ENGINEERING COUNCIL

CERTIFICATE LEVEL

THERMODYNAMIC, FLUID AND PROCESS ENGINEERING C106

TUTORIAL 1 - MOMENTS OF AREA

This tutorial should be skipped if you are already familiar with the topic.

In this section you will do the following.

- Define the centre of area.

- Define and calculate 1st. moments of areas.

- Define and calculate 2nd moments of areas.

- Derive standard formulae.

1.   CENTROIDS AND FIRST MOMENTS OF AREA

A moment about a given axis is something multiplied by the distance from that axis measured at 90° to the axis.

The moment of force is hence force times distance from an axis.
The moment of mass is mass times distance from an axis.
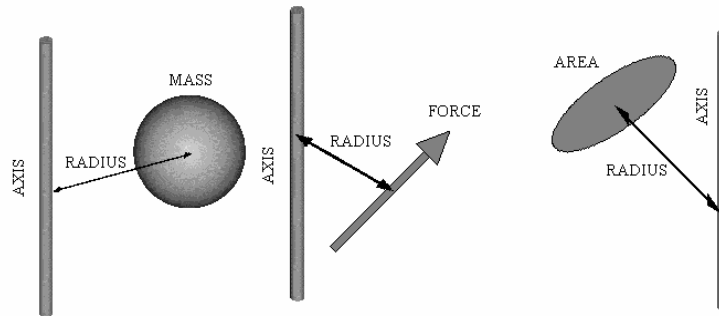The moment of area is area times the distance from an axis.

Fig.1

In the case of mass and area, the problem is deciding the distance since the mass and area are not concentrated at one point.

The point at which we may assume the mass concentrated is called the centre of gravity.

The point at which we assume the area concentrated is called the centroid.

Think of area as a flat thin sheet and the centroid is then at the same place as the centre of gravity. You may think of this point as one where you could balance the thin sheet on a sharp point and it would not tip off in any direction.

This section is mainly concerned with moments of area so we will start by considering a flat area at some distance from an axis as shown in Fig.1.2
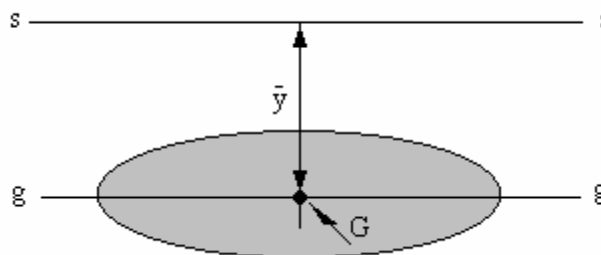

Fig..2

The centroid is denoted G and its distance from the axis s-s is y. The axis drawn through G parallel to s-s is the axis g-g. The first moment of area about the axis s-s is the product of area A and distance.

1st moment of area = A $\overline{y}$

From this we may define the distance y.

$\overline{y}$ = *1st moment of area/Area.*

For simple symmetrical shapes, the position of the centroid is obvious.

WORKED EXAMPLE 1

Find the formula for the first moment of area for rectangle about its longer edge given the dimensions are B and D.

SOLUTION

The centroid is at the middle of the rectangle and may be found at the point where the two diagonals cross. In other words it is half way from either edge. The distance from the long edge is hence D/2.
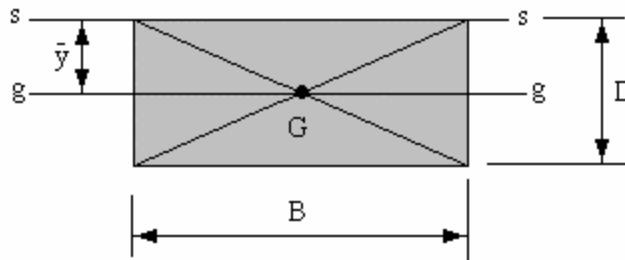

Figure 1.3

y = D/2
A = BD
1st moment = $A\bar{y}$ = $BD^2/2$

WORKED EXAMPLE 2

Find the formula for the 1st moment of area of a circular area about an axis touching its edge in terms of its diameter d.

SOLUTION

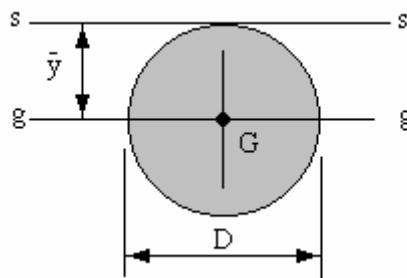The centroid is at the geometric centre distance one half diameter from the edge.

Fig.4

$\bar{y} = D/2$ $A = \pi D^2/4$ 1st moment = $A\bar{y} = (\pi D^2/4)/(D/2) = \pi D^3/8$

COMPLEX AREAS

In order to find the moment of area of more complex shapes we divide them up into sections, solve each section separately and then add them together.

WORKED EXAMPLE 3

Calculate the 1st. moment of area for the shape shown about the axis s-s and find the position of the centroid.

Fig.5

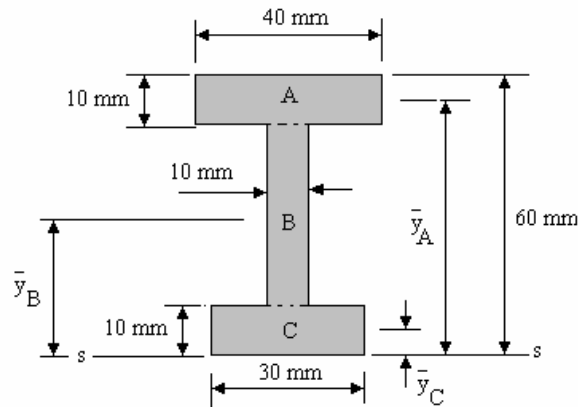### SOLUTION

The shape is not symmetrical so the centroid is not half way between the top and bottom edges. First determine the distance from the axis s-s to the centre of each part A, B and C. A systematic tabular method is recommended.

Part	Area	$\bar{y}$	Ay
A	400	55	22000
B	400	30	12000
C	300	5	1500
Total	1100		35500

The total first moment of area is 35 500 mm^3.
This must also be given by A $\bar{y}$ for the whole section hence
$\bar{y}$ = 35 500/1100 = 32.27 mm.

The centroid is 32.77 mm from the bottom edge.

SELF ASSESSMENT EXERCISE 1

1. Find the distance of the centroid from the axis s – s. All dimensions are in metres. (0.549 m).
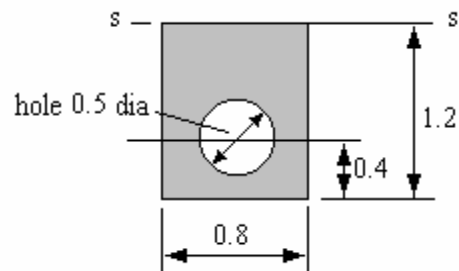

fig.6

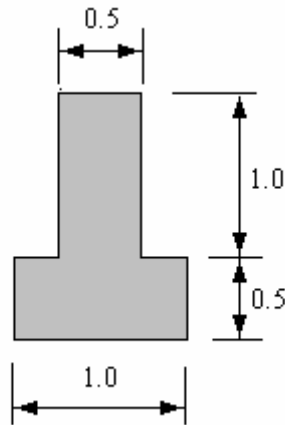2. Find the distance of the centroid from the bottom edge. All dimensions are in metres.   (0.625 m)

Fig.7

2. SECOND MOMENTS OF AREAS

2.2 GENERAL THEORY

If any quantity is multiplied by the distance from the axis s-s twice, we have a second moment. Mass multiplied by a distance twice is called the moment of inertia but is really the second moment of mass. We are concerned here with area only and the area multiplied by a distance twice is the second moment of area. The symbol for both is confusingly a letter I.

The above statement is over simplified. Unfortunately, both the mass and area are spread around and neither exists at a point. We cannot use the position of the centroid to calculate the 2nd. moment of area. Squaring the distance has a greater effect on parts further from the axis than those nearer to it. The distance that gives the correct answer is called the **RADIUS OF GYRATION** and is denoted with a letter k. This is not the same as $\bar{y}$.

The simplest definition of the 2nd. moment of area is $I = A k^2$

Whilst standard formulae exist for calculating the radius of gyration of various simple shapes, we should examine the derivations from first principles. We do this by considering the area to be made up of lots of elementary strips of width b and height dy. The distance from the axis s-s to the strip is y.
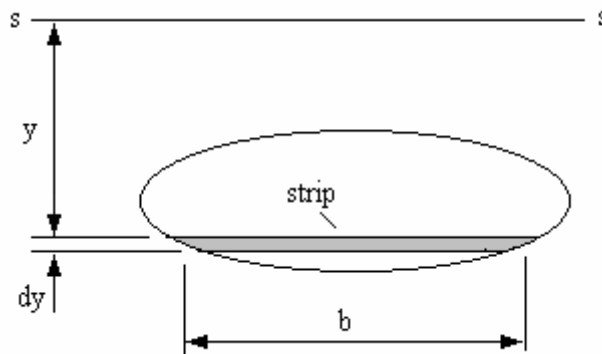
Fig.8

The area of the strip $= dA = b\,dy$

1st moment of area of strip $= y\,dA = by\,dy$

2nd moment of area of strip $= y^2\,dA = b\,y^2\,dy$

For the whole area, the 2nd moment of area is the sum of all the strips that make up the total area. This is found by integration.

$$I = \int b\,y^2\,dy$$

The limits of integration are from the bottom to the top of the area. This definition is important because in future work, whenever this expression is found, we may identify it as I and use standard formulae when it is required to evaluate it. We should now look at these.

WORKED EXAMPLE 4

Derive the standard formula for the second moment of area and radius of gyration for a rectangle of width B and depth D about an axis on its long edge.

SOLUTION
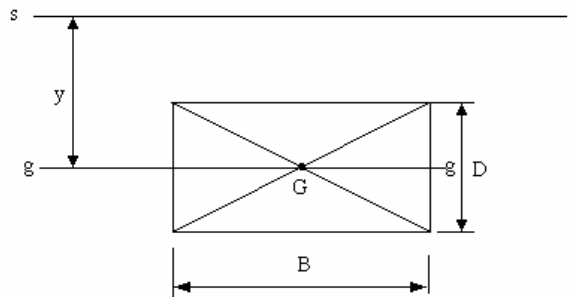
Fig.9

b = constant = B

$$I = \int_{0}^{\frac{D}{2}} by^2 dy = B \int_{0}^{\frac{D}{2}} y^2 dy$$

$$I = B\left[\frac{y^3}{3}\right]_{0}^{D} = \frac{BD^3}{3}$$

$$I = Ak^2 \qquad k = \sqrt{\frac{I}{A}} = \sqrt{\frac{BD^3}{3BD}} = 0.577D$$

Note $\bar{y}$ is 0.5 D and is not the same as k.

WORKED EXAMPLE 5

Derive the standard formula for the second moment of area and radius of gyration for a rectangle of width B and depth D about an axis through its centroid and parallel to the long edge.
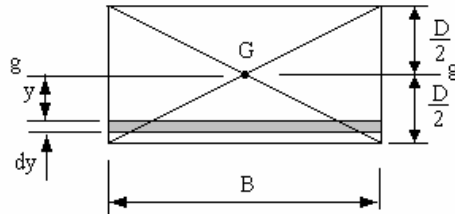
SOLUTION

Fig.1.10

$b = \text{constant} = B$

$$I = \int_{-\frac{D}{2}}^{\frac{D}{2}} by^2 dy = B \int_{-\frac{D}{2}}^{\frac{D}{2}} y^2 dy$$

$$I = B \left[\frac{y^3}{3} \right]_{-\frac{D}{2}}^{\frac{D}{2}} = \frac{BD^3}{12}$$

$$I = Ak^2 \qquad k = \sqrt{\frac{I}{A}} = \sqrt{\frac{BD^3}{12BD}} = 0.289D$$

Note $\bar{y}$ is zero and not the same as k.

CIRCLES

The integration involved for a circle is complicated because the width of the strip b varies with distance y.
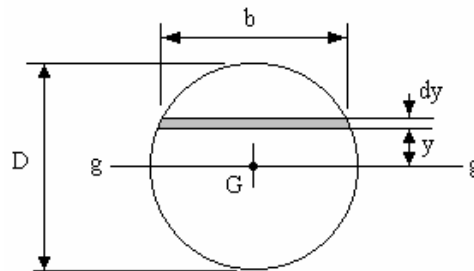
Fig.11

The solution yields the following result.   **I= $\pi D^4/64$ k = D/4**

2.2   PARALLEL AXIS THEOREM

If we wish to know the 2nd moment of area of a shape about an axis parallel to the one through the centroid (g-g), then the parallel axis theorem is useful.

The parallel axis theorem states $I_{ss} = I_{gg} + A\,(\bar{y})^2$

Consider a rectangle B by D and an axis s-s parallel to axis g-g.
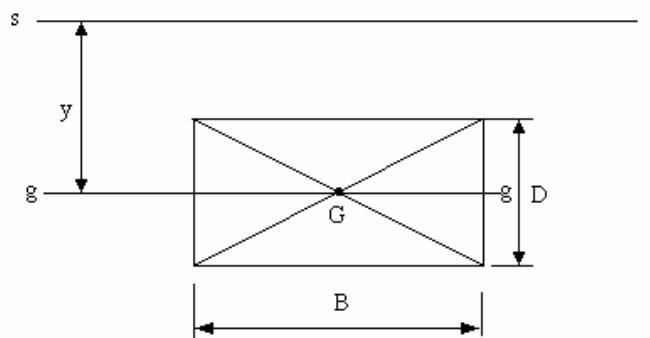
Fig.12

$$I_{gg} = \frac{BD^3}{12} \qquad A = BD \qquad I_{ss} = \frac{BD^3}{12} + BD\bar{y}^2$$

Consider when s-s is the top edge.

$$I_{ss} = \frac{BD^3}{12} + BD\bar{y}^2 \quad \text{but } \bar{y} = \frac{D}{2} \quad \text{so } I_{ss} = \frac{BD^3}{12} + BD\left(\frac{D}{2}\right)^2 = \frac{BD^3}{12}$$

This is the result obtained previously and confirms the method.

WORKED EXAMPLE 6

Calculate the 2nd moment of area for the same shape as in worked example 1.3. about the axis s-s
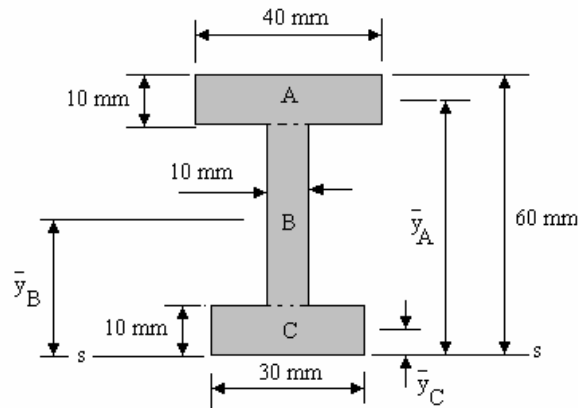
Fig.13

SOLUTION

The table shows the previous solution with extra columns added to calculate the second moment of area using the parallel axis theorem. In the new column calculate the second moment of area for each part (A, B and C) about each's own centroid using $BD^3/12$. In the next column calculate Ay^2.

Part	Area	$\bar{y}$	$A\bar{y}$	$I_{gg}=BD^3/12$	$A\bar{y}^2$	I_{ss}
A	400	55	22000	3333	1210000	1213333
B	400	30	12000	53333	360000	413333
C	300	5	1500	22500	7500	30000
Total	1100		35500			1656666

The total 2nd moment of area is 1656666 mm^4 about the bottom. We require the answer about the centroid so we now use the parallel axis theorem to find the 2nd moment about the centroid of the whole section.

