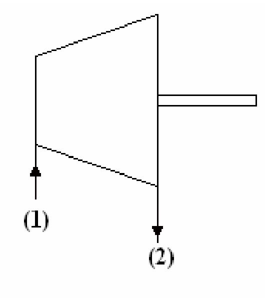
APPLIED THERMODYNAMICS D201

SELF ASSESSMENT SOLUTIONS
TUTORIAL 1

SELF ASSESSMENT EXERCISE No.1

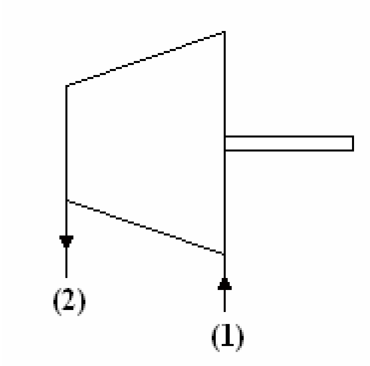
1. Steam is expanded adiabatically in a turbine from 100 bar and 600^{o}C to 0.09 bar with an isentropic efficiency of 0.88. The mass flow rate is 40 kg/s.
Calculate the enthalpy at exit and the power output.



h_1 = 3624 kJ/kg
s_1 = 6.902 kJ/kg K
$s_2 = s_f + x_2' s_{fg}$ at 0.09 bar
$s_2 = s_1$ =6.902 = 0.662 + x_2' 7.564
x_2'= 0.83
$h_2' = h_f + x_2' h_{fg}$ at 0.09 bar
h_2' = 183 + 0.83 (2397) =2173.1 kJ/kg
Ideal Power = 30(3624 – 2173.1 = 58 MW
Actual Power = η x 58 = 0.88 x 58 = 51 MW

2. A gas compressor compresses gas adiabatically from 1 bar and 15^{o}C to 10 bar with an isentropic efficiency of 0.89. The gas flow rate is 5 kg/s.

Calculate the temperature after compression and the power input.
(Ans. -1.513 MW)



Take c_V = 718 J/kg K and c_p = 1005 J/kg K

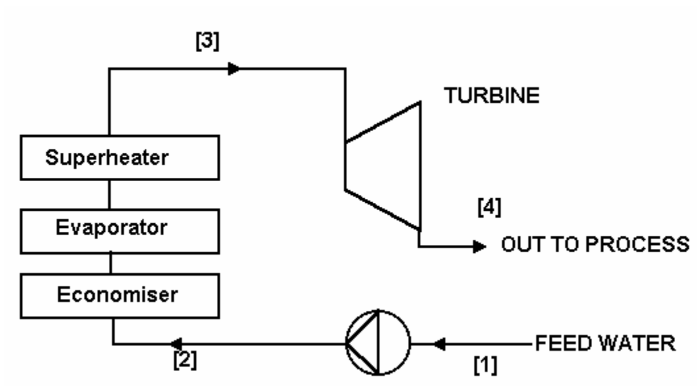
$\gamma = c_p/c_v = 1.4 \quad T_{2'} = T_1\left(\frac{p_2}{p_1}\right)^{1-1/\gamma} \qquad T_{2'} = 288(10)^{-0.286}$ =556 K

Ideal Power = 5 x 1.005 x (556 – 288) = 1.347 MW
Actual Power = P/ η = 1.347/0.89 = 1.513 MW

SELF ASSESSMENT EXERCISE No.2

A back pressure steam cycle works as follows. The boiler produces 8 kg/s of steam at 40 bar and 500°C. This is expanded to 2 bar with an isentropic efficiency of 0.88. The pump is supplied with feed water at 0.5 bar and 30°C and delivers it to the boiler at 31°C and 40 bar.

Calculate the net power output of the cycle. (Answer 5.24 MW)



From tables h_3= 3445 kJ/kg s_3 = 7.089 kJ/kg K

For an ideal expansion

$S_3 = s_4 = 7.089 = 1.53 + x's_{fg}$ at 3 bar

$6.646 = 1.672 + x_4'(5.597)$ at 2 bar

$x_4' = 0.993$

$h_4' = h_f + x'h_{fg}$ at 2 bar

$h_4' = 505 + 0.993(2202) = 2693$ kJ/kg

Ideal Power Out = 8(3445 –2693) = 6.024 MW

Actual Power = 0.88(6.024 = 5.3 MW

Next we examine the enthalpy change at the pump.

$h_1 = 125.7$ kJ/kg h_f at 30°C

$h_2 = mc\theta + pv = 1 \text{ x } 4186 \text{ x } 31 + 40 \text{ x } 10^5 \text{ x } 0.001 = 133.7$ kJ/kg

The power input to the pump is 8(133.7 – 125.7) = 64 kW

Net Power output of the cycle = 5300 – 64 = 5236 kW

<u>SELF ASSESSMENT EXERCISE No.3</u>

1. A steam turbine plant is used to supply process steam and power. The plant comprises an economiser, boiler, superheater, turbine, condenser and feed pump. The process steam is extracted between intermediate stages in the turbine at 2 bar pressure. The steam temperature and pressure at outlet from the superheater are 500°C and 70 bar, and at outlet from the turbine the pressure is 0.1 bar. The overall isentropic efficiency of the turbine is 0.87 and that of the feed pump is 0.8.
Assume that the expansion is represented by a straight line on the h-s chart. The make-up water is at 15°C and 1 bar and it is pumped into the feed line with an isentropic efficiency 0.8 to replace the lost process steam.
If due allowance is made for the feed pump-work, the net mechanical power delivered by the plant is 30 MW when the process steam load is 5 kg/s. Calculate the rate of steam flow leaving the superheater and the rate of heat transfer to the boiler including the economiser and superheater. Sketch clear T- s and h-s and flow diagrams for the plant. (29.46 kg/s   95.1 MW)

From the tables

h_3 = 3410 kJ/kg

s_3 = 6.796 kJ/kg K

$s_5' = s_3 = 0.649 + x_5'$ (7.5)

x_5'= 0.8196

h_5' = 192 + 0.8196(2392) = 2152.5 kJ/kg

Δh(ideal) = 3410 – 2152.5 =1257.5 kJ/kg

Δh(actual) = 1257.5 x 0.87 =1094 kJ/kg

h_5 = 3410 – 1094 = 2316 kJ/kg

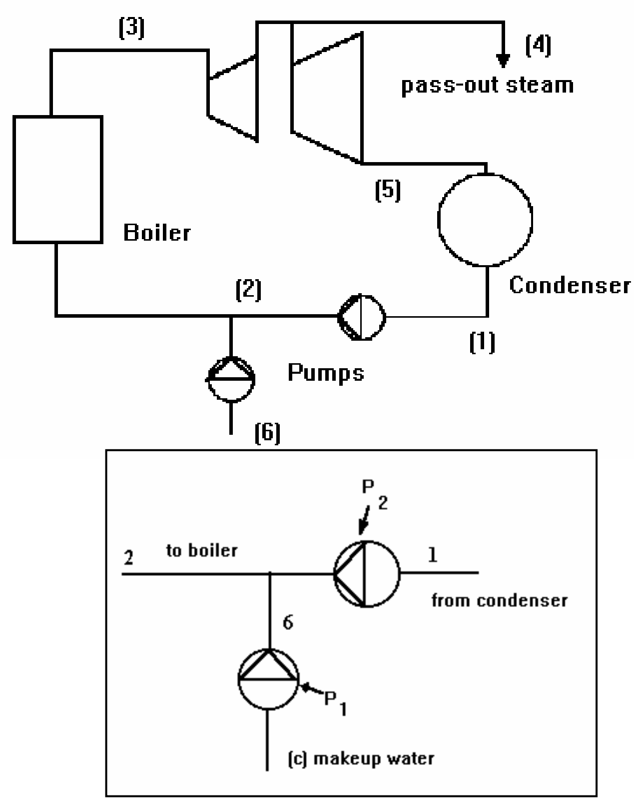
From the T – s chart h_4 = 2710 kJ/kg

P(out) = m($h_3 – h_4$) + (m - s)($h_4 – h_5$)

P(out) =m(3410 – 2710)+(m-s)(2710 – 2316)

P(out) =700m + 394m - 1970

P(out) =1094 m - 1970



Now consider the pumps.

$h_1 = h_f$ =192 kJ/kg

$h_6 = h_f$ at 15°C = 62.9 kJ/kg

P1(ideal) = 5 x 0.001 x (70 – 0.1)x10^5 = 34.5 kW

P1(actual) = 34.5/0.8 = 43.125 kW

P2(ideal) = (m – 5)(0.001)((70 – 0.1)x10^5

P2(ideal) = 6.99m – 34.95 kW

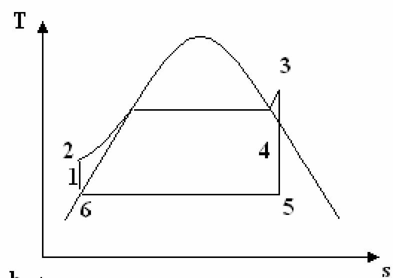
P2(actual) =(6.99m – 34.95)/0.8 = 8.7375m – 43.6875

P(net) = 30 000 kW = (1094 m – 1970) - 43.125 - 8.7375m – 43.6875

30 000 = 1085.3m -1926.9 + 43.68

31970.6 = 1085.3 m

m = 29.45 kg/s



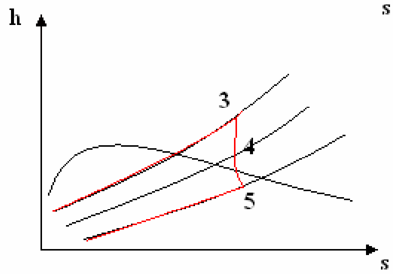
P2 = 8.7375 x 29.45 – 43.6875 = 213.63 kW

P1 = 43.125 kW

P1 + P2 = 256 = mh_2 – 5h_6 – (m – 5)h_1

256 = 29.45 h_2 – 5 x 62.9 – 24.45 x 192

h_2 = 5255.3/29.45 = 178.4 kJ/kg



Φ = m($h_3 – h_2$) = 29.45(3410 – 178.4) = 95169 kW

η = 30MW/95.169 MW = 31.5%

2. The demand for energy from an industrial plant is a steady load of 60 MW of process heat at 117°C and a variable demand of up to 30 MW of power to drive electrical generators. The steam is raised in boilers at 70 bar pressure and superheated to 500°C. The steam is expanded in a turbine and then condensed at 0.05 bar. The process heat is provided by the steam bled from the turbine at an appropriate pressure, and the steam condensed in the process heat exchanger is returned to the feed water line.

Calculate the amount of steam that has to be raised in the boiler. Assume an overall isentropic efficiency of 0.88 in the turbine. The expansion is represented by a straight line on the h-s diagram. Neglect the feed pump work.

From the tables

h_3 = 3410 kJ/kg

s_3 = 6.796 kJ/kg K

$s_5' = s_3 = 0.476 + x_5'\,(7.918)$

$x_5' = 0.7982$

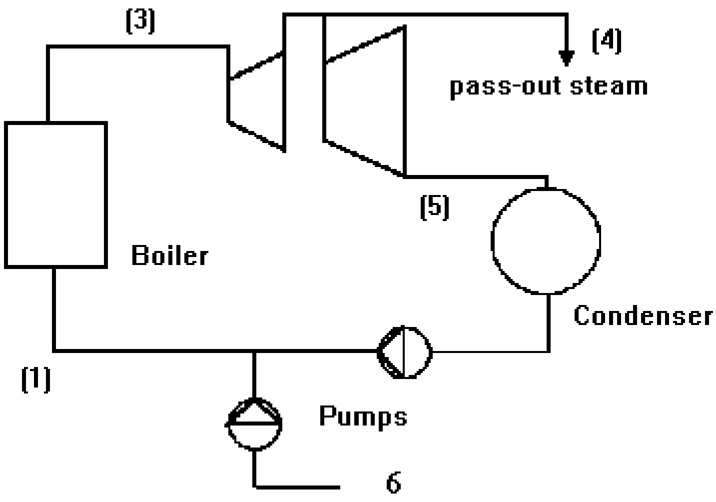
$h_5' = 138 + 0.7982(2423)$

h_5' = 2072 kJ/kg

Δh(ideal) = 3410 – 2072 =1338 kJ/kg

Δh(actual) = 1338 x 0.88 =1177 kJ/kg

h_5 = 3410 – 1174 = 2233 kJ/kg



From the T – s chart 117°C is in the wet region at 1.8 bar so

h_4 = 2680 kJ/kg

Process Flow

$m_p(h_4 - h_6) = 60\,000$

Assume $h_6 = h_f$ at 1.8 bar =491 kJ/kg $\quad m_p = 60\,000/(h_4 - h_6) = 60\,000/(2680 - 491) = 27.4$ kg/s

TURBINE

$30000 = m(h_3 - h_4) + (m - 27.4)(h_4 - h_5)$

$30000 = m(3410 - 2680) + (m - 27.4)(2680 - 2233)$

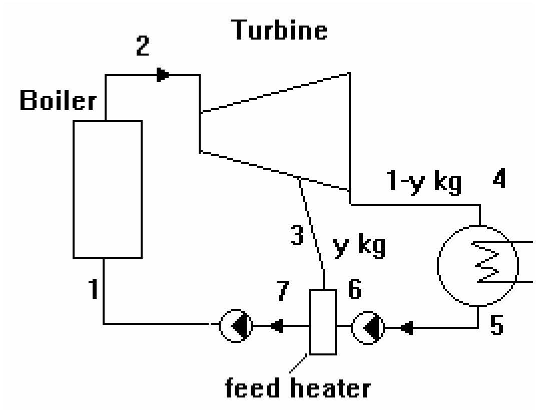
m = 36 kg/s

SELF ASSESSMENT EXERCISE No.4

A simple steam plant uses a Rankine cycle with one regenerative feed heater. The boiler produces steam at 70 bar and 500^oC. This is expanded to 0.1 bar isentropically. Making suitable assumptions, calculate the cycle efficiency. (41.8%)

$T_{bleed} = \{t_s(hp) - t_s(lp)\}/2$
$T_{bleed} = (285.8 + 45.8)/2 = 165.8^oC$

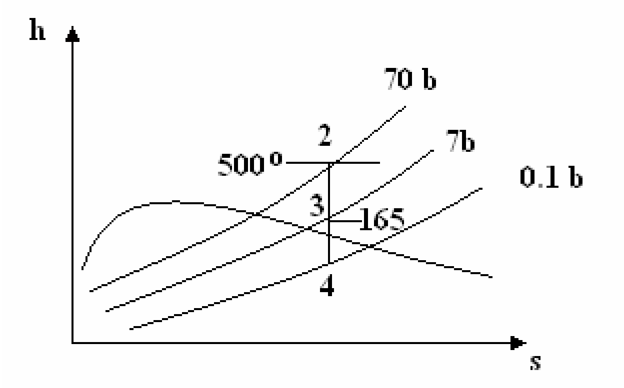
The corresponding pressure is 7 bar



From the h – s chart

$h_2 = 3410$ kJ/kg
$h_3 = 2800$ kJ/kg
$h_4 = 2150$ kJ/kg

From tables
$h_5 = h_f = 192$ kJ/kg
$h_7 = h_f = 697$ kJ/kg



HEATER
$2800y + 192(1-y) = h_7 = 697$ $\quad$ $y = 0.193$

BOILER
$\Phi(in) = 3410 – 697 = 2713$ kJ/kg

CONDENSER
$\Phi(out) = (1 – 0.193)(2150 – 192) = 1579$ kJ/kg

$P(net) = \Phi(in) - \Phi(out) = 2713 - 1579 = 1133$ kJ/kg

$\eta = 1133/2713 = 41.8\%$

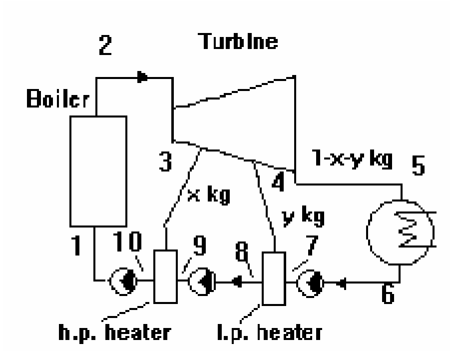
SELF ASSESSMENT EXERCISE No.5

1. Explain how it is theoretically possible to arrange a regenerative steam cycle which has a cycle efficiency equal to that of a Carnot cycle.
In a regenerative steam cycle steam is supplied from the boiler plant at a pressure of 60 bar and a temperature of 500°C. Steam is extracted for feed heating purposes at pressures of 30 bar and 3.0 bar and the steam turbine exhausts into a condenser operating at 0.035 bar.
Calculate the appropriate quantities of steam to be bled if the feed heaters are of the open type, and find the cycle efficiency; base all calculations on unit mass leaving the boiler.
Assume isentropic expansion in the turbine and neglect the feed pump work.

(Answers 0.169 kg/s, 0.145 kg/s and 45 %)

If regenerative feed heating is conducted with an infinite number of heaters, the heat transfer would be isothermal thus producing the Carnot ideal. The only way this might be done is arranging a heat exchanger inside the turbine casing so that the temperature is the same on both sides at all points. (see the diagram in the next question).

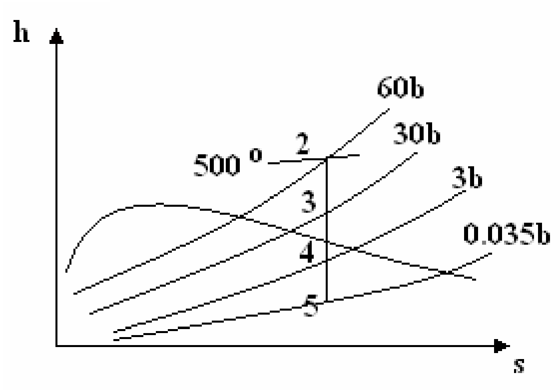
$h_2 = 3421$ kJ/kg



From the h – s chart

$h_3 = 3210$ kJ/kg
$h_4 = 2680$ kJ/kg
$h_5 = 2050$ kJ/kg

From tables
$h_6 = h_f$ at 0.035b =112 kJ/kg
$h_7 = h_f$ at 3b = 561 kJ/kg
$h_1 = h_f$ at 30b = 1008 kJ/kg



HEATER A
$3200x + 561(1-x) = 1008$ $x = 0.169$
HEATER B
$2680y + 112(0.831-y) = 0.831(561)$ $y= 0.145$

BOILER
$\Phi(in) = 3421 – 1008 = 2413$ kJ/kg

CONDENSER
$\Phi(out) = 0.686(2050 – 112) = 1329.5$ kJ/kg

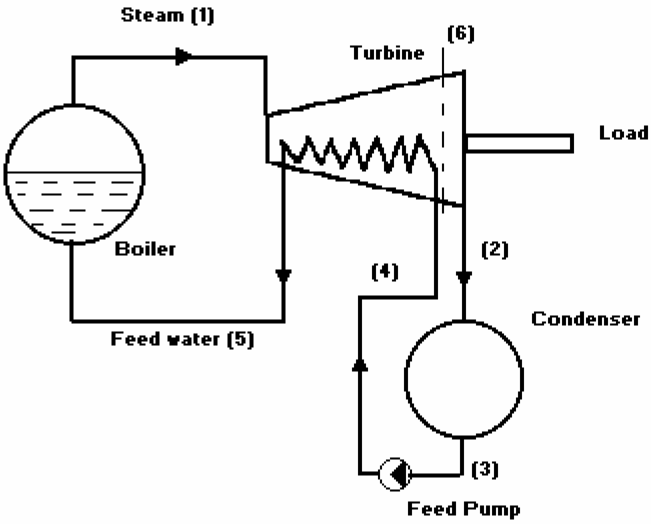
$P(net) = \Phi(in) - \Phi(out) = 1083.5$ kJ/kg

$\eta = 1083.5/2413 = 45\%$

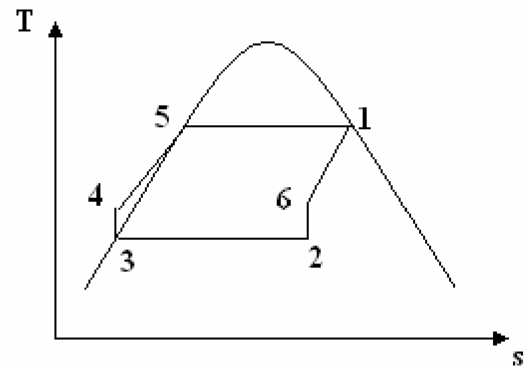
2. The sketch shows an idealised regenerative steam cycle in which heat transfer to the feed water in the turbine from the steam is reversible and the feed pump is adiabatic and reversible. The feed water enters the pump as a saturated liquid at 0.03 bar, and enters the boiler as a saturated liquid at 100 bar, and leaves as saturated steam.

Draw a T-s diagram for the cycle and determine, not necessarily in this order, the dryness fraction in state 2, the cycle efficiency and the work per unit mass.

Outline the practical difficulties which are involved in realising this cycle and explain how regenerative cycles are arranged in practice.



Note point (6) is the point in the steam expansion where the feed water enters and presumably the temperatures are equal. There is further expansion from (6) to (2).



Φ_T = heat transfer inside the turbine from the steam to the feed water.

If the Carnot Efficiency is achieved

$$\eta = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{273 + 24.1}{273 + 311} = 50\%$$

$h_1 = h_g$ at 100 bar = 2725 kJ/kg
$h_3 = h_f$ at 0.03 bar = 101 kJ/kg
$h_5 = h_f$ at 100 bar = 1408 kJ/kg

$\Phi(in) = h_1 - h_5 = 1317$ kJ/kg

$P(net) = \eta \times \Phi(in) = 0.5 \times 1317 = 658.5$ kJ/kg

$\Phi(out) = \Phi(in) - P(net) = 658.5 = h_2 - h_3$

$h_2 = 658.5 + 101 = 759.5$ kJ/kg

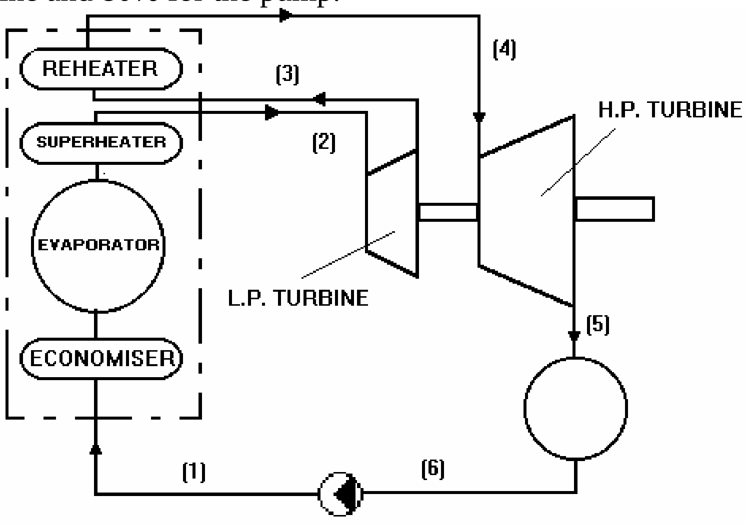
$759.5 = h_f + x h_{fg} = 101 + x(2444)$

x = 0.269 at point (2)

It is not practical to make a turbine with a heat exchanger inside the casing. In practice a series of regenerative feed heaters is used that raises the feed temperature in steps.

