

Fig. 12 shows this approximate relationship between C_D and Re.

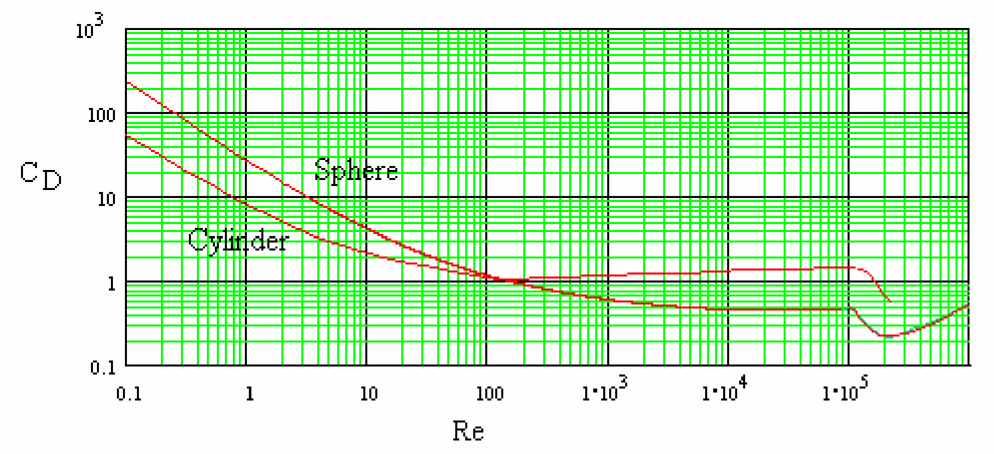


Fig.12

WORKED EXAMPLE No.3

A sphere diameter 40 mm moves through a fluid of density 750 kg/m^3 and dynamic viscosity 50 cP with a velocity of 0.6 m/s. Note 1 cP = 0.001 Ns/m^2.

Calculate the drag on the sphere.

Use the graph to obtain the drag coefficient.

SOLUTION

$$Re = \frac{\rho u d}{\mu} = \frac{750 \text{ x } 0.6 \text{ x } 0.04}{0.05} = 360$$

from the graph $C_D = 0.8$

$$C_D = \frac{2R}{\rho u^2 \text{ x projected Area}} \qquad \text{Projected area} = \pi\frac{d^2}{4} = \pi\frac{0.04^2}{4} = 1.2566 \text{ x } 10^{-3}\,m^2$$

$$R = \frac{C_D \rho u^2 A}{2} = \frac{0.8 \text{ x } 750 \text{ x } 0.6^2 \text{ x } 1.2566 \text{ x } 10^{-3}}{2} = 0.136 \text{ N}$$

1.6 TERMINAL VELOCITY

When a body falls under the action of gravity, a point is reached, where the drag force is equal and opposite to the force of gravity. When this condition is reached, the body stops accelerating and the terminal velocity reached. Small particles settling in a liquid are usually modelled as small spheres and the preceding work may be used to calculate the terminal velocity of small bodies settling in a liquid. A good application of this is the falling sphere viscometer described in chapter one.

For a body immersed in a liquid, the buoyant weight is W and this is equal to the viscous resistance R when the terminal velocity is reached.

R = W = volume x gravity x density difference $= \frac{\pi d^3 g(\rho_s - \rho_f)}{6}$

ρ_s = density of the sphere material
ρ_f = density of fluid
d = sphere diameter

STOKES' FLOW

For R_e<0.2 the flow is called Stokes flow and Stokes showed that $R = 3\pi d\mu u_t$

For a falling sphere viscometer, Stokes flow applies. Equating the drag force and the buoyant weight we get

$$3\pi d \mu u_t = \frac{\pi d^3 g(\rho_s - \rho_f)}{6} \qquad \mu = \frac{d^2 g(\rho_s - \rho_f)}{18 u_t}$$

The terminal velocity for Stokes flow is $u_t = \frac{d^2 g(\rho_s - \rho_f)}{18\mu}$

This formula assumes a fluid of infinite width but in a falling sphere viscometer, the liquid is squeezed between the sphere and the tube walls and additional viscous resistance is produced. The Faxen correction factor F is used to correct the result.