The centroid is 32.77 mm from the bottom edge.

$$I_{gg} = I_{ss} - A\bar{y}^2$$

$$I_{gg} = 1656666 - 1100 \times 32.77^2$$

$$I_{gg} = 475405.8 \text{ mm}^4 = 475.4 \times 10^{-9} \text{ m}^4$$

Note $1 \text{ m}^4 = 10^{12} \text{ mm}^4$

WORKED EXAMPLE 7

Calculate the second moment of area of the shape shown about the axis s –s.
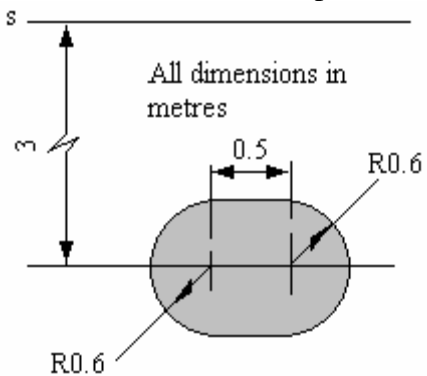
Fig.14

SOLUTION

The shape is equivalent to a circle 1.2 m diameter and a rectangle 0.5 by 1.2. All have their centroids located 3 m from the axis s – s.

$$I_{gg} = \frac{\pi d^4}{64} + \frac{BD^3}{12} = \frac{\pi \times 1.2^4}{64} + \frac{0.5 \times 1.2^3}{12} = 0.1018 + 0.072 = 0.1738 \, m^4$$

$$Area = A = \frac{\pi d^2}{4} + BD = \frac{\pi \times 1.2^2}{4} + 0.5 \times 1.2 = 1.131 + 0.6 = 1.731 \, m^2$$

$$\bar{y} = 3 \, m$$

$$I_{ss} = I_{gg} + A\bar{y}^2 = 0.1738 + 1.731 \times 3^2 = 15.75 \, m^4$$

SELF ASSESSMENT EXERCISE 2

1. Find the second moment of area of a rectangle 3 m wide by 2 m deep about an axis parallel to the longer edge and 5 m from it. (218 m^4).

2. Find the second moment of area of a rectangle 5 m wide by 2m deep about an axis parallel to the longer edge and 3 m from it. (163.33 m^4).

3. Find the second moment of area of a circle 2 m diameter about an axis 5 m from the centre. (79.3 m^4).

4. Find the second moment of area of a circle 5 m diameter about an axis 4.5 m from the centre. (428.29 m^4).

5. Find the 2nd moment of area for the shape shown the about the axis s – s. All the dimensions are in metres. $(35.92 \times 10^{-3} \text{ m}^4)$
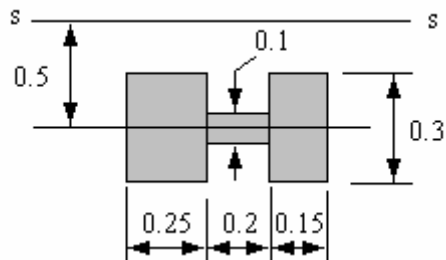
Fig.15

6. Find the 2nd moment of area for the shape shown the about the axis s – s. All the dimensions are in metres.. $(79.33 \times 10^{-3} \text{ m}^4)$
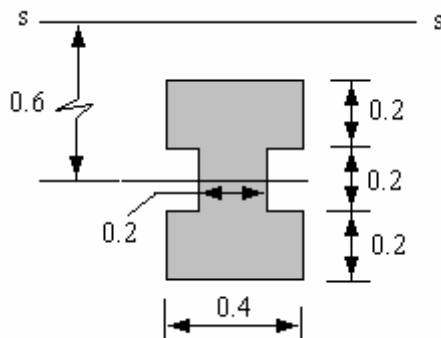
Fig.16

7. Find the position of the centroid for the shape shown and the 2nd moment of area about the bottom edge. (28.33 mm from the bottom and 2.138 x 10^{-6} m^4)
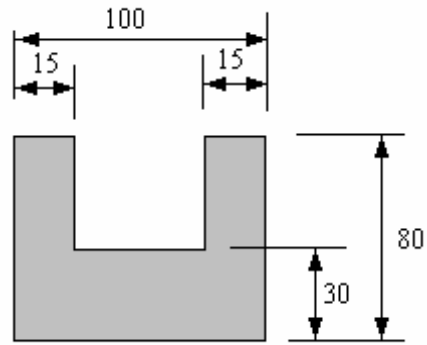
Fig.17

ENGINEERING COUNCIL

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THERMODYNAMIC, FLUID AND PROCESS ENGINEERING C106

TUTORIAL 2 – LIQUIDS, VAPOURS AND GASES

INTRODUCTION

This tutorial is designed to teach you the basic concepts of thermodynamics and the properties of fluids. On completion of this tutorial you should be able to the following.

- Use the correct thermodynamic symbols.

- Determine the properties of a gas.

- Determine the properties of vapours.

- Determine the properties of liquids.

We will start by examining the symbols used.

1. SYMBOLS

The symbols used throughout in these tutorials are S.I. (International System). These are not always the ones used by the examiner. These are the main S.I. Symbols. The S.I. also recommends the use of / to mean divide rather than the use of negative indices favoured by examiners. For example ms^{-1} becomes m/s.

Quantity	**units**	**Derived Unit**	**S.I. symbol**
Length	m		various
Mass	kg		m
Time	s		t
Volume	m^3		V or Q
Specific Volume	m^3/kg		v
Volume Flow Rate	m^3/s		
Density	kg/m^3		ρ
Force	kg m/s^2	N	F
Weight	kg m/s^2	N	W
Pressure Head	m		h
Altitude	m		z
Area	m^2		A
Speed of Sound	m/s		a
Specific Heat Cap.	N m/kg K	Joule/kg K	c
Energy	N m	Joule	
Enthalpy	N m	Joule	H
Internal Energy	N m	Joule	U
Specific Enthalpy	N m/kg	J/kg	h
Specific Int. Energy	N m/kg	J/kg	u
Mass flow rate	kg/s		
Polytropic Index			n
Adiabatic Index			γ
Pressure	N/m^2	Pascal	p
Heat Transfer	N m	Joule	Q
Work	N m	Joule	W
Heat Transfer Rate	N m/s	Watt	Φ
Work Rate (power)	N m/s	Watt	P
Char. Gas Const	N m/kg K	J/kg K	R
Universal Gas Constant	J/kmol K		R_O
Entropy	J/K		S
Specific Entropy	J/kg K		s
Absolute Temperature	K		T
Celsius Temperature	^oC		θ
Velocity	m/s^2		v or u
Dynamic Viscosity	N s/m^2	Pa s	η or μ
Kinematic Viscosity	m^2/s		ν

Now we will examine the basic concepts required to do this tutorial successfully.

2.   BASIC CONCEPTS

Throughout these tutorials you will use properties which are either EXTENSIVE or INTENSIVE.

An extensive property is one which is divisible. For example Volume when divided by a number becomes smaller. Other examples are mass and energy.

An intensive property is a property of a mass which remains the same value when the mass is divided into smaller parts. For example the temperature and density of a substance is unchanged if it is divided into smaller masses.

Throughout the tutorials you will use TOTAL and SPECIFIC quantities which relate only to extensive properties. A total quantity is always denoted by a higher case letter such as V for volume (m^3) and H for enthalpy (J). A specific quantity represents the quantity per kg and is obtained by dividing the property by the mass. Such properties are always designated by lower case letters such as v for specific volume (m^3/kg) and h for specific enthalpy (J/kg).

Specific volume is mainly used for gas and vapours. The inverse of specific volume is density (ρ) (kg/m^3) and this is mainly used for liquids and solids but also for gases.
Note $\rho = 1/v$.

Because the same letters are used to designate more than one property, often alternative letters are used. For example v for specific volume may occur in the same work as v for velocity so often u or c is used for velocity. h is used for height, head and specific enthalpy so z is often used for height instead.

The unit of Force and Weight is the kg m/s^2. This comes from Newton's Second Law of Motion (Force = mass x acceleration). The derived name for the unit is the Newton. In the case of Weight, the acceleration is that of gravity and in order to convert mass in kg into weight in Newtons, you must use $W = mg$ where g is normally 9.81 m/s^2.

Now we will examine forms of energy a fluid may have.

3.   ENERGY FORMS

A fluid may possess several forms of energy. All fluids possess energy due to their temperature and this is called INTERNAL ENERGY. All possess GRAVITATIONAL or POTENTIAL ENERGY due to their elevation relative to some datum level. If the fluid is moving it will possess KINETIC ENERGY due to its velocity. If it has pressure then it will possess FLOW ENERGY. Often pressure and temperature are the main two governing factors and we add internal energy to flow energy in order to produce a single entity called ENTHALPY. Let us look at each in more detail.

3.1.   GRAVITATIONAL or POTENTIAL ENERGY

In order to raise a mass m kg a height z metres, a lifting force is required which must be at least equal to the weight mg.

The work done raising the mass is as always, force x distance moved so

$$\text{Work} = mgz$$

Since energy has been used to do this work and energy cannot be destroyed, it follows that the energy must be stored in the mass and we call this gravitational energy or potential energy P.E. There are many examples showing how this energy may be got back, e.g. a hydro-electric power station.

$$\textbf{P.E.} = \textbf{mgz}$$
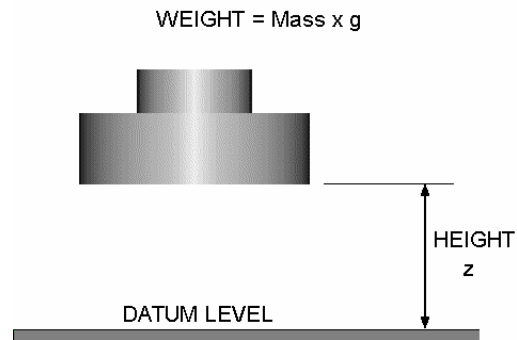
3.2   KINETIC ENERGY

When a mass m kg is accelerated from rest to a velocity of v m/s, a force is needed to accelerate it. This is given by Newton's 2nd Law of Motion   F= ma.

After time t seconds the mass travels x metres and reaches a velocity v m/s. The laws relating these quantities are

$a = v/t$ and $x = vt/2$

The work done is $W = Fx = max = mv^2/2$
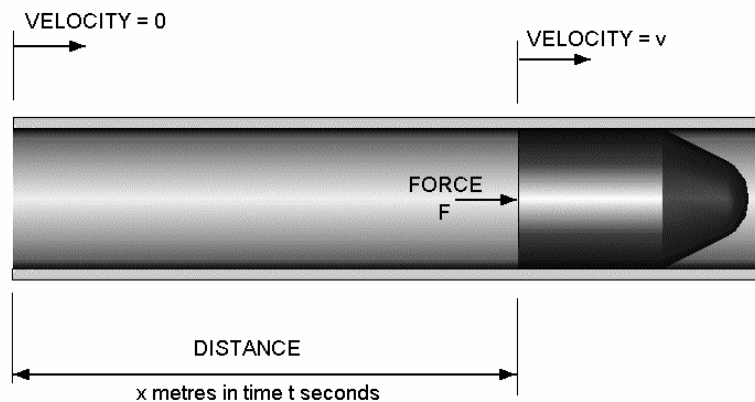
Energy has been used to do this work and this must be stored in the mass and carried along with it. This is KINETIC ENERGY.

$$\textbf{K.E.} = \textbf{mv}^2\textbf{/2}$$

3.3 FLOW ENERGY

When fluid is pumped along a pipe, energy is used to do the pumping. This energy is carried along in the fluid and may be recovered (as for example with an air tool or a hydraulic motor). Consider a piston pushing fluid into a cylinder.
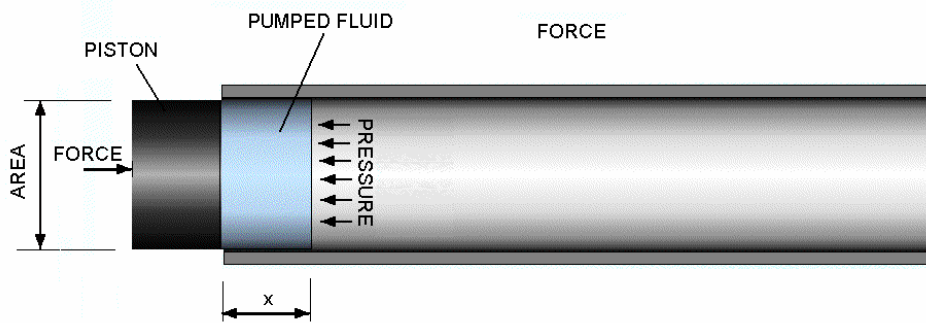
The fluid pressure is p N/m^2. The force needed on the piston is

$$F = pA$$

The piston moves a distance x metres. The work done is

$$W = Fx = pAx$$

Since Ax =V and is the volume pumped into the cylinder the work done is

$$W = pV$$

Since energy has been used doing this work, it must now be stored in the fluid and carried along with it as FLOW ENERGY.

$$\textbf{F.E.} = \textbf{pV}$$

3.4 INTERNAL ENERGY

This is covered in more detail later. The molecules of a fluid possess kinetic energy and potential energy relative to some internal datum. Usually this is regarded simply as the energy due to the temperature and very often the change in internal energy in a fluid which undergoes a change in temperature is given by

$$\mathbf{\Delta U = mc\Delta T}$$

The symbol for internal energy is U kJ or u kJ/kg. Note that a change in temperature is the same in degrees Celsius or Kelvin. The law which states internal energy is a function of temperature only is known as **JOULE'S LAW.**

3.5 ENTHALPY

When a fluid has pressure and temperature, it must possess both flow and internal energy. It is often convenient to add them together and the result is ENTHALPY. The symbol is H kJ or h kJ/kg.

$$\textbf{H = F.E. + U}$$

Next you need to study the properties of fluids and the laws relating them.

4   GAS LAWS

In this section you will do the following.

- Derive basic gas laws.
- Examine the characteristic gas law.
- Examine the universal gas law.
- Define the mol.
- Solve gas law problems.

4.1   THEORY

A gas is made of molecules which move around with random motion. In a perfect gas, the molecules may collide but they have no tendency at all to stick together or repel each other. In other words, a perfect gas in completely inviscid. In reality there is a slight force of attraction between gas molecules but this is so small that gas laws formulated for an ideal gas work quite well for real gas.

Each molecule in the gas has an instantaneous velocity and hence has kinetic energy. The sum of this energy is the internal energy U. The velocity of the molecules depends upon the temperature. When the temperature changes, so does the internal energy. The internal energy is for all intents and purposes zero at - 273° C. This is the absolute zero of temperature. Remember that to convert from Celsius to absolute, add on 273. For example
$\qquad$ 40°C is   40 + 273= 313 Kelvins.

4.2   PRESSURE

If a gas is compressed it obtains pressure. This is best explained by considering a gas inside a vessel as shown.

The molecules bombard the inside of the container. Each produces a momentum force when it bounces. The force per unit area is the pressure of the gas.   Remember that pressure = Force/area
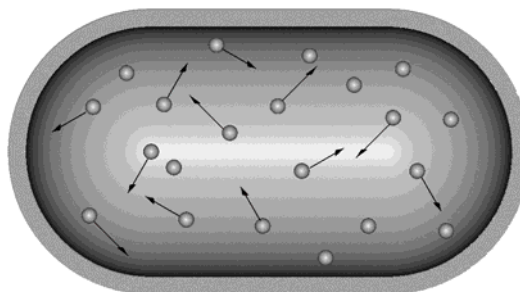
$\qquad$ $p = F/A$ N/m^2 or Pascals

Note that 10^3 Pa = 1 kPa $\qquad$ 10^6 Pa = 1 MPa $\qquad$ 10^5 Pa = 1 bar

4.3   CONSTANT VOLUME LAW

If the gas is heated the velocity of the molecules increases. If the container is rigid, then the molecules will hit the surface more often and with greater force so we expect the pressure to rise proportional to temperature.

$$p = c\,T \quad \text{when V is constant.}$$

WORKED EXAMPLE No.1

A mass of gas has a pressure of 500 kPa and temperature of 150°C. The pressure is changed to 900 kPa but the volume is unchanged. Determine the new temperature.

SOLUTION

Using constant volume law find $p_1/T_1 = c = p_2/T_2$ where

$T_1 = 150 + 273 = 423$ K
$p_1 = 500\ 000 \qquad p_2 = 900\ 000$
$T_2 = p_2 T_1/p_1 = 900\ 000 \times 423/500\ 000 \qquad T_2 = 761.4$ K

4.4   CHARLES'S LAW

If we kept the pressure constant and increased the temperature, then we would have to make the volume bigger in order to stop the pressure rising. This gives us Charles's Law:

$$V = c\,T \quad \text{when p is constant}$$

WORKED EXAMPLE No.2

A mass of gas has a temperature of 150°C and volume of 0.2 m³. The temperature is changed to 50°C but the pressure is unchanged. Determine the new volume.

SOLUTION

Using Charles's law we find $V_1/T_1 = c = V_2/T_2$ where

$T_1 = 150 + 273 = 423$ K
$V_1 = 0.2$
$T_2 = 50 + 273 = 323$ K
$V_2 = T_2 V_1/T_1 = 323 \times 0.2/523$
$V_2 = 0.123$ m³

4.5 BOYLE'S LAW

If we keep the temperature constant and increase the volume, then the molecules will hit the surface less often so the pressure goes down. This gives Boyle's Law:

$$p = c/V \quad \text{when T is constant.}$$

WORKED EXAMPLE No.3

A mass of gas has a pressure of 800 kPa and volume of 0.3 m^3. The pressure is changed to 100 kPa but the temperature is unchanged. Determine the new volume.