SELF ASSESSMENT EXERCISE No.6

1. Repeat worked example No.10 but this time do not ignore the feed pump term and assume an isentropic efficiency of 90% for each turbine and 80% for the pump.



h_2 =3097 kJ/kg
h_6 =251 kJ/kg
h_4 =3196 kJ/kg

HP TURBINE
P1 = $0.9(h_2 – h_3')$ = 150.3 kJ/kg
h_3' = 2930 kJ/kg
$h_3 = h_2 – 150.3$ = 2947 kJ/kg

LP TURBINE
P2 = $0.9(h_4 – h_5')$ = 906 kJ/kg
h_5' = 2189 kJ/kg
$h_5 = h_4 – 906$ = 2290 kJ/kg

CONDENSER
Φ(out) = $30(h_5 – h_6)$ = 60.4 MW

PUMP
P3 = $0.001(100 – 0.02) \times 10^5/0.8$ = 12470 J/kg or 12.47 kJ/kg
h_1 = 251 + 12.5 = 263.5 kJ/kg

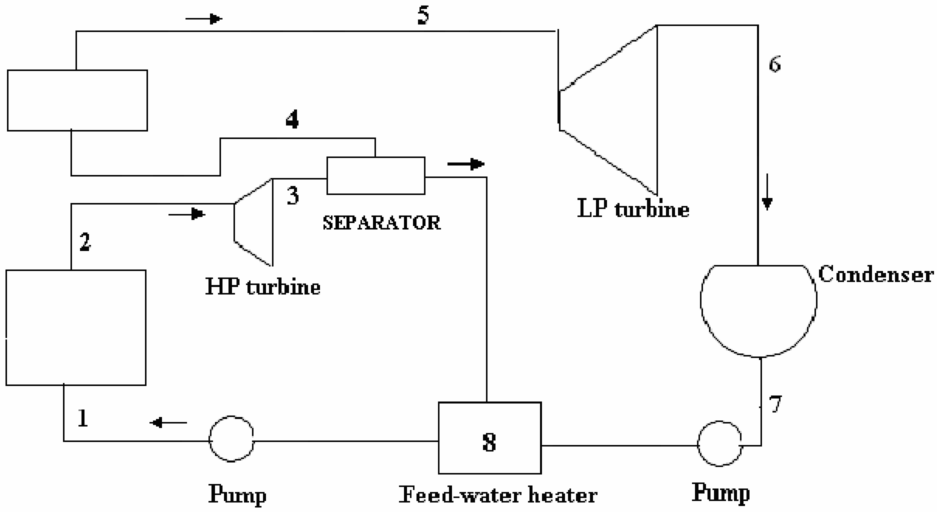
BOILER
Φ(in) = 30(3097 – 263.5) + 30(3196 – 2947) = 92.5 MW

TURBINES
P1 = 30(150.3) = 4.51 mw
P2 = 30(906) = 27.18 mw
P(NET) = 4.51 + 27.18 - 0.374 = 31.3 mw

η = 31.3/92.5 = 33.8 %

2. A water-cooled nuclear reactor supplies dry saturated steam at a pressure of 50 bar to a two-cylinder steam turbine. In the first cylinder the steam expands with an isentropic efficiency of 0.85 to a pressure of 10 bar, the power generated in this cylinder being 100 MW. The steam then passes at a constant pressure of 10 bar through a water separator from which all the water is returned to the reactor by mixing it with the feed water. The remaining dry saturated steam then flows at constant pressure through a reheater in which its temperature is raised to 250^{o}C before it expands in the second cylinder with an isentropic efficiency of 0.85 to a pressure of 0.1 bar, at which it is condensed before being returned to the reactor.

Calculate the cycle efficiency and draw up an energy balance for the plant. Neglect the feed pump work.



Start with known points.

Point 2	50 bar	dss	h_2 = 2794 kJ/kg	s_2 = 5.973 kJ/kg
Point 4	10 bar	dss	h_4 = 2778 kJ/kg	
Point 5	10 bar	250^{o}C	h_5 = 2944 kJ/k	s_5 =6.926 kJ/kg
Point 7	0.1 bar	sw	h_7 = 192 kJ/kg	
Point 9	10 bar	sw	h_9 = 763 kJ/kg	

HP Turbine

Ideal expansion $s_2 = s_3' = 5.973 = 2.138 + x_3'(4.448)$ $x_3' = 0.862$

$h_{3'} = 763 + 0.862 \times 2015$ at 10 bar = 2500 kJ/kg

$\eta = 0.85 = \dfrac{2794 - h_3}{2794 - 2500}$ h_3 = 2544 kJ/kg – the actual enthalpy

Power output = 100 000 kW = $m_1(2794 - 2544)$ m_1= 400.6 kg/s

$h_3 = 2544 = 763 + x_3(2015)$ $x_3 = 0.884$

$m_4 = 0.884 \times 400.6 = 354.1$ kg/s $m_9 = 400.6 - 354.1 = 46.52$ kg/s

LP TURBINE

Ideal expansion $s_5 = s_6' = 6.926 = 0.649 + x_6'(7.5)$ $x_6' = 0.837$

$h_{6'} = 192 + 0.837 \times 2392 = 2194$ kJ/kg

Power out = 354.1 (2944 - 2194) x 85% = 225700 kW

Power out = m(2648.1 - 2134.6) = 513.45 m kW m = mass flowing to condenser.

FEED HEATER

$46.52 h_9 + 354.1 h_7 = 400.6 h_1$

$46.52 \times 763 + 354.1 \times 192 = 400.6 h_1$ h_1= 258.1 kJ/kg

BOILER

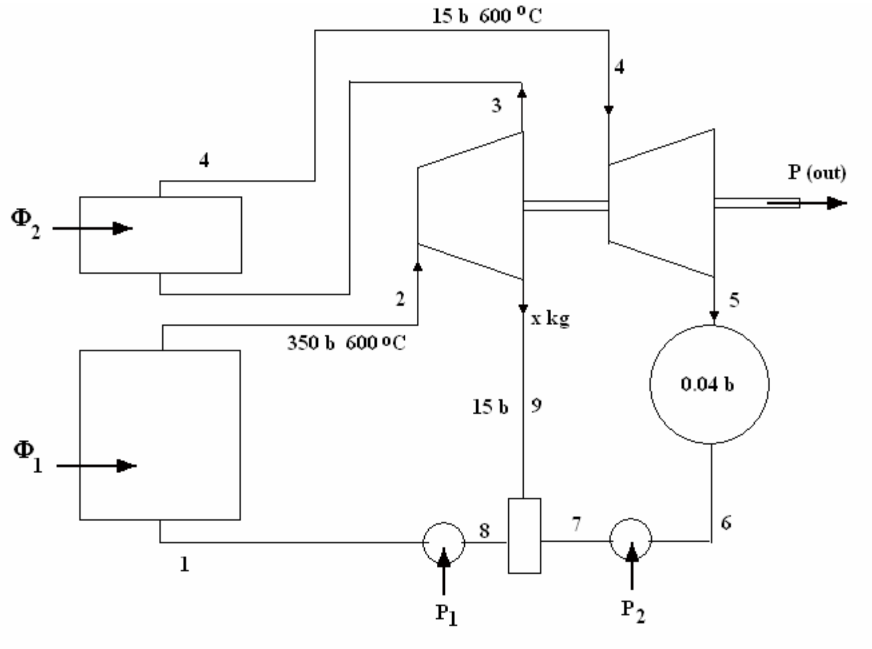
$\Phi(1) = 400.6(2794 - 258.3)$ $\Phi(2) = 354.1(2944 - 2778)$ Total Φ = 1075 MW

Total Power Out = 100 + 225.7 = 325.7 MW

$\eta = P/\Phi = 325.7/1075 = 30.3\%$ (pump powers ignored)

3. Steam is raised in a power cycle at the supercritical pressure of 350 bar and at a temperature of 600°C. It is then expanded in a turbine to 15 bar with an overall isentropic efficiency of 0.90. At that pressure some steam is bled to an open regenerative feed heater, and the remainder of the steam is, after reheating to 600°C, expanded in a second turbine to the condenser pressure of 0.04 bar, again with an isentropic efficiency of 0.90. The feed pumps each have an overall isentropic efficiency of 0.90.

Calculate the amount of steam to be bled into the feed heater, making the usual idealising assumptions. Also calculate the cycle efficiency. Use the h-s chart wherever possible and do not neglect feed pump work.



Known Points
h_2 = 3397 kJ/kg
h_6 = 121 kJ/kg
(h_f at 0.04b)
h_4 = 3694kJ/kg
h_8 = 845 kJ/kg
(h_f at 15b)

LP PUMP
$P_2 = 0.001(15-0.04)\text{x}10^5$
P_2 = 1.496 kJ/kg
Actual P_2 =1.496/0.9
Actual P_2 = 1.662 kJ/kg

Energy Balance
$h_7(1-x) = P_2(1-x)+ h_6(1-x)$
$h_7 = P_2 + h_6$ = 122.62 kJ/kg

HP TURBINE
h_2 = 3397 kJ/kg $\quad$ s_2 = 6.12 kJ/kg K = s_3 = 2.315 + x_3' (4.13) $\quad$ x_3'= 0.921
h_3'= 845 + 0.921(1947) = 2638.8 kJ/kg
Δh(ideal) = 3397 - 2638.8 = 758.2 kJ/kg
Δh(actual) = 758.2 x 90% = 682.4 kJ/kg
h_3 = 3397 - 682.4 = 2714.6 kJ/kg

LP TURBINE
h_4 = 3694 kJ/kg $\quad$ s_4 = 7.838 kJ/kg K = s_5 = 0.422 + x_5' (8.051) $\quad$ x_5'= 0.921
h_5'= 121 + 0.921(2423) = 2362 kJ/kg
Δh(ideal) = 3694 - 2362 = 1331.9 kJ/kg
Δh(actual) = 1331.9 x 90% = 1198.7 kJ/kg
h_5 = 3694 – 1198.7 = 2795.3 kJ/kg

HEATER $\quad$ $h_8 = xh_3 + (1-x)h_7$ $\quad$ 845 = 2714.6x + (1-x)1222.62 $\quad$ x = 0.2787 kg
HP PUMP $\quad$ $P_1 = 0.001(350-15)\text{x}10^5/90\%$ = 37220 J/kg or 37.22 kJ/kg
$h_1 = P_1 + h_8$ = 37.22 + 845 = 882.22 kJ/kg
BOILER $\quad$ Φ(in) = (3397 – 882.22) + (0.721(3694 – 2714.6) = 3221 kJ/kg
CONDENSER $\quad$ Φ(out) = $(1-x)(h_5 - h_6)$ = 0.721(2495.5 – 121) = 1712.6 kJ/kg
EFFICIENCY $\quad$ $\eta = 1 - \Phi(\text{out})/\Phi(\text{in})$ = 1 - 1712.6/3221 = 46.8%

APPLIED THERMODYNAMICS D201

SELF ASSESSMENT SOLUTIONS
TUTORIAL 2
SELF ASSESSMENT EXERCISE No. 1

Show how the volumetric efficiency of an ideal single stage reciprocating air compressor may be represented by the equation

$$\eta_{vol} = 1 - c\left[\left(\frac{p_H}{pL}\right)^{(1/n)} - 1\right]$$

Where c is the clearance ratio, p_H the delivery pressure and p_L the induction pressure.
A reciprocating air compressor following the ideal cycle has a free air delivery of 60 dm^3/s. The clearance ratio is 0.05. The inlet is at atmospheric pressure of 1 bar. The delivery pressure is 7 bar and the compression is polytropic with an index of 1.3. Calculate the following.

i. The ideal volumetric efficiency. (82.7%)
ii. The ideal indicated power. (14.7 kW)

Swept volume = $V_1 - V_3$ Induced volume = $V_1 - V_4$
Clearance volume = V_3
Swept volume = $V_1 - V_3$ Induced volume = $V_1 - V_4$
Clearance volume = V_3

$$\eta_{vol} = \frac{V_1 - V_4}{V_1 - V_3} \quad c = \frac{V_3}{V_1 - V_3}$$

$$V_1 - V_3 = \frac{V_3}{c} \quad \frac{V_1}{V_3} = \frac{(1+c)}{c}$$

$$\eta_{vol} = \frac{c(V_1 - V_3)}{V_3} = c\left\{\left(\frac{V_1}{V_3}\right) - \left(\frac{V_4}{V_3}\right)\right\}$$

$$\frac{V_4}{V_3} = \left(\frac{p_3}{p_4}\right)^{1/n} = \left(\frac{p_H}{p_L}\right)^{1/n}$$

$$\eta_{vol} = c\left[\left\{\left(\frac{1+c}{c}\right)\right\} - \left(\frac{p_H}{p_L}\right)^{1/n}\right]$$

$$\eta_{vol} = 1 + c - c\left(\frac{p_H}{p_L}\right)^{1/n}$$

$$\eta_{vol} = 1 - c\left\{\left(\frac{p_H}{p_L}\right)^{1/n} - 1\right\}$$

$$\eta_{vol} = 1 - c\left\{\left(\frac{p_H}{p_L}\right)^{1/n} - 1\right\} = 1 - 0.05\left\{(7)^{1/1.3} - 1\right\} = 82.7\%$$

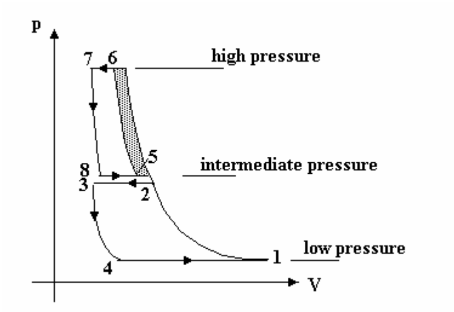
$$W = \left(\frac{n}{n-1}\right)p_1\left\{r^{\frac{n-1}{n}} - 1\right\}x F.A.D = \left(\frac{1.3}{1.3-1}\right)1x10^5\left\{7^{\frac{1.3-1}{1,3}} - 1\right\}[0.06] = 14.73\,kW$$

SELF ASSESSMENT EXERCISE No. 2

1. A single acting 2 stage compressor draws in 8.5 m^3/min of free air and compresses it to 40 bar. The compressor runs at 300 rev/min. The atmospheric conditions are 1.013 bar and 15°C. There is an intercooler between stages which cools the air back to 15°C. The polytropic index for all compressions is 1.3. The volumetric efficiency is 90% for the low pressure stage and 85% for the high pressure stage. Ignore the effect of the clearance volume. Calculate the following.

   The intermediate pressure for minimum indicated work. (6.365 bar)
   The theoretical indicated power for each stage. (32.85 kW)
   The heat rejected in each cylinder. (6.31 kW)
   The heat rejected by the intercooler. (26.53 kW)
   The swept volumes of both stages. (31.4 dm^3 and 5.3 dm^3)

   What advantage is there in using an after-cooler?
   State the effect on your answers of not ignoring the clearance volume and leakages.



$V = 8.5\ m^3/min = 0.14166\ m^3/s$ $N = 5$ rev/s

Induced volume $= V_1 – V_4$ $0.14166/5 = 0.02833\ m^3$/stroke

$k = \sqrt{(40/1.103)} = 6.283$

$p_1 = 1.013$ b $p_2 = 6.365$ bar

$$IP = \frac{n}{n-1} mR(T_2 - T_1)$$

$$T_2 = 288(6.283)^{\frac{0.3}{1.3}} = 440.1\,K$$

$$m = \frac{p_1V_1}{RT_1} = \frac{1.103x10^5\ x\ 0.14166}{287\ x\ 288} = 0.1736\ kg/s$$

Note that there is a conflict here. If the clearance volume is neglected then
$V_1 – V_4 = 90\%(V_1 – V_3)$ and $V_3 = 0$ so $V_1 = 0.03148\ m^3/s$ and this will give different answers.

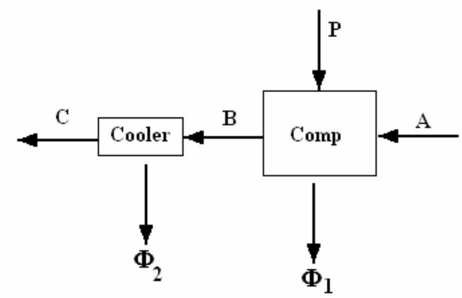
$$IP = \frac{1.3}{0.3} x0.1736\ x\ 287\ x\ (440.1 - T288) = 32.85\ kW$$

Apply the SFEE to the compressor

$h_A + P = h_B + \Phi_1$

$\Phi_1 = mc_p(T_A – T_B) + P$

$\Phi_1 = 0.1736\ x\ 1.005\ (288 – 440.1) + 32.85 = 6.31$ kW



Apply the SFEE to the whole system

$h_A + P - \Phi_1 – \Phi_2 = h_C$ $h_A = h_C$

$\Phi_2 = 32.85 – 6,31 = 26.53$ kW

LP CYLINDER

Vol/Stroke = 0.0283 m^3

η_{vol} = Actual volume/Swept Volume SV = 0.0283/0.9 = 0.0314 m^3

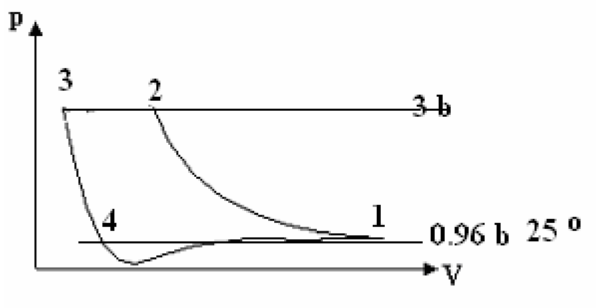
HP CYLINDER

$$\frac{p_1(V_1 - V_4)}{RT_1} = m = \frac{p_5(V_5 - V_8)}{RT_5}$$ But since $T_1 = T_5$ $$1.013\,x\,10^5\,\frac{0.02833}{6.365} = (V_5 - V_8)$$

$$(V_5 - V_8) = 0.004504\,m^3$$

SV = 0.004504/0.85 = 0.0053 m^3

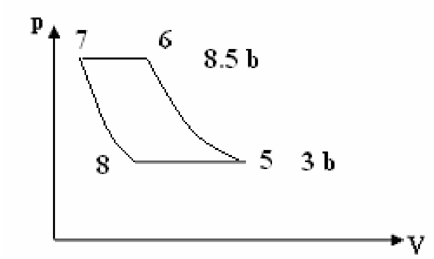
2. A single acting 2 stage compressor draws in free air and compresses it to 8.5 bar. The compressor runs at 600 rev/min. The atmospheric conditions are 1.013 bar and 15°C.
The interstage pressure is 3 bar and the intercooler cools the air back to 30°C. The polytropic index for all compressions is 1.28.
Due to the effect of warming from the cylinder walls and the pressure loss in the inlet valve, the pressure and temperature at the start of the low pressure compression stroke is 0.96 bar and 25°C. The high pressure cycle may be taken as ideal.
The clearance volume for each stages is 4% of the swept volume of that stage. The low pressure cylinder is 300 mm diameter and the stroke for both stages is 160 mm. Calculate the following.
The free air delivery. The volumetric efficiency of the low pressure stage.
The diameter of the high pressure cylinder. The indicated power for each stage.



LOW PRESSURE STAGE
$N = 600$ rev/min $D_1 = 300$ mm
$L_1 = L_2 = 160$ mm CV = 4% SV
SV = $\pi \times 0.3^2/4 \times 0.16$
SV = 0.01131 m^3 = 11.31 dm^3
CV = 4% x 11.31= 0.452 m^3
For an ideal cycle $V_1 - V_4$ = induced volume
Find V_4
$p_3V_3^{1.28} = p_4V_4^{1.28}$ $3 \times 0.452^{1.28} = 0.96 \times V_4^{1.28}$ $V_4 = 1.1$ dm^3
$V_1 - V_3 = 11.31$ $V_3 = 0.452$ $V_1 = 11.31 + 0.452 = 11.762$ dm^3 $V_1 - V_4 = 10.662$ dm^3
Change this to FAD (volume at 1.103 b and 15°C)
FAD/stroke = 10.662 x (288/298) x (0.96/1.013) = 9.765 dm^3/stroke
FAD = 9.765 x 600 = 5859 dm^3/min or 5.859 m^3/min
Ideal FAD = 11.31 x 600 = 6786 dm^3/min or 6.786 m^3/min
η_{vol} = 5.859/6.786 = 86.3% If we neglect the work of the induction stroke

$$W = \left(\frac{n}{n-1}\right)p_1\left\{r^{\frac{n-1}{n}} - 1\right\} x F.A.D = \left(\frac{1.28}{0.28}\right) x\ 0.96 \times 10^5 \left\{(3/0.96)^{\frac{1.28-1}{1.28}} - 1\right\} 0.010662 = 1324 \text{ J/cycle}$$

IP = W N/60 = 1324 x 600/60 = 13240 W or 13.24 kW
HIGH PRESSURE STAGE



Induced mass = mass of the FAD
m = pV/RT = $1.013 \times 10^5 \times 9.765 \times 10^{-3}/(283 \times 288)$
m = 0.01197 kg/stroke

$$T_6 = 303\left(\frac{8.5}{3}\right)^{\frac{0.28}{1.28}} = 380.5 \text{ K}$$

$p_5V_5^{1.28} = p_6V_6^{1.28}$ $V_6/V_5 = (3/8.5)1^{1/1.28} = 0.4432$ $V_6 = 0.4452\ V_5$ ....(A)
The mass expelled in process 6 to 7 is 0.01197 kg

$$0.01197 = \frac{p_6(V_6 - V_7)}{RT_6} = \frac{8.5 x 10^5 x (V_6 - V_7)}{287 \times 380.5}$$ $V_6 - V_7 = 0.001537$ m^3 = 1.537 dm^3.......(B)

$V_7 = 0.04(V_5 - V_7) = 0.04\ V_5 - 0.04\ V_7$ $1.04V_7 = 0.04\ V_5$ $V_5 = 26\ V_7$ ....(C)
Put (A) into (C) $V_6 = 0.4452 \times 26\ V_7 = 11.523\ V_7$ ....(D)
Put (D) into (B) $11.523\ V_7 - V_7 = 0.001537$ $V_7 = 0.146$ dm^3
Put V_7 into (C) $V_5 = 26 \times 0.146 = 3.798$ dm^3
Put V_5 into (A) $V_6 = 1.683$ dm^3
SV = $V_5 - V_7 = 0.00365$ m^3 = $\pi D^2L/4$ hence D = 0.17 m or 170 mm

$$V_8 = V_7\left(\frac{p_7}{p_8}\right)^{\frac{1}{1.28}} = 0.146\left(\frac{8.5}{3}\right)^{\frac{1}{1.28}} = 0.333\,dm^3$$

$$IP = \left(\frac{n}{n-1}\right)p_5\left\{r^{\frac{n-1}{n}} - 1\right\} x(V_5 - V_8)\frac{N}{60} = \left(\frac{1.28}{0.28}\right) x\ 3 \times 10^5 \left\{\left(\frac{8.5}{3}\right)^{\frac{1.28-1}{1.28}} - 1\right\} 0.003469 x \frac{600}{60} = 12.17 kW$$

3. A 2 stage reciprocating air compressor has an intercooler between stages. The induction and expulsion for both stages are at constant pressure and temperature. All the compressions and expansions are polytropic.

Neglecting the effect of the clearance volume show that the intermediate pressure, which gives minimum, indicated work is $p_M = (p_L p_H)^{½}$

Explain with the aid of a sketch how the delivery temperature from both cylinders varies with the intermediate pressure as it changes from p_L to p_H.

$W = W_1 + W_2$ where W_1 is the work done in the low pressure stage and W_2 is the work done in the high pressure stage.

$$W = \frac{mRn(T_2 - T_1)}{(n-1)} + \frac{mRn(T_6 - T_5)}{(n-1)} \qquad \text{Since } T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(1-1/n)} \text{ and } T_6 = T_5\left(\frac{p_6}{p_5}\right)^{(1-1/n)}$$

then assuming the same value of n for each stage

$$W = mR\left[\left\{\frac{nT_1}{(n-1)}\right\}\left\{\frac{p_2}{p_1}\right\}^{1-(1/n)} - 1\right] + mR\left[\left\{\frac{nT_6}{(n-1)}\right\}\left\{\frac{p_6}{p_5}\right\}^{1-(1/n)} - 1\right]$$

Since $p_2 = p_5 = p_m$ and $p_6 = p_H$ and $p_1 = p_L$

$$W = mR\left[\left\{\frac{nT_1}{(n-1)}\right\}\left\{\frac{p_M}{p_L}\right\}^{1-(1/n)} - 1\right] + mR\left[\left\{\frac{nT_6}{(n-1)}\right\}\left\{\frac{p_H}{p_M}\right\}^{1-(1/n)} - 1\right]$$

For a minimum value of W we differentiate with respect to p_M and equate to zero.

$$\frac{dW}{dp_M} = mRT_1 p_L^{(1-n)/n} p_M^{-1/n} \quad - mRT_5 p_H^{(n-1)/n} p_M^{(1-2n)/n}$$

If the intercooler returns the air to the original inlet temperature so that $T_1 = T_5$, then equating to zero reveals that for minimum work

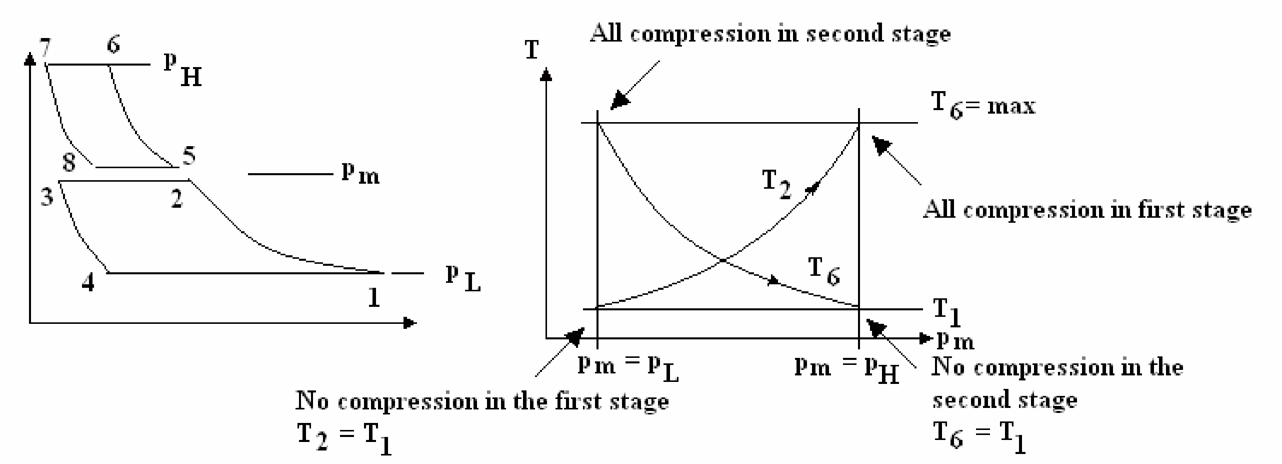
$$\mathbf{p_M = (p_L p_H)^{½}}$$

It can further be shown that when this is the case, the work done by both stages is equal.