WORKED EXAMPLE No.4

The terminal velocity of a steel sphere falling in a liquid is 0.03 m/s. The sphere is 1 mm diameter and the density of the steel is 7830 kg/m^3. The density of the liquid is 800 kg/m^3. Calculate the dynamic and kinematic viscosity of the liquid.

SOLUTION

Assuming Stokes' flow the viscosity is found from the following equation.

$$\mu = \frac{d^2 g(\rho_s - \rho_f)}{18u_t} = \frac{0.001^2 \text{ x } 9.81 \text{ x } (7830 - 800)}{18 \text{ x } 0.03} = 0.1277 \text{ Ns/m}^2 = 127.7 \text{ cP}$$

$$\nu = \frac{\mu}{\rho_s} = \frac{0.1277}{800} = 0.0001596 \text{ m}^2\text{/s} = 159.6 \text{ cSt}$$

Check the Reynolds number. $R_e = \frac{\rho_f ud}{\mu} = \frac{800 \text{ x } 0.03 \text{ x } 0.001}{0.0547} = 0.188$

As this is smaller than 0.2 the assumption of Stokes' flow is correct.

ALLEN FLOW

For $0.2 < R_e < 500$ the flow is called Allen flow and the following expression gives the empirical relationship between drag and Reynolds number. $\mathbf{C_D = 18.5 R_e^{-0.6}}$

Equating for C_D gives the following result. $C_D = \dfrac{8R}{\rho_f u_t^2 \ \pi \ {}^2} = 18.5 R_e^{-0.6}$

Substitute $R = \dfrac{\pi d^3 g(\rho_s - \rho_f)}{6}$

$$C_D = \frac{8dg(\rho_s - \rho_f)}{6\rho_f u_t^2} = 18.5 R_e^{-0.6} = 18.5\left(\frac{\rho_f u_t d}{\mu}\right)^{-0.6}$$

$$\frac{8dg(\rho_s - \rho_f)}{6\rho_f u_t^2} = 18.5\left(\frac{\rho_f u_t d}{\mu}\right)^{-0.6}$$

From this equation the velocity u_t may be found.

NEWTON FLOW

For $500 < R_e < 10^5$ C_D takes on a constant value of 0.44.

Equating for C_D gives the following. $C_D = \dfrac{8R}{\rho_f u_t^2 \ \pi \ {}^2} = 0.44$

Substitute $R = \dfrac{\pi d^3 g(\rho_s - \rho_f)}{6}$

$$\frac{8dg(\rho_s - \rho_f)}{6\rho_f u_t^2} = 0.44 \qquad u_t = \sqrt{\frac{29.73 dg(\rho_s - \rho_f)}{\rho_f}}$$

When solving the terminal velocity, you should always check the value of the Reynolds number to see if the criterion used is valid.

WORKED EXAMPLE No.5

Small glass spheres are suspended in an up wards flow of water moving with a mean velocity of 1 m/s. Calculate the diameter of the spheres. The density of glass is 2630 kg/m^3. The density of water is 1000 kg/m^3 and the dynamic viscosity is 1 cP.

SOLUTION

First, try the Newton flow equation. This is the easiest.

$$u_t = \sqrt{\frac{29.73 d\, g(\rho_s - \rho_f)}{\rho_f}}$$

$$d = \frac{u_t^2 \rho_f}{29.73\, g(\rho_s - \rho_f)} = \frac{1^2 \text{ x } 1000}{29.73 \text{ x } 9.81 \text{ x } (2630 - 1000)} = 0.0021 \text{ m or } 2.1 \text{ mm}$$

Check the Reynolds number.

$$R_e = \frac{\rho_f u_t d}{\mu} = \frac{1000 \text{ x } 1 \text{ x } 0.0021}{0.001} = 2103$$

The assumption of Newton flow was correct so the answer is valid.

WORKED EXAMPLE No.6

Repeat the last question but this time with a velocity of 0.05 m/s. Determine the type of flow that exists.