SOLUTION

Using Boyle's law we find $p_1 V_1 = c = p_2 V_2$ where

$p_1 = 800 \times 10^3$ $V_1 = 0.3$ $p_2 = 100 \times 10^3$
$V_2 = p_1 V_1/p_2 = 800 \times 10^3 \times 0.3/100 \times 10^3$
$V_2 = 2.4 m^3$.

4.6 GENERAL GAS LAW

Consider a gas which undergoes a change in pV and T from point (1) to point (2) as shown. It could have gone from (1) to (A) and then from (A)to (2) as shown.

Process (1) to (A) is constant volume
$$p_A/T_A = p_1/T_1$$

Process (A) to (2) is constant temperature
$$T_2 = T_A$$
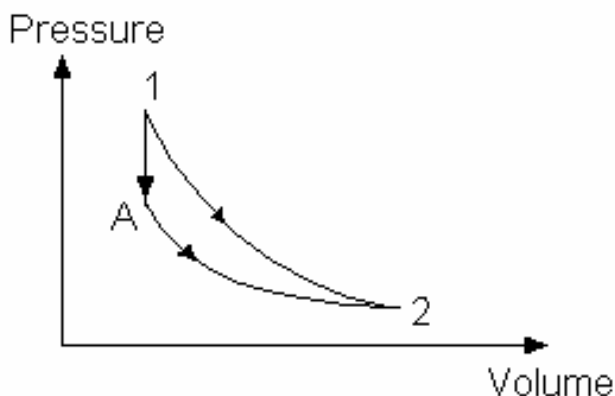
Hence $p_A/T_2 = p_1/T_1$ and $p_A = p_1 T_2/T_1$ ....(1)

For the process (A) to (2) Boyle's Law applies so $p_A V_A = p_2 V_2$

Since $V_A = V_1$ then we can write $p_A V_1 = p_2 V_2$

So $p_A = p_2 V_2/V_1$............(2)

Equating (1) and (2) we get $\mathbf{p_1 V_1/T_1 = p_2 V_2/T_2 = constant}$

This is the GENERAL GAS LAW to be used to calculate one unknown when a gas changes from one condition to another.

WORKED EXAMPLE No.4

A mass of gas has a pressure of 1.2 MPa, volume of 0.03 m^3 and temperature of 100°C. The pressure is changed to 400 kPa and the volume is changed to 0.06 m^3. Determine the new temperature.

SOLUTION

Using the general gas law we find $p_1V_1/T_1 = p_2V_2/T_2$ where
$p_1 = 1.2 \times 10^6$
$V_1 = 0.03$
$p_2 = 400 \times 10^3$

$T_1 = 100 + 273 = 373$ K
$T_2 = p_2V_2 \, T_1/p_1V_1 = 400 \times 10^3 \times 0.06 \times 373/(1.2 \times 10^6 \times 0.03)$

$T_2 = 248.7$ K

4. 7 CHARACTERISTIC GAS LAW

The general gas law tells us that when a gas changes from one pressure, volume and temperature to another, then

$$pV/T = \text{constant}$$

Thinking of the gas in the rigid vessel again, if the number of molecules was doubled, keeping the volume and temperature the same, then there would be twice as many impacts with the surface and hence twice the pressure. To keep the pressure the same, the volume would have to be doubled or the temperature halved. It follows that the constant must contain the mass of the gas in order to reflect the number of molecules. The gas law can then be written as $pV/T = mR$

where m is the mass in kg and R is the remaining constant which must be unique for each gas and is called the CHARACTERISTIC GAS CONSTANT. If we examine the units of R they are J/kg K.

The equation is usually written as $\qquad$ **pV = mRT**

Since m/V is the density ρ, it follows that $\qquad$ **$\rho = p/RT$**

Since V/m is the specific volume ,v, then $\qquad$ **$v = RT/p$**

WORKED EXAMPLE No.5

A mass of gas has a pressure of 1.2 MPa, volume of 0.03 m^3 and temperature of 100ºC. Given the characteristic gas constant is 300 J/kg K find the mass.

SOLUTION

From the characteristic gas law we have $pV = mRT$ where
$p = 1.2 \times 10^6$ N/m^2 $V = 0.03$ m^3 $T = 100 + 273 = 373$ K
$m = pV/RT = 1.2 \times 10^6 \times 0.03/300 \times 373 = 0.322$ kg

---

SELF ASSESSMENT EXERCISE No.1

All pressures are absolute.

1. Calculate the density of air at 1.013 bar and 15 ºC if R = 287 J/kg K.
   (1.226 kg/m^3)

2. Air in a vessel has a pressure of 25 bar, volume 0.2 m^3 and temperature 20ºC. It is connected to another empty vessel so that the volume increases to 0.5 m^3 but the temperature stays the same. Taking R = 287 J/kg K. Calculate

   i. the final pressure. (10 bar)
   ii. the final density. (11.892 kg/m^3)

3. 1 dm^3 of air at 20ºC is heated at constant pressure of 300 kPa until the volume is doubled. Calculate
   i. the final temperature. (586 K)
   ii. the mass. (3.56 g)

4. Air is heated from 20ºC and 400 kPa in a fixed volume of 1 m^3. The final pressure is 900 kPa. Calculate
   i. the final temperature.(659 K)
   ii. the mass. (4.747 kg)

5. 1.2 dm^3 of gas is compressed from 1 bar and 20ºC to 7 bar and 90ºC.
   Taking R = 287 J/kg K calculate
   i. the new volume. (212 cm^3)
   ii. the mass. (1.427 g)

4.8   THE UNIVERSAL GAS LAW

The Characteristic Gas Law states $pV = mRT$
where R is the characteristic constant for the gas.
This law can be made universal for any gas because $R = R_O/M_m$.
where M_m is the mean molecular mass of the gas (numerically equal to the relative molecular mass).

The formula becomes $pV = mR_OT/M_m$.

R_O is a universal constant with value 8314.3 J/kmol K. It is worth noting that in the exam, this value along with other useful data may be found in the back of your fluids tables.

The kmol is defined as the number of kg of substance numerically equal to the mean molecular mass. Typical values are

GAS	Symbol	M_m.
Hydrogen	H_2	2
Oxygen	O_2	32
Carbon Dioxide	CO_2	44
Methane	CH_4	16
Nitrogen	N_2	28
Dry Air		28.96

Hence   1 kmol of hydrogen (H_2) is 2 kg
       1 kmol of oxygen (O_2) is 32 kg
       1 kmol of Nitrogen is 28 kg and so on.

For example if you had 3 kmol of nitrogen (N_2) you would have
       3 x 28 = 84 kg

It follows that the M_m must have units of kg/kmol

In order to calculate the characteristic gas constant we use
       $R = R_O/M_m$

For example the characteristic gas constant for air is

      $R = 8314.3/28.96 = 287$

Examine the units

      $R = R_O/M_m = (J/kmol\ K)/(kg/kmol) = J/kg\ K$

WORKED EXAMPLE No.6

A vessel contains 0.2 m^3 of methane at 60°C and 500 kPa pressure. Calculate the mass of Methane.

SOLUTION

$pV = mR_OT/M_m.$ $M_m = 16$

500 000 x 0.2 = m x 8314.3 x (273 + 60)/16   m = 0.578 kg

SELF ASSESSMENT EXERCISE No. 2

1. A gas compressor draws in 0.5 m^3/min of Nitrogen at 10°C and 100 kPa pressure. Calculate the mass flow rate.
    (0.595 kg/min)

2. A vessel contains 0.5 m^3 of Oxygen at 40°C and 10 bar pressure. Calculate the mass.
    (6.148 kg)

Next we will examine the meaning of specific heat capacities.

5.   SPECIFIC HEAT CAPACITIES

In this section you will do the following.

- Learn how to calculate the change in internal energy of gases and liquids.
- Learn how to calculate the change in enthalpy of gases and liquids.
- Define the specific heats of fluids.
- Relate specific heat capacities to the characteristic gas constant.

5.1   SPECIFIC HEAT CAPACITIES

The specific heat capacity of a fluid is defined in two principal ways as follows:

1. Constant Volume

The specific heat which relates change in specific internal energy 'u' and change in temperature 'T' is defined as :
$$c_V = du/dT$$
If the value of the specific heat capacity c_V is constant over a temperature range ΔT then we may go from the differential form to the finite form

$$c_V = \Delta u/\Delta T \quad J/kg$$
Hence $\qquad \Delta u = c_V \Delta T \quad J/kg$

For a mass m kg the change is $\qquad \mathbf{\Delta U = mc_V\Delta T}$ Joules

This law indicates that the internal energy of a gas is dependant only on its temperature. This was first stated by Joule and is called JOULE'S LAW.

2. Constant Pressure.

The specific heat which relates change in specific enthalpy 'h' and change in temperature 'T' is defined as :
$$c_p = dh/dT$$
If the value of the specific heat capacity c_p is constant over a temperature range ΔT then we may go from the differential form to the finite form

$$c_p = \Delta h/\Delta T \quad J/kg$$
Hence $\qquad \Delta h = c_p \Delta T \quad J/kg$

For a mass m kg the change is $\qquad \mathbf{\Delta H = mc_p\Delta T}$ Joules

The reasons why the two specific heats are given the symbols c_V and c_p will be explained next. They are called the PRINCIPAL SPECIFIC HEATS.

5.2   CONSTANT VOLUME HEATING

When a fluid is heated at constant volume, the heat transfer 'Q' must be same as the increase in internal energy the fluid ΔU since no other energy is involved. It follows that :

$$Q = \Delta U = mc_V\Delta T \text{ Joules}$$

The change in internal energy is the same as the heat transfer at constant volume so the symbol c_V should be remembered as applying to constant volume processes as well as internal energy.
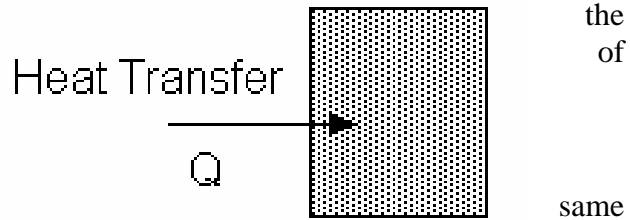
5.3 CONSTANT PRESSURE HEATING
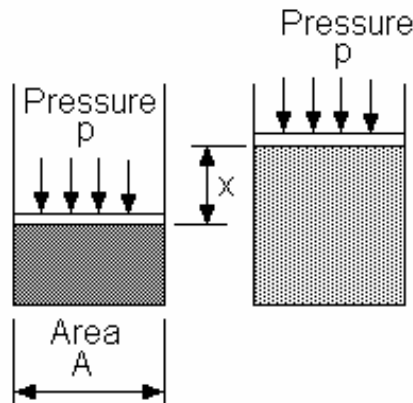
When a fluid is heated at constant pressure, the volume must increase against a surrounding pressure equal and opposite to the fluid pressure p.

The force exerted on the surroundings must be $F = pA$ Newtons
The work done is force x distance moved hence :
Work Done = F x = pAx = p ΔV where ΔV is the volume change.

The heat transfer Q must be equal to the increase in internal energy plus the work done against the external pressure. The work done has the same formula as flow energy p V
Enthalpy was defined as $\Delta H = \Delta U + p\Delta V$

The heat transfer at constant pressure is also: $Q = \Delta U + p\Delta V$
Since specific heats are used to calculate heat transfers, then in this case the heat transfer is by definition : $Q = mc_p\Delta T$

It follows that $\Delta H = Q = mc_p\Delta T$

For the same temperature change ΔT it follows that the heat transfer at constant pressure must be larger than that at constant volume. The specific heat capacity c_p is remembered as linked to constant pressure.

5.4 LINK BETWEEN c_v, c_p AND R.

From the above work it is apparent that $\Delta H = mc_p\Delta T = \Delta U + p\Delta V$
We have already defined $\Delta U = mc_v\Delta T$
Furthermore for a gas only, $p\Delta V = mR\Delta T$
Hence $mc_p\Delta T = mc_v\Delta T + mR\Delta T$
Hence $\mathbf{c_p = c_v + R}$

5.5 LIQUIDS

Since the volume of a liquid does not change much when heated or cooled, very little work is done against the surrounding pressure so it follows that c_v and c_p are for all intents and purposes the same and usually the heat transfer to a liquid is given as :
$$Q = mc\ \Delta T$$
Where c is the specific heat capacity.

5.6 VAPOURS

Vapour is defined as a gaseous substance close to the temperature at which it will condense back into a liquid. In this state it cannot be considered as a perfect gas and great care should be taken applying specific heats to them. We should use tables and charts to determine the properties of vapours and this is covered in the next section.

WORKED EXAMPLE No.7

Calculate the change in enthalpy and internal energy when 3 kg of gas is heated from 20°C to 200°C. The specific heat at constant pressure is 1.2 kJ/kg K and at constant volume is 0.8 kJ/kg K. Also determine the change in flow energy.

SOLUTION

i. Change in enthalpy.

$\Delta H = mc_p\Delta T = 3 \times 1.2 \times 180 = 648$ kJ

ii. Change in internal energy.

$\Delta H = mc_v\Delta T = 3 \times 0.8 \times 180 = 432$ kJ

iii. Change in flow energy

$\Delta FE = \Delta H - \Delta = 216$ kJ

WORKED EXAMPLE No.8

A vertical cylinder contains 2 dm^3 of air at 50°C. One end of the cylinder is closed and the other end has a frictionless piston which may move under the action of weights placed on it. The weight of the piston and load is 300 N. The cylinder has a cross sectional area of 0.015 m^2. The outside is at atmospheric conditions.

Determine

i. the gas pressure.

ii. the gas mass.

iii. the distance moved by the piston when the gas is heated to 150°C.

For air take c_p= 1005 J/kg K and c_v= 718 J/kg K. Atmospheric pressure = 100 kPa

SOLUTION

The pressure of the gas is constant and always just sufficient to support the piston so
p = Weight/Area + atmospheric pressure

p = 300/0.015 + 100 kPa = 20 kPa + 100 kPa = 120 kPa

T_1 = 50 + 273 = 323 K

V_1 = 0.002 m^3

R = c_p - c_v = 1005 - 718 = 213 J/kg K

m = pV/RT   = 120 000 x 0.002/(213 x 323) = 0.00348 kg

T_2 = 150 + 273 = 423 K

V_2= $p_1V_1T_2/p_2T_1$ but p_1 = p_2

V_2= V_1T_2/T_1 = 0.02 x 423/323 = 0.0262 m^3

Distance moved = Volume change/Area = (0.0262 - 0.02)/0.015 = 0.412 m

WORKED EXAMPLE No.9

Convert the principal specific heats and characteristic gas constant for dry air into molar form.

SOLUTION

The normal values for dry air are found in the back of your fluids tables and are:
c_p= 1.005 kJ/kg K c_v= 0.718 kJ/kg K   R = 0.287 kJ/kg K

In order to convert these into molar form we must multiply them by the molar mass. The molar mass for dry air is a mean value for a gas mixture and is found on the back page of your fluids tables and is 28.96 kg/kmol.

In molar form
c_p= 1.005 [kJ/kg K] x 28.96 [kg/kmol] = 29.1 kJ/kmol K

c_v= 0.718 [kJ/kg K] x 28.96 [kg/kmol] = 20.8 kJ/kmol K

R = 0.287 [kJ/kg K] x 28.96 [kg/kmol] = 8.31 kJ/kmol K

Note that the last value is the universal gas constant R_O.

SELF ASSESSMENT EXERCISE No. 3

For air take c_p= 1005 J/kg K and c_v= 718 J/kg K unless otherwise stated.

1. 0.2 kg of air is heated at constant volume from 40°C to 120°C. Calculate the heat transfer and change in internal energy. (11.49 kJ for both)

2. 0.5 kg of air is cooled from 200°C to 80°C at a constant pressure of 5 bar. Calculate the change in internal energy, the change in enthalpy, the heat transfer and change in flow energy. (-43 kJ), (-60.3 kJ), (-17.3 kJ)

3. 32 kg/s of water is heated from 15°C to 80°C.
    Calculate the heat transfer given c = 4186J/kg K.
    (8.7 MW)

4. Air is heated from 20°C to 50°C at constant pressure. Using your fluid tables (pages 16 and 17) determine the average value of c_p and calculate the heat transfer per kg of air. (30.15 kJ)

5. The diagram shows a cylinder fitted with a frictionless piston. The air inside is heated to 200°C at constant pressure causing the piston to rise. Atmospheric pressure outside is 100 kPa. Determine :

    i. the mass of air. (11.9 g)

ii. the change in internal energy. (1.537 kJ)

iii. the change in enthalpy. (2.1553 kJ)

iv. the pressure throughout. (500 kPa)

v. the change in volume. (1.22 dm^3)
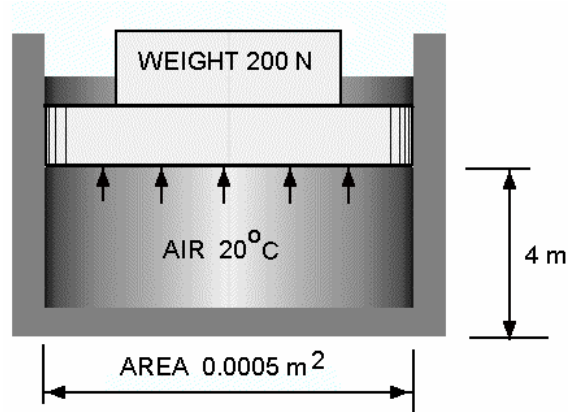
6. Define the meaning of a mole as a means of measuring the amount of a substance.