The relationship between the temperatures and pressures are:

$T_2 = T_1 (p_m/p_L)^{1-1/n}$ $\qquad$ $T_6 = T_1 (p_H/p_m)^{1-1/n}$

This produces the plot shown.



4.a. Prove that the ideal volumetric efficiency of a single stage reciprocating compressor is

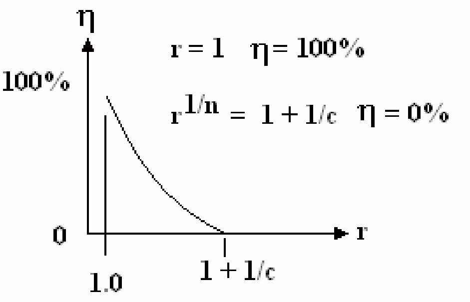
$$\eta_{vol} = 1 - c(r^{1/n}-1)$$

r is the pressure ratio, n is the polytropic index and c the clearance ratio.
Sketch curves of η_{vol} against r for typical values of n and c.

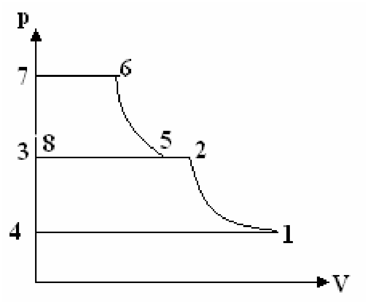
b. A two stage reciprocating air compressor works between pressure limits of 1 and 20 bar. The inlet temperature is 15°C and the polytropic index is 1.3. Intercooling between stages reduces the air temperature back to 15°C.
Find the free air delivery and mass of air that can be compressed per kW h of work input.

Find the ratio of the cylinder diameters if the piston have the same stroke. Neglect the effect of the clearance volume.

Part (a) Derivation as in Q3 The sketch is shown below.



Part (b)



$p_2 = \sqrt{(20 \times 1)} = 4.472$ bar
Since the work in both stages is equal

$$W = 2\left(\frac{n}{n-1}\right)p_1V_1\left\{r^{\frac{n-1}{n}} - 1\right\}$$

1 kWh = 100 x 60 x 60 = 3.6×10^6 J
$3.6 \times 10^6 = 357800V$ V = 10.06 m^3 per kWh

$$m = \frac{pV}{RT} = \frac{100 \times 10^3 \times 10.06}{287 \times 288} = 2.2513\ \text{kg/kWh}$$

$$V_5 = \frac{mRT_5}{p_5} = \frac{12.171 \times 287 \times 288}{4.47 \times 10^5} 2.2513\ \text{m}^3/\text{kWh}$$

V_5 = swept volume of HP stage $V_5 = V_1/4.472 = 2.25\ m^3$/kWh (confirmation)
V_1 = swept volume of LP stage

$$\frac{V_5}{V_1} = \frac{\pi d^2L/4}{\pi D^2L/4} = \frac{d^2}{D^2} = \frac{2.2513}{10.06} = 0.224 \qquad d/D = 0.473$$

SELF ASSESSMENT EXERCISE No. 3

1. Show that for any compression process the overall efficiency is given by $\eta_O = \dfrac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\frac{\gamma-1}{\gamma\eta_\infty}} - 1}$

where η_∞ is the polytropic efficiency.

Determine the index of compression for a gas with an adiabatic index of 1.4 and a polytropic efficiency of 0.9. (1.465)
Determine the overall efficiency when the pressure compression ratio is 4/1 and 8/1. (0.879 and 0.866)

Gas Laws plus Compression Laws are pV/T = V and $pV^\gamma = C$

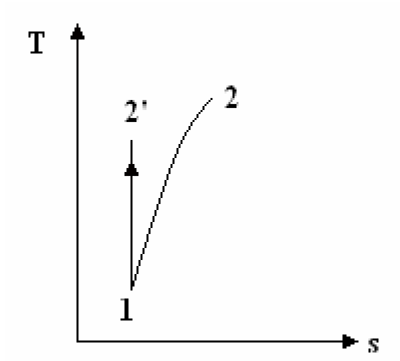
Combining these we have $T = C \text{ x } p^{1-1/\gamma}$ $\quad p^{1-1/\gamma} = \dfrac{T}{C}$

Differentiate $dT = C\left(\dfrac{\gamma-1}{\gamma}\right)p^{-1/\gamma}dp$ Next divide by p $\quad \dfrac{dT}{p^{1-1/\gamma}} = C\left(\dfrac{\gamma-1}{\gamma}\right)\dfrac{dp}{p}$

Substitute $p^{1-1/\gamma} = \dfrac{T}{C}$ $\quad \dfrac{dT}{T} = \left(\dfrac{\gamma-1}{\gamma}\right)\dfrac{dp}{p}$

For an isentropic process designate the final temperature as T' and the differential as dT'

$$\frac{dT'}{T} = \left(\frac{\gamma-1}{\gamma}\right)\frac{dp}{p}\text{...........(1)}$$



Isentropic Efficiency is defined as

$\eta_{is} = \dfrac{T_{2'} - T_1}{T_2 - T_1}$ Let the change be infinitesimally small.

$T_2 = T_1 + dT$ and $T_{2'} = T_1 + dT'$ $\quad \eta_\infty = \dfrac{dT_{2'}}{dT}$

Suppose the compression is made from many tiny step changes.

$\eta_\infty = \dfrac{dT'}{dT}$ $\quad dT' = \eta_\infty \dfrac{dT}{T}$ Substitute (1) into this.

$\left(\dfrac{\gamma-1}{\gamma}\right)\dfrac{dp}{p} = \eta_\infty \dfrac{dT}{T}$ Integrate and $\left(\dfrac{\gamma-1}{\gamma}\right)\displaystyle\int_{p_1}^{p_2}\dfrac{dp}{p} = \eta_\infty \int_{T_1}^{T_2}\dfrac{dT}{T}$

$\left(\dfrac{\gamma-1}{\gamma}\right)[\ln p]_{p_1}^{p_2} = \eta_\infty [\ln T]_{T_1}^{T_2}$ $\quad \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\dfrac{T_2}{T_1}\right)^{\frac{\gamma-1}{\gamma}}$ $\quad \dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\eta_\infty}}$

η_∞ is called the Polytropic Efficiency $\quad$ The overall efficiency is $\eta_o = \dfrac{T_{2'} - T_1}{T_2 - T_1}$

adiabatic process $\quad T_{2'} = T_1 r^{\frac{\gamma-1}{\gamma}}$ $\quad$ polytropic process $T_2 = T_1 r^{\frac{n-1}{n}}$

Substitution gives $\quad \eta_O = \dfrac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\frac{\gamma-1}{\gamma\eta_\infty}} - 1}$

Compare $\dfrac{T_2}{T_1} = r^{\frac{n-1}{n}} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\eta_\infty}}$ and it follows that $\eta_\infty = 1$

Now put $\eta_\infty = 0.9$ and $\gamma = 1.4$ $\quad \dfrac{n-1}{n} = \dfrac{\gamma-1}{\gamma\eta_\infty} = \dfrac{1.4-1}{1.4 \text{ x } 0.9} = 0.3175$

$n - 1 = 0.3175n \qquad 0.6825\,n = 1 \quad n = 1.465$

Now put r = 4

$$\frac{\gamma - 1}{\gamma} = 0.2857 \quad \frac{n-1}{n} = 0.3175$$

$$\eta_O = \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\frac{\gamma-1}{\gamma\eta_\infty}} - 1} = \frac{4^{0.2857} - 1}{4^{0.3175} - 1} = 0.879$$

Now put r = 8

$$\eta_O = \frac{8^{0.2857} - 1}{8^{0.3175} - 1} = 0.866$$

2. A compressor draws in air at 223.3 K temperature and 0.265 bar pressure. The compression ratio is 6. The polytropic efficiency is 0.86. Determine the temperature after compression. Take $\gamma = 1.4$

From the previous question we have

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma\eta_\infty}} = 222.3 (6)^{\frac{1.4-1}{1.4 \text{ x } 0.86}} = 405 \text{ K} \quad 1$$

APPLIED THERMODYNAMICS D201

SELF ASSESSMENT SOLUTIONS
TUTORIAL 3
SELF ASSESSMENT EXERCISE No. 1

1. A gas turbine expands 6 kg/s of air from 8 bar and 700ºC to 1 bar isentropically. Calculate the exhaust temperature and the power output. $\gamma = 1.4$ $c_p = 1005$ J/kg K

$T_2 = T_1\,(1/12)^{1-1/1.4} = 973\,(1/8)^{0.2958} = 537.1$ K
$P = \Delta H/s = m\,c_p\,\Delta T = 6 \times 1005 \times (537.1 - 973)$
$P = -2.628 \times 10^6$ W (Leaving the system) **P(out) = 2.628 MW**

2. A gas turbine expands 3 kg/s of air from 10 bar and 920ºC to 1 bar adiabatically with an isentropic efficiency of 92%. Calculate the exhaust temperature and the power output.
$\gamma = 1.41$ $c_p = 1010$ J/kg K

$T_{2'} = T_1\,(1/12)^{1-1/1.4} = 1193\,(1/10)^{0.2958} = 610.7$ K
P(out) = $\eta\,m\,c_p\,\Delta T = 0.92 \times 3 \times 1.01 \times (1610.7 - 1193)$
P(out) = 1.682×10^6 W **P(out) = 1.682 MW**

3. A gas turbine expands 7 kg/s of air from 9 bar and 850ºC to 1 bar adiabatically with an isentropic efficiency of 87%. Calculate the exhaust temperature and the power output.
$\gamma = 1.4$ $c_p = 1005$ J/kg K

$T_{2'} = T_1\,(1/12)^{1-1/1.4} = 1123\,(1/9)^{0.2958} = 599.4$ K
P(out) = $\eta\,m\,c_p\,\Delta T = 0.87 \times 7 \times 1.005 \times (1123 – 599.4)$
P(out) = 3.2×10^6 W
$3.2 \times 10^6 = 7 \times 1.005 \times (1123 – T_2)$ $T_2 = 667.5$ K (The true temperature)

SELF ASSESSMENT EXERCISE No.2

A gas turbine uses the Joule cycle. The inlet pressure and temperature to the compressor are respectively 1 bar and -10ºC. After constant pressure heating, the pressure and temperature are 7 bar and 700ºC respectively. The flow rate of air is 0.4 kg/s. Calculate the following.

1. The cycle efficiency.
2. The heat transfer into the heater.
3. the net power output.

$\gamma = 1.4$ $c_p = 1.005$ kJ/kg K
(Answers 42.7 % , 206.7 kW and 88.26 kW)

$\eta = 1 – r_p^{\,1/\gamma -1} = 1 – 7^{-0.286} = 42.7\%$

$T_2 = 263 \times 7^{0.286} = 459$ K $T_3 = 973$ K

$\Phi = m\,c_p\,\Delta T = 0.4 \times 1.005 \times (973 – 459) = 206.7$ kW (into the system)

P(net) = $\eta \times \Phi$in = 0.427 x 206.7= 88.26 kW

SELF ASSESSMENT EXERCISE No. 3

A gas turbine uses a standard Joule cycle but there is friction in the compressor and turbine. The air is drawn into the compressor at 1 bar 15°C and is compressed with an isentropic efficiency of 94% to a pressure of 9 bar. After heating, the gas temperature is 1000°C. The isentropic efficiency of the turbine is also 94%. The mass flow rate is 2.1 kg/s. Determine the following.

1. The net power output.
2. The thermal efficiency of the plant.

$\gamma = 1.4$ and $c_p = 1.005$ kJ/kg K.

COMPRESSOR $T_{2'} = 288 \times 9^{0.286} = 539.9$ K $\eta_{IS} = 0.94 = \frac{T_{2'} - T_1}{T_2 - T_1} = \frac{539.9 - 288}{T_2 - 288}$ $T_2 = 556\,\text{K}$

TURBINE $T_{4'} = 1273 \times 9^{-0.286} = 679.1$ K $\eta_{IS} = 0.94 = \frac{T_3 - T_4}{T_3 - T_{4'}} = \frac{1273 - T_4}{1273 - 679.1}$ $T_4 = 714.7\,\text{K}$

HEAT REJECTED $\Phi(\text{out}) = m\, c_p\, \Delta T = 2.1 \times 1.005 \times (714.7 - 288) = 900$ kW

HEAT SUPPLY $\Phi(\text{in}) = m\, c_p\, \Delta T = 2.1 \times 1.005 \times (1273 - 679.1) = 1513$ kW

THERMAL EFFICIENCY $\eta_{th} = 1 - \frac{\Phi(\text{out})}{\Phi(\text{in})} = 1 - \frac{900}{1513} = 40.5\%$

Net Power P(net) = $\Phi(\text{in}) - \Phi(\text{out}) = 613.2$ kW

You could calculate the turbine and compressor powers and obtain the same answer.

SELF ASSESSMENT EXERCISE No. 4

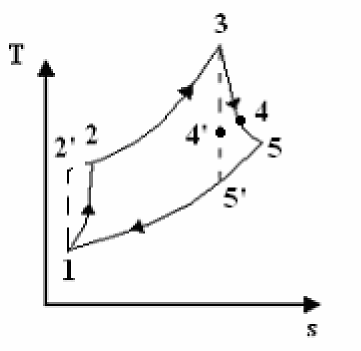
A gas turbine draws in air from atmosphere at 1 bar and 15°C and compresses it to 4.5 bar with an isentropic efficiency of 82%. The air is heated to 1100 K at constant pressure and then expanded through two stages in series back to 1 bar. The high pressure turbine is connected to the compressor and produces just enough power to drive it. The low pressure stage is connected to an external load and produces 100 kW of power. The isentropic efficiency is 85% for both stages.

For the compressor $\gamma = 1.4$ and for the turbines $\gamma = 1.3$. The gas constant R is 0.287 kJ/kg K for both.

Neglect the increase in mass due to the addition of fuel for burning.

Calculate the mass flow of air, the inter-stage pressure of the turbines and the thermal efficiency of the cycle.

(Answers 0.642 kg/s and 20.1 %)



COMPRESSOR $T_{2'} = 288 \times 4.5^{0.286} = 442.8$ K

$$\eta_{IS} = 0.82 = \frac{T_{2'} - T_1}{T_2 - T_1} = \frac{442.8 - 288}{T_2 - 288} \quad T_2 = 476.8\,\text{K}$$

$c_p = R/(1-1/\gamma) = 287/(1-1/1.4) = 1005$ J/kg K

P(in) = $m\, c_p\, \Delta T = m \times 1.005\,(476.8 - 288) = 189.65m$ kW

HP TURBINE $c_p = R/(1-1/\gamma) = 287/(1-1/1.3) = 1243.7$ J/kg K

P(out) = $m\, c_p\, \Delta T = m \times 1.243\,(1100 - T_4) = 189.65m$ kW

Hence $T_4 = 947.5$ K

$$\eta_{IS} = 0.85 = \frac{T_3 - T_4}{T_3 - T_{4'}} = \frac{1100 - 947.5}{1100 - T_{4'}} \quad T_{4'} = 920.6\,\text{K}$$

$$\frac{T_3}{T_{4'}} = \frac{1100}{920.6} = \left(\frac{p_3}{p_4}\right)^{0.2307} = \left(\frac{4.5}{p_4}\right)^{0.2307} \quad p_4 = 2.08\,\text{bar}$$

L.P. TURBINE $\frac{947.5}{T_{5'}} = \left(\frac{2.08}{1}\right)^{0.2307}$ $T_{5'} = 800.2$ K $\eta_{IS} = 0.85 = \frac{947.5 - T_5}{947.5 - 800.2}$ $T_5 = 822.3\,\text{K}$

P(out) = 100 kW = $m \times 1.243\,(947.5 - 822.3)$ m = 0.6422 kg/s

HEAT SUPPLY $\Phi(\text{in}) = m\, c_p\, \Delta T = 0.6422 \times 1.243 \times (1100 - 476.8) = 497.7$ kW

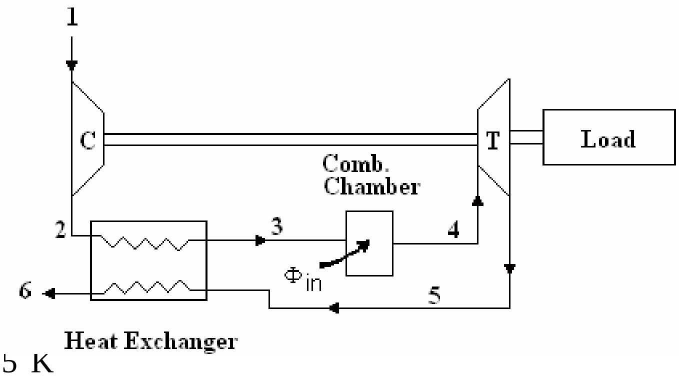
THERMAL EFFICIENCY $\eta_{th} = \frac{\text{P)(net)}}{\Phi(\text{in})} = 497.7 = 20.1\%$

SELF ASSESSMENT EXERCISE No. 5

1. A gas turbine uses a pressure ratio of 7/1. The inlet temperature and pressure are respectively 10^oC and 100 kPa. The temperature after heating in the combustion chamber is 1000 ^oC. The specific heat capacity c_p is 1.005 kJ/kg K and the adiabatic index is 1.4 for air and gas. Assume isentropic compression and expansion. The mass flow rate is 0.7 kg/s.

   Calculate the net power output and the thermal efficiency when an exhaust heat exchanger with a thermal ratio of 0.8 is used.

   Referring to the numbers used on diagram the solution is as follows.



$T_2 = T_1 r_p^{(1-1/\gamma)} = 283\ (7)^{0.286} = 493.4$ K

$T_5 = T_4/r_p^{(1-1/\gamma)} = 1273/(7)^{0.25} = 730$ K

Use the thermal ratio to find T_3.

$$0.8 = \frac{(T_3 - T_2)}{(T_5 - T_2)} = \frac{(T_3 - 493.4)}{(730 - 493.4)} \qquad T_3 = 682.5\ \text{K}$$

$\Phi(in) = m\, c_{pg}\, (T_4\text{-}T_3) = 0.7 \times 1.005\ (1273\text{-}682.5) = 415$ kW

In order find the thermal efficiency, it is best to solve the energy transfers.

$P(in) = mc_{pa}(T_2\text{-}T_1) = 0.7 \times 1.005\ (493.4\text{-}283) = 148$ kW
$P(out) = mc_{pg}(T_4\text{-}T_5) = 0.7 \times 1.005\ (1273\text{-}730) = 382$ kW
P(nett) = P(out) - P(in) = 234 kW
η_{th} = P(nett)/Φ(in) = 234/415 = **0.56 or 56%**

2. A gas turbine uses a pressure ratio of 6.5/1. The inlet temperature and pressure are respectively 15^oC and 1 bar. The temperature after heating in the combustion chamber is 1200 ^oC. The specific heat capacity c_p for air is 1.005 kJ/kg K and for the exhaust gas is 1.15 kJ/kg K. The adiabatic index is 1.4 for air and 1.333 for the gas. The isentropic efficiency is 85% for both the compression and expansion process. The mass flow rate is 1kg/s.
Calculate the thermal efficiency when an exhaust heat exchanger with a thermal ratio of 0.75 is used.Refer to the same circuit diagram as Q1.

COMPRESSOR $T_{2'} = 288 \times 6.5^{0.286} = 491.6$ K $\eta_{IS} = 0.85 = \dfrac{T_{2'} - T_1}{T_2 - T_1} = \dfrac{491.6 - 288}{T_2 - 288}$ $T_2 = 527.5$ K

TURBINE $T_{5'} = 1473 \times 6.5^{-0.286} = 922.8$ K $\eta_{IS} = 0.85 = \dfrac{T_4 - T_5}{T4 - T_{5'}} = \dfrac{1473 - T_5}{1473 - 922.8}$ $T_5 = 1005$ K

Use the thermal ratio to find T_3. $T_2 = T_6$

$$0.75 = \frac{1.005(T_3 - T_2)}{1.15(T_5 - T_2)} = \frac{1.005(T_3 - 527.5)}{1.15(1005 - 527.5)} \qquad T_3 = 937.3\ \text{K}$$

$P(in) = mc_{pa}(T_2\text{-}T_1) = 1 \times 1.005\ (527.5\text{-}288) = 240.7$ kW
$P(out) = mc_{pg}(T_4\text{-}T_5) = 1 \times 1.15\ (1473\text{-}1005) = 538.2$ kW
P(nett) = P(out) - P(in) = 297.5 kW
$\Phi(in) = mc_{pg}(T_4\text{-}T_3) = 1 \times 1.15(1473\text{-}37.3) = 616$ kW

η_{th} = P(nett)/Φ(in) = 297.5/616 = **0.483 or 48.3%**

SELF ASSESSMENT EXERCISE No. 6

1. List the relative advantages of open and closed cycle gas turbine engines.

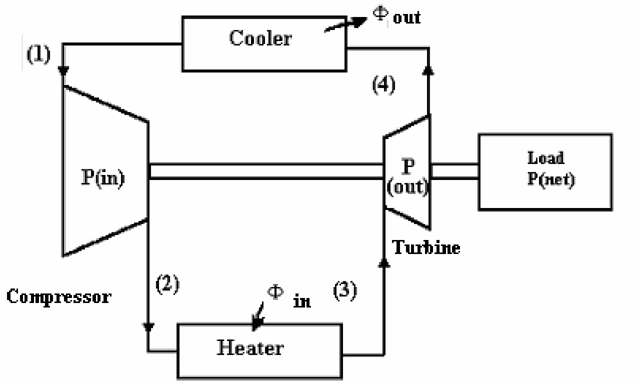
   Sketch the simple gas turbine cycle on a T-s diagram. Explain how the efficiency can be improved by the inclusion of a heat exchanger.

   In an open cycle gas turbine plant, air is compressed from 1 bar and 15°C to 4 bar. The combustion gases enter the turbine at 800°C and after expansion pass through a heat exchanger in which the compressor delivery temperature is raised by 75% of the maximum possible rise. The exhaust gases leave the exchanger at 1 bar. Neglecting transmission losses in the combustion chamber and heat exchanger, and differences in compressor and turbine mass flow rates, find the following.
   (i) The specific work output.
   (ii) The work ratio
   (iii) The cycle efficiency
   The compressor and turbine polytropic efficiencies are both 0.84.
   Compressor $c_p = 1.005$ kJ/kg K $\gamma = 1.4$
   Turbine $c_p = 1.148$ kJ/kg K $\gamma = 1.333$

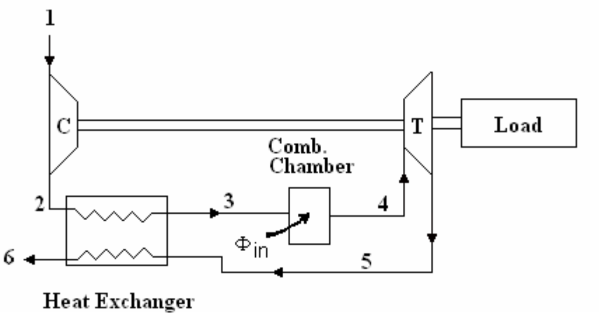
   Note for a compression $T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\eta_\infty}}$ and for an expansion $T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{(\gamma-1)\eta_\infty}{\gamma}}$

An open cycle can burn any convenient fuel in the air and requires simpler plant.