SOLUTION

If no assumptions are made, we should use the general formula $C_D = \frac{24}{R_e} + \frac{6}{1+\sqrt{R_e}} + 0.4$

$$R_e = \frac{\rho_f u_t d}{\mu} = \frac{1000 \text{ x } 0.05 \text{ x } d}{0.001} = 50\,000 d$$

$$C_D = \frac{24}{R_e} + \frac{6}{1+\sqrt{R_e}} + 0.4 = \frac{24}{50000d} + \frac{6}{1+\sqrt{50000d}} + 0.4 = 0.00048 d^{-1} + \frac{6}{1+223.6 d^{0.5}} + 0.4$$

$$C_D = \frac{8dg(\rho_s - \rho_f)}{6\rho_f u^2} = \frac{8d \text{ x } 9.81 \text{ x } (2630 - 1000)}{6 \text{ x } 1000 \text{ x } 0.05^2} = 8528.16d$$

$$8528.16d = 0.00048 d^{-1} + \frac{6}{1+223.6 d^{0.5}} + 0.4$$

This should be solved by any method known to you such as plotting two functions and finding the point of interception.

$$f1(d) = 8528.16d$$

$$f2(d) = 0.00048 d^{-1} + \frac{6}{1+223.6 d^{0.5}} + 0.4$$

The graph below gives an answer of d = 0.35 mm.

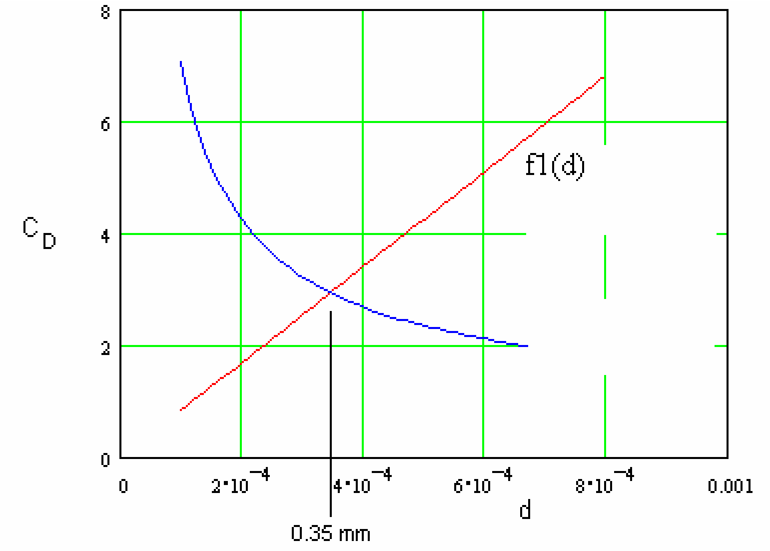


Fig. 13

Checking the Reynolds' number $R_e = \frac{\rho_f u_t d}{\mu} = \frac{1000 \text{ x } 0.05 \text{ x } 0.00035}{0.001} = 17.5$

This puts the flow in the Allen's flow section.

SELF ASSESSMENT EXERCISE No.3

1. a. Explain the term Stokes flow and terminal velocity.

b. Show that the terminal velocity of a spherical particle with Stokes flow is given by the formula
$u = d^2g(\rho_s - \rho_f)/18\mu$

Go on to show that $C_D = 24/R_e$

2. Calculate the largest diameter sphere that can be lifted upwards by a vertical flow of water moving at 1 m/s. The sphere is made of glass with a density of 2630 kg/m^3. The water has a density of 998 kg/m^3 and a dynamic viscosity of 1 cP. (20.7 mm)

3. Using the same data for the sphere and water as in Q2, calculate the diameter of the largest sphere that can be lifted upwards by a vertical flow of water moving at 0.5 m/s. (5.95 mm).

4. Using the graph (fig. 12) to obtain the drag coefficient of a sphere, determine the drag on a totally immersed sphere 0.2 m diameter moving at 0.3 m/s in sea water. The density of the water is 1025 kg/m^3 and the dynamic viscosity is 1.05 x 10^{-3} Ns/m^2. (0.639 N).

5. A glass sphere of diameter 1.5 mm and density 2 500 kg/m^3 is allowed to fall through water under the action of gravity. The density of the water is 1000 kg/m^3 and the dynamic viscosity is 1 cP.

Calculate the terminal velocity assuming the drag coefficient is
$C_D = 24\, R_e^{-1}(1 + 0.15 R_e^{0.687})$ (Ans. 0.215 m/s