Calculate the volume occupied at a temperature of 25°C and a pressure of 3 bar, by 60 kg of (i) Oxygen gas, O_2, (ii)atomic oxygen gas,O, and (iii) Helium gas, H_e. The respective molar masses ,M, and the molar heats at constant volume, C_v, of the three gases, and the molar (universal) gas constant, R_M, are as follows:

	M (kg/kmol)	c_v(kJ/kmol K)	R_M(kJ/kmol K)
O_2	32	20.786	8.3144
O	16	12.4716	8.3144
H_e	4	12.4716	8.3144

Go on to calculate the values of the specific heats C_p and C_v. Using these values, calculate the specific gas constant R for all three gases. Show not only numerical work, but also the manipulation of units in arriving at your results.

You should now be able to determine the properties of gases. Next we will examine the properties of liquids and vapours.

6.   PROPERTIES OF LIQUIDS AND VAPOURS

In this section you will do the following.

- Learn about the properties and definitions concerning vapours.

- Learn how to find the properties of vapours and liquids from your tables and charts.

You should ensure that you have a copy of 'Thermodynamic and Transport Properties of Fluids' by Mayhew and Rogers.

6.1   GENERAL THEORY

When a liquid changes into a vapour by the process of evaporation, it undergoes a change of state or phase. The reverse process is called liquefaction or condensing. The following work should lead you to an understanding of this process and by the end of it you should be able to find the same quantities and do the same type of problems as you have already done for gas.

When a liquid is heated, the temperature rises directly proportional to the heat transferred, 'Q'. Q is given by $Q = mc\Delta T$
The specific heat c is reasonably constant but changes significantly if the pressure   or temperature change is very large.

When discussing heat transfer and energy of a fluid, we may wish to consider the internal energy U or the enthalpy H. In the following, the energy of the fluid may be construed as either. In tables this is tabulated as specific internal energy or enthalpy u and h.

When the liquid receives enough heat to bring it to the boiling point , the energy it contains is called SENSIBLE ENERGY. In tables this is denoted as u_f or hf.

A liquid starts to evaporate because it becomes saturated with heat and can absorb no more without changing state (into a vapour and hence gas). For this reason the boiling point is more correctly described as the SATURATION TEMPERATURE and the liquid in this state is called SATURATED LIQUID. The saturation temperature is denoted as t_s in tables.

If a boiling liquid is supplied with more heat, it will evaporate and vapour is driven off. The vapour, still at the saturation temperature is called DRY SATURATED VAPOUR.

A vapour is a gas near to the temperature at which it will condense. In order to convert liquid into vapour, extra heat must be transferred into it. The amount of enthalpy and internal energy required to evaporate 1 kg is denoted h_{fg} and u_{fg} in tables and this is called the LATENT ENTHALPY and LATENT INTERNAL ENERGY respectively.

The energy contained in 1 kg of dry saturated vapour must be the sum of the sensible and latent energy and this is denoted h_g and u_g. It follows that :

$$h_g   = h_f + h_{fg}$$
$$u_g   = u_f + u_{fg}$$

## TABLE FOR THE ENTHALPY OF HIGH PRESSURE WATER

Temp °C	Pressure in bar										
	0	25	50	75	100	125	150	175	200	221	250
0	0	2.5	5	7.5	10	12.6	15	17.5	20	22	25
20	84	86	87	91	93	96	98	100	103	105	107
40	168	170	172	174	176	179	181	183	185	187	190
60	251	253	255	257	259	262	264	266	268	270	272
80	335	337	339	341	343	345	347	349	351	352	355
100	419	421	423	425	427	428	430	432	434	436	439
120	504	505	507	509	511	512	514	516	518	519	521
140	589	591	592	594	595	597	599	600	602	603	605
160	675	677	678	680	681	683	684	686	687	688	690
180	763	764	765	767	767	769	790	772	773	774	776
200	852	853	854	855	856	857	858	859	861	862	863

All enthalpy values are given in kJ/kg

The temperature at which evaporation occurs 't_S' depends upon the pressure at which it takes place. For example we all know that water boils at 100°C at atmospheric pressure (1.013 bar). At pressure below this, the boiling point is less. At higher pressures the boiling point is higher. If we look up the values of t_S and p for water in the tables and plot them we get the graph below. It should also be noted that if the temperature of a liquid is kept constant, it may be made to boil by changing the pressure. The pressure at which it boils is called the SATURATION PRESSURE and is denoted as p_S in the tables.

The graph below also shows the freezing point of water plotted against pressure (pressure has little effect on it).
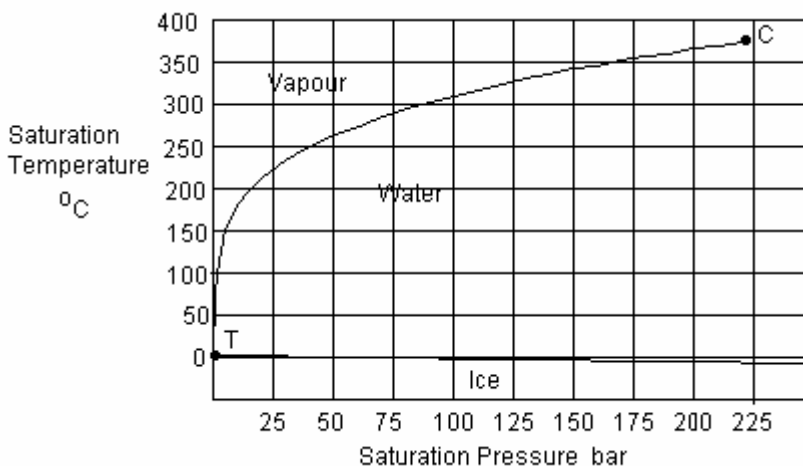

The two graphs cross at 0.01°C and 0.006112 bar. This point is called the TRIPLE POINT. The graph shows the three phases of ice, water and steam. At the triple point, all three can occur together. Below the triple point, ice can change into steam without a liquid stage (and vice versa). All substances have a triple point.

If you did the exercise of plotting the graph of t_S against p for water/steam you would find that the tables stop at 221.2 bar and 374.15°C. Above this pressure and temperature, the phenomenon of evaporation does not occur and no latent energy stage exists. This point is called the CRITICAL POINT and every substance has one.

If vapour is heated, it becomes hotter than the boiling point and the more it is heated, the more it becomes a gas. Such vapour is referred to as SUPERHEATED VAPOUR, except when it is a substance at pressures and temperatures above the critical point when it is called SUPERCRITICAL VAPOUR.

6.2 CONTINUOUS EVAPORATION
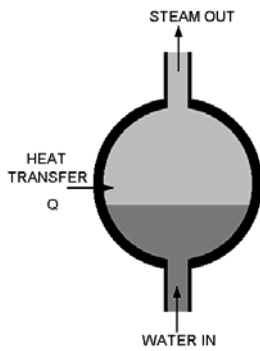
A simple boiler or evaporator as shown is needed to continuously produce vapour from liquid. The liquid is pumped in at the same rate at which the vapour is driven off. The heat transfer rate needed to do this must supply the internal energy to the process and the flow energy. In other words, the heat transfer is equal to the increase in the enthalpy from liquid to vapour. This is why enthalpy is such an important property.

6.3 WET VAPOUR

Wet vapour is a mixture of dry saturated vapour and liquid droplets. It may also be thought of as a partially evaporated substance. In order to understand its properties, consider the evaporation of 1 kg of water illustrated with a temperature - enthalpy graph. Starting with water at atmospheric pressure and 0.01°C, the enthalpy is arbitrarily taken as zero. Keeping the pressure constant and raising the temperature, the enthalpy of the water rises to 419.1 kJ/kg at 100°C. At this point it is saturated water and the sensible enthalpy is h_f=419.1 kJ/kg. The addition of further heat will cause the water to evaporate. During evaporation, the temperature remains at 100°C. When the latent enthalpy h_{fg} (2256.7 kJ/kg) has been added, the substance is dry saturated vapour and its specific enthalpy h_g is 2675.8 kJ/kg. Further addition of heat will cause the temperature to rise and the substance becomes superheated vapour.
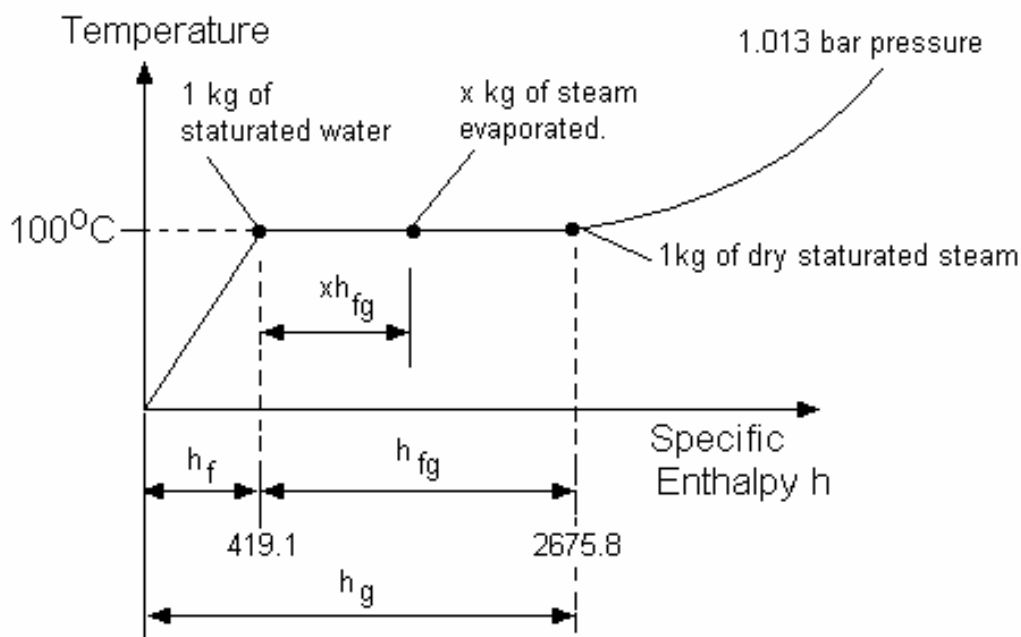
This graph may be drawn for any pressure and the same basic shape will be obtained but of course with different values. At the critical pressure it will be found that h_{fg} is zero.

The point of interest is the enthalpy value at some point along the evaporation line. Any point on this line represents wet steam. Suppose only fraction x kg has been evaporated. The latent enthalpy added will only be xh_{fg} and not h_{fg}. The enthalpy of the water/steam mixture is then   **$h = h_f + xh_{fg}$**

The fraction x is called the DRYNESS FRACTION but it is rarely given as a fraction but rather as a decimal. If no evaporation has started, then x = 0. If all the liquid is evaporated then x = 1. x cannot be larger than 1 as this would mean the vapour is superheated.

The same logic applies to internal energy and it follows that $\quad\quad\quad \mathbf{u = u_f + x u_{fg}}$

6.4 VOLUMES

The specific volume of saturated water is denoted v_f. The specific volume of dry saturated steam is denoted v_g. The change in volume from water to steam is v_{fg}. It follows that the specific volume of wet steam is $\quad v = v_f + x v_{fg}$

Since the value of v_f is very small and the specific volume of dry steam is very large (in all but the extreme cases), then v_{fg} is practically the same as v_g and v_f is negligible. The specific volume of steam is then usually calculated from the formula $\quad \boldsymbol{v = x v_g}$

6.5 TOTAL VALUES

All the formula above represent the values for 1 kg (specific values). When the mass is m kg, the values are simply multiplied by m . For example the volume of m kg of wet steam becomes $\mathbf{V = m x v_g}$

6.6 SATURATION CURVE

If we plot a graph of h_f and h_g against either temperature or pressure, we get a property chart. The graph itself is the SATURATION CURVE. Taking the p-h graph as an example, temperatures and dryness fractions may be drawn on it and with the resulting graph, the enthalpy of water, wet, dry or superheated steam may be found. The pressure - enthalpy chart is popular for refrigerants but not for steam. A p-h chart is enclosed for arcton 12.
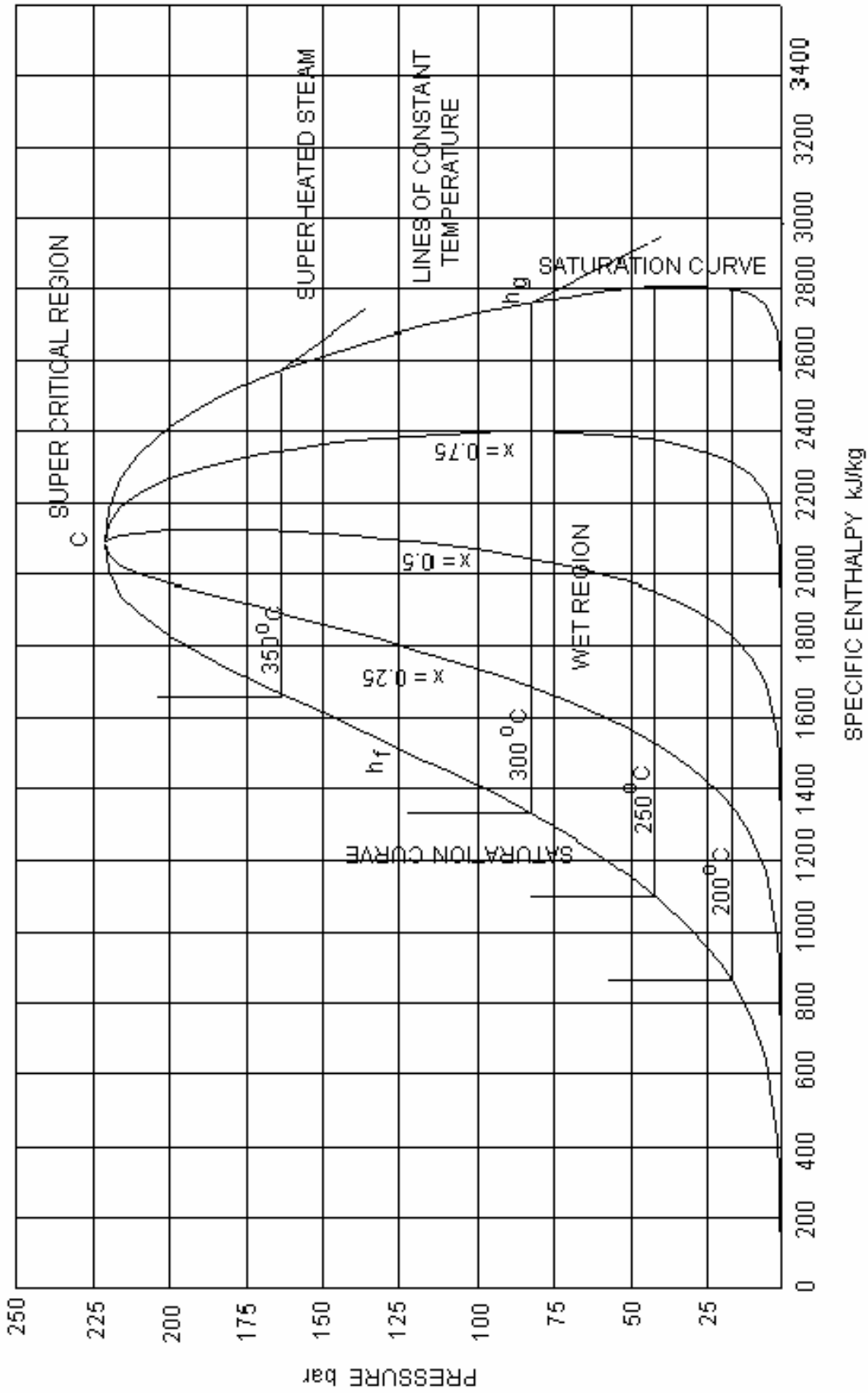
6.7   USE OF TABLES

It is vitally important for you to be able to use the fluid tables in order to find the properties of steam. The tables are supplied in the exam but you must have a copy and become completely proficient in their use. Regarding water/steam, the tables contain a section on saturated water/steam and a section on superheated/supercritical steam.