A closed cycle may use a gas with a higher specific heat capacity than air, such as helium and so produce a better performance. The fluid must be contained so heat must be supplied with a heat exchanger instead of a combustion chamber and this produces limitations.



In a simple gas turbine, the temperature at point (4) is usually hotter than the temperature at point (2) so it is possible to reduce the heat supply to the heater or combustion chamber by transferring heat from point (4) to point (3).



$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\eta_\infty}} = 288(4)^{\frac{1.4-1}{1.4 \times 0.84}} = 461.5\text{ K}$$

$$T_5 = T_4\left(\frac{p_5}{p_4}\right)^{\frac{(\gamma-1)\eta_\infty}{\gamma}} = 1073\left(\frac{1}{4}\right)^{\frac{(1.33-1)0.84}{1.33}} = 803.7\text{K}$$

HEAT EXCHANGER

$$0.75 = \frac{1.005(T_3 - T_2)}{1.148(T_5 - T_2)} = \frac{1.005(T_3 - 461.6)}{1.148(802.6 - 461.5)} \qquad T_3 = 754.7\text{ K}$$

P(in)= $mc_{pa}(T_2-T_1)$ = 1 x 1.005 (461.5-288) = 174.4 kW
P(out) = $mc_{pg}(T_4-T_5)$ = 1 x 1.148 (1073 - 802.6) = 310 kW
P(nett) = P(out) - P(in) = 135 kW

Φ(in) = $mc_{pg}(T_4-T_3)$ = 1 x 1.148(1073-754.7) = 365 kW

Work Ratio = P(nett)/P(in) = 135/310 = 0.44
η_{th} = P(nett)/Φ(in) = 135/365 = **0.371 or 37.1%**

2. A gas turbine for aircraft propulsion is mounted on a test bed. Air at 1 bar and 293K enters the compressor at low velocity and is compressed through a pressure ratio of 4 with an isentropic efficiency of 85%. The air then passes to a combustion chamber where it is heated to 1175 K. The hot gas then expands through a turbine which drives the compressor and has an isentropic efficiency of 87%. The gas is then further expanded isentropically through a nozzle leaving at the speed of sound. The exit area of the nozzle is 0.1 m^2. Determine the following.

(i) The pressures at the turbine and nozzle outlets.

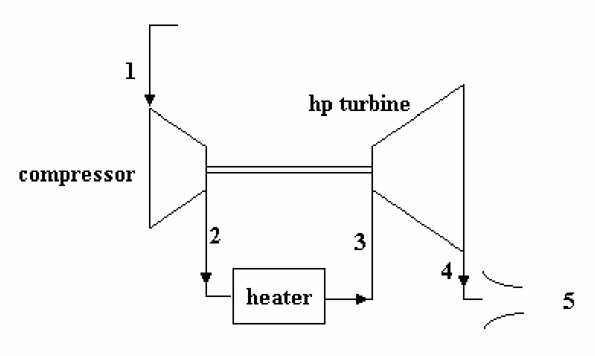
(ii) The mass flow rate.

(iii) The thrust on the engine mountings.

Assume the properties of air throughout.

The sonic velocity of air is given by $a = (\gamma RT)^{1/2}$.

The temperature ratio before and after the nozzle is given by T(in)/T(out) = 2/(γ+1)



$$T_{2'} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\eta_\infty}} = 293(4)^{\frac{1.4-1}{1.4 \times 0.84}} = 435.4\,\text{K}$$

$$\eta_{IS} = 0.85 = \frac{T_{2'} - T_1}{T_2 - T_1} = \frac{435.4 - 293}{T_2 - 293} \quad T_2 = 460.5\,\text{K}$$

P(in) = $mc_p(T_2 - T_1)$ = 1 x 1.005 (167.5)

P(out) = P(in) = $mc_p(T_3 - T_4)$ hence

$(T_3 - T_4)$ =167.5 and T_4 = 1175 – 167.5 = 1007.5 K

$$\eta_{IS} = 0.85 = \frac{T_3 - T_4}{T_3 - T_{4'}} = \frac{1175 - 1007.5}{1175 - T_{4'}} \quad T_{4'} = 982.5\,\text{K}$$

$$T_{4'} = 1175(r)^{\frac{\gamma-1}{\gamma}} = 1175(r)^{\frac{1.4-1}{1.4 \times 0.84}} \qquad r = 0.5346 = \frac{p_{4'}}{p_3} = \frac{p_{4'}}{4} \qquad p_{4'} = p_4 = 2.138\,\text{bar}$$

$$\frac{T_5}{T_4} = \frac{2}{\gamma+1} = 0.833 \qquad T_5 = 839.6\,\text{K}$$

$$\frac{839.6}{1007.5} = 0.833 = \left(\frac{p_5}{4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_5}{4}\right)^{0.286} \qquad p_5 = 1.13\,\text{bar}$$

When the nozzle is choked, $v = \sqrt{(\gamma\, \rho\, T)} = \sqrt{(1.4 \times 287 \times 839.6)} = 580.8$ m/s

Volume flow rate = V = A v = 0.1 x 580.8 = 58.08 m^3/s

m = pV/RT = 1.13 x 10^5 x 55.08/(287 x 839.6) = 27.23 kg/s

F = A Δp + m Δv = 0.1 x 0.13 x 10^5 + 27.23 x 580.8 = 17 kN

3. (A). A gas turbine plant operates with a pressure ratio of 6 and a turbine inlet temperature of 927°C. The compressor inlet temperature is 27°C. The isentropic efficiency of the compressor is 84% and of the turbine 90%. Making sensible assumptions, calculate the following.

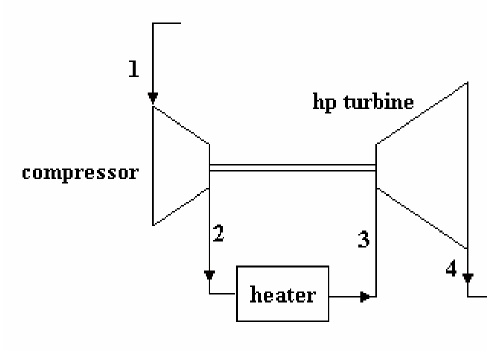
   (i) The thermal efficiency of the plant.
   (ii) The work ratio.

   Treat the gas as air throughout.

   (B). If a heat exchanger is incorporated in the plant, calculate the maximum possible efficiency which could be achieved assuming no other conditions are changed.

   Explain why the actual efficiency is less than that predicted.

PART A



$$T_{2'} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 300(6)^{\frac{1.4-1}{1.4 \times 0.84}} = 500.8\,\mathrm{K}$$

$$\eta_{IS} = 0.84 = \frac{T_{2'} - T_1}{T_2 - T_1} = \frac{500.8 - 300}{T_2 - 300} \quad T_2 = 539\,\mathrm{K}$$

$$T_{4'} = 1200(1/6)^{\frac{\gamma-1}{\gamma}} = 1200(1/6)^{0.286} = 719\,\mathrm{K}$$

$$\eta_{IS} = 0.9 = \frac{T_3 - T_4}{T_3 - T_{4'}} = \frac{1200 - T_4}{1200 - 719} \quad T_4 = 767\,\mathrm{K}$$

$\Phi(\mathrm{in}) = mc_{pg}(T_3 - T_2)$ = m x 1.005(1200 - 539) = 664m kW

$\Phi(\mathrm{out}) = mc_{pg}(T_4 - T_1)$ = m x 1.005(767 - 300) = 469.3m kW

$$\eta_{th} = 1 - \frac{\Phi(\mathrm{out})}{\Phi(\mathrm{in})} = 1 - \frac{469.3\mathrm{m}}{664\mathrm{m}} = 0.293 \text{ or } 29.3\%$$

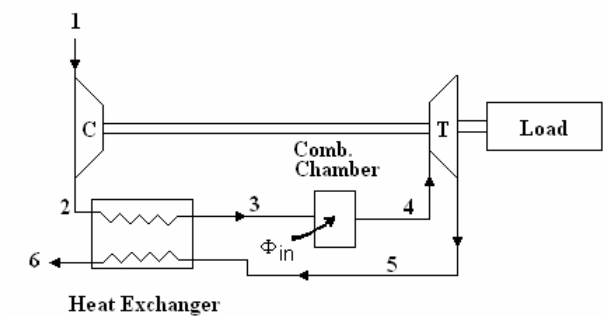
P(in)= $mc_{pa}(T_2 - T_1)$ = m x 1.005 (539 - 300) = 340.2 m kW

P(out) = $mc_{pg}(T_3 - T_4)$ = m x 1.005 (1200 - 767) = 435.2 m kW

P(nett) = P(out) - P(in) = 95 kW

Work Ratio = P(nett)/P(in) = 95/340.2 = 0.279

PART B



HEAT EXCHANGER

For max possible efficiency the thermal ratio is 1 hence

$$1 = \frac{(T_3 - T_2)}{(T_5 - T_2)}$$

$T_5 = T_3 = 767$ K $T_6 = T_2 = 539$ K

$\Phi(\mathrm{in}) = mc_{pg}(T_4 - T_3)$ = m x 1.005(1200 - 300) = 435.2 m kW

$\Phi(\mathrm{out}) = mc_{pg}(T_6 - T_1)$ = m x 1.005(539 - 300) = 240.2 m kW

$$\eta_{th} = 1 - \frac{\Phi(\mathrm{out})}{\Phi(\mathrm{in})} = 1 - \frac{240.2\,\mathrm{m}}{435.2\,\mathrm{m}} = 0.448 \text{ or } 44.8\%$$

In practical cases $T_6 > T_2$ and $T_5 > T_2$so a smaller efficiency is obtained.

APPLIED THERMODYNAMICS D201

SELF ASSESSMENT SOLUTIONS
TUTORIAL 4

SELF ASSESSMENT EXERCISE No. 1

1. A 4 stroke carburetted engine runs at 3 000 rev/min. The engine capacity is 4 litres. The air is supplied at 0.7 bar and 10°C with an efficiency ratio of 0.5. The air fuel ratio is 13/1. The calorific value is 45 MJ/kg. Calculate the heat released by combustion. (149 KW)

Induced volume = (3000/2) x 0.004 = 6 m^3/min or 0.1 m^3/s
Actual volume = 0.1 x 0.5 = 0.05 m^3/s
Mass = pV/RT = 0.7 x 10^5 x 0.05/(287 x 283) = 0.0431 kg/s
Fuel used = 0.0431/13 = 0.00331 kg/s
Heat released = 0.00331 x 45 000 = 149.2 kW

SELF ASSESSMENT EXERCISE No.2

1. A 4 stroke spark ignition engine gave the following results during a test.

Number of cylinders	6
Bore of cylinders	90 mm
Stroke	80 mm
Speed	5000 rev/min
Fuel consumption rate	0.3 dm^3/min
Fuel density	750 kg/m^3
Calorific value	44 MJ/kg
Nett brake load	180 N
Torque arm	0.5 m
Nett indicated area	720 mm^2
Base length of indicator diagram	60 mm
Pressure scale	40 kPa/mm

Calculate

i) the Brake Power. (47.12 kW)
ii) the Mean effective Pressure. ((480 kPa).
iii) the Indicated Power. (61 kW).
iv) the Mechanical Efficiency. (77.2%).
v) the Brake Thermal efficiency. (28.6 %).

BP = 2π(5000/60)x 180 x 0.5 = 47.12 kW
MEP = (720/60) x 40 = 480 kPa
IP = PLAN = 480 x 0.08 x (π x 0.09^2/4) x (5000/60) x (6/2) = 61 kW
Fuel flow rate = 0.3 l/min = 0.00036 m^3/min
Mass flow rate = (0.0003/60) x 750 = 0.00375 kg/s
FP = 0.00375 x 44000 = 165 kW
η_m = 47.12/61.07 = 77.1%
η_{BTh} = 47.12/165 = 28.6%

2. A two stroke spark ignition engine gave the following results during a test.

Number of cylinders	4
Bore of cylinders	100 mm
Stroke	100 mm
Speed	2000 rev/min
Fuel consumption rate	5 g/s
Calorific value	46 MJ/kg
Net brake load	500 N
Torque arm	0.5 m
Net indicated area	1 500 mm^2
Base length of indicator diagram	66 mm
Pressure scale	25 kPa/mm

Calculate i) the Indicated thermal efficiency. (26.3 %)
ii) the Mechanical Efficiency. (87%).
iii) the Brake Thermal efficiency. (22.8%).

MEP = (1500/66) x 25 = 568 kPa
IP = PLAN = 568 x 0.1 x (π x 0.1^2/4) x (2000/60) x 4 = 59.5 kW
FP = 0.005 x 46000 = 230 kW
BP = 2πNT = 2π(3000/60)x 500 x 0.5 = 52.4 kW
η_{ITh} = 59.5/230= 26%
η_m = 52.4/59.5 = 88%

3. A two stroke spark ignition engine gave the following results during a test.

Number of cylinders	4
Bore of cylinders	80 mm
Stroke	80 mm
Speed	2 200 rev/min
Fuel consumption rate	1.6 cm^3/s
Fuel density	750 kg/m^3
Calorific value	60 MJ/kg
Nett brake load	195 N
Torque arm	0.4 m
Nett indicated area	300 mm^2
Base length of indicator diagram	40.2 mm
Pressure scale	50 kPa/mm

Calculate i) the Indicated thermal efficiency. (30.5 %)
ii) the Mechanical Efficiency. (81.7%).
iii) the Brake Thermal efficiency. (25%).

BP = 2πNT = 2π(2200/60)x 195 x 0.4 = 17.9 kW
FP = 1.6 x 10^{-6} x 750 x 60 000 = 72 kW
MEP = (300/40.2) x 50 = 373.1 kPa
IP = PLAN = 373.1 x 0.08 x (π x 0.08^2/4) x (2200/60) x 4 = 22 kW
η_{ITh} = 22/72= 30.5%
η_m = 17.9/22 = 81.4%
η_{BTh} = 17.9/72= 24.9%

4. A four stroke spark ignition engine gave the following results during a test.

Number of cylinders	4
Bore of cylinders	90 mm
Stroke	80 mm
Speed	5 000 rev/min
Fuel consumption rate	0.09 kg/min
Calorific value	44 MJ/kg
Nett brake load	60 N
Torque arm	0.5 m
MEP	280 kPa

Calculate i) the Mechanical Efficiency. (66.1%).
ii) the Brake Thermal efficiency. (23.8%).

BP = $2\pi(5000/60) \times 60 \times 0.5 = 15.7$ kW
FP= $(0.09/60) \times 44000 = 66$ kW
MEP = 280 kPa
IP = PLAN = $280 \times 0.08 \times (\pi \times 0.09^2/4) \times (5000/60) \times (4/2) = 23.75$ kW
$\eta_m = 15.7/23.75 = 66.1\%$
$\eta_{BTh} = 15.7/66 = 23.8\%$

5. Define Indicated Mean Effective Pressure and Brake Mean Effective Pressure.

The BMEP for a 4 cylinder, 4 stroke spark ignition engine is 8.4 bar. The total capacity is 1.3 dm^3 (litres). The engine is run at 4 200 rev/min.

Calculate the Brake Power. (38.22 kW)

There are 10 kW of mechanical losses in the engine.

Calculate the Indicated Mean effective Pressure. (10.6 bar).

The Volumetric Efficiency is 85% and the Brake Thermal Efficiency of the engine is 28%. The air drawn in to the engine is at 5^{o}C and 1.01 bar. The fuel has a calorific value of 43.5 MJ/kg.
Calculate the air/fuel ratio. (12.3/1).

MEP is based on indicated power.
BMEP is based on brake power.
BP = (BMEP)(LA)N/2 = $8.5 \times 10^5 \times 1.3 \times 10^{-3} \times (4200/60)(1/2) = 38.22$ kW
IP = 38.22 + 10 = 48.22
IMEP = IP x 60 x 2/LAN = $48220 \times 60 \times 2 /(1.3 \times 10^{-3} \times 4200) = 10.6$ bar
$\eta_{BTh} = 0.28$ = BP/FP
Q(in) – 38.22/0.28 = 136.5 kJ
Fuel Heat released = m x 43500 m = 136.5/43500 = 0.00313 kg/s
Swept Volume = 1.3 litre $\eta_{vol} = 85\%$
Actual air = 0.85 x 1.3 = 1.105 litre
Air flow = 0.0011 x 4200/(60 x 2) = 0.038675 m^3/s
Mass flow = pV/RT = $1.01 \times 10^5 \times 0.038675/(287 \times 278) = 0.04874$ kg/s
Air/Fuel ratio = 0.04874/0.00313 = 15.6/1

SELF ASSESSMENT EXERCISE No.3

Take Cv = 0.718 kJ/kg K, R = 287 J/kg K and γ= 1.4 throughout.

1. In an Otto cycle air is drawn in at 20°C. The maximum cycle temperature is 1500°C. The volume compression ratio is 8/1. Calculate the following.

   i. The thermal efficiency. (56.5%)
   ii. The heat input per kg of air. (789 kJ/kg).
   iii. The net work output per kg of air. (446 kJ/kg).

$$\eta = 1 - r_v^{-0.4} = 1 - 8^{-0.4} = 56.5\%$$

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-1/\gamma} = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 193 \text{ x } 8^{0.4} = 673.1\,\text{K}$$

Q(in) = 1 x 0.718 x (1773 – 673.1) = 789 kJ/kg
W(net) = η x Q(in) = 0.565 x 789 = 446 kJ/kg

2. An Otto cycle has a volume compression ratio of 9/1. The heat input is 500kJ/kg. At the start of compression the pressure and temperature are 100 kPa and 40°C respectively. Calculate the following.

   i. The thermal efficiency. ((58.5%)
   ii. The maximum cycle temperature. (1450 K).
   iii. The maximum pressure. (4.17 MPa).
   iv. The net work output per kg of air. (293 kJ/kg).

$$\eta = 1 - r_v^{-0.4} = 1 - 9^{-0.4} = 58.5\%$$

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-1/\gamma} = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 313 \text{ x } 9^{0.4} = 753.8\,\text{K}$$

Q(in) = 500 = 1 x 0.718 x (T_3 – 673.1) $\qquad$ T_3 = 1450 K

$$p_3 = \left(\frac{p_1 V_1 T_3}{T_1 V_3}\right) = \frac{100 \text{ x } 9 \text{ x } 1450}{313} = 4169\,\text{kPa}$$

W(net) = η x Q(in) = 0.585 x 500 = 292.5 kJ/kg

3. Calculate the volume compression ratio required of an Otto cycle which will produce an efficiency of 60%. (9.88/1)

The pressure and temperature before compression are 105 kPa and 25ºC respectively. The net work output is 500 kJ/kg). Calculate the following.

i. The heat input. (833 kJ/kg).
ii. The maximum temperature. (1 906 K)
iii. The maximum pressure. (6.64 MPa).

$\eta = 0.6 = 1 - r^{-0.4} \quad 0.4 = r^{-0.4} \quad r = 9.882$

Q(in)= 500/0.6 = 833.3 kJ/kg

$T_2 = 298(9.882)^{0.4} = 745\ K$

$833.3 = 0.718\ (T_3 - 745) \quad T_3 = 1906\ K$

$$p_3 = \left(\frac{p_1 V_1 T_3}{T_1 V_3}\right) = \frac{105 \text{ x } 9.882 \text{ x } 1906}{298} = 6635\ \text{kPa}$$

4. An Otto cycle uses a volume compression ratio of 9.5/1. The pressure and temperature before compression are 100 kPa and 40ºC respectively. The mass of air used is 11.5 grams/cycle. The heat input is 600 kJ/kg. The cycle is performed 3 000 times per minute. Determine the following.

i. The thermal efficiency. (59.4%).
ii. The net work output. (4.1 kJ/cycle)
iii. The net power output. (205 kW).

$\eta = 1 - r^{-0.4} = 1 - 9.5^{-0.4} = 59.4\%$

Q(in)= 600 x 0.0115 = 6.9 kJ

W = 0.594 x 6.9 = 4.098 KJ

$T_2 = 298(9.882)^{0.4} = 745\ K$

Power = 4.098 x 3000/60 = 204.9 kW

5. An Otto cycle with a volume compression ratio of 9 is required to produce a net work output of 450 kJ/cycle. Calculate the mass of air to be used if the maximum and minimum temperatures in the cycle are 1300ºC and 20ºC respectively.
(1.235 kg).

$\eta = 1 - r^{-0.4} = 1 - 9^{-0.4} = 58.5\%$

$T_2 = 293(9)^{0.4} = 705.6\ K$

Q(in)= 450/0.585 = 769 kJ

769 = m x 0.718 (1573-705.6)

m = 1.235 kg

6. The air standard cycle appropriate to the reciprocating spark ignition engine internal-combustion engine is the Otto. Using this, find the efficiency and output of a 2 litre (dm^3), 4 stroke engine with a compression ratio of 9 running at 3000 rev/min. The fuel is supplied with a gross calorific value of 46.8 MJ/kg and an air fuel ratio of 12.8.
Calculate the answers for two cases.
a. The engine running at full throttle with the air entering the cylinder at atmospheric conditions of 1.01 bar and 10°C with an efficiency ratio of 0.49.
b. The engine running at part throttle with the air entering the cylinder at 0.48 bar and efficiency ratio 0.38. (

PART (A)
p = 1.10 bar V = 0.002 m^3 T = 283 K Eff Ratio = 0.49
Allowing for the efficiency ratio V = 0.002 x 0.49 = 0.00098 m^3

$$m_{air} = \left(\frac{p_1 V_1}{RT_1}\right) = \frac{1.01 \times 10^5 \text{ x } 0.00098}{287 \text{ x } 283} = 0.001218 \text{ kg/rev}$$

m_{fuel} = 0.001218/12.8 = 95.2 x 10^{-6} kg/rev
Q(in) =95.2 x 10^{-6} x 46800 = 4.45 kJ/cycle   Φ(in) = 4.45 x 3000/(60 x 2) = 111.39 kW
$\eta = 1 - 9^{0.4}$ = 58.5%   P(net) = 111.39 x 58.5% = 65.16 kW

PART (B)
p = 0.48 bar V = 0.002 m^3 T = 283 K Eff Ratio = 0.38
Allowing for the efficiency ratio V = 0.002 x 0.38 = 0.00076 m^3

$$m_{air} = \left(\frac{p_1 V_1}{RT_1}\right) = \frac{0.48 \times 10^5 \text{ x } 0.00076}{287 \text{ x } 283} = 0.000449 \text{ kg/rev}$$

m_{fuel} = 0.000449/12.8 = 35.09 x 10^{-6} kg/rev   Q(in) =35.09 x 10^{-6} x 46800 = 1.642 kJ/cycle
Φ(in) = 1.642 x 3000/(60 x 2) = 41 kW $\eta = 1 - 9^{0.4}$ = 58.5%
P(net) = 41 x 58.5% = 24 kW

7 The working of a petrol engine can be approximated to an Otto cycle with a compression ratio of 8 using air at 1 bar and 288 K with heat addition of 2 MJ/kg. Calculate the heat rejected and the work done per kg of air.