The saturated water/steam tables are laid out as follows with an example of values. Check this out for yourself on page 4.

p	t_s	v_g	u_f	u_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
10	179.9	0.1944	762	2584	763	2015	2778			

Don't worry about the columns headed s at this stage. This is the property called entropy which is dealt with later.

The latent internal energy u_{fg} is not listed because of lack of room. However you do need to remember that it is the difference between u_f and u_g. Note that in all cases the value of h_{fg} is the difference between the values on either side of it.

The superheat tables are laid out differently. In this case the property value depends upon the pressure and temperature since the steam can exist at any pressure and temperature above the saturation values. This by necessity makes the tables very concise. More detailed tables are published. Interpolation is required to find values between those tabulated.

In the superheat tables (e.g. page 6), you must locate the temperature along the top and the pressure down the side. This results in set of values at these co-ordinates giving v, u, h and s.

WORKED EXAMPLE No.10

Find the specific enthalpy, internal energy and volume of steam at 3 bar and 200°C .

SOLUTION

On page 6 of your tables locate the column with 200°C at the top and come down the page to the row with 3 bar at the left side. At this point you have a block of 4 figures. The enthalpy value is the third figure down and is   2866 kJ/kg. The second figure down is the internal energy and is 2651 kJ/kg. The first figure is the volume and is 0.7166 m^3/kg. You don't need the fourth figure at this stage.

p/bar	t	50	100	150	200	250
3					0.7166.......volume	
(133.5)					2651...........int. energy	
					2866...........enthalpy	
					7.312..........entropy	

WORKED EXAMPLE No.11

Find the enthalpy, internal energy and volume of 3 kg of steam at 11 bar and dryness 0.75.

SOLUTION

From page 4 of the steam tables determine the row corresponding to 11 bar and look up the following values.

h_f = 781 kJ/kg
h_{fg}=2000 kJ/kg
h_g=2781 kJ/kg

u_f=780 kJ/kg
u_g=2586 kJ/kg

v_g=0.1774 m^3/kg

Next deduce u_{fg} = 2586 - 780 = 1806 kJ/kg

Now find the enthalpy.
$H = m(h_f + xh_{fg}) = 3(781 + 0.75 \times 2000) = 6843$ kJ

Next find the internal energy in the same way.
$U = m(u_f + xu_{fg}) = 3(780 + 0.75 \times 1806) = 6403.5$ kJ

Finally the volume.
$V = mxv_g = 3 \times 0.75 \times 0.1774 = 0.399$ m^3

SELF ASSESSMENT EXERCISE NO. 4

1. Using your steam tables, plot a graph of h_f and h_g against pressure horizontally and mark on the graph the following:

    i. the superheat region
    ii. the wet steam region.
    iii. the liquid region.
    iv. the critical point.

    Also label the saturation curve with dry saturated steam and saturated water.

2. Using your steam tables, plot a graph of v_g horizontally against pressure vertically. Also plot v_f

    Show on the graph:

    i. the superheated steam region.
    ii. the wet vapour region.
    iii. the liquid region.
    iv. the critical point.

    Also label the saturation curve with dry saturated steam and saturated water.

SELF ASSESSMENT EXERCISE No. 5

Use tables and charts to do the following.

1. What is the saturation temperature at 32 bars ?

2. What is the specific enthalpy and internal energy of saturated water at 16 bars?

3. What is the specific enthalpy and internal energy of dry saturated steam at 16 bars?

4. Subtract the enthalpy in 2 from that in 3 and check that it is the latent enthalpy h_{fg} at 16 bars in the tables.

5. What is the specific enthalpy and internal energy of superheated steam at 10 bar and 400°C ?

6. What is the specific volume of dry saturated steam at 20 bars ?

7. What is the volume of 1 kg of wet steam at 20 bars with dryness fraction x=0.7?

8. What is the specific enthalpy and internal energy of wet steam at 20 bars with a dryness fraction of 0.7 ?

9. What is the specific volume of superheated steam at 15 bars and 500°C.

10. What is the volume and enthalpy of 3 kg of wet steam at 5 bar with dryness fraction 0.9.

11. Using the p-h chart for arcton 12 (freon 12) determine
a. the specific enthalpy at 2 bar and 70% dry. (x = 0.7).
b. the specific enthalpy at 5 bar and 330 K
c. the specific enthalpy of the liquid at 8 bars and 300 K.

12. What is the enthalpy of 1.5 kg of superheated steam at 8 bar and 350°C ?

13. What is the internal energy of 2.2 kg of dry saturated steam at 11 bars ?

14. What is the volume of 0.5 kg of superheated steam at 15 bar and 400°C ?

Answers to Assignment 5.

Compare your answers with those below. If you find your answers are different, go back and try again referring to the appropriate section of your notes.

1. 237.4^{o}C.
2. 859 and 857 kJ/kg.
3. 2794 and 2596kJ/kg.
3. 1935 kJ/kg and 1739 kJ/kg.
5. 3264 and 2957 kJ/kg.
6. 0.09957 m^3/kg.
7. 0.0697 m^3/kg.
8. 2232 and 2092.1 kJ/kg.
9. 0.2351 m^3

10. 1.012 m^3, 7.61 MJ, 7.11 MJ.
11 a. 190 kJ/kg. b.286 kJ/kg. c. 130 kJ/kg
12. 4.74 MJ.
13. 5.69 MJ.
14. 0.101m^3.

ENGINEERING COUNCIL

CERTIFICATE LEVEL

THERMODYNAMIC, FLUID AND PROCESS ENGINEERING C106

TUTORIAL 2 – THERMODYNAMIC PROCESSES

When you have completed this tutorial you should be able to do the following.

- Explain and use the First Law of Thermodynamics.

- Solve problems involving various kinds of thermodynamic systems.

- Explain and use polytropic expansion and compression processes.
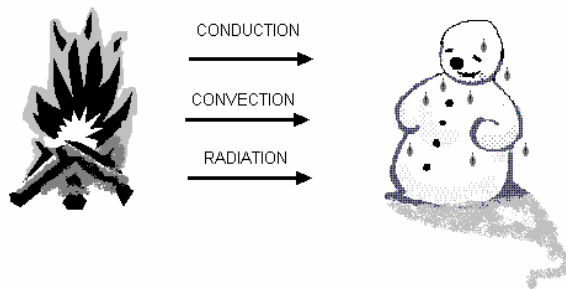
1. ENERGY TRANSFER

There are two ways to transfer energy in and out of a system, by means of work and by means of heat. Both may be regarded as a quantity of energy transferred in Joules or energy transfer per second in Watts.

When you complete section one you should be able to explain and calculate the following.

❑ Heat transfer.
❑ Heat transfer rate.
❑ Work transfer
❑ Work transfer rate (Power)

1.1. HEAT TRANSFER

Heat transfer occurs because one place is hotter than another. Under normal circumstances, heat will only flow from a hot body to a cold body by virtue of the temperature difference. There are 3 mechanisms for this, *Conduction, convection and radiation.* You do not need to study the laws governing conduction, convection and radiation in this module.
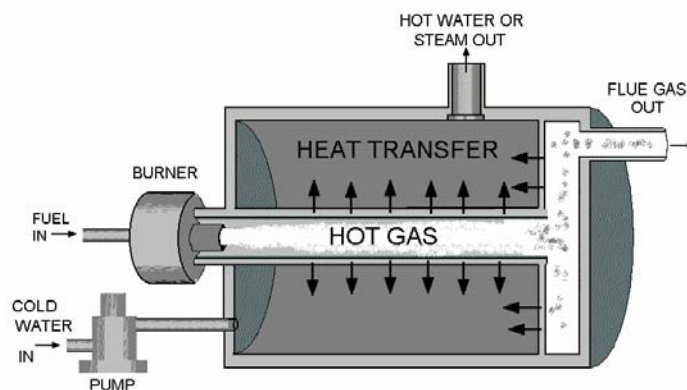

Fig.1

A quantity of energy transferred as heat is given the symbol Q and it's basic unit is the Joule. The quantity transferred in one second is the heat transfer rate and this has the symbol Φ and the unit is the Watt.

An example of this is when heat passes from the furnace of a steam boiler through the walls separating the combustion chamber from the water and steam. In this case, conduction, convection and radiation all occur together.

SELF ASSESSMENT EXERCISE No.1

1. 1 kg/s of steam flows in a pipe 40 mm bore at 200 bar pressure and 400ºC.

i. Look up the specific volume of the steam and determine the mean velocity in the pipe.
(7.91 m/s)

ii. Determine the kinetic energy being transported per second.
(31.3 W)
iii. Determine the enthalpy being transported per second.
(2819 W)

1.2.   WORK TRANSFER

Energy may be transported from one place to another mechanically. An example of this is when the output shaft of a car engine transfers energy to the wheels. A quantity of energy transferred as work is 'W' Joules but the work transferred in one second is the Power 'P' Watts.

An example of power transfer is the shaft of a steam turbine used to transfer energy from the steam to the generator in an electric power station.
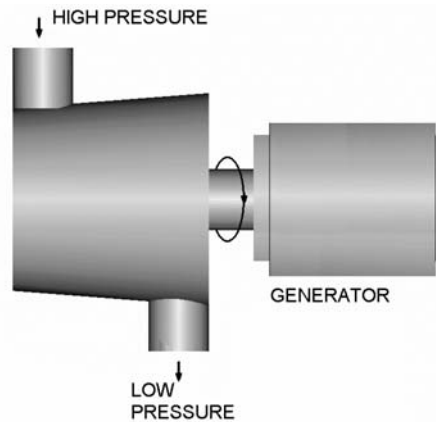

Fig.3

It is useful to remember that the power transmitted by a shaft depends upon the torque and angular velocity. The formulae used are $P = \omega T$ or $P = 2\pi N T$

ω is the angular velocity in radian per second and N is the angular velocity in revolutions per second.

WORKED EXAMPLE No. 1

A duct has a cross section of 0.2 m x 0.4 m. Steam flows through it at a rate of 3 kg/s with a pressure of 2 bar. The steam has a dryness fraction of 0.98. Calculate all the individual forms of energy being transported.

SOLUTION

Cross sectional area = 0.2 x 0.4 = 0.08 m^2.
Volume flow rate = mxv_g at 2 bar

Volume flow rate = 3 x 0.98 x 0.8856 = 2.6 m^3/s.
velocity = c = Volume/area = 2.6/0.08 = 32.5 m/s.
Kinetic Energy being transported = $mc^2/2$ = 3 x 32.5^2 /2 = 1 584 Watts.
Enthalpy being transported = $m(h_f + x\, h_{fg})$

H= 3(505 + 0.98 x 2202) = 7988.9kW
Flow energy being transported = pressure x volume
Flow Energy = 2x10^5 x 2.6 = 520 000 Watts
Internal energy being transported = $m(u_f + x\, u_{fg})$

U = 3(505 + 0.98 x 2025)=7468.5 kW
Check flow energy = H - U = 7988.9 - 7468.5 = 520 kW

---

2.   THE FIRST LAW OF THERMODYNAMICS

When you have completed section two, you should be able to explain and use the following terms.

❑   The First Law of Thermodynamics.
❑   Closed systems.
❑   The Non-Flow Energy Equation.
❑   Open systems.
❑   The Steady Flow Energy Equation.

2.1   THERMODYNAMIC SYSTEMS

In order to do energy calculations, we identify our system and draw a boundary around it to separate it from the surroundings. We can then keep account of all the energy crossing the boundary. The first law simply states that

The nett energy transfer = nett energy change of the system.
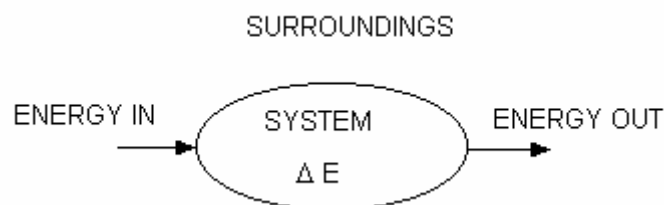
Fig. 4

Energy transfer into the system = E(in)

Energy transfer out of system = E (out)

Nett change of energy inside system = E(in) - E (out) = ΔE

This is the fundamental form of the first law.

Thermodynamic systems might contain only static fluid in which case they are called **NON-FLOW or CLOSED SYSTEMS.**

Alternatively, there may be a steady flow of fluid through the system in which case it is known as a **STEADY FLOW or OPEN SYSTEM.**

The energy equation is fundamentally different for each because most energy forms only apply to a fluid in motion. We will look at non-flow systems first.

2.2   NON-FLOW SYSTEMS

The rules governing non-flow systems are as follows.

❑   The volume of the boundary may change.
❑   No fluid crosses the boundary.
❑   Energy may be transferred across the boundary.

When the volume enlarges, work (-W) is transferred from the system to the surroundings. When the volume shrinks, work (+W) is transferred from the surroundings into the system. Energy may also be transferred into the system as heat (+Q) or out of the system (-Q). This is best shown with the example of a piston sliding inside a cylinder filled with a fluid such as gas.
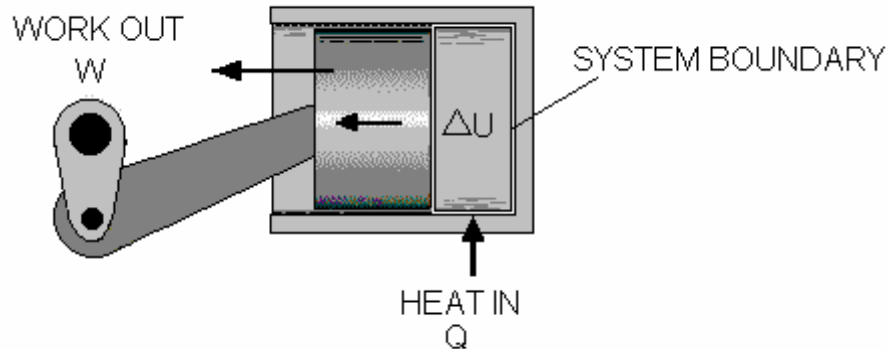
Fig.5

The only energy possessed by the fluid is internal energy (U) so the net change is ΔU. The energy equation becomes

$$Q + W = \Delta U$$

This is known as the **NON-FLOW ENERGY EQUATION (N.F.E.E.)**

2.3   STEADY FLOW SYSTEMS

The laws governing this type of system are as follows.

❑   Fluid enters and leaves through the boundary at a steady rate.
❑   Energy may be transferred into or out of the system.

A good example of this system is a steam turbine. Energy may be transferred out as a rate of heat transfer Φ or as a rate of work transfer P.
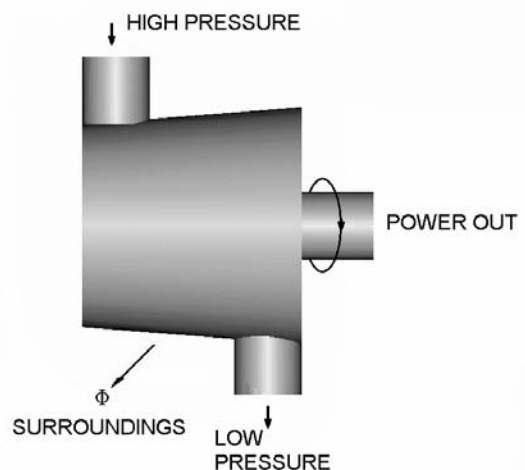
Fig.6.

The fluid entering and leaving has potential energy (PE), kinetic energy (KE) and enthalpy (H). The first law becomes Φ + P = Nett change in energy of the fluid.

$$\Phi + P = \Delta(PE)/s + \Delta(KE)/s + \Delta(H)/s$$

This is called the **STEADY FLOW ENERGY EQUATION (S.F.E.E.)**

Again, we will use the convention of positive for energy transferred into the system. Note that the term Δ means '*change of*' and if the inlet is denoted point (1) and the outlet point (2). The change is the difference between the values at (2) and (1). For example ΔH means (H_2-H_1).

WORKED EXAMPLE No.3

A steam turbine is supplied with 30 kg/s of superheated steam at 80 bar and 400°C with negligible velocity. The turbine shaft produces 200 kNm of torque at 3000 rev/min. There is a heat loss of 1.2 MW from the casing. Determine the thermal power remaining in the exhaust steam.

SOLUTION

Shaft Power = $2\pi NT = 2\pi(3000/60)$ x 200 000 = 62.831 x 10^6 W = 62.831 MW
Thermal power supplied = H at 80 bar and 400°C
H = 30(3139) = 94170 kW = 94.17 MW
Total energy flow into turbine = 94.17 MW
Energy flow out of turbine = 94.17 MW = SP + Loss + Exhaust.
Thermal Power in exhaust = 94.17 -1.2 - 62.831 = **30.14 MW**

SELF ASSESSMENT EXERCISE No.3

1. A non-flow system receives 80 kJ of heat transfer and loses 20 kJ as work transfer. What is the change in the internal energy of the fluid?
   (60 kJ)

2. A non-flow system receives 100 kJ of heat transfer and also 40 kJ of work is transferred to it. What is the change in the internal energy of the fluid?
   (140 kJ)

3. A steady flow system receives 500 kW of heat and loses 200 kW of work. What is the net change in the energy of the fluid flowing through it?
   (300 kW)

4. A steady flow system loses 2 kW of heat also loses 4 kW of work. What is the net change in the energy of the fluid flowing through it?
   (-6 kW)

5. A steady flow system loses 3 kW of heat also loses 20 kW of work. The fluid flows through the system at a steady rate of 70 kg/s. The velocity at inlet is 20 m/s and at outlet it is 10 m/s. The inlet is 20 m above the outlet. Calculate the following.
   i.   The change in K.E./s (-10.5 kW)
   ii. The change in P.E/s (-13.7 kW)
   iii. The change in enthalpy/s (1.23 kW)

3. MORE EXAMPLES OF THERMODYNAMIC SYSTEMS

When we examine a thermodynamic system, we must first decide whether it is a non-flow or a steady flow system. First, we will look at examples of non-flow systems.

3.1 PISTON IN A CYLINDER

Fig. 7

There may be heat and work transfer. The N.F.E.E. is, $Q + W = \Delta U$

Sometimes there is no heat transfer (e.g. when the cylinder is insulated).

$Q = 0$ so $W = \Delta U$

If the piston does not move, the volume is fixed and no work transfer occurs. In this case $Q = \Delta U$

For a GAS ONLY the change in internal energy is $\Delta U = mC_V \Delta T$.

3.2. SEALED EVAPORATOR OR CONDENSER.

Fig. 8

Since no change in volume occurs, there is no work transfer so $Q = \Delta U$
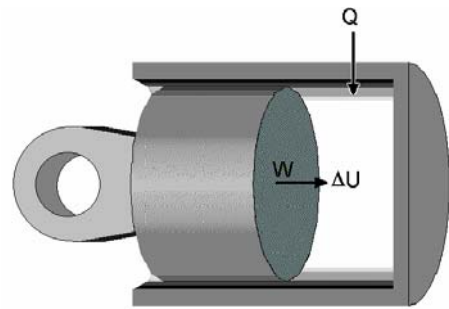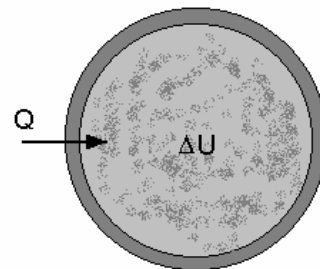
WORKED EXAMPLE No.4

30 g of gas inside a cylinder fitted with a piston has a temperature of 15°C. The piston is moved with a mean force of 200 N so that that it moves 60 mm and compresses the gas. The temperature rises to 21°C as a result.

Calculate the heat transfer given $c_V = 718$ J/kg K.

SOLUTION

This is a non flow system so the law applying is $Q + W = \Delta U$
The change in internal energy is $\Delta U = mc_V \Delta T = 0.03 \times 718 \times (21 - 15)$
$\Delta U = 129.24$ J
The work is transferred into the system because the volume shrinks.
$W = $ force $\times$ distance moved $= 200 \times 0.06 = 12$ J
$Q = \Delta U - W = 117.24$ J

Now we will look at examples of steady flow systems.

3.3. PUMPS AND FLUID MOTORS

The diagram shows graphical symbols for hydraulic pumps and motors.
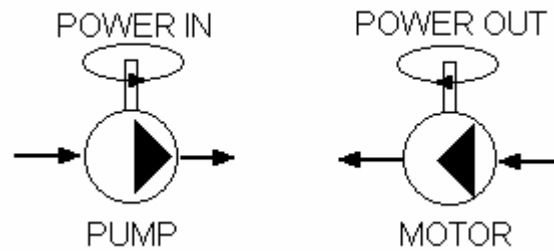
Fig.9

The S.F.E.E. states,

$$\Phi + P = \Delta KE/s + \Delta PE/s + \Delta H/s$$

In this case, especially if the fluid is a liquid, the velocity is the same at inlet and outlet and the kinetic energy is ignored. If the inlet and outlet are at the same height, the PE is also neglected. Heat transfer does not usually occur in pumps and motors so Φ is zero.

The S.F.E.E. simplifies to **$P = \Delta H/s$**

Remember that enthalpy is the sum of internal energy and flow energy. The enthalpy of gases, vapours and liquids may be found. In the case of liquids, the change of internal energy is small and so the change in enthalpy is equal to the change in flow energy only.

The equation simplifies further to $P = \Delta FE/s$

Since FE = pV and V is constant for a liquid, this becomes **$P = V\Delta p$**

WORKED EXAMPLE No.5

A pump delivers 20 kg/s of oil of density 780 kg/m^3 from atmospheric pressure at inlet to 800 kPa gauge pressure at outlet. The inlet and outlet pipes are the same size and at the same level. Calculate the theoretical power input.

SOLUTION

Since the pipes are the same size, the velocities are equal and the change in kinetic energy is zero. Since they are at the same level, the change in potential energy is also zero. Neglect heat transfer and internal energy.

P= V Δp
V = m/ρ = 20/780 = 0.0256 m^3/s
Δp = 800 - 0 = 800 kPa

P = 0.0256 x 800000 = 20 480 W or 20.48 kW

WORKED EXAMPLE No.6

A feed pump on a power station pumps 20 kg/s of water. At inlet the water is at 1 bar and 120°C. At outlet it is at 200 bar and 140°C. Assuming that there is no heat transfer and that PE and KE are negligible, calculate the theoretical power input.

SOLUTION

In this case the internal energy has increased due to frictional heating.

The SFEE reduces to $\qquad$ $P = \Delta H/s = m(h_2 - h_1)$

The h values may be found from tables. $\qquad$ $h_1 = 504$ kJ/kg

This is near enough the value of h_f at 120°C bar in steam tables. $\qquad$ $h_2 = 602$ kJ/kg

P = 20 (602 - 504) = 1969 kW or 1.969 MW

If water tables are not to hand the problem may be solved as follows. $\Delta h = \Delta u + \Delta f.e.$

$\Delta u = c \Delta T$ where c = 4.18 kJ/kg K for water

$\Delta u = 4.18 (140 - 120) = 83.6$ kJ/kg $\qquad$ $\Delta f.e. = V\Delta p$

The volume of water is normally around 0.001 m³/kg.

$\Delta f.e. = 0.001 \times (200 - 1) \times 10^5 = 19\ 900$ J/kg or 19.9 kJ/kg

hence $\Delta h = \Delta u + \Delta fe = 83.6 + 19.9 = 103.5$ kJ/kg

P = mΔh = 20 x 103.5 = 2070 kW or 2.07 MW

The discrepancies between the answers are slight and due to the fact the value of the specific heat and of the specific volume are not accurate at 200 bar.

3.4. GAS COMPRESSORS AND TURBINES.

Figure 10 shows the basic construction of an axial flow compressor and turbine. These have rows of aerofoil blades on the rotor and in the casing. The turbine passes high pressure hot gas or steam from left to right making the rotor rotate. The compressor draws in gas and compresses it in stages.
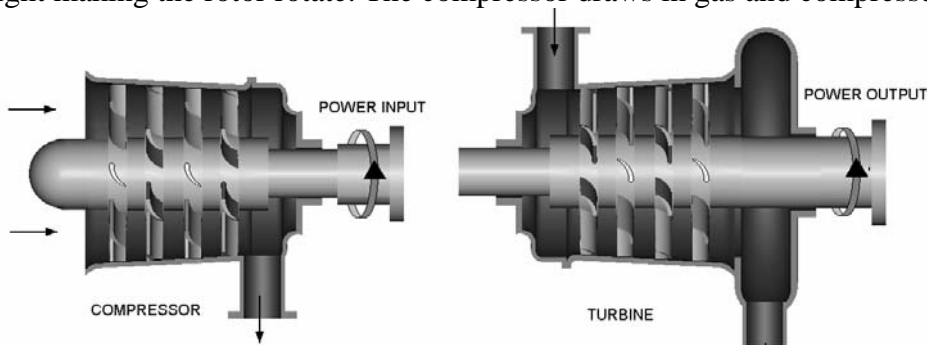
Fig. 10

Compressing a gas normally makes it hotter but expanding it makes it colder. This is because gas is compressible and unlike the cases for liquids already covered, the volumes change dramatically with pressure. This might cause a change in velocity and hence kinetic energy. Often both kinetic and potential energy are negligible. The internal energy change is not negligible. Figure 11 shows graphical symbols for turbines and compressors. Note the narrow end is always the high pressure end.
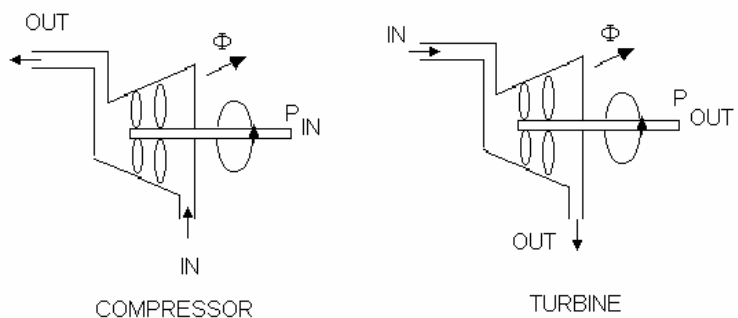
Fig.11

A gas turbine uses 5 kg/s of hot air. It takes it in at 6 bar and 900°C and exhausts it at 450°C. The turbine loses 20 kW of heat from the casing. Calculate the theoretical power output given that c_p = 1005 J/kg K.

SOLUTION

First identify this as a steady flow system for which the equation is

$\Phi + P = \Delta K.E./s + \Delta P.E./s + \Delta H/s$

For lack of further information we assume K.E. and PE to be negligible. The heat transfer rate is -20 kW.

The enthalpy change for a gas is $\Delta H = mC_p\Delta T$

$\Delta H = 5 \times 1005 \times (450 - 900) = -2261000$ W or -2.261 MW

$P = \Delta H - \Phi = -2261 - (-20) = -2241$ kW

The minus sign indicates that the power is leaving the turbine. Note that if this was a steam turbine, you would look up the h values in the steam tables.

3.5   STEADY FLOW EVAPORATORS AND CONDENSERS

A refrigerator is a good example of a thermodynamic system. In particular, it has a heat exchanger inside that absorbs heat at a cold temperature and evaporates the liquid into a gas. The gas is compressed and becomes hot. The gas is then cooled and condensed on the outside in another heat exchanger.
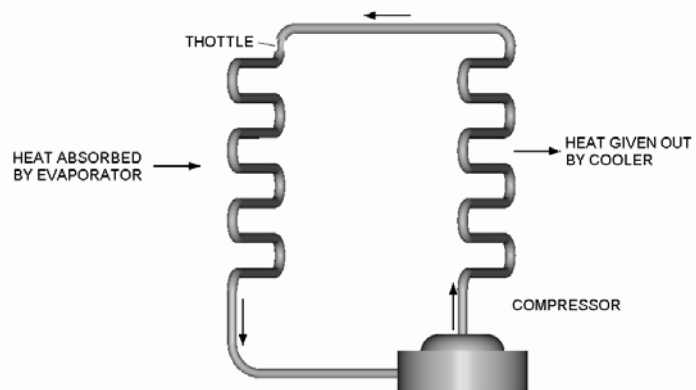
Fig. 12

For both the evaporator and condenser, there is no work transferred in or out. K.E. and P.E. are not normally a feature of such systems so the S.F.E.E. reduces to   **$\Phi = \Delta H/s$**

On steam power plant, boilers are used to raise steam and these are examples of large evaporators working at high pressures and temperatures. Steam condensers are also found on power stations. The energy equation is the same, whatever the application.

A steam condenser takes in wet steam at 8 kg/s and dryness fraction 0.82. This is condensed into saturated water at outlet. The working pressure is 0.05 bar.
Calculate the heat transfer rate.

SOLUTION

$\Phi = \Delta H/s = m(h_2 - h_1)$ $h_1 = h_f + x\, h_{fg}$ at 0.05 bar

from the steam tables we find that $h_1 = 138 + 0.82(2423) = 2125$ kJ/kg

$h_2 = h_f$ at 0.05 bar = 138 kJ/kg   hence   **$\Phi = 8(138 - 2125) = -15896$ kW**

The negative sign indicates heat transferred from the system to the surroundings.

SELF ASSESSMENT EXERCISE No.4

1. Gas is contained inside a cylinder fitted with a piston. The gas is at 20°C and has a mass of 20 g. The gas is compressed with a mean force of 80 N which moves the piston 50 mm. At the same time 5 Joules of heat transfer occurs out of the gas. Calculate the following.

    i. The work done.(4 J)
    ii. The change in internal energy. (-1 J)
    iii. The final temperature. (19.9^{o}C)   Take c_V as 718 J/kg K

2. A steady flow air compressor draws in air at 20°C and compresses it to 120°C at outlet. The mass flow rate is 0.7 kg/s. At the same time, 5 kW of heat is transferred into the system. Calculate the following.
    i. The change in enthalpy per second. (70.35 kW)
    ii. The work transfer rate. (65.35 kW)   Take c_p as 1005 J/kg K.

3. A steady flow boiler is supplied with water at 15 kg/s, 100 bar pressure and 200°C. The water is heated and turned into steam. This leaves at 15 kg/s, 100 bar and 500°C. Using your steam tables, find the following.

    i. The specific enthalpy of the water entering. (856 kJ/kg)
    ii. The specific enthalpy of the steam leaving. (3373 kJ/kg)
    iii. The heat transfer rate. (37.75 kW)

4. A pump delivers 50 dm^3/min of water from an inlet pressure of 100 kPa to an outlet pressure of 3 MPa. There is no measurable rise in temperature. Ignoring K.E. and P.E, calculate the work transfer rate. (2.42 kW)

5. A water pump delivers 130 dm^3/minute (0.13 m^3/min) drawing it in at 100 kPa and delivering it at 500 kPa. Assuming that only flow energy changes occur, calculate the power supplied to the pump. (860 W)

6. A steam condenser is supplied with 2 kg/s of steam at 0.07 bar and dryness fraction 0.9. The steam is condensed into saturated water at outlet. Determine the following.

    i. The specific enthalpies at inlet and outlet. (2331 kJ/kg and 163 kJ/kg)
    ii. The heat transfer rate. (4336 kW)

7. 0.2 kg/s of gas is heated at constant pressure in a steady flow system from 10°C to 180°C. Calculate the heat transfer rate Φ. (37.4 kW) C_p = 1.1 kJ/kg K

8. 0.3 kg of gas is cooled from 120°C to 50°C at constant volume in a closed system. Calculate the heat transfer. (-16.8 kJ) C_V = 0.8 kJ/kg.

4.   POLYTROPIC PROCESSES.

When you complete section four you should be able to do the following.

❑ Use the laws governing the expansion and compression of a fluid.
❑   State the names of standard processes.
❑ Derive and use the work laws for closed system expansions and compressions.
❑ Solve problems involving gas and vapour processes in closed systems.

We will start by examining expansion and compression processes.

4.1   COMPRESSION AND EXPANSION PROCESSES.

A compressible fluid (gas or vapour) may be compressed by reducing its volume or expanded by increasing its volume. This may be done inside a cylinder by moving a piston or by allowing the pressure to change as it flows through a system such as a turbine. For ease of understanding, let us consider the change as occurring inside a cylinder. The process is best explained with a pressure - volume graph.