$\eta = 1 - 8^{0.4}$ = 56.5%

W = 0.565 x 2000 = 1129 kJ/kg

η = 1 – Q(out)/Q(in)

Q(out) = (1 – 0.565) x 2000 = 870 kJ

ASSIGNMENT 4

1. A Dual Combustion Cycle uses a compression ratio of 20/1. The cut off ratio is 1.6/1. The temperature and pressure before compression is 30ºC and 1 bar respectively. The maximum cycle pressure is 100 bar. Calculate

i. the maximum cycle temperature.
ii. the nett work output per cycle.
iii. the thermal efficiency.

r = 20 β = 1.6 $T_2 = T_1 r^{0.4} = 303 \text{ x } 20^{0.4} = 1004.3\,K$

$$T_3 = \frac{p_3 V_3 T_1}{p_1 V_1} = \frac{100 \text{ x } 1 \text{ x } 303}{1 \text{ x } 20} = 1515\,K$$

T_4 =1.6 T_3 = 2424 K

$$T_5 = T_4 \left(\frac{1.6}{20}\right)^{0.4} = 2424 \text{ x } 0.08^{0.4} = 882.6\,K$$

Q(in) = 1 x 1.005 x (2424 – 1515) + 1 x 0.718 x (1515 – 1004.3) = 1280 kJ
Q(out) = 0.718 x (882.6 – 303) = 416 kJ
W(net) = Q(net) = 1280 – 416 = 864 kJ
η = W(net)/Q(in) = 67.5 %

2. A Dual Combustion Cycle uses a compression ratio of 12/1. The cut off ratio is 2/1. The temperature and pressure before compression is 280 K and 1 bar respectively. The maximum temperature 2000 K. Calculate

i. the nett work output per cycle. (680 kJ/kg).
ii. the thermal efficiency. (57.6 %).

r = 12 β = 2 $T_2 = T_1 r^{0.4} = 280 \text{ x } 12^{0.4} = 756\,K$

$$T_3 = \frac{V_4}{V_3} T_4 = \frac{1}{2} 2000 = 1000\,K$$

T_4 =1.6 T_3 = 2424 K

$$T_5 = T_4 \left(\frac{V_4}{V_5}\right)^{0.4} = 2000 \text{ x } \left(\frac{1}{6}\right)^{0.4} = 977\,K$$

Q(out) = 500 kJ Q(in) = 1180 kJ

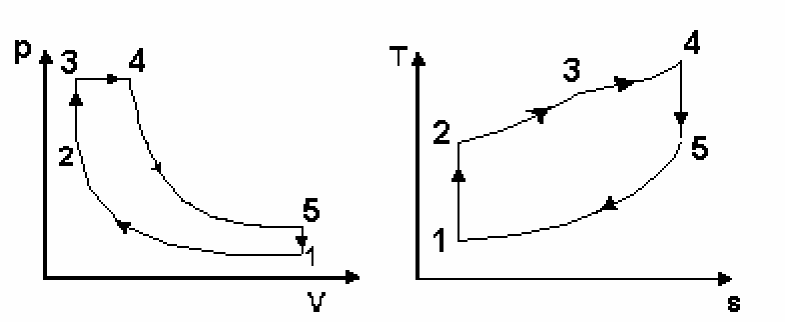
W(net) = Q(net) = 1180 –500 = 680 kJ

η = W(net)/Q(in) = 57.6 %

3. Similar to Q8 1988
Draw a p - V and T - s diagram for the Dual Combustion Cycle.

A reciprocating engine operates on the Dual Combustion Cycle. The pressure and temperature at the beginning of compression are 1 bar and 15ºC respectively. The compression ratio is 16. The heat input is 1800 kJ/kg and the maximum pressure is 80 bar. Calculate

i. the pressure, volume and specific volume at all points in the cycle.
ii. the cycle efficiency.62.8%).
iii. the mean effective pressure. (14.52 bar).



T_1 = 288 K $\quad$ p_1 = 1 bar $\quad$ $p_3 = p_4$ = 80 bar $\quad$ Q(in) = 1800 kJ/kg
r = 16 $\quad$ $V_2 = V_3$

$T_2 = 298 \times 18^{(\gamma-1)} = 947$ K

$p_2 = p_1 r^{\gamma} = 1 \times 16^{1.4} = 48.5$ bar

$$T_2 = \frac{p_2 V_2}{p_1 V_1} T_1 = \frac{48.5 \text{ x } 1}{1 \text{ x } 16} \text{ x } 288 = 873 \text{ K}$$

$$T_3 = \frac{p_3}{p_2} T_2 = \frac{80}{48.5} \text{ x } 873 = 1440 \text{ K}$$

Q(in) =1800 = 0.718(1440 -873) + 1.005(T_4-1440) $\quad$ T_4 = 2826 K

$V_4 / V_3 = T_4 / T_3$ = 2826/1440 = 1.9625

V_3 / V_4 = 1/1.9625 = 0.509

$V_5 / V_4 = (V_3 / V_4)(V_5 / V_3)$ = 0.509 x 16 = 8.15

$P_4 V_4^{\gamma} = P_5 V_3^{\gamma}$ $\quad$ $P_5 = 80(1/8.15)^{1.4} = 4.238$ bar

$$\frac{p_5}{T_5} = \frac{p_1}{T_4} \qquad T_5 = \frac{4.238 \text{ x } 288}{1} = 1221 \text{ K}$$

Q(out) = 0.718(1221-288) = 670 kJ/g
W(nett) = 1800-670 = 1130 kJ/kg

η =1130/1800 = 62.8%

$$V_1 = \frac{mRT_1}{p_1} = \frac{1 \text{ x } 287 \text{ x } 288}{1 \text{ x } 10^5} = 0.83 \text{ m}^3/\text{kg}$$

$$V_2 = \frac{mRT_2}{p_2} = \frac{V_1}{16} = 0.0519 \text{ m}^3/\text{kg}$$

MEP = W(nett)/SV = W(nett)/($V_1 - V_2$) = 1130 x 10^3/(0.778) = 1452 kPa

APPLIED THERMODYNAMICS D201

SELF ASSESSMENT SOLUTIONS
TUTORIAL 5

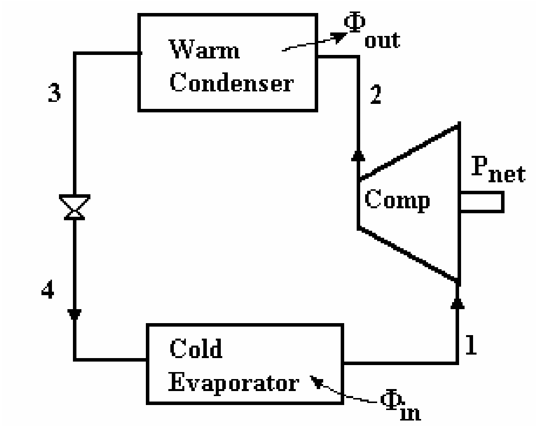
SELF ASSESSMENT EXERCISE No. 1

1. A simple vapour compression refrigerator comprises an evaporator, compressor, condenser and throttle. The condition at the 4 points in the cycle are as shown.

Point	Pressure	Temperature
After evaporator	0.8071 bar	$-20^{o}C$
After compressor	5.673 bar	$50^{o}C$
After condenser	5.673 bar	$15^{o}C$
After throttle	0.8071 bar	$-35^{o}C$

The refrigerant is R12 which flows at 0.05 kg/s. The power input to the compressor is 2 kW. Compression is reversible and adiabatic.
Calculate the following.
i. The theoretical power input to the compressor.
ii. The heat transfer to the evaporator.
iii. The coefficient of performance based answer (i.)
iv. The mechanical efficiency of the compressor.
v. The coefficient of performance based on the true power input.
Is the compression process isentropic?

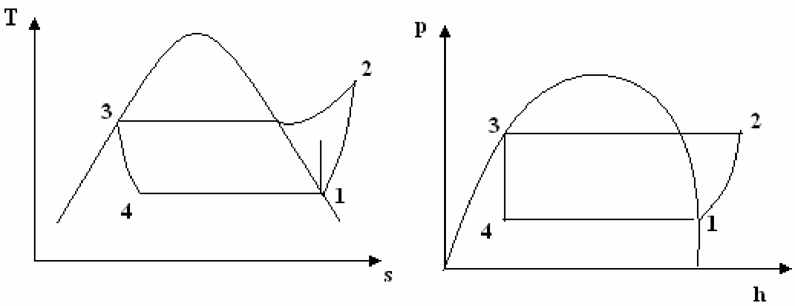
h_1 = 180.45 kJ/kg h_2 = 216.75 kJ/kg
$h_3 = h_4$ =50.1 kJ/kg
$\Phi(in) = 0.05(180.45 - 50.1) = 6.517$ kW
$P(in) = 0.05(216.75 - 180.45) = 1.815$ kW
C of P = 6.517/1.815 = 3.59
$\eta = 1.818/2 = 90.75\%$
C of P = 6.517/2 = 3.25
s_1 = 0.7568 kJ/kg K
s_2 = 0.7567 kJ/kg K
The compression is Isentropic since $s_1 = s_2$



2. A vapour compression cycle uses R12. the vapour is saturated at $-20^{o}C$ at entry to the compressor. At exit from the compressor it is at 10.84 bar and $75^{o}C$. The condenser produces saturated liquid at 10.84 bar. The liquid is throttled, evaporated and returned to the compressor.

Sketch the circuit and show the cycle on a p-h diagram.
Calculate the coefficient of performance of the refrigerator. (2.0)
Calculate the isentropic efficiency of the compressor. (71%)

Using the same diagram as Q1 for the point numbers the T – s and p –h diagrams are as shown.



Point 1 p = 1.509 bar θ = -20^{o}C h = h_g = 178.73 kJ/kg
Point 2 p = 10.84 bar ts = 45 ^{o}C θ = 75^{o}C hence there is 30K of superheat h = 228.18 kJ/kg
Point 3 p = 10.84 bar θ = ts = 45 ^{o}C h = h_f = 79.71 kJ/kg
Point 4 p = 1.509 bar θ = ts = -20 ^{o}C h = h_f = 79.71 kJ/kg

$h_4 = h_f + xh_{fg}$ = 79.71 + 17.82 + x(178.73 – 17.82) x = 0.385

Φ(in) = $h_1 - h_4$ = 99.02 kJ/kg
P(in) = $h_2 - h_1$ = 49.45 kJ/kg
C of P = 99.02/49.45 = 2.0
Compression process $s_1 = s_g$ = 0.7087 kJ/kg K
Ideally $s_1 = s_2$ at 10.84 bar
Interpolation from tables gives

$$\frac{0.7087-0.6811}{0.7175-0.6811}=\frac{h_2'-204.87}{216.74-204.87} \quad h_2' = 213.87 \text{ kJ/kg}$$

Ideal P(in) = 213.87 – 178.87 = 35.14 kJ/kg

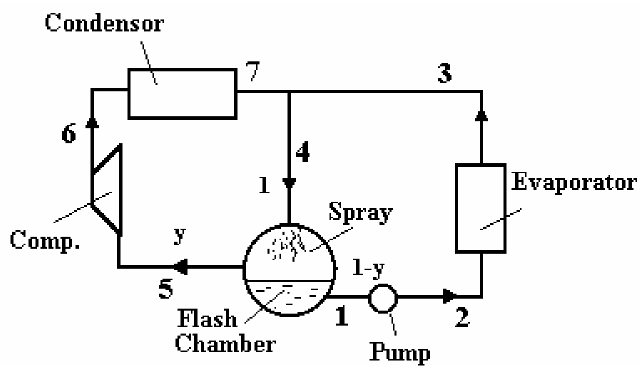
η_{is} = 35.14/49.45 = 71%

SELF ASSESSMENT EXERCISE No.2

1. A refrigerator operates with ammonia. The plant circuit is shown below. The conditions at the relevant points of the cycle are as follows.

1	saturated liquid at -30ºC
3,4 and 7	saturated liquid at 10ºC
5	saturated vapour at -30ºC

The pump and compressor have an isentropic efficiency of 80%.
There are no heat losses. The specific volume of ammonia liquid is 0.0015 m3/kg.
Determine the coefficient of performance and the mass flow rate if the refrigeration effect is 10 kW.

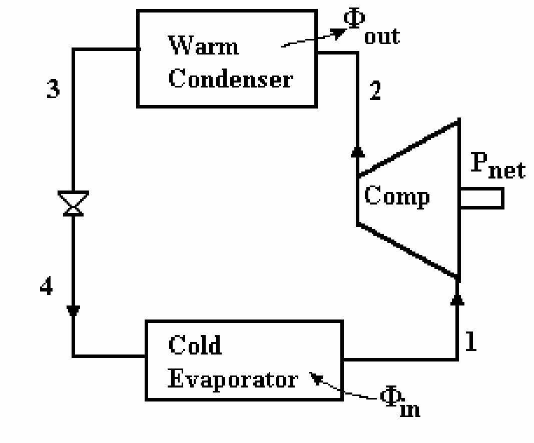


Point 1 p = 1.196 bar θ = -30^{o}C $h_1 = h_f$ = 44.7 kJ/kg
Points 3,4 and 7 p = 6.149 bar h = h_f = 227.8 kJ/kg
Point 5 h = 1405.6 kJ/kg
PUMP P(in) = V Δp/η = 0.0015(6.149-1.196)x10^5/0.8 = 928 W per unit mass flow rate.
$h_2 = h_1$ + energy added = 44.7 + 0.928 = 45.628 kJ/kg
Φ(in) = $h_3 - h_2$ = 227.8 – 45.628 = 182.17 kW per unit mass flow rate.
$s_5 = s_6'$ = 5.785 kJ/kg K

Interpolate at 6.149 bar $$\frac{5.785-5.634}{5.967-5.634}=\frac{h_6'-1583.1}{1702.2-1583.1} \quad h_6' = 1637.2 \text{ Kj/kg}$$

COMPRESSOR P(in) = (h_6'- h_5)/η = (1637.2 – 1405.6)/0.8 = 289.54 kW per unit mass flow rate.
FLASH VESSEL
$h_4 = yh_5 + (1-y)h_1$ 227.8 = 1405.6y + (1-y)44.7 y = 0.134 kg 1 – y = 0.865 kg
For a total flow of 1 kg/s
Pump Power = 0.928 x 0.865 = 0.803 kW
Compressor Power = 289.54 x 0.134 = 38.95 kW
Φ(in) = 182.17 x 0.865 = 157.58 kW
C of P = 157.58/39.75 = 3.964
Φ(in) = 10 kW = m x 182.17 hence m = 0.05489 kg/s but this is 0.865 of the total flow
Mass flow = 0.05489/0.865 = 0.06346 kg/s

2. A heat pump consists of a compressor, condenser, throttle, and evaporator. The refrigerant is R12. The refrigerant is at 0^{o}C at entry to the compressor and 80^{o}C at exit. The condenser produces saturated liquid at 50^{o}C. The throttle produces wet vapour at -10^{o}C. The mass flow rate is 0.02 kg/s. The indicated power to the compressor is 1 kW.
Sketch the T - s diagram and p - h diagram for the cycle.
Calculate the coefficient of performance for the heat pump.
Calculate the rate of heat loss from the compressor.
Calculate the coefficient of performance again for when the refrigerant is sub cooled to 45^{o}C at exit from the condenser.
Calculate the temperature at exit from the compressor if the compression is reversible and adiabatic.



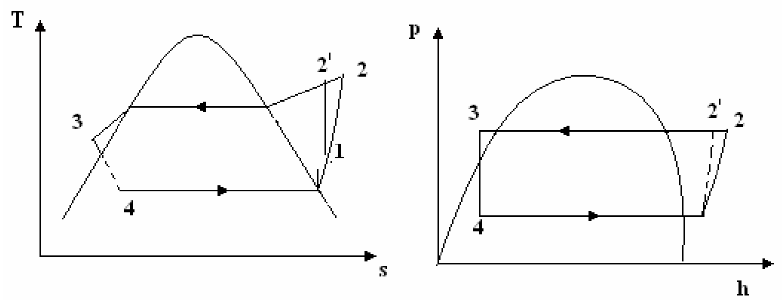
h_3=h_f at 50^{o}C = 84.94 kJ/kg p_3 = p_2 = p_s at50^{o}C=12.19 b
h_1 with 10 K superheat is 190 kJ/kg
s_1 is 0.725 kJ/kg K
P(in) = 0.02(230 – 190) = 0.8 kW Loss is 0.2 kW
Φ(out) = 0.02(230 – 85) = 2.9 kW
Φ(in) = 0.02(190 – 85) = 2.1 kW
C of P = 2.9/0.8 = 3.625 based on the cycle.
C of P = 2.9/1 = 2.9 based on the Indicated Power.
At 45^{o}C h_3 = 79.71 kJ/kg
Φ(out) = 0.02(230 – 79.71)= 3 kW
C of P = 3/0.8 = 3.75 based on the cycle and 3/1 = 3 based on the IP

REVERSIBLE COMPRESSION
s_2 = 0.725 kJ/kg K h_2 = 230 kJ/kg

Interpolation $\frac{0.725-0.7166}{0.7503-0.7166}=\frac{\Delta T}{30-15}$ ΔT = 3.73 θ = 50 + 3.73 + 15 = 68.7^{o}C

3. A refrigeration cycle uses R12. The evaporator pressure is 1.826 bar and the condenser pressure is 10.84 bar. There is 5K of superheat at inlet to the compressor. The compressor has an isentropic efficiency of 90%. the condensed liquid is under-cooled by 5K and is throttled back to the evaporator.
Sketch the cycle on a T-s and p-h diagram.
Calculate the coefficient of performance. (3.04)
Explain why throttles are used rather than an expansion engine.

POINT 1 θ = -10^{o}C
p = 1.826 bar ts = -15 ^{o}C
hence 5 K superheat.



Interpolation

$$h_1=\frac{5}{15}(190.15-180.97)+180.97=184.03\text{ kJ/kg}$$

$$s_1=\frac{5}{15}(0.7397-0.7051)+0.7051=0.7166\text{ kJ/kg K}$$

POINT 2 p = 10.84 bar s = 0.7166 kJ/kg K $\frac{\Delta T}{15}=\frac{0.7166-0.6811}{0.7175-0.6811}$ $\Delta T=14.64\text{ K}$

$$\frac{14.64}{15}=\frac{h_2'-204.87}{216.74-204.87}\quad h_2'=216.46\text{ kJ/kg}$$

$$\eta_{IS} = 0.9 = \frac{h_2' - h_1}{h_2 - h_1} = \frac{216.46 - 184.03}{h_2 - 184.03} \quad h_2 = 220 \text{ kJ.kg}$$

POINT 3 $\theta = 40^oC$ (liquid) $h_3 = h_f = 74.59$ kJ/kg

POINT 4 $h_4 = h_3$ $\text{C of P} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{184.03 - 74.59}{220 - 184.03} = 3$

Throttles are used because they are simple with no moving parts. Expansion engines are expensive and difficult to use with wet vapour being present.

SELF ASSESSMENT EXERCISE No. 3

1. Why is it preferable that vapour entering a compressor superheated?

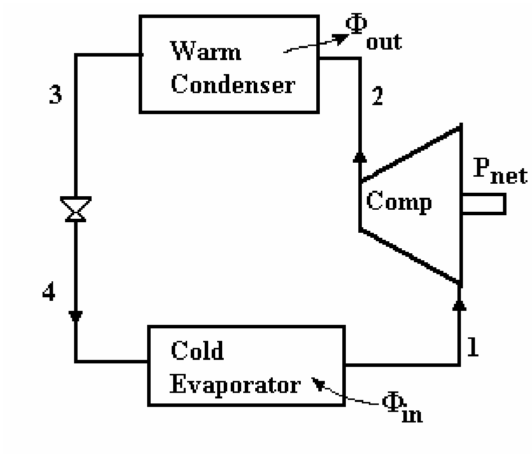
A vapour compression refrigerator uses R12. The vapour is evaporated at -10ºC and condensed at 30ºC. The vapour has 15 K of superheat at entry to the compressor. Compression is isentropic. The condenser produces saturated liquid.

The compressor is a reciprocating type with double action. The bore is 250 mm and the stroke is 300 mm. The speed is 200 rev/min. The volumetric efficiency is 85%. You may treat superheated vapour as a perfect gas. Determine

i. the mass flow rate (0.956 kg/s)
ii. the coefficient of performance. (5.51)
iii. the refrigeration effect. (122.7 kW)

(Note that double acting means it pumps twice for each revolution. The molecular mass for R12 is given in the tables.)

(a) Liquid refrigerant must be prevented from entering the compressor as it would damage the piston and cylinders and contaminate the lubricant.



(b) $h_1 = 192.53$ kJ/kg $s_1 = 0.7365$ kJ/kg K

$s_1 = s_2$ at 7.449 bar

$$\frac{\Delta T}{15} = \frac{0.7365 - 0.7208}{0.754 - 0.7208} \quad \Delta T = 7.09 \text{ K}$$

$$\frac{7.09}{15} = \frac{h_2 - 210.63}{221.44 - 210.63} \quad h_2 = 215.74 \text{ kJ/kg}$$

P(in) = 215.74 – 192,53=23.21 kJ/kg

$h_3 = h_4 = h_f$ at 7.449 bar = 64.59 kJ/kg

Φ(in) = $h_1 - h_4$ = 129.9 kJ/kg

C of P = 127.9/23.21 = 5.51

Swept Volume = π x $(0.25^2/4)$ x 0.3 = 0.014726 m^3

η_{vol} = 85%

Induced volume = 2 x 0.014726 x 0.85 x 200/60 = 0.08344 m^3/s

$$\text{At inlet } m = \frac{pV\tilde{N}}{R_oT} = \frac{2.191 \text{x} 10^5 \text{ x } 0.08344 \text{ x } 120.92}{8314.4 \text{ x } 278} = 0.956 \text{ kg/s}$$

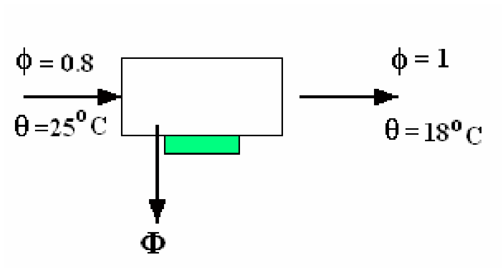
Φ(in) = 127.9 x 0.956 = 122.27 kW

APPLIED THERMODYNAMICS D201

SELF ASSESSMENT SOLUTIONS
TUTORIAL 6

SELF ASSESSMENT EXERCISE No. 1

1. Repeat the worked example 2 but this time the relative humidity 0.8 at inlet. Is water condensed or evaporated? (m_{s1} = 0.01609 kg so water is condensed)



$\phi_1 = 0.8 = p_{s1}/p_{g1}$ $\quad p_{s1} = 0.8\ p_{g1}$

p_{s1} = 0.8 x 0.03166 = 0.025328 bar
p_{a1} = 1 - 0.025328 = 0.974672 bar

$\omega_1 = 0.622\ p_{s1}/\ p_{a1} = 0.622(0.025328/0.974672) = 0.01616$
Condensate formed = 0.01616 – m_{s2}
$\omega_2 = 0.622\ p_{s2}\ /\ p_{a2}$

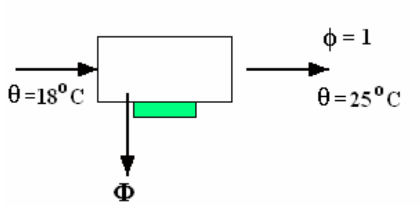
p_{s2} at 18°C = 0.02063 bar $\quad p_{a2}$ = 1 - 0.02063 = 0.97937 bar
$\omega_2 = 0.622\ (0.02063/\ 0.97937) = 0.013$
Condensate formed = 0.01616 – 0.013 = 0.00316 kg

2. Define specific humidity ω and prove that $\omega = p_s\tilde{N}_s/\tilde{N}_a(p - p_s)$
Humid air at 1 bar flows through an insulated vessel over a pool of water and emerges saturated. The temperatures are 25°C and 18°C at inlet and outlet respectively. The mass of water is maintained constant at 18°C all the time. Calculate the relative humidity at inlet assuming constant pressure throughout. (Ans. 0.651)

Specific humidity ω refers to a mixture of dry air and steam ω = mass of steam/mass of air.
Relative humidity ϕ refers to the mass of steam as a % of the maximum possible.
Maximum steam = m_g $\quad \phi = m_s/\ m_g \quad m = V/v \quad \phi = v_g/v_s$ $\quad$ ω = mass of water vapour/mass of dry air

Starting with the gas law

$$m = \frac{pV\tilde{N}}{R_oT}$$



$$\omega = \frac{p_s V R_o T \tilde{N}_s}{p_a V R_o T \tilde{N}_a} = \frac{p_s \tilde{N}_s}{p_a \tilde{N}_a} = \frac{p_s}{p_a} \text{ x } \frac{18}{28.96} = 0.622\frac{p_s}{p_a}$$

$$\omega = 0.622\frac{p_s}{p - p_s}$$

Assuming no evaporation or condensation if p is constant then m_s is constant and it follows that $\omega_1 = \omega_2$
$p_s = p_g$ at 18°C = 0.02063 bar $\quad$ Pa = 1 - 0.02063 = 0.97937 bar

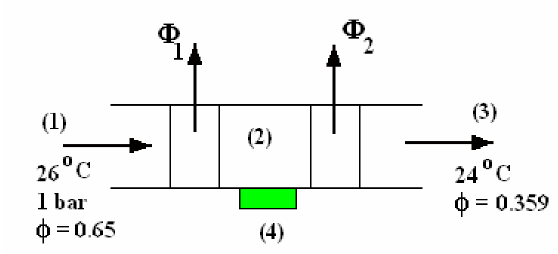
$$\omega_1 = 0.622\frac{p_s}{p - p_s} = 0.622\frac{0.02063}{0.97937} = 0.0131$$

$p_{s1} = p_{s2}$

At inlet $p_g = p_s$ at 25 °C = 0.03166 bar
$\phi_1 = p_{s1}/\ p_g = 0.02063/0.03166 = 0.6516$

SELF ASSESSMENT EXERCISE No. 2

Air having a pressure, temperature and relative humidity of 1 bar, 26°C and 0.65 respectively, flows into an air conditioner at a steady rate and is dehumidified by cooling and removing water from it. The air is then heated to produce an outlet temperature and relative humidity of 24°C and 0.359 respectively. The pressure is constant throughout. Determine the heat transfers in the cooler and heater per kg of conditioned air at exit. Draw up a complete mass balance.
(36.22 kJ/kg and 16.24 kJ/kg)



INLET
p_g at 26°C = 0.0336 bar p_{s1} = 0.02184 bar
$p_{a1} = 1 - 0.02184 = 0.97816$ bar
$\phi_1 = 0.65 = p_{s1}/p_g$
$\omega_1 = 0.622\ (p_{s1}/p_{a1}) = 0.0138878$

OUTLET
p_g at 24°C = 0.02982 bar p_{s3} = 0.010705 bar
$p_{a3} = 1 - 0.010705 = 0.98929$ bar
$\phi_3 = 0.359 = p_{s3}/p_g$
$\omega_3 = 0.622\ (p_{s3}/p_{a3}) = 0.0067306$

$m_{s1} = 0.0138878\ m_a$ $m_{s3} = 0.0067306\ m_a$
Condensate formed at (2) = $m_{s1} - m_{s3} = 0.0071572\ m_a$
Temperature of the condensate is t_s at 0.010705 bar and is 8°C
$m_{s2} = m_{s3} = 0.0067306\ m_a$

COOLER
$m_a \times 1.005 \times 26 + m_{s1} h_{s1} = m_a \times 1.005 \times 8 + m_{s2} h_{s2} + m_w \times 4.186 \times 8 + \Phi_1$
h_{s1} = 2545 kJ/kg (from the h – s chart) $h_{s2} = h_g$ at 8°C = 2515.5 kJ/kg
$2613\ m_a + 35.325\ m_a = 8.04\ m_a + 16.93\ m_a + 0.24\ m_a + \Phi_1$

$\Phi_1 = 36.245\ m_a$ kJ per kg of dry air
m_a is 1 kg dry air. The mass of conditioned air is more.
$\omega_3 = 0.0067306 = m_a / m_{s3}$ $m_{s3} = 0.0067306\ m_a$

Mass of conditioned air = $1.0067306\ m_a$ $m_a = 0.9933$ M

Φ_1 = 36 kJ per kg of conditioned air.