When the volume changes, the pressure and temperature may also change. The resulting pressure depends upon the final temperature. The final temperature depends on whether the fluid is cooled or heated during the process. It is normal to show these changes on a graph of pressure plotted against volume. (p-V graphs). A typical graph for a compression and an expansion process is shown in fig.13.
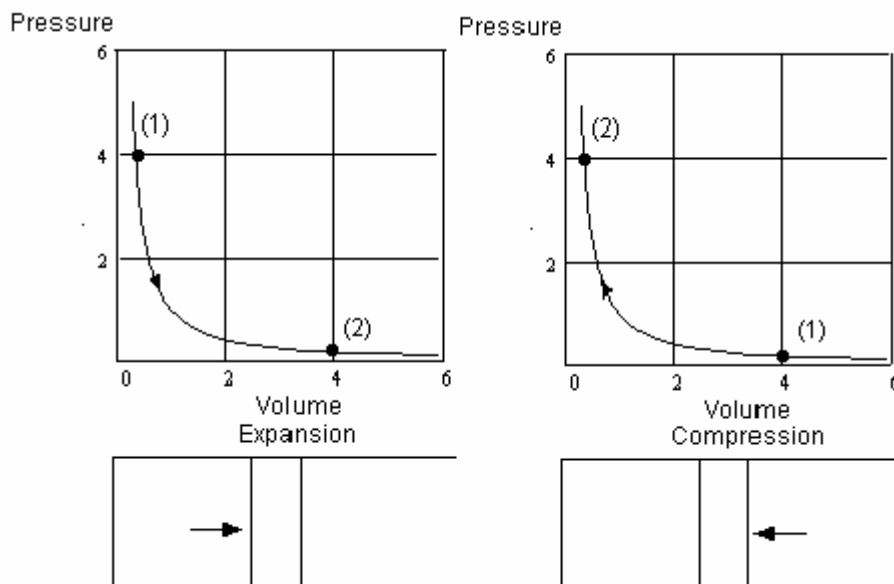

Fig. 13

It has been discovered that the resulting curves follows the mathematical law

$$pV^n = \text{constant}.$$

Depending on whether the fluid is heated or cooled, a family of such curves is obtained as shown (fig.14). Each graph has a different value of n and n is called the index of expansion or compression.
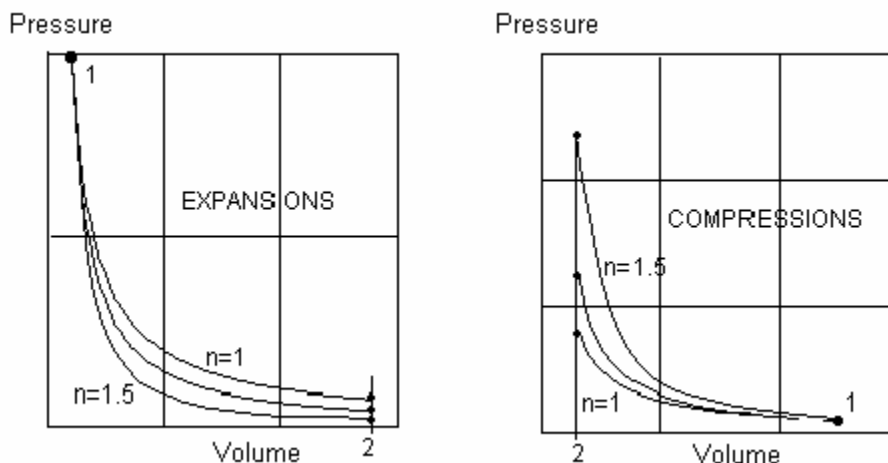
Fig.14

The most common processes are as follows.

CONSTANT VOLUME also known as ISOCHORIC

A vertical graph is a constant volume process and so it is not a compression nor expansion. Since no movement of the piston occurs no work transfer has taken place. Nevertheless, it still fits the law with n having a value of infinity.

CONSTANT PRESSURE also known as ISOBARIC

A horizontal graph represents a change in volume with no pressure change (constant pressure process). The value of n is zero in this case.

CONSTANT TEMPERATURE also known as ISOTHERMAL

All the graphs in between constant volume and constant pressure, represent processes with a value of n between infinity and zero. One of these represents the case when the temperature is maintained constant by cooling or heating by just the right amount.

When the fluid is a gas, the law coincides with Boyle's Law pV= constant so it follows that n is 1.

When the fluid is a vapour, the gas law is not accurate and the value of n is close to but not equal to 1.

ADIABATIC PROCESS

When the pressure and volume change in such a way that no heat is added nor lost from the fluid (e.g. by using an insulated cylinder), the process is called adiabatic. This is an important process and is the one that occurs when the change takes place so rapidly that there is no time for heat transfer to occur. This process represents a demarcation between those in which heat flows into the fluid and those in which heat flows out of the fluid. In order to show it is special, the symbol γ is used instead of n and the law is

$$pV^{\gamma} = C$$

It will be found that each gas has a special value for γ (e.g. 1.4 for dry air).

POLYTROPIC PROCESS

All the other curves represent changes with some degree of heat transfer either into or out of the fluid. These are generally known as polytropic processes.

HYPERBOLIC PROCESS

The process with n=1 is a hyperbola so it is called a hyperbolic process. This is also isothermal for gas but not for vapour. It is usually used in the context of a steam expansion.

WORKED EXAMPLE No.9

A gas is compressed from 1 bar and 100 cm^3 to 20 cm^3 by the law $pV^{1.3}$=constant. Calculate the final pressure.

SOLUTION.

If $pV^{1.3} = C$ then $p_1V_1^{1.3} = C = p_2V_2^{1.3}$
hence $1 \times 100^{1.3} = p_2 \times 20^{1.3}$ $1 \times (100/20)^{1.3} = p_2 = 8.1$ bar

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WORKED EXAMPLE No.10

Vapour at 10 bar and 30 cm^3 is expanded to 1 bar by the law $pV^{1.2} = C$. Find the final volume.

SOLUTION.

$p_1V_1^{1.2} = C = p_2V_2^{1.2}$ $10 \times 30^{1.2} = 1 \times V_2^{1.2}$ $V_2 = (592.3)^{1/1.2} = 204.4$ cm^3

WORKED EXAMPLE No.11

A gas is compressed from 200 kPa and 120 cm^3 to 30 cm^3 and the resulting pressure is 1 MPa. Calculate the index of compression n.

SOLUTION.

$200 \times 120^n = 1000 \times 30^n$ $(120/30)^n = 1000/200 = 5$ $4^n = 5$

$n\log 4 = \log 5$ $n = \log 5/\log 4 = 1.6094/1.3863 = 1.161$

Note this may be solved with natural or base 10 logs or directly on suitable calculators.

SELF ASSESSMENT EXERCISE No. 5

1. A vapour is expanded from 12 bar and 50 cm^3 to 150 cm^3 and the resulting pressure is 6 bar. Calculate the index of compression n.
   (0.63)

2.a. A gas is compressed from 200 kPa and 300 cm^3 to 800 kPa by the law $pV^{1.4}=C$. Calculate the new volume. (111.4 cm^3)

2.b. The gas was at 50°C before compression. Calculate the new temperature using the gas law $pV/T = C$. (207°C)

3.a. A gas is expanded from 2 MPa and 50 cm^3 to 150 cm^3 by the law $pV^{1.25} = C$. Calculate the new pressure. (506 kPa)

3.b. The temperature was 500°C before expansion. Calculate the final temperature.
     (314°C)

4.2.   COMBINING THE GAS LAW WITH THE POLYTROPIC LAW.

For gases only, the general law may be combined with the law of expansion as follows.

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad \text{and so} \quad \frac{T_2}{T_1} = \frac{p_2 V_2}{p_1 V_1}$$

Since for an expansion or compression

$$p_1 V_1^n = p_2 V_2^n$$

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^n$$

Substituting into the gas law we get

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

and further since

$$\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = \frac{V_2}{V_1}$$

substituting into the gas law gives

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}$$

To summarise we have found that

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}$$

In the case of an adiabatic process this is written as

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}}$$

For an isothermal process $n = 1$ and the temperatures are the same.

WORKED EXAMPLE No.12

A gas is compressed adiabatically with a volume compression ratio of 10. The initial temperature is 25°C. Calculate the final temperature given $\gamma = 1.4$

SOLUTION

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 298(10)^{1.4-1} = 748.5K \quad \text{or} \quad 475.5°C$$

WORKED EXAMPLE No.13

A gas is compressed polytropically by the law $pV^{1.2} = C$ from 1 bar and 20°C to 12 bar. calculate the final temperature.

SOLUTION

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}} \quad T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}} = 293(12)^{1-\frac{1}{1.2}}$$

$$T_2 = 293(12)^{0.167} = 293(1.513) = 443.3K$$

WORKED EXAMPLE No.14

A gas is expanded from 900 kPa and 1100°C to 100 kPa by the law $pV^{1.3} = C$. Calculate the final temperature.

SOLUTION

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}} \quad\quad T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}$$

$$T_2 = 1373\left(\frac{100}{900}\right)^{1-\frac{1}{1.3}} = 1373(0.111)^{0.2308} = 1373(0.602) = 826.9K$$

SELF ASSESSMENT EXERCISE No. 6

1. A gas is expanded from 1 MPa and 1000°C to 100 kPa. Calculate the final temperature when the process is

    i. Isothermal (n=1) (1000°C)
    ii Polytropic (n=1.2) (594°C)
    iii. Adiabatic (γ =1.4) (386°C)
    iv. Polytropic (n= 1.6) (264°C)

2. A gas is compressed from 120 kPa and 15°C to 800 kPa. Calculate the final temperature when the process is

    i. Isothermal (n=1) (15°C)
    ii. Polytropic (n=1.3) (173°C)
    iii Adiabatic (γ=1.4) (222°C)
    iv. Polytropic (n= 1.5) (269°C)

3. A gas is compressed from 200 kPa and 20°C to 1.1 MPa by the law $pV^{1.3}$=C. The mass is 0.02 kg. c_p=1005 J/kg K. c_v = 718 J/kg K. Calculate the following.

   i.   The final temperature. (434 K)
   ii. The change in internal energy (2.03 kJ)
        iii.   The change in enthalpy (2.84 kJ)

4. A gas is expanded from 900 kPa and 1200°C to 120 kPa by the law $pV^{1.4}$ = C. The mass is 0.015 kg. c_p=1100 J/kg K c_v = 750 J/kg K
   Calculate the following.

   i.   The final temperature. (828 K)
   ii. The change in internal energy (-7.25 kJ)
        iii.   The change in enthalpy (-10.72 kJ)

4.3. EXAMPLES INVOLVING VAPOUR

Problems involving vapour make use of the formulae pV^n = C in the same way as those involving gas. You cannot apply gas laws, however, unless it is superheated into the gas region. You must make use of vapour tables so a good understanding of this is essential. This is best explained with worked examples.

WORKED EXAMPLE No.15

A steam turbine expands steam from 20 bar and 300°C to 1 bar by the law $pV^{1.2}$= C. Determine for each kg flowing:

a. the initial and final volume.
b. the dryness fraction after expansion.
c. the initial and final enthalpies.
d. the change in enthalpy.

SOLUTION

The system is a steady flow system in which expansion takes place as the fluid flows. The law of expansion applies in just the same way as in a closed system.

The initial volume is found from steam tables. At 20 bar and 300°C it is superheated and from the tables we find   v= 0.1255 m^3/kg

Next apply the law $pV^{1.2}$ = C $p_1V_1^{1.2} = p_2V_2^{1.2}$ $20 \times 0.1255^{1.2} = 1 \times V_2^{1.2}$

Hence V_2= 1.523m^3/kg

Next, find the dryness fraction as follows. Final volume = 1.523 m^3/kg = xv_g at 1 bar.

From the tables we find v_g is 1.694 m^3/kg hence   1.523 = 1.694x   x = 0.899

We may now find the enthalpies in the usual way.

h_1 at 20 bar and 300°C is 3025 kJ/kg

$h_2 = h_f + xh_{fg}$ at 1 bar (wet steam)

$h_2 = 417 + (0.899)(2258) = 2447$ kJ/kg

The change in enthalpy is $h_2 - h_1$ = -578 kJ/kg

SELF ASSESSMENT EXERCISE No.7

1. 3 kg/s of steam is expanded in a turbine from 10 bar and 200°C to 1.5 bar by the law $pV^{1.2}=C$. Determine the following.

    i. The initial and final volumes. (0.618 m^3 and 3 m^3)

    ii. The dryness fraction after expansion. (0.863)

    iii. The initial and final enthalpies. (2829 kJ/kg and 2388 kJ/kg)

    iv. The change in enthalpy. -1324 kW)

2. 1.5 kg/s of steam is expanded from 70 bar and 450°C to 0.05 bar by the law $pV^{1.3} = C$. Determine the following.

    i. The initial and final volumes. (0.066 m^3/kg and 17.4 m^3/kg)

    ii. The dryness fraction after expansion. (0.411)

    iii. The initial and final enthalpies. (3287 kJ/kg and 1135 kJ/kg)

    iv. The change in enthalpy. (-3228 kW)

3. A horizontal cylindrical vessel is divided into two sections each 1m^3 volume, by a non-conducting piston. One section contains steam of dryness fraction 0.3 at a pressure of 1 bar, while the other contains air at the same pressure and temperature as the steam. Heat is transferred to the steam very slowly until its pressure reaches 2 bar.

    Assume that the compression of the air is adiabatic ($\gamma=1.4$) and neglect the effect of friction between the piston and cylinder. Calculate the following.

    i.   The final volume of the steam. (1.39 m^3)

    ii.   The mass of the steam. (1.97 kg)

    iii.   The initial internal energy of the steam. (2053 kJ)

    iv   The final dryness fraction of the steam. (0.798)

    v.   The final internal energy of the steam. (4172 kJ)

    vi.   The heat added to the steam. (2119 kJ)

4.4. CLOSED SYSTEM WORK LAWS

4.4.1.   EXPANSION OF PRESSURE WITH VOLUME

We will start by studying the expansion of a fluid inside a cylinder against a piston which may do work against the surroundings.

A fluid may expand in two ways.

a)   It may expand rapidly and uncontrollably doing no useful work. In such a case the pressure could not be plotted against volume during the process. This is called an **UNRESISTED EXPANSION**

b)   It may expand moving the piston. The movement is resisted by external forces so the gas pressure changes in order to overcome the external force and move the piston. In this case the movement is controlled and the variation of pressure with volume may be recorded and plotted on a p-V graph. Work is done against the surroundings. This process is called a **RESISTED EXPANSION.**

Consider the arrangement shown in fig. 15. Assume that there is no pressure outside. If the string holding the weight was cut, the gas pressure would slam the piston back and the energy would be dissipated first by acceleration of the moving parts and eventually as friction. The expansion would be unresisted.

If the weights were gradually reduced, the gas would push the piston and raise the remaining weights. In this way, work would be done against the surroundings (it ends up as potential energy in the weights). The process may be repeated in many small steps, with the change in volume each time being dV.

The pressure although changing, is p at any time.

Fig. 15

This process is characterised by two important factors.

1.   The process may be reversed at any time by adding weights and the potential energy is transferred back from the surroundings as work is done on the system. The fluid may be returned to its original pressure, volume, temperature and energy.

2.   The fluid force on one side of the piston is always exactly balanced by the external force (in this case due to the weights) on the other side of the piston.

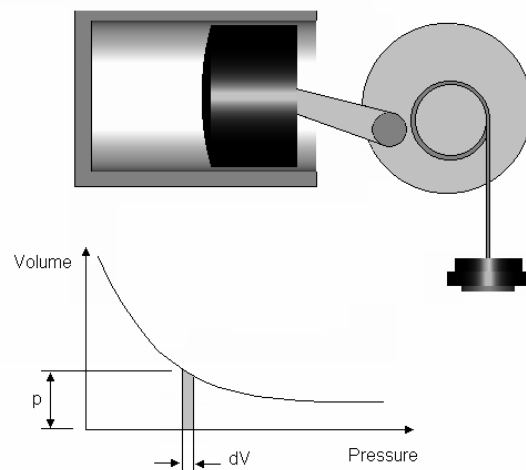The expansion or compression done in this manner is said to be **REVERSIBLE and CARRIED OUT IN EQUILIBRIUM.**

4.4.2. WORK AS AREA UNDER THE p - V DIAGRAM

If the expansion is carried out in equilibrium, the force of the fluid must be equal to the external force F. It follows that F = pA.

When the piston moves a small distance dx, the work done is dW

$$dW = -F\,dx = -pA\,dx = -pdV.$$

The minus sign is because the work is leaving the system.

For an expansion from points 1 to 2 it follows that the total work done is given by

$$W = -\int_{V_1}^{V_2} pdV$$

We must remember at this stage that our sign convention was that work leaving the system is negative.

It should be noted that some of the work is used to overcome any external pressure such as atmospheric and the useful work is reduced. Consider the system shown in fig.15 again but this time suppose there is atmospheric pressure on the outside p_a.

In this case It follows that

F + p_aA = pA.   F = pA. - p_aA

When the piston moves a small distance dx, the the useful work done is –F dx

- F dx = - (pAdx - p_aAdx) = - (p - p_a)dV.