HEATER

$m_a \times 1.005 \times 24 + m_{s3} h_{s3} + \Phi_2 = m_a \times 1.005 \times 8 + m_{s2} h_{s3}$
h_{s3} = 2540 (from the h – s chart) $m_{s3} = 0.0067306\ m_a$

$\Phi_2 = 24.12\ m_a + 17.09\ m_a - 8.04\ m_a - 16.93\ m_a$

Φ_2 = 16.24 kJ per kg of dry air or 16.16 kJ per kg of conditioned air.

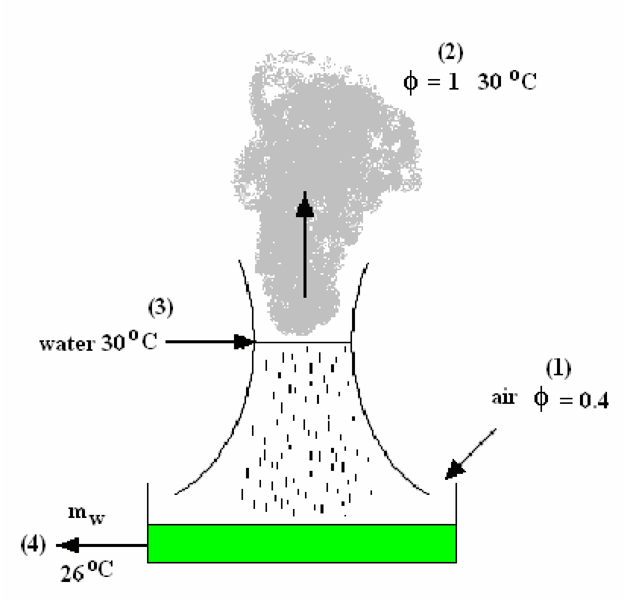
SELF ASSESSMENT EXERCISE No. 3

1. Derive the expression for specific humidity $\omega = 0.622(p_s/p_a)$

Water flows at 5 000 kg/h and 40°C into a cooling tower and is cooled to 26°C. The unsaturated air enters the tower at 20°C with a relative humidity of 0.4. It leaves as saturated air at 30°C. The pressure is constant at 1 bar throughout. Calculate

i. the mass flow of air per hour. (4 636 kg/h)

ii. the mass of water evaporated per hour. (100.5 kg/h)



INLET AIR

p_{g1} = 0.02337 bar at 20°C

$\phi_1 = 0.4 = p_{s1} / p_g$

p_{s1} = 0.4 x 0.02337 = 0.009348 bar

hence p_{a1} = 1.0 - 0.009348 = 0.99065 bar

$$\omega_1 = 0.622\frac{0.009348}{0.99065} = 0.0058693$$

$m_{s1} = 0.0058693\ m_a$

OUTLET AIR

$\phi_2 = 1.0$

$p_{s2} = p_{g2}$ = 0.0424242 bar hence p_{a2} = 0.95758 bar

$$\omega_2 = 0.622\frac{0.004242}{0.95758} = 0.02755$$

$m_{s2} = 0.02755 \times m_a$

MASS BALANCE

$m_{w4} = m_{w3} - (m_{s2} - m_{s1}) = 5000 - (0.02755\ m_a - 0.0058693\ m_a) = 5000 - 0.02168\ m_a$

ENERGY BALANCE

$h_{s2} = h_g$ = 2555.7 kJ/kg

h_{s1} = 2535 kJ/kg h (from h-s chart)

Balancing energy we get

(5000 x 4.86 x 40) + (m_a x 1.005 x 20) + (0.0058693 x m_a x 2538) =

{(5000 – 0.02168 m_a) x 4.186 x 26} + (0.02755 x 2555.7 m_a) + (m_a x 1.005 x 30)

837200 + 20.1 m_a + 14.896 m_a = 544180 – 2.359 m_a + 70.4 m_a + 30.15 m_a

293020 = 63.195 m_a

m_a = 4636 kg/hour

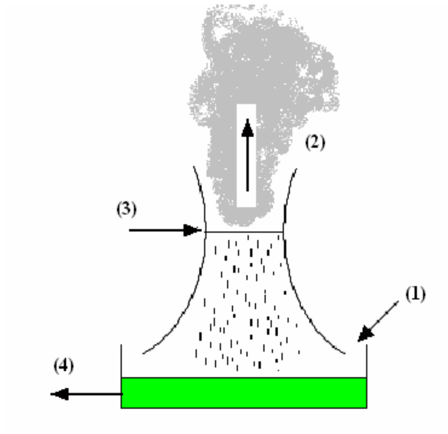
m_{s2} = 127.74 kg/hour

m_{s1} = 27.21 kg/hour

Evaporation rate is 100.5 kg/hour

2. The cooling water for a small condenser is sent to a small cooling tower. 7 m^3/s of air enters the tower with a pressure, temperature and relative humidity of 1.013 bar, 15°C and 0.55 respectively. It leaves saturated at 32°C. The water flows out of the tower at 7.5 kg/s at 13°C. Using a mass and energy balance, determine the temperature of the water entering the tower.

(Answer 33.9°C)



INLET AIR

p_{g1} = 0.01704 bar at 15°C

$\phi_1 = 0.55 = p_{s1}/p_{g1}$ $\quad p_{s1}$ = 0.009372 bar

hence p_{a1} = 1.01325 - 0.009372= 1.003878 bar

$\omega_1 = 0.622\ p_{s1}/\ p_{a1} = 0.005807$

$m_a = pV/RT = 1.003125 \times 10^5 \times 7/(287 \times 288) = 8.5$ kg/min

$m_{s1} = 0.005807 \times 8.5 = 0.04937$ kg/s

OUTLET AIR

p_{g2} = 0.04754 bar at 32 °C

p_{a2} = 0.96571 bar

$\omega_2 = 0.622\ p_{s2}/\ p_{a2} = 0.03062 = m_{s2}/\ m_a$

$m_{s2} = 0.03062 \times 8.5 = 0.26$ kg/s

MASS BALANCE $\qquad m_{s1} + m_3 = m_{s2} + m_4$

$m_3 = 7.5 + 0.26 – 0.04937 = 7.71$ kg/s

ENERGY BALANCE

$m_{s1}\ h_{s1} + m_a\ h_{a1} + m_3\ h_{w3} = m_{s2}\ h_{s2} + m_a\ h_{a2} + m_4\ h_{w4}$

h_{s1} = h @ 0.009372 bar & 15°C = 2525 kJ/kg (from h-s chart)

h_{s2} = h @ 0.04759 bar and 32°C = 2559.3 kJ/kg

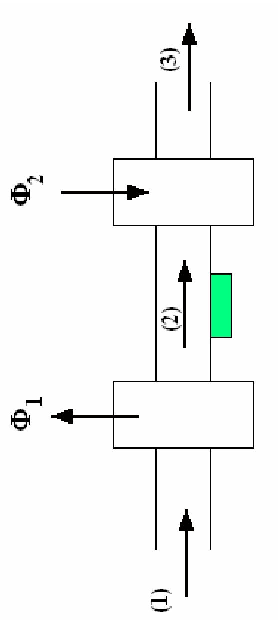
Balancing energy we get

$$(0.04937 \times 2525) + (8.5 \times 1.005 \times 15) + (7.71 \times 4.186 \times \theta_3)$$
$$= (0.26 \times 2559) + (8.51 \times 1.005 \times 32) + (7.5 \times 4.186 \times 13)$$

Hence $\theta_3 = 33.9^oC$

3. A fan supplies 600 dm^3/s of air with a relative humidity of 0.85, temperature 30°C and pressure 1.04 bar into an air conditioner. Moisture is removed from the air by cooling and both the air and condensate leave at the same temperature. The air is then heated to 20°C and has a relative humidity of 0.6. Determine the following.

i. The mass of dry air and water at entrance to the conditioner.
ii. The mass of water vapour delivered at exit.
iii. The mass of water extracted from the cooler.
iv. The temperature at exit from the cooler.
v. The heat transfer in the cooler.



INLET

$V_1 = 0.6\ m^3$ $T_1 = 303$ K $p_1 = 1.04$ bar $\phi_1 = 0.85$

$\phi_1 = 0.85 = p_{s1}/p_g$

p_g at $30^oC = 0.04242$ bar

$p_{s1} = 0.03606$ bar

$p_{a1} = 1.04 - 0.03606 = 1.0039$ bar

$m_a = pV/RT = 1.0039 \times 10^5 \times 0.6/(287 \times 303) = 0.6927$ kg

$m_S = pV/RT = 0.03606 \times 10^5 \times 0.6/(462 \times 303) = 0.01546$ kg

$\omega_1 = m_S / m_a = 0.02232$

$\omega_1 = 0.622\ (p_{s1}/p_{a1}) = 0.02232$ (checks out)

OUTLET

$\phi_3 = 0.6 = p_{s3}/p_{g3} = p_{s3}/0.02337$ $p_{s3} = 0.014$ bar

$p_{a3} = 1.04 - 0.014 = 1.026$ bar

$\omega_3 = 0.622\ (p_{s3}/p_{a3}) = 0.622\ (0.014/1.026) = 0.00849$

$m_{s3} = 0.00849\ m_a = 0.00849 \times 0.6927 = 0.00588$ kg

Condensate = 0.01546 - 0.00588 = 0.00958 kg

ENERGY BALANCE

$p_{S2} = p_{S3} = p_{g2} =$ at $12^oC = 0.04$ bar

$h_{S1} = 2555$ (from chart) $h_{S2} = 2523$

$m_a\, c_a\, (T_{a1} - T_{a2}) - m_w\, c_w\, T_w + m_{s1}\, h_{s1} - m_{s2}\, h_{s2} = \Phi_1$

$\{0.6927 \times 1.005\ (30 – 12)\} – (0.00958 \times 4.186 \times 12) + (0.01546 \times 2555) – (0.00588 \times 2523) = \Phi_1$

$\Phi_1 = 36.2$ kJ

SELF ASSESSMENT EXERCISE No. 4

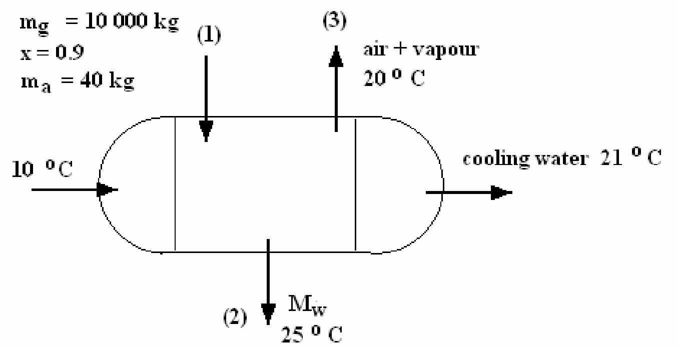
a. Discuss the reasons why air mixed with steam in a condenser is not desirable.

b. Wet steam with a dryness fraction of 0.9 enters a condenser at 0.035 bar pressure at a rate of 10 000 kg/h. The condensate leaves at 25^oC. Air also enters with the steam at a rate of 40 kg/h. The air is extracted and cooled to 20^oC. The partial pressure of the air at inlet is negligible and the process is at constant pressure. The cooling water is at 10^oC at inlet and 21^oC at outlet.

i. Determine the mass of vapour extracted with the air. (50 kg/h)

ii. Calculate the flow rate of the cooling water required. (475 484 kg/h)

Air mixed with steam changes the partial pressure of the steam and lowers the saturation temperature thus reducing the temperature of the steam and reducing the cycle efficiency. Air dissolved in water is also a problem in the feed water plant and needs to be removed.



Air + vapour is saturated $p_s = p_g$ at 20_oC
$p_s = 0.02337$ bar $\phi = 1$

$p_a + p_s = 0.035$ $p_a = 0.0116$ bar

$\omega_3 = 0.622(p_a / p_a) = 1.25$
$\omega_3 = 1.25 = m_s / m_a$ $m_s = 50$ kg

ALTERNATIVELY
Volume of vapour removed = mRT/p
Volume of air removed =mRT/p = 40 x 287 x 293/(0.0116 x 10^5) = 2900 m^3
Volumes are the same v_g at 0.02337 bar = 57.84 m^3/kg
Mass of steam = V/ v_g = 2900/57.84 = 50.1 kg

M_w = 10 000 – 50 9950 kg

ENERGY BALANCE

$m_{s1} h_{s1} + c_p T_1 m_a – c_w T_2 m_w - c_p T_2 m_a - m_{s3} h_{s3} = \Phi$

$h_{s1} = h_f + x h_{fx}$ at 0.035 bar = 112 + 0.9 x 2438 = 2306 kJ/kg
$h_{s3} = h_g$ at 20^oC = 2537.6 kJ/kg
$T_1 = T_s$ at 0.035 bar = 26.7 = ^oC

10000 x 2306 + 1.005 x 26.7 x 40 – 9950 x 4.186 x 25 – 1.005 x 20 x 40 – 50 x 2537.6 = Φ

Φ = 21.894 kJ/hour = M(cooling water) x 4.186 x 11

Mass of cooling water = 475484 kg/hour

SELF ASSESSMENT SOLUTIONS
TUTORIAL 7
SELF ASSESSMENT EXERCISE No. 1

1. Calculate the specific entropy change when a perfect gas undergoes a reversible isothermal expansion from 500 kPa to 100 kPa. R = 287 J/kg K.

T is constant so $\Delta s = mR\ln(p_1/p_2) = 1 \times 287 \times \ln(5/1) = 462$ J/kg K

2. Calculate the total entropy change when 2 kg of perfect gas is compressed reversibly and isothermally from 9 dm^3 to 1 dm^3. R=300 J/kg K.

$\Delta s = mR\ln(V_2/V_1) = 1 \times 300 \times \ln(1/9) = 470$ J/kg K

3. Calculate the change in entropy when 2.5 kg of perfect gas is heated from 20^{o}C to 100^{o}C at constant volume. Take c_v= 780 J/kg K (Answer 470 J/K)

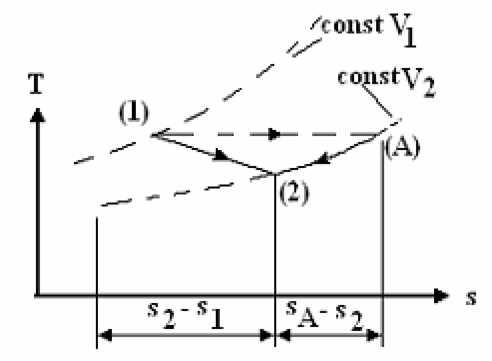
$\Delta s = m\, c_v \ln(T_2/T_1) = 2.5 \times 780 \times \ln(373/293) = -1318$ J/kg K

4. Calculate the total entropy change when 5 kg of gas is expanded at constant pressure from 30^{o}C to 200^{o}C. R = 300 J/kg K c_v= 800 J/kg K (Answer 2.45 kJ/K)

$\Delta s = m\, c_p \ln(T_2/T_1)$ $\quad$ $c_p = R + c_v = 1100$ J/kg K

$\Delta s = = 5 \times 1100 \times \ln(473/303) = 2450$ J/kg K

5. Derive the formula for the specific change in entropy during a polytropic process using a constant volume process from (A) to (2).



$s_2 - s_1 = (s_A - s_1) - (s_A - s_2)$
$s_2 - s_1 = (s_A - s_1) + (s_2 - s_A)$
For the constant temperature process
$(s_A - s_1) = R\ln(p_1/p_A)$
For the constant volume process
$(s_2 - s_A) = (c_v/R)\ln(T_2/T_A)$
Hence

$$\Delta s = R\ln\frac{p_1}{p_A} + C_p\ln\frac{T_2}{T_A} + s_2 - s_1 \quad T_A = T_1$$

Then $\Delta s = s_2 - s_1 = \Delta s = R\ln\left(\frac{p_1}{p_A}\right) + c_v\ln\left(\frac{T_2}{T_A}\right)$

Divide through by R $\quad \Delta s/R = \ln\left(\frac{p_1}{p_A}\right) + \frac{c_v}{R}\ln\left(\frac{T_2}{T_A}\right)$

From the relationship between c_p, c_v, R and γ we have $c_p/R = \gamma/(\gamma-1)$
From the gas laws we have $p_A/T_A = p_2/T_2$ $\quad p_A = p_2 T_A/T_2 = p_2 T_1/T_2$
Hence

$$\frac{\Delta s}{R} = \ln\left(\frac{p_1}{p_2}\right) + \frac{1}{\gamma-1}\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right)^{1+\frac{1}{\gamma-1}} = \ln\left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

6. A perfect gas is expanded from 5 bar to 1 bar by the law $pV^{1.6} = C$. The initial temperature is 200°C. Calculate the change in specific entropy.

R = 287 J/kg K γ =1.4.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-1/n} = 473\left(\frac{1}{5}\right)^{1-1/1.6} = 258.7\ \mathrm{K}$$

$$\Delta s = R\ln\left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = 287\ \ln(5)\left(\frac{258.7}{473}\right)^{\frac{1.4}{0.4}} = -144\ \mathrm{J/K}$$

7. A perfect gas is expanded reversibly and adiabatically from 5 bar to 1 bar by the law $pV^{\gamma} = C$. The initial temperature is 200°C. Calculate the change in specific entropy using the formula for a polytropic process. R = 287 J/kg K γ =1.4.

$$T_2 = 473\left(\frac{1}{5}\right)^{1-1/1.4} = 298.6\ \mathrm{K}$$

$$\Delta s = R\ln\left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = 287\ \ln(5)\left(\frac{298.6}{473}\right)^{\frac{1.4}{0.4}} = 0$$

SELF ASSESSMENT EXERCISE No. 2

Take γ = 1.4 and R = 283 J/kg K in all the following questions.

1. An aeroplane flies at Mach 0.8 in air at 15º C and 100 kPa pressure. Calculate the stagnation pressure and temperature. (Answers 324.9 K and 152.4 kPa)

$$\frac{\Delta T}{T} = M^2 \frac{k-1}{2} = 0.8^2 \frac{1.4}{2} = 0.128 \quad \Delta T = 0.128 \text{ x } 288 = 36.86 \text{ K}$$

$$\frac{p_2}{p_1} = \left(M^2 \frac{k-1}{2} + 1 \right)^{\frac{k}{k-1}} = 1.128^{3.5} = 1.5243 \qquad p_2 = 100 \text{ x } 1.5243 = 152.43 \text{ kPa}$$

2. Repeat problem 1 if the aeroplane flies at Mach 2.

$$\frac{\Delta T}{T} = M^2 \frac{k-1}{2} = 2^2 \frac{1.4}{2} = 0.8 \qquad \Delta T = 0.8 \text{ x } 288 = 230.4 \text{ K}$$

T_2 = 288 + 230.4 = 518.4 K

$$\frac{p_2}{p_1} = \left(M^2 \frac{k-1}{2} + 1 \right)^{\frac{k}{k-1}} = 1.8^{3.5} = 7.824 \qquad p_2 = 100 \text{ x } 7.824 = 782.4 \text{ kPa}$$

3. The pressure on the leading edges of an aircraft is 4.52 kPa more than the surrounding atmosphere. The aeroplane flies at an altitude of 5 000 metres. Calculate the speed of the aeroplane.(Answer 109.186 m/s)

From fluids tables, find that a = 320.5 m/s p_1 = 54.05 kPa γ = 1.4

$$\frac{p_2}{p_1} = \frac{58.57}{54.05} = 1.0836 = \left(M^2 \frac{k-1}{2} + 1 \right)^{\frac{k}{k-1}}$$

$$1.0836 = \left(M^2 \frac{1.4-1}{2} + 1 \right)^{\frac{1.4}{1.4-1}} = \left(0.2\, M^2 + 1\right)^{3.5}$$

$1.0232 = 0.2\, M^2 + 1$ M = 0.3407 = v/a v = 109.2 m/s

4. An air compressor delivers air with a stagnation temperature 5 K above the ambient temperature. Determine the velocity of the air. (Answer 100.2 m/s)

$$\frac{\Delta T}{T_1} = \frac{v_1^2 (k-1)}{2\gamma R T_1} \qquad \Delta T = \frac{v_1^2 (1.4-1)}{2 \text{ x } 1.4 \text{ x } 287} = 5\,\text{K} \qquad v_1 = 100.2 \text{ m/s}$$

SELF ASSESSMENT EXERCISE No. 3

1. A Venturi Meter must pass 300g/s of air. The inlet pressure is 2 bar and the inlet temperature is 120°C. Ignoring the inlet velocity, determine the throat area. Take C_d as 0.97.
Take γ =1.4 and R = 287 J/kg K (assume choked flow)

$$m = C_d A_2 \sqrt{\left[\frac{2\gamma}{\gamma-1}\right] p_1 \rho_1 \left\{ (r_c)^{\frac{2}{\gamma}} - (r_c)^{1+\frac{1}{\gamma}} \right\}} \qquad r_c = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{2.4}\right)^{3.5} = 0.528$$

$\rho_1 = p_1/RT_1 = 2 \times 10^5/(287 \times 393) = 1.773 \text{ kg/m}^3$

$$0.3 = 0.97 A_2 \sqrt{7 \times 2 \times 10^5 \times 1.773 \left\{ (0.528)^{1.428} - (0.528)^{1.714} \right\}} = 0.97 A_2 \sqrt{166307}$$

$A_2 = 758 \times 10^{-6} \text{ m}^2$ and the diameter = 31.07 mm

2. Repeat problem 1 given that the inlet is 60 mm diameter and the inlet velocity must not be neglected.

$$m = C_d A_2 \sqrt{\frac{\left[\frac{2\gamma}{\gamma-1}\right] p_1 \rho_1 \left\{ (r_c)^{\frac{2}{\gamma}} - (r_c)^{1+\frac{1}{\gamma}} \right\}}{1 - \left(\frac{A_2}{A_1}\right)^2 \left(\frac{p_2}{p_1}\right)^{2/\gamma}}} \qquad 0.3 = C_d A_2 \sqrt{\frac{166307}{1 - \left(\frac{A_2}{A_1}\right)^2 (0.4)}}$$

$1 - (A_2/A_1)^2 \times 0.4 = 1738882 A_2^2$ $\qquad A_1^2 = (\pi \times 0.06^2/4)^2 = 7.99 \times 10^{-6} \text{ m}^2$

$1 - 50062\, A_2^2 = 1738882 A_2^2$ $\qquad A_2^2 = 559 \times 10^{-9}$ $\qquad A_2 = 747.6 \times 10^{-6} \text{ m}^2$

The diameter is 30.8 mm. Neglecting the inlet velocity made very little difference.

3. A nozzle must pass 0.5 kg/s of steam with inlet conditions of 10 bar and 400°C. Calculate the throat diameter that causes choking at this condition. The density of the steam at inlet is 3.263 kg/m^3. Take γ for steam as 1.3 and C_d as 0.98.

$$m = C_d A_2 \sqrt{\left[\frac{2\gamma}{\gamma-1}\right] p_1 \rho_1 \left\{ (r_c)^{\frac{2}{\gamma}} - (r_c)^{1+\frac{1}{\gamma}} \right\}} \qquad r_c = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{2.3}\right)^{4.33} = 0.5457$$

$$0.5 = 0.98 A_2 \sqrt{8.667 \times 3.2626 \times 10 \times 10^5 \left\{ (0.5457)^{1.538} - (0.5457)^{1.538} \right\}} = 0.98 A_2 \sqrt{1.4526 \times 10^6}$$

$A_2 = 423 \times 10^{-6} \text{ m}^2$ and the diameter =23.2 mm

4. A Venturi Meter has a throat area of 500 mm^2. Steam flows through it, and the inlet pressure is 7 bar and the throat pressure is 5 bar. The inlet temperature is 400°C. Calculate the flow rate. The density of the steam at inlet is 2.274 kg/m^3. Take γ= 1.3. R = 462 J/kg K. C_d = 0.97.
From the steam tables $v_1 = 0.4397 \text{ m}^3/\text{kg}$ so $\rho_1 = 1/0.4397 = 2.274 \text{ kg/m}^3$

$$m = C_d A_2 \sqrt{\left[\frac{2\gamma}{\gamma-1}\right] p_1 \rho_1 \left\{ (r_c)^{\frac{2}{\gamma}} - (r_c)^{1+\frac{1}{\gamma}} \right\}}$$

$$m = 0.97 \times 500 \times 10^{-6} \sqrt{\left[\frac{2 \times 1.3}{1.3-1}\right] 7 \times 10^5 \times 2.274 \left\{ (5/7)^{1.538} - (5/7)^{1.764} \right\}}$$

$m = 485 \times 10^{-6} \times 783$ $\qquad m = 0.379 \text{ kg/s}$

5. A pitot tube is pointed into an air stream which has an ambient pressure of 100 kPa and temperature of 20ºC. The pressure rise measured is 23 kPa. Calculate the air velocity. Take γ = 1.4 and R = 287 J/kg K.

$$\frac{p_2}{p_1} = \frac{123}{100} = 1.23 = \left(M^2 \frac{\gamma - 1}{2} + 1 \right)^{\frac{\gamma}{\gamma - 1}} \qquad 1.23 = \left(0.2M^2 + 1\right)^{3.5}$$

$1.0609 = 0.2M^2 + 1 \qquad 0.0609 = 0.2M^2 \qquad M\ 0.5519$
$a = \gamma RT^{1/2} = (1.4 \times 287 \times 293)^{1/2} = 343.1$ m/s
$v = 0.5519 \times 343.1 = 189.4$ m/s

6. A fast moving stream of gas has a temperature of 25ºC. A thermometer is placed into it in front of a small barrier to record the stagnation temperature. The stagnation temperature is 28ºC. Calculate the velocity of the gas. Take γ= 1.5 and R = 300 J/kg K. (Answer 73.5 m/s)

$\Delta T = 3$ K $\quad \Delta T/T_1 = v^2/\gamma RT \qquad c_p = \gamma R/(\gamma - 1)$

$\Delta T = 3 = v^2/2\,c_p = v^2(\gamma - 1)/(2\,\gamma R) = v^2(1.5 - 1)/(2 \times 1.5 \times 300)$

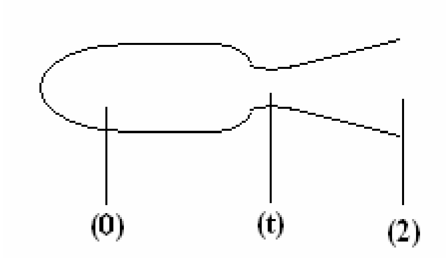
$v^2 = 5400 \quad v = 73.48$ m/s

SELF ASSESSMENT SOLUTIONS
TUTORIAL 7

SELF ASSESSMENT EXERCISE No. 4

1. Similar to Q6 1990
A nozzle is used with a rocket propulsion system. The gas is expanded from complete stagnation conditions inside the combustion chamber of 20 bar and 3000K. expansion is isentropic to 1 bar at exit. The molar mass of the gas is 33 kg/kmol. The adiabatic index is 1.2. The throat area is 0.1 m^2. Calculate the thrust and area at exit. (0.362 m^2 and 281.5 kN)
Recalculate the thrust for an isentropic efficiency of 95%. (274.3 kN)
Note that expansion may not be complete at the exit area.

$$p_t/p_o = \{2/(\gamma+1)\}^{\gamma/(\gamma-1)}$$



STAGNATION CONDITIONS
$p_o = 20$ bar $\quad T_o = 3000$ K
$\gamma = 1.4 \quad p_2 = 1$ bar $\quad R = R_o/\check{N} = 8314.4/33 = 251.94$ J/kg K
$c_p = \gamma R/(\gamma-1) = 1511$ J/kg K

$$\frac{T_o}{T_t} = 1 + \frac{\gamma-1}{2}M_t^2 \qquad M_t = 1 \qquad T_t = \frac{3000}{1.1} = 2727\ \text{K}$$

ENERGY BALANCE
$c_p T_o = c_p T_t + v_t^2/2 \quad 2c_p (3000 - 2727) = v_t^2 \quad v_t = 908.6$ m/s
or $v_t = \sqrt{(\gamma R T_t)} = 908.6$ m/s

$$\frac{p_o}{p_t} = \left(1 + \frac{\gamma-1}{2}M_t^2\right)^{\frac{\gamma}{\gamma-1}} = 0.564 \quad p_t = 11.289\ \text{bar} \quad \text{or} \quad \frac{p_o}{p_t} = \left(\frac{2}{\gamma-1}\right)^{\frac{\gamma}{\gamma-1}} = 0.564$$

$\rho = p/RT = 11.289 \times 10^5 /(251.94 \times 2727) = 1.643$ kg/m^3

Volume = Area x velocity = 0.1 x 908.6 = 90.86 m^3/s
Mass = volume x density = 149.3 kg/s

EXIT

$$\frac{p_o}{p_2} = \left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma}{\gamma-1}} \qquad \frac{20}{1} = \left(1 + 0.1M_2^2\right)^6 \qquad M_2 = 2.5447\ \text{(supesonic)}$$

$$\frac{T_o}{T_2} = \frac{3000}{T_2} = 1 + \frac{\gamma-1}{2}M_2^2 \qquad T_2 = 1821\ \text{K}$$

$c_p T_o = c_p T_2 + v_2^2/2$
$2c_p (3000 - 1821) = v_2^2 \quad v_2 = 1888$ m/s

Volume = mRT_2/p_2 = 149.3 x 251.94 x 1821/1 x 10^5 = 683.9 m^3/s
A_2 = Volume/velocity = 683.9/1888 = 0.362 m^2

THRUST
Assume no pressure force

Thrust = Δ(mv)/second = 149.3 (1880 - 0) = 281.5 kN
$\eta_{is} = v_2^2/(v_2')^2 \quad v_2^2 = 0.95 \times 1888^2 \quad v_2 = 1840$ m/s

Thrust = Δ(mv)/second = 149.3 (1840 - 0) = 274 kN

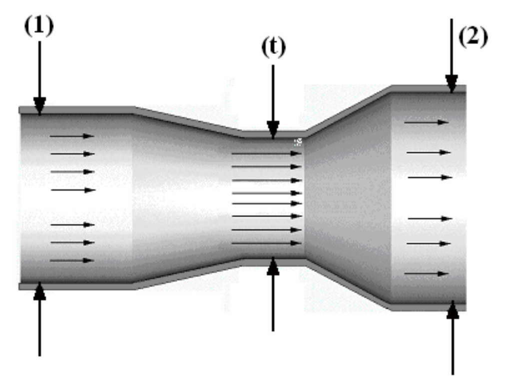
2. Similar to Q3 1988

A perfect gas flows through a convergent-divergent nozzle at 1 kg/s. At inlet the gas pressure is 7 bar, temperature 900 K and velocity 178 m/s. At exit the velocity is 820m/s. The overall isentropic efficiency is 85%. The flow may be assumed to be adiabatic with irreversibility's only in the divergent section.

c_p = 1.13 kJ/kg K R = 287 J/kg K.

Calculate the cross sectional areas at the inlet, throat and exit.

Calculate the net force acting on the nozzle if it is stationary. The surrounding pressure is 1 bar. You may assume $p_t/p_o = \{2/(\gamma+1)\}^{\gamma/(\gamma-1)}$



STAGNATION CONDITIONS

$c_p T_o = c_p T_o + v_1^2/2$ $T_o = 914$ k

$$p_o = p_1\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 7.44\text{ bar}$$

$$p_t = p_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 4\text{ bar}$$

$$T_t = T_0\left(\frac{p_t}{p_o}\right)^{\frac{\gamma}{\gamma-1}} = 780\text{ K} \qquad v_t = \sqrt{\gamma R T_t} = 548\text{ m/s}$$

$$\rho_1 = \frac{p_1}{RT_1} = 2.7\text{ kg/m}^3 \qquad \rho_t = \frac{p_t}{RT_t} = 1.785\text{ kg/m}^3$$

$c_p T_o = c_p T_2 + v_2^2/2$ 1130 x 914 = 1130 T_2 + $820^2/2$ T_2 = 616.5 K

Had the flow been isentropic $T_2 = T_2'$

With friction $\eta_{is} = 0.85 = \dfrac{T_1 - T_2'}{T_1 - T_2} = \dfrac{900 - T_2'}{900 - 616.5}$ $T_2' = 659$ K

This seems irrelevant since T_2 = is the actual outlet temperature.

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = 7\left(\frac{616.5}{900}\right)^{\frac{\gamma}{\gamma-1}} = 1.576\text{ bar}$$

$$\rho_2 = \frac{p_2}{RT_2} = 0.8907\text{ kg/m}^3$$

$A_2 = \text{mass}/(\rho_2 v_2) = 1/(0.8907 \times 820) = 0.001369\text{ m}^2$

FORCES

Pressure force = $p_1 A_1 - p_2 A_2 - p_a(A_1 - A_2)$ p_a is atmospheric

Pressure force = 7 x 10^5 x 0.00208 – 1.57 x 10^5 x 0.001369 - 1 x 10^5 x(0.00208 - 0.001369)

Pressure force = 1169 N

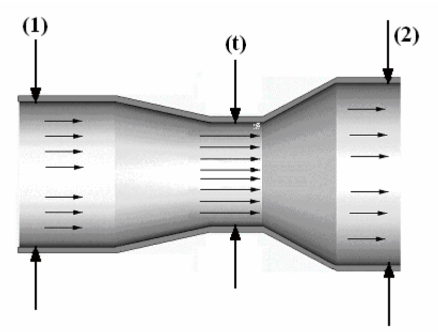
Momentum force = m Δv = 1(820 – 178) = 642 N and this acts in opposite direction

Resultant force = 1169 – 642 = 527 N

3. Similar to Q4 1989

Dry saturated steam flows at 1 kg/s with a pressure of 14 bar. It is expanded in a convergent-divergent nozzle to 0.14 bar. Due to irreversibility's in the divergent section only, the isentropic efficiency 96%. The critical pressure ratio may be assumed to be 0.571. Calculate:

(i) The dryness fraction, specific volume and specific enthalpy at the throat.
(0.958, 0.23 m^3/kg and 2683 kJ/kg)
(ii) The velocity and cross sectional area at the throat and exit.
(462.6 m/s , 497 mm^2, 1163 m/s and 73.2 cm^2.)
(iii) The overall isentropic efficiency.(96.6%)



p_1 = 14 bar dry saturated steam.
p_2 = 0.14 bar $\qquad \eta_{is} = 0.96$
$p_t/p_1 = 0.571$ $\qquad p_t$ = 8 bar

$h_1 = h_g$ at 14 bar = 2790 kJ/kg
$v_1 = v_g$ at 14 bar = 0.1408 m^3/kg
$s_1 = s_g$ at 14 bar = 6.469 kJ/kg K
$s_1 = s_t = s_f + x_t\, s_{fg}$ $\qquad 6.469 = 2.046 + x_t\, 4.617$ $\qquad x_t = 0.958$

At the throat assume the moisture has negligible volume and that steam takes up all the volume.
v_g at 8 bar = 0.2403 m^3/kg $\quad v_t = x_t\, v_g$ = 0.958 x 0.2403 = 0.23 m^3/kg

$h_t = h_f + x_t\, h_{fg}$ at 8 bar $\quad h_t$ = 721 + 0.958 x 2048 = 2683 kJ/kg

ENERGY EQUATION
$h_1 + c_1^2/2 = h_t + c_t^2/2$

2790 x 10^3 + 0 = 2683 x 10^3 + $c_t^2/2$ $\qquad c_t$ = 462.6 m/s

mass = ρ A c = A c/v = 1 kg/s $\qquad A_t = v_t/c_t$ = 0.23 /462.6 = 497 x 10^{-6} m^2

h_2' = 2087 (from the h – s chart) $\qquad \eta_{is} = 0.96 = \dfrac{2683 - h_2}{2683 - 2087}$

h_2 = 2114 kJ/kg = $h_f + x_2\, h_{fg}$ at 0.14 bar

2114 = 220+ x_2 2376 $\qquad x_2$ = 0.797

ENERGY EQUATION

$h_t + c_t^2/2 = h_2 + c_2^2/2$

2683 x 10^3 + $463.6^2/2$ = 2114 x 10^3 + $c_2^2/2$ $\qquad c_2$ = 1163 m/s

$v_2 = x_2\, v_g$ = at 0.14 bar = 0.797 x 10.69 = 8.52 m^3/kg

$A_2 = v_2/c_2$ = 8.52/ 1163 = 0.00732 m^2

$$\eta(\text{overall}) = \frac{h_1 = h_2}{h_1 - h_2'} = \frac{2790 - 2114}{2790 - 2090} = 0.9657$$

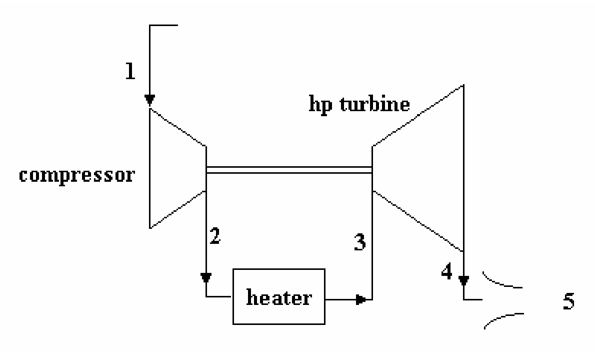
4. Similar to Q2 1989

A jet engine is tested on a test bed. At inlet to the compressor the air is at 1 bar and 293 K and has negligible velocity. The air is compressed adiabatically to 4 bar with an isentropic efficiency of 85%. The compressed air is heated in a combustion chamber to 1175 K. It is then expanded adiabatically in a turbine with an isentropic efficiency of 87%. The turbine drives the compressor. The gas leaving the turbine is expanded further reversibly and adiabatically through a convergent nozzle. The flow is choked at exit. the exit area is 0.1 m2.

Determine the pressures at the outlets of the turbine and nozzle. (2.38 bar and 1.129 bar)
the mass flow rate. (27.2 kg/s)
the thrust produced. (17 kN)

It may be assumed that $T_t/T_o = 2/(\gamma+1)\}$ $a = (\gamma R T)^{1/2}$

$$T_{2'} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma\eta_\infty}} = 293(4)^{\frac{1.4-1}{1.4 \times 0.84}} = 435.4\,\mathrm{K}$$



$$\eta_{IS} = 0.85 = \frac{T_{2'} - T_1}{T_2 - T_1} = \frac{435.4 - 293}{T_2 - 293} \quad T_2 = 460.5\,\mathrm{K}$$

$P(in) = mc_p(T_2 - T_1) = 1 \times 1.005\ (167.5)$

$P(out) = P(in) = mc_p(T_3 - T_4)$

$(T_3 - T_4) = 167.5$ and $T_4 = 1175 - 167.5 = 1007.5$ K

$$\eta_{IS} = 0.85 = \frac{T_3 - T_4}{T_3 - T_{4'}} = \frac{1175 - 1007.5}{1175 - T_{4'}} \quad T_{4'} = 982.5\,\mathrm{K}$$

$$T_{4'} = 1175(r)^{\frac{\gamma-1}{\gamma}} = 1175(r)^{\frac{1.4-1}{1.4 \times 0.84}} \quad r = 0.5346 = \frac{p_{4'}}{p_3} = \frac{p_{4'}}{4} \quad p_{4'} = p_4 = 2.138\,\mathrm{bar}$$

$$\frac{T_5}{T_4} = \frac{2}{\gamma + 1} = 0.833 \qquad T_5 = 839.6\ \mathrm{K}$$

$$\frac{839.6}{1007.5} = 0.833 = \left(\frac{p_5}{4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_5}{4}\right)^{0.286} \quad p_5 = 1.13\ \mathrm{bar}$$

When the nozzle is choked, $v = \sqrt{(\gamma\ \rho\ T)} = \sqrt{(1.4 \times 287 \times 839.6)} = 580.8$ m/s

Volume flow rate = $V = A\,v = 0.1 \times 580.8 = 58.08\ m^3/s$

$m = pV/RT = 1.13 \times 10^5 \times 55.08/(287 \times 839.6) = 27.23$ kg/s

$F = A\,\Delta p + m\,\Delta v = 0.1 \times 0.13 \times 10^5 + 27.23 \times 580.8 = 17$ kN

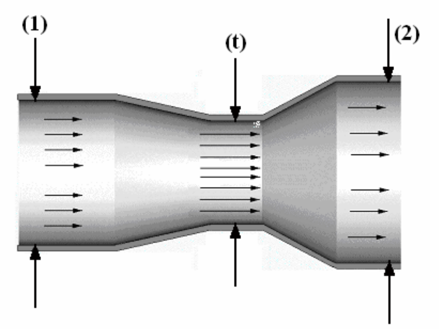
5. Similar to Q4 1986

Dry saturated steam expands through a convergent-divergent nozzle. The inlet and outlet pressures are 7 bar and 1 bar respectively at a rate of 2 kg/s. The overall isentropic efficiency is 90% with all the losses occurring in the divergent section. It may be assumed that $\gamma = 1.135$ and

$$p_t/p_o = \{2/(\gamma+1)\}^{\gamma/(\gamma-1)}$$

Calculate the areas at the throat and exit. (19.6 cm^2 and 38.8 cm^2).

The nozzle is horizontal and the entry is connected directly to a large vessel containing steam at 7 bar. The vessel is connected to a vertical flexible tube and is free to move in all directions. Calculate the force required to hold the receiver static if the ambient pressure is 1.013 bar.



Critical ratio $r_c = \left(\dfrac{2}{\gamma-1}\right)^{\frac{\gamma}{\gamma-1}} = 0.5774$

$p_t = 7 \times 0.5774 = 4$ bar

$h_1 = h_g = 2764$ kJ/kg $\quad s_1 = s_g = 6.709$ kJ/kg K

$s_1 = s_t = s_f + x_t\, s_{fg} \quad 6.709 = 2.046 + x_t\, 5.201$

$x_t = 0.963$

Neglecting the inlet velocity

$h_1 = h_t + c_t^2/2 \quad 2764 \times 10^3 = 2661 \times 10^3 + c_t^2/2 \quad c_t = 453.8$ m/s

v_t = specific volume = $x_t\, v_g = 0.963 \times 0.4623 = 0.445$ m^3/kg

mass = $\rho A c = A c/v = 2$ kg/s $\quad A_t = m\, v_t/c_t = 2 \times 0.445/453.8 = 0.00196$ m^2

For isentropic flow $s_1 = s_t = s_2'$

Assuming $p_2 = 1.013$ bar $\quad 6.709 = 1.307 + x_2'\, 6.048 \quad x_2' = 0.893$

$h_2' = h_f + x_2\, h_{fg} = 419.1 + 0.893 \times 2256.7 = 2435$ kJ/kg

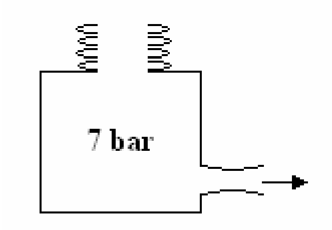
η(overall) = $0.9 = \dfrac{h_1 = h_2}{h_1 - h_2'} = \dfrac{2764 - h_2}{2764 - 2435} \quad h_2 = 2467$ kJ/kg

$h_1 = h_2 + c_2^2/2$

$2764 \times 10^3 = 2467 \times 10^3 + c_2^2/2 \quad c_2 = 770$ m/s

$v_2 = x_2\, v_g = 0.893 \times 1.673 = 1.494$ m^3/kg

$A_2 = m v_2/c_2 = 2 \times 1.494/770 = 0.00388$ m^2



$F = A_2(p_2 - p_{atm}) + m\,\Delta v$

$F = 0.00388(7 - 1) \times 10^5 + 2 \times (770 - 0) = 3.868$ kN

If atmospheric pressure is 0.95 bar $F = 0.00388(7 - 0.95) \times 10^5 + 2 \times (770 - 0) = 3.887$ kN

SELF ASSESSMENT SOLUTIONS
TUTORIAL 8
SELF ASSESSMENT EXERCISE No. 1

1. A boiler burns fuel oil with the following analysis by mass :

80% C   18% H_2 2%S

30% excess air is supplied to the process. Calculate the stoichiometric ratio by mass and the % Carbon Dioxide present in the dry products.

$C + O_2 \rightarrow CO_2$
12 32 48
0.8 2.133 2.933

$2\ H_2 + O_2 \rightarrow 2\ H_2O$
4 32 36
0.18 1.44 1.62

$S + O_2 \rightarrow SO_2$
32 32 64
0.02 0.02 0.04

Total O_2 = 3.593 kg   Air needed = 3.593/23% = 15.62 kg
30% Excess air so air supplied = 1.3 x 15.62 = 20.308 kg
Contents
N_2 = 0.77x 20.308 =   15.637 kg or 79%
SO_2 =   0.04 kg or 0.2%
CO_2 =   2.933 kg or 15%
O_2 =   1.078 kg or 5.8%
Total mass of products   19.69 kg

2. A boiler burns coal with the following analysis by mass :

75% C   15% H_2 7% S   remainder ash

Calculate the % Carbon Dioxide present in the dry products if 20% excess air is supplied.

$C + O_2 \rightarrow CO_2$
12 32 48
0.75 2 2.75

$2\ H_2 + O_2 \rightarrow 2\ H_2O$
4 32 36
0.15 1.2 1.35

$S + O_2 \rightarrow SO_2$
32 32 64
0.07 0.07 0.14

Total O_2 = 3.27 kg   Air needed = 3.27/23% = 14.217 kg
20% Excess air so air supplied = 1.2 x 14.217 = 17.06 kg
Contents
N_2 = 0.77x 17.06 =   13.137kg or 78.7%
SO_2 =   0.14 kg or 0.8%
CO_2 =   2.75 kg or 16.5%
O_2 =   0.654 kg or 4%
Total mass of products   16.681 kg

3. Calculate the % of each dry product when coal is burned stoichiometrically in air. The analysis of the coal is: 80% C 10% H_2 5% S and 5% ash.

$C + O_2 \rightarrow CO_2$
12 32 48

0.8 2.133 2.933

$2 H_2 + O_2 \rightarrow 2 H_2O$
4 32 36
0.1 0.8 0.9

$S + O_2 \rightarrow SO_2$
32 32 64
0.05 0.05 0.1

Total O_2 = 2.983 kg
Air needed = 2.983/23% = 12.97 kg
Contents

N_2 = 0.77x 12.97 =	9.986 kg or 76.7%
SO_2 =	0.1 kg or 0.8%
CO_2 =	2.933 kg or 22.5%
O_2 =	0
Total mass of products	13.02 kg

SELF ASSESSMENT EXERCISE No. 2

1. Find the air fuel ratio for stoichiometric combustion of Ethene by volume. (26.19/1)

$C_2H_4 + 3O_2 \rightarrow 2CO_2 + 2H_2O$
1 m^3 3 m^3 2 m^3 2 m^3
Stoichiometric ratio =3/21% = 14.28/1

2. Find the air fuel ratio for stoichiometric combustion of Butane by volume.(30.95/1). Calculate the % carbon dioxide present in the dry flue gas if 30% excess air is used. (10.6%)

$C_4H_{10} + 6½ O_2 \rightarrow 4CO_2 + 5H_2O$
1 m^3 6½m^3 4 m^3 5 m^3
Stoichiometric ratio =6.5/21% = 30.95/1

30% excess air so air supplied = 30.95 x 1.3 = 40.238
Contents

N_2 = 0.79 x 40.238 =	31.79 m^3
CO_2 =	4 m^3
O_2 = 0.3 x 6.5 =	1.95 m^3
Total	37.74 m^3

% CO_2 = 4/37.74 = 10.6%

3. Find the air fuel ratio for stoichiometric combustion of Propane by volume. Calculate the % oxygen present in the dry flue gas if 20% excess air is used.

$C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O$
$1\ m^3$ $5\ m^3$ $3\ m^3$ $4\ m^3$
Stoichiometric ratio =5/21% = 23.8/1
20% Excess Air Air supplied = 1.2 x 23.8 = 28.571 m^3
Contents

N_2 = 0.79 x 28.571 =	22.57 m^3
CO_2 =	3 m^3
O_2 = 0.2 x 5 =	1 m^3
Total	26.57 m^3

% O_2 = 1/26.57 = 3.76%

4. A gaseous fuel contains by volume :

5% CO_2, 40% H_2 , 40% CH_4, 15% N_2

Determine the stoichiometric air and the % content of each dry product.

0.4 m^3 H_2 needs 0,2 m^3 O_2 and makes .05 m^3 CO_2
0.4 m^3 CH_4 needs 0,8 m^3 O_2 and makes .4 m^3 CO_2
Total O_2 needed is 1 m^3
Stoichiometric ratio = 1/0.21 = 4.762 m^3

N_2 in air is .79 x 4.762 3.762
N_2 in gas is 0.25 m^3
Total N_2 = 3.912 m^3
Total dry gas = 0.05 + 0.4 + 3.912 = 4.362 m^3

% N_2 = 3.912/4.362 x 100 = 89.7 %
% CO_2 = 0.45/4.362 x 100 = 10.3 %

SELF ASSESSMENT EXERCISE No. 3

1. C_2H_6 is burned in a boiler and the dry products are found to contain 8% CO_2 by volume. Determine the excess air supplied. (59%)

$C_2H_6 + 3\frac{1}{2} O_2 \rightarrow 2CO_2 + 3H_2O$
$2C_2H_6 + 7 O_2 \rightarrow 4CO_2 + 6H_2O$
Let the excess air be x Stoichiometric Ratio
$O_2 \rightarrow 3.5\ m^3$
Air $\rightarrow$ 3.5/21% = 16.67 m^3
Actual Air = 16.67(1 + x)

DRY PRODUCTS

$N_2 \rightarrow$ 0.79 x 16.67(1 + x) =	13.167 +	13.167 x
$O_2 \rightarrow$		3.500 x
$CO_2 \rightarrow$	2.000	
TOTAL	15.167 +	16.667 x

% CO_2 is 8 so 8/100 = 2/(15.167 + 16.667 x)
200/8 = 15.167 + 16.667 x
25 – 15.167 = 16.667 x
9.833/16.667 = x = 0.59 or 59%

2. The analysis of the dry exhaust gas from a boiler shows 10% carbon dioxide. Assuming the rest is oxygen and nitrogen determine the excess air supplied to the process and the % excess air. The fuel contains 85% C and 15% H_2

$C + O_2 \rightarrow CO_2$
12 32
0.85kg needs (32/12) x 0.85 = 11.2 kg O_2
$2H_2 + O_2 \rightarrow 2H_2O$
4 32
0.15kg needs (32/8) x 0.15 = 1.2 kg O_2
Total O_2 = 3.467 kg
Stoichiometric ratio = 3.467/0.233 = 14.88/1

DEG Consider 1 kmol
This contains 0.1 kmol of CO_2
y kmol of O_2
1 – 0.1 – y kmol of N_2
0.1 kmol of CO_2 is 0.1 x 44 = 4.4 kg of CO_2
Carbon in it is 12/44 x 4.4 = 1.2 kg Carbon
This came from the fuel.

FUEL consider 1 kmol
It contains 0.85 kmol of carbon.
0.85/12 = 0.708 kmol of Dry Exhaust Gas

NITROGEN
(0.9 – y) x 28 = 25.2 – 28y kg per kmol of DEG
OXYGEN
(25.2 – 28y)x 23.3/76.7 = 7.666 – 8.506 y per kmol of DEG (Oxygen supplied per kmol DEG)

Oxygen in CO_2 (32/44) x 4.4 = 3.2 kg per kmol DEG
Excess oxygen 32y kg per kmol DEG
Total Oxygen excluding that in the water is 32y + 3.2 kg per kmol DEG

Oxygen to burn H_2
Subtract from Oxygen supplied (7.655 – 8.506 y) – (32y + 3.2)

4.455 – 40.506y kg per kmol DEG

There are 0.708 kmol DEG

Oxygen to burn H_2 = 0.708(4.455 – 40.506y) = 3.154 – 28.678 y kg

$2H_2$ + O2 $\rightarrow$
4 32 ratio 8/1
1 8 for 0.1 kg H_2 0.8 kg O_2
0.8 = 3.145 – 28.678y y = 0.0818 kg

NITROGEN
25.2 – 28 x 0.0818 = 22.91 kg per kmol DEG
But there are 0.708 kmol so 25.91 x 0.708 = 16.22 kg N_2
AIR
16.22/0.767 = 21.1 kg
Excess air is 21.1 – 14.88 = 6.27 kg
% Excess = 6.27/14.88 = 42%

SELF ASSESSMENT EXERCISE No. 4

1. Similar to Q8 1991

The gravimetric analysis of a fuel is Carbon 78%, hydrogen 12%, oxygen 5% and ash 5%. The fuel is burned with 20% excess air. Assuming complete combustion, determine the composition of the products, the dew point of the products and the mass of water condensed when the products are cooled to 30°C.

C +	O_2 =	CO_2		$2H_2$ +	O_2 =	$2H_2O$
0.78	2.08	2.86		0.12	0.96	1.08
0.065 kmol C				0.03 kmol H_2		

Total O_2 needed = 2.08 + 0.96 – 0.05 = 2.99 kg

Air needed = 2.99/0.23 = 13 kg

Actual air = 13 x 1.2 = 15.6 kg

Nitrogen = 0.77 x 15.6 = 12.012 kg

	kmol	mass	%
N_2	0.429	12.012	72.6
CO_2	0.065	2.860	17.3
H_2O	0.06	1.080	6.5
O_2	0.019	0.598	3.6
Totals	0.574	16.55	100

If everything ends up as gas then the partial pressure of H_2O is $p_{H_2O} = \frac{0.06}{0.574} \text{x1bar} = 0.1045 \text{ bar}$

The corresponding saturation temperature is 47°C. If cooled to 30 °C some will condense and the vapour left will be dry saturated.

p_s at 30 °C = 0.04242 bar

The process is constant pressure so the volume is not constant.

Let the kmol of H_2O vapour be X

$p_{N_2} = \frac{0.429}{0.574 + X}$ $\quad p_{CO_2} = \frac{0.019}{0.574 + X}$ $\quad p_{H_2O} = \frac{X}{0.574} = 0.04242$

X = 0.04242(0.574 + X) X = 0.02277

Mass of vapour = M M/18 = 0.02277 M = 0.4098 kg

Condensate formed = 1.08 – 0.41 = 0.67 kg

2. Similar to Q8 1989

Carbon monoxide is burned with 25% excess oxygen in a fixed volume of 0.2 m^3. The initial and final temperature is 25°C. The initial pressure is 1 bar. Calculate the final pressure and the heat transfer. Use your thermodynamic tables for enthalpies of reaction.

$CO + \frac{1}{2} O_2 \rightarrow CO_2$ Heat released = 282990 kJ/kmol (page 21 of tables 3rd edition)

125% oxygen

For 1 kmol of CO we have 0.5 x 1.25 = 0.625 kmol of O_2

We also have 0.625 x 79/21 = 2.35 kmol of N_2

kmol Ratio $1\ CO + 0.625\ O_2 + 2.35\ N_2 \rightarrow 1\ CO_2 + 0.125\ O_2 + 2.35\ N_2$

We don't know the actual number of kmol of CO_2

p_2 = (3.475/3.975) x 1 = 0.874

$p_1V_1 = \check{N}\, R_o\, T_1$ $\check{N} = (1 \times 10^5 \times 0.2)/(8314.4 \times 298) = 0.00807$ (the total kmol of reactants)

Let the number of kmol of CO be X

X CO + 0.625X O_2 + 2.35X N_2 = 0.00807

3.975X = 0.00807 X = 0.00203 kmol (CO)

ΔH = 0.00203 x 282990 = 574.5 kJ

Final kmol of products = (3.475/3.975) x 0.00203 = 0.007055

0.007055 = (p_2 x 0.2)/(8314 x 298) p_2 = 0.874 bar

3. Similar to Q9 1984

Prove that the enthalpy and the internal energy of reaction are related by

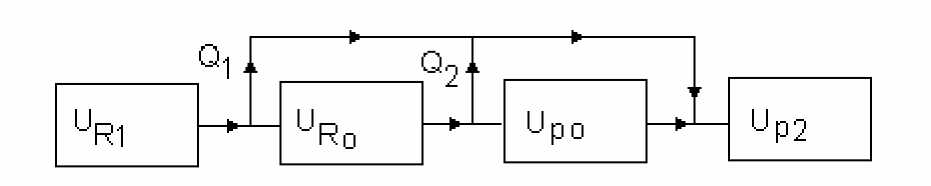
$\Delta H_O = \Delta U_O + R_O T_O(n_P - n_R)$

where n_P and n_R are the kmols of products and reactants respectively.

Ethylene (C_2H_4) and 20% excess air at 77^oC are mixed in a vessel and burned at constant volume. Determine the temperature of the products. You should use your thermodynamic tables to find ΔU_O or ΔH_O and the table below. (Answer 2263 K)

	C_2H_4	O_2	N_2	CO_2	H_2O
T(K)		U (kJ/kmol)			
298.15	- 2479	-2479	-2479	-2479	-2479
300	- 2415	-2440	-2440	-2427	-2432
400	-1557	-297	-355	683	126
2400		54537	50696	95833	73650
2600		60657	50696	95833	73650
2800		66864	62065	117160	92014
3000		73155	67795	127920	101420

Consider a mixture of reactants at condition (1) which is burned and the resulting products are at condition (2). In order to solve problems we consider that the reactants are first cooled to a reference condition (0) by removing energy Q_1. The reaction then takes place and energy is released. The products are then brought back to the same reference conditions (0) by removing energy Q_2. The energy Q_1 and Q_2 are then returned so that the final condition of the products is reached (2).



For constant volume combustion (closed system), we use Internal Energy. Balancing we have

$$U_{p2} - U_{R1} = (U_{Ro} - U_{R1}) + (U_{po} - U_{Ro}) + (U_{p2} - U_{po})$$

The energy released by combustion is in this case the Internal Energy of combustion and this occurs at standard conditions of 1 bar and 25^oC. This pressure is designated p^θ and the internal energy of combustion is designated ΔU^θ. When this is based on 1 kmol it is designated Δu^θ

$$U_{p2} - U_{R1} = (U_{Ro} - U_{R1}) + \Delta U_o^\theta + (U_{p2} - U_{po})$$

The standard conditions chosen for the combustion are 1 bar and 25^oC. At this temperature the internal energy of all gases is the same (-2 479 kJ/kmol). The figure is negative because the zero value of internal energy arbitrarily occurs at a higher temperature.

If the process is conducted in a steady flow system, enthalpy is used instead of internal energy. The reasoning is the same but U is replaced by H.

$$H_{p2} - H_{R1} = (H_{Ro} - H_{R1}) + \Delta H_o^\theta + (H_{p2} - H_{po})$$

Δh_o^θ may be found in the thermodynamic tables for some fuels. The figures are quoted in kJ per kmol of substance.

For the products In terms of kmol fractions $h_{po} = u_{po} + n_p R_o T_o$

For the reactants In terms of kmol fractions $h_{Ro} = u_{Ro} + n_R R_o T_o$

where n is the kmols.

$\Delta h_o^\theta = (u_{po} + n_p R_o T_o) - (u_{Ro} + n_R R_o T_o)$

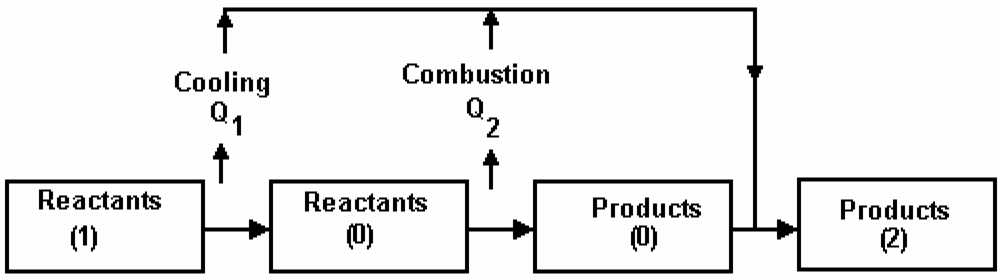
$\Delta h_o^\theta = (u_{po} - u_{Ro}) - n_R R_o T_o + n_p R_o T_o$

$\Delta h_o^\theta = (u_{po} - u_{Ro}) + (n_p - n_R) R_o T_o$

$\Delta u_o^\theta = \Delta h_o^\theta + (n_p - n_R) R_o T_o$

The combustion equation is:
$C_2H_4 + 3.6\ O_2 + 13.54\ N_2 = 2CO_2 + 2H_2 + 0.6O_2 + 13.54\ N2$

The process may be idealised as follows



Mean temperature = (77 + 25) + 273 = 324 K
From the tables at 325 K is very close.

Table of values

C_2H_4 C_p = 1.621 kJ/kg Kmass = 1 kmol x 28 = 28 kg
Q_1 = 28 x 1.621 x (77 – 25) = 2360 kJ

O_2 C_p = 0.923 kJ/kg Kmass = 3.6 kmol x 32 = 115.2 kg
Q_1 = 115.2 x 0.923 x (77 – 25) = 5529 kJ

N_2 C_p = 1.04 kJ/kg K mass = 13.563 kmol x 28 = 379.12 kg
Q_1 = 379.12 x 1.04 x (77 – 25) = 20503 kJ

Total Q_1 =28392 kJ per kmol of fuel (leaving system)
$Q_2 = \Delta H_o$ = 1323170 kJ (leaving system)

Next we repeat the process for the products to find $Q_1 + Q_2$
In order to use a mean specific heat we must guess the approximate final temperature of the products. A good guess is always 1150 K.

CO_2 C_p = 1.270 kJ/kg Kmass = 2 kmol x 44 = 88 kg
Q = 88 x 1.270 x ΔT = 111.76ΔT

H_2O C_p = 2.392 kJ/kg Kmass = 2 kmol x 18 = 36 kg
Q = 36 x 2.392 x ΔT = 86.112ΔT

O_2 C_p = 1.109 kJ/kg Kmass = 0.6 kmol x 32 = 19.2 kg
Q_1 = 19.2 x 1.109 x ΔT = 21.293ΔT

N_2 C_p = 1.196 kJ/kg Kmass = 13.54 kmol x 28 = 379.12 kg
Q = 379.12 x 1.196 x ΔT = 453.428ΔT

Total $Q_1 + Q_2$ = 672.593ΔT = 1323170 + 28392
ΔT = 2009 K t2 = 2009 + 298 = 2307 k
New mean temperature is (2307 + 298)/2 = 1302.5 K

Second Iteration

Next we must repeat the last stage with this mean temperature. 1300 K is the closest in the table.

CO_2	C_p = 1.298 kJ/kg K	Q = 88 x 1.298 x ΔT = 114.22ΔT
H_2O	C_p = 2.490 kJ/kg K	Q = 36x 2.490 x ΔT = 89.64ΔT
O_2	C_p = 1.125 kJ/kg K	Q_1 = 19.2 x 1.125 x ΔT = 21.5ΔT
N_2	C_p = 1.219 kJ/kg K	Q = 379.12 x 1.219 x ΔT = 462.147ΔT

Total $Q_1 + Q_2$ = 687.611ΔT kJ per kmol of fuel (positive entering system)

687.611ΔT = 28392 + 1323170 ΔT = 1965.6 K

T_2 = 1965.6 + 298 = 2263.6 K

Q 4 Similar to Q8 1993

An engine burns hexane (C_6H_{14}) in air. At a particular running condition the volumetric analysis of the dry products are

CO2	8.7%
CO	7.8 %
N2	83.5%

Calculate the air-fuel ratio by mass and find the stoichiometric ratio.

$$C_6H_{14} + X\left\{9.5O_2 + \frac{79}{21} x\, 9.5N_2\right\} \rightarrow aCO_2 + bCO + 7H_2O + \frac{79}{21} x\, 9.5N_2X$$

X is the fraction of air.

Carbon balance 6 = a + b

Dry Analysis $N_2 = \frac{35.738X}{6 + 35.738X} = 0.835$ X = 0.85

Mass C_6H_{14} + $8.075O_2$ + 30.377 N_2
86 258.4 850.556

Air/Fuel = (850.556/86) x (100/76.7) = 12.9/1 or (258.4/86) x (100/23.3) = 12.9/1

Stoichiometric ratio
C_6H_{14} + $9.5O_2$ =
86 304

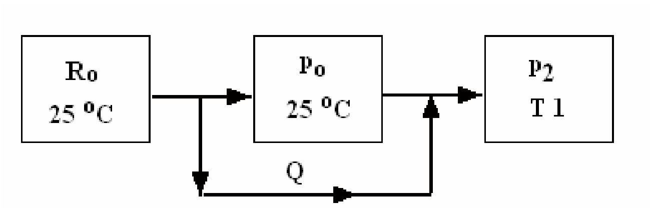
Air/Fuel = (304/86) x (100/23.3) = 15.17/1

SELF ASSESSMENT EXERCISE No. 4

Q1 Similar to Q1 1992

Hydrogen is mixed with stoichiometric air at 25°C and burned adiabatically at constant volume. After combustion 6% of the hydrogen remains unburned. Determine the temperature and pressure of the products. (Answer the temperature is 2344K after two approximations)

You need to find K^θ in the tables. Also find ΔH_O=241800 kJ/kmol. Deduce the partial pressures of the products as a fraction of p and then use K^θ to solve p.



$H_2 + ½ O_2 \leftrightarrow H_2O$

$N_2 = 0.5 \times 79/21 = 1.881$ kmol

REACTANTS
1 kmol H_2 only 94% burns
0.5 kmol O_2 Q = 241830 x 0.94 = 227320 kJ/kmol H_2
1.88 kmol N_2

PRODUCTS 1200 K

	0.06 H_2	c_p = 15.34	m = 0.12 kg	Q = 1.8408ΔT
	1.881 N_2	c_p = 1.204	m = 52.668 kg	Q = 63.412ΔT
	0.96 H_2	c_p = 2.425	m = 17.28 kg	Q = 41.904ΔT
	0.03 O_2	c_p = 1.115	m = 0.96 kg	Q =1.0704ΔT
Totals	2.931 kmol			Q = 108.22ΔT

227320 = 108.22ΔT ΔT = 2100 K
T_2 = 2100 + 298 = 2398 K
Mean temperature = (2398 + 298)/2 = 1348 K say 1350 for tables.

2nd iteration
PRODUCTS 1350 K

	0.06 H_2	c_p = 15.65	m = 0.12 kg	Q = 1.878ΔT
	1.881 N_2	c_p = 1.226	m = 52.668 kg	Q = 64.571ΔT
	0.96 H_2	c_p = 2.521	m = 17.28 kg	Q = 43.562ΔT
	0.03 O_2	c_p = 1.130	m = 0.96 kg	Q =1.0848ΔT
Totals	2.931 kmol			Q = 111.1ΔT

ΔT = 2046 T_2 = 2344 K

Q2 Similar to Q7 1985

A mixture of air and CO is burned adiabatically at constant volume. The air is 90% of the stoichiometric requirement. The mixture is initially at 5 bar and 400K. The only dissociation that occurs is $CO_2 \rightarrow CO + \frac{1}{2}O_2$. Show that the equilibrium constant at the final temperature T_2 is

$$K_p = \frac{1.121a}{(1-a)(0.9-a)^{1/2}\left(\frac{T_p}{T_R}\right)^{1/2}}$$

where a is the amount of CO_2 kmol in the products per kmol of CO in the reactants.

If it assumed that initially T_2 = 2900 K for which log K_p = 0.649, the solution of the above equation gives a=0.784. Check the assumed value of T_2 given that the internal energy of reaction at T_o= 298.15 K is -281750 kJ/kmol.

T (K)	U kJ/kmol			
	CO	O_2	N_2	CO_2
298.15	-2479	-2479	-2479	-2479
400	- 351	- 297	- 355	+ 683
2900	+65715	+69999	+64924	+122530

STOICHIOMETRIC

$CO + \frac{1}{2} O_2 + (79/21) \times \frac{1}{2} N_2 \rightarrow CO_2 + (79/21) \times \frac{1}{2} N_2$

$CO + 0.5\, O_2 + 1.88\, N_2 \rightarrow CO_2 + 1.88\, N_2$

90% air

$CO + 0.45\, O_2 + 1.692\, N_2 \rightarrow a\, CO_2 + b\, CO + d\, O_2 + 1.692\, N_2$

CARBON BALANCE

1 = a + b   b = 1 -a

OXYGEN BALANCE

0.5 + 0.45 = a + b/2 + d

0.95 = a + b/2 + d   multiply by 2

1.9 = 2a + b + 2d = 2a + 1 - a + 2d

0.9 = a + 2d   d = (0.9 – a)/2 = (0.9 – a)2

N_P = a + b + d + 1.692 = a + 1 - a + (0.9 – a)2 + 1.692 = 1 + 0.45 – a/2 + 1.692

N_R = 1 + 0.5 + 1.692 = 3.142

$$K_P = \frac{(a/N_p)p}{(b/N_p)p(d/N_p)^{1/2}p^{1/2}} = \frac{a}{b(d/N_p)^{1/2}p^{1/2}} \qquad K_P = \frac{a}{(1-a)\left(\frac{9-a}{2N_p}\right)^{1/2}p^{1/2}}$$

p = pressure of the products

$$p_P = \frac{N_p R_o T_p}{V} \qquad V = \frac{N_p R_o T_p}{p_P} \qquad p_R = \frac{N_R R_o T_R}{V} \qquad V = \frac{N_R R_o T_R}{p_R}$$

Equate V

$$\frac{N_p R_o T_p}{p_P} = \frac{N_R R_o T_R}{p_R} \qquad \frac{N_p T_p p_R}{N_R T_R} = p_P$$

$$K_P = \frac{a}{(1-a)\left(\frac{0.9-a}{2N_p}\right)^{1/2}\left(\frac{N_p T_p p_R}{N_R T_R}\right)^{1/2}} = \frac{a}{(1-a)\left(\frac{0.9-a}{2}\right)^{1/2}\left(\frac{T_p p_R}{N_R T_R}\right)^{1/2}} \qquad p_R = 5 \text{ bar}$$

$$K_p = \frac{a\sqrt{2 \times 3.142/5}}{(1-a)(0.9-a)^{1/2}\left(\frac{T_p}{T_R}\right)^{1/2}} = \frac{1.121a}{(1-a)(0.9-a)^{1/2}\left(\frac{T_p}{T_R}\right)^{1/2}}$$