For an expansion from points 1 to 2 it follows that the useful work done is given by

$$W = -\int_{V_1}^{V_2} (p - p_a)dV$$

4.4.3. WORK LAWS FOR CLOSED SYSTEMS

If we solve the expression $W = -\int_{V_1}^{V_2} pdV$ we obtain the work laws for a closed system. The solution depends upon the relationship between p and V. The formulae now derived apply equally well to a compression process and an expansion process. Let us now solve these cases.

CONSTANT PRESSURE   **CONSTANT VOLUME**

$$W = -\int_{V_1}^{V_2} pdV$$

If V is constant t hen dV = 0

$$W = -p\int_{V_1}^{V_2} dV$$

W = - p (V$_2$-V$_1$)   **W = 0.**

HYPERBOLIC

This is an expansion which follows the law $pV^1 = C$ and is **ISOTHERMAL** when it is a gas. Substituting $p = CV^{-1}$ the expression becomes

$$W = -\int_{V_1}^{V_2} pdV = -C\int_{V_1}^{V_2} V^{-1}dV = -C\ln\left[\frac{V_2}{V_1}\right]$$

Since $pV = C$ then

$$W = -pV\ln\left[\frac{V_2}{V_1}\right]$$

$$\text{since } \frac{V_2}{V_1} = \frac{p_1}{p_2} \quad W = -pV\ln\left[\frac{p_1}{p_2}\right]$$

In the case of gas we can substitute $pV = mRT$ and so

$$W = -mRT\ln\left[\frac{V_2}{V_1}\right] = -mRT\ln\left[\frac{p_1}{p_2}\right]$$

POLYTROPIC

In this case the expansion follows the law $pV^n = C$. The solution is as follows.

$$W = -\int_{V_1}^{V_2} pdV \quad \text{but } p = CV^{-n} \qquad W = -C\int_{V_1}^{V_2} V^{-n}dV = -C\frac{\left[V_2^{-n+1} - V_1^{-n+1}\right]}{-n+1}$$

$$\text{Since } C = p_1V_1 \text{ or } p_2V_2 \quad W = \frac{\left[p_2V_2 - p_1V_1\right]}{n-1}$$

For gas only we may substitute $pV = mRT$ and so $\qquad W = mR\frac{\left[T_2 - T_1\right]}{n-1}$

ADIABATIC

Since an adiabatic case is the special case of a polytropic expansion with no heat transfer, the derivation is identical but the symbol γ is used instead of n.

$$W = \frac{\left[p_2V_2 - p_1V_1\right]}{\gamma - 1}$$

For gas only we may substitute $pV = mRT$ and so $W = mR\frac{\left[T_2 - T_1\right]}{\gamma - 1}$

This is the special case of the polytropic process in which Q=0. $Q = 0 \quad W = \frac{mR\Delta R}{\gamma - 1}$

Substituting for Q and ΔU in the NFEE we find

$$Q + W = \Delta U \qquad 0 + \frac{mR\Delta R}{\gamma - 1} = mCv\Delta C \qquad \frac{R}{\gamma - 1} = Cv$$

Since $R = Cp - Cv \qquad Cp - Cv = Cv(\gamma - 1) \quad \frac{Cp}{Cv} = \gamma$

This shows that the ratio of the principal specific heat capacities is the adiabatic index. It was shown earlier that the difference is the gas constant R. These important relationships should be remembered.

$$\textbf{Cp} - \textbf{Cv} = \textbf{R}$$

$$\mathbf{\gamma = Cp/Cv}$$

WORKED EXAMPLE No.15

Air at a pressure of 500 kPa and volume 50 cm^3 is expanded reversibly in a closed system to 800 cm^3 by the law $pV^{1.3} = C$. Calculate the following.

a. The final pressure.

b. The work done.

SOLUTION

$p_1 = 500$ kPa $V_1 = 50 \times 10^{-6}$ m^3 $V_2 = 800 \times 10^{-6}$ m^3

$p_1 V_1^{1.3} = p_2 V_2^{1.3}$ $500 \times 10^3 (50 \times 10^{-6})^{1.3} = p_2 (800 \times 10^{-6})^{1.3}$

$p_2 = 13.6 \times 10^3$ or 13.6kPa

$$W = \frac{(p_2 V_2 - p_1 V_1)}{n-1} = \left(\frac{13.6 \times 10^3 \times 800 \times 10^{-6} - 500 \times 10^3 \times 50 \times 10^{-6}}{1.3 - 1} \right)$$

$W = -47$Joules

WORKED EXAMPLE No.16

Steam at 6 bar pressure and volume 100 cm^3 is expanded reversibly in a closed system to 2 dm^3 by the law $pV^{1.2} = C$. Calculate the work done.

SOLUTION

$p_1 = 6$ bar $V_1 = 100 \times 10^{-6}$ m^3 $V_2 = 2 \times 10^{-3}$ m^3

$$p_2 = \frac{p_1 V_1^{1.2}}{V_2^{1.2}} = 6x \left(\frac{100 \times 10^{-6}}{2 \times 10^{-3}} \right)^{1.2} = 0.1648 \text{bar}$$

$$W = \frac{(p_2 V_2 - p_1 V_1)}{n-1} = \frac{\left(0.1648 \times 10^5 \times 2 \times 10^{-3} - 6 \times 10^5 \times 100 \times 10^{-6} \right)}{1.2 - 1}$$

$W = -135.2$Joules

SELF ASSESSMENT EXERCISE No.8

1. 10 g of steam at 10 bar and 350ºC expands reversibly in a closed system to 2 bar by the law $pV^{1.3}=C$. Calculate the following.

   i. The initial volume. (0.00282 m^3)

   ii. The final volume. (0.00974 m^3)

   iii. The work done. (-2.92 kJ)

2. 20 g of gas at 20ºC and 1 bar pressure is compressed to 9 bar by the law $pV^{1.4} = C$. Taking the gas constant R = 287 J/kg K calculate the work done. (Note that for a compression process the work will turn out to be positive if you correctly identify the initial and final conditions). (3.67 kJ)

3. Gas at 600 kPa and 0.05 dm^3 is expanded reversibly to 100 kPa by the law $pV^{1.35} = C$. Calculate the work done. (-31.8 kJ)

4. 15 g of gas is compressed isothermally from 100 kPa and 20ºC to 1 MPa pressure. The gas constant is 287 J/kg K. Calculate the work done. (2.9 kJ)

5. Steam at 10 bar with a volume of 80 cm^3 is expanded reversibly to 1 bar by the law pV=C. Calculate the work done. (-184.2 kJ)

6. Gas fills a cylinder fitted with a frictionless piston. The initial pressure and volume are 40 MPa and 0.05 dm^3 respectively. The gas expands reversibly and polytropically to 0.5 MPa and 1 dm^3 respectively. Calculate the index of expansion and the work done. (1.463 and -3.24 kJ)

7. An air compressor commences compression when the cylinder contains 12 g at a pressure is 1.01 bar and the temperature is 20ºC. The compression is completed when the pressure is 7 bar and the temperature 90ºC. (1.124 and 1944 J)

   The characteristic gas constant R is 287 J/kg K. Assuming the process is reversible and polytropic, calculate the index of compression and the work done.

WORKED EXAMPLE No.17

0.2 kg of gas at 100 °C is expanded isothermally and reversibly from 1 MPa pressure to 100 kPa. Take C_V = 718 J/kg K and R = 287 J/kg K.
Calculate

i. The work transfer.
ii. The change in internal energy.
iii. The heat transfer.

SOLUTION

$$W = -pVln\left(\frac{V_2}{V_1}\right) = -mRTln\left(\frac{V_2}{V_1}\right) = -mRTln\left(\frac{p_1}{p_2}\right)$$

$$W = -0.2x287x373ln\left(\frac{1x10^6}{1x10^5}\right) = -49300\,J \text{ or } -49.3 \text{ kJ}$$

The work is leaving the system so it is a negative work transfer.

Since T is constant $\Delta U = 0$ Q - 49.3 = 0   Q = 49.3 kJ

Note that 49.3 kJ of heat is transferred into the gas and 49.3 kJ of work is transferred out of the gas leaving the internal energy unchanged.

WORKED EXAMPLE No.18

Repeat worked example 17 but for an adiabatic process with γ = 1.4
Calculate

SOLUTION

$$T_2 = 373 \text{ x}\left(\frac{100 \times 10^3}{1 \times 10^6}\right)^{1-\frac{1}{\gamma}} = 193\,K$$

$$W = -mRT(T_2 - T_1) = -0.2 \text{ x } 287 \text{ x } \frac{(193 - 373)}{0.4}$$

$$W = -25\,830\,J$$

For an adiabatic process Q = 0

$Q + W = \Delta U$ hence ΔU = -25 830 J

Check $\Delta U = mC_V\Delta T = 0.2 \text{ x } 718 \text{ x } (193 - 373) = -25\,848\,J$

WORKED EXAMPLE No.19

Repeat worked example 17 but for a polytropic process with n=1.25
Calculate

SOLUTION

$$T_2 = 373 \text{ x} \left(\frac{100 \text{ x } 10^3}{1 \text{ x } 10^6} \right)^{1-\frac{1}{n}} = 235.3 \text{ K}$$

$$W = -mRT(T_2 - T_1) = -0.2 \text{ x } 287 \text{ x } \frac{(235.3 - 373)}{0.4}$$

$$W = -31605 \text{ J}$$

$$\Delta U = mC_V \Delta T = 0.2 \text{ x } 718 \text{ x } (235.3 - 373) = -19773.7 \text{ J}$$

$$Q = \Delta U - W$$

$$Q = -19773.7 - (-31603) = 11831.3 \text{ J}$$

SELF ASSESSMENT EXERCISE No.9

Take C_V = 718 J/kg K and R = 287 J/kg K throughout.

1. 1 dm^3 of gas at 100 kPa and 20ºC is compressed to 1.2 MPa reversibly by the law $pV^{1.2}$ = C. Calculate the following.

    i. The final volume. (0.126 dm^3)
    ii. The work transfer. (257 J)
    iii. The final temperature. (170oC)
    iv. The mass. (1.189 g)
    v. The change in internal energy. (128 J)
    vi. The heat transfer. (-128 J)

2. 0.05 kg of gas at 20 bar and 1100ºC is expanded reversibly to 2 bar by the law $pV^{1.3}$ =C in a closed system. Calculate the following.

    i. The initial volume. (9.85 dm^3)
    ii. The final volume. (58 dm^3)
    iii. The work transfer. (-27 kJ)
    iv. The change in internal energy. (-20.3 kJ)
    v. The heat transfer. (6.7 kJ)

3. 0.08 kg of air at 700 kPa and 800ºC is expanded adiabatically to 100 kPa in a closed system. Taking γ = 1.4 calculate the following.

    i. The final temperature. (615.4 K)
    ii. The work transfer. (26.3 kJ)
    iii. The change in internal energy. (-26.3 J)

4. A horizontal cylinder is fitted with a frictionless piston and its movement is restrained by a spring as shown (Figure 16.)
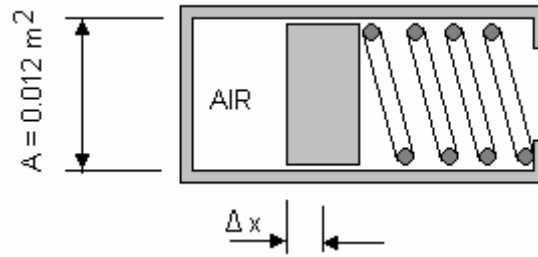

Figure 16

a. The spring force is directly proportional to movement such that $\Delta F/\Delta x = k$

   Show that the change in pressure is directly proportional to the change in volume such that $\Delta p/\Delta V = k/A^2$

b. The air is initially at a pressure and temperature of 100 kPa and 300 K respectively. Calculate the initial volume such that when the air is heated, the pressure – volume graph is a straight line that extends to the origin. (0.5 ^{dm3})

c. The air is heated making the volume three times the original value. Calculate the following.
   i. The mass. (0.58 g)
   ii. The final pressure. (300 kPa)
   iii. The final temperature. (2700 K)
   iv. The work done. (-200 kJ)
   v. The change in internal energy. (917 J)
   vi. The heat transfer. (1.12 kJ)

4.5. CLOSED SYSTEM PROBLEMS INVOLVING VAPOUR

The solution of problems involving steam and other vapours is done in the same way as for gases with the important proviso that gas laws must not be used. Volumes and internal energy values should be obtained from tables and property charts. This is best illustrated with a worked example.

WORKED EXAMPLE No.20

1kg of steam occupies a volume of 0.2 m^3 at 9 bar in a closed system. The steam is heated at constant pressure until the volume is 0.3144 m^3. Calculate the following.
   i. The initial dryness fraction.
   ii. The final condition.
   iii. The work transfer.
   iv. The change in internal energy.
   v. The heat transfer.

SOLUTION

First find the initial dryness fraction.
$V_1 = 0.2 = mx_1v_g$ at 9 bar $x_1 = 0.2/(1 \times 0.2149)$ **$x_1 = 0.931$ (initial dryness fraction)**.
Now determine the specific volume after expansion.

$p_2 = 9$ bar (constant pressure) $V_2 = 0.3144$ m^3.

$V_2 = mv_2$ $v_2 = 0.3144/1 = 0.3144 m^3/kg$

First, look in the superheat tables to see if this value exists for superheat steam. We find that at 9 bar and 350°C, the specific volume is indeed 0.3144 m³/kg.

The final condition is superheated to 350°C .
Note that if v_2 was less than v_g at 9 bar the steam would be wet and x_2 would have to be found. Next find the work.
$W = -p(V_2 - V_1) = -9 \times 10^5 (0.3144 - 0.2) = -102950$ J
W = -102.95 kJ (Energy leaving the system)

Next determine the internal energy from steam tables.
$U_1 = m u_1$ and $u_1 = u_f + x_1 u_{fg}$ at 9 bar
u_{fg} at 9 bar = $u_g - u_f = 2581 - 742 = 1839$ kJ/kg
$U_1 = 1\{742 + 0.931(1839)\} = 2454$ kJ
$U_2 = m u_2$ and $u_2 = u$ at 9 bar and 350 °C = 2877 kJ/kg
$U_2 = m u_2 = 1(2877) = 2877$ kJ.
The change in internal energy = $U_2 - U_1$ = 423 kJ (increased)

Finally deduce the heat transfer from the NFEE
$Q + W = \Delta U$
hence $Q = \Delta U - W = 423 - (-102.95)$ **Q = 526 kJ (energy entering the system)**

SELF ASSESSMENT EXERCISE No.10

1. 0.2 kg of dry saturated steam at 10 bar pressure is expanded reversibly in a closed system to 1 bar by the law $pV^{1.2} = C$. Calculate the following.

   i. The initial volume. (38.9 dm³)
   ii. The final volume. (264 dm³)
   iii. The work transfer. (-62 kJ)
   iv. The dryness fraction. (0.779)
   v. The change in internal energy. (-108 kJ)
   vi. The heat transfer. (-46 kJ)

2. Steam at 15 bar and 250°C is expanded reversibly in a closed system to 5 bar. At this pressure the steam is just dry saturated. For a mass of 1 kg calculate the following.

   i. The final volume. (0.375 m³)
   ii. The change in internal energy. (-165 kJ)
   iii. The work done. (-187 kJ)
   iv. The heat transfer. (22.1 kJ)

ENGINEERING COUNCIL

CERTIFICATE LEVEL

THERMODYNAMIC, FLUID AND PROCESS ENGINEERING C106

TUTORIAL 3 – HEAT ENGINE THEORY

When you have completed this tutorial, you should be able to do the following.

❑ Explain the basic idea behind the Second Law of Thermodynamics.

❑ Define the property called Entropy

❑ Define an Isentropic process.

❑ Solve basic problems involving isentropic expansions.

❑ Explain the Carnot Principle.

1. **THE SECOND LAW OF THERMODYNAMICS**

The Second Law of Thermodynamics is not something that can be written as a simple statement or formulae. It is a set of observations concerning the way that things flow or run as time progresses forward. It encompasses many observations such as "water normally flows from high levels to low levels" and "heat normally flows from hot to cold". In this module, you must concern yourself only with how the second law relates to heat engines and the efficiency of a heat engine.

In the context of heat engines, the second law may be summed as :

"No heat engine can be 100% efficient".

This should become apparent in the following sections.

1.1 **HEAT ENGINES**

Nearly all motive power is derived from heat using some form of heat engine. Here are some examples.

- ❑ Steam Power Plant.

- ❑ Gas Turbines.

- ❑ Jet Engines.

- ❑ Internal Combustion Engines.

A heat engine requires a source of hot energy. We get this by burning fossil fuel or by nuclear fission. The main sources of natural heat are solar and geothermal. In order to understand the basic theory, it might help to draw an analogy with a hydraulic motor and an electric motor. All motors require a high level source of energy and must exhaust at a low level of energy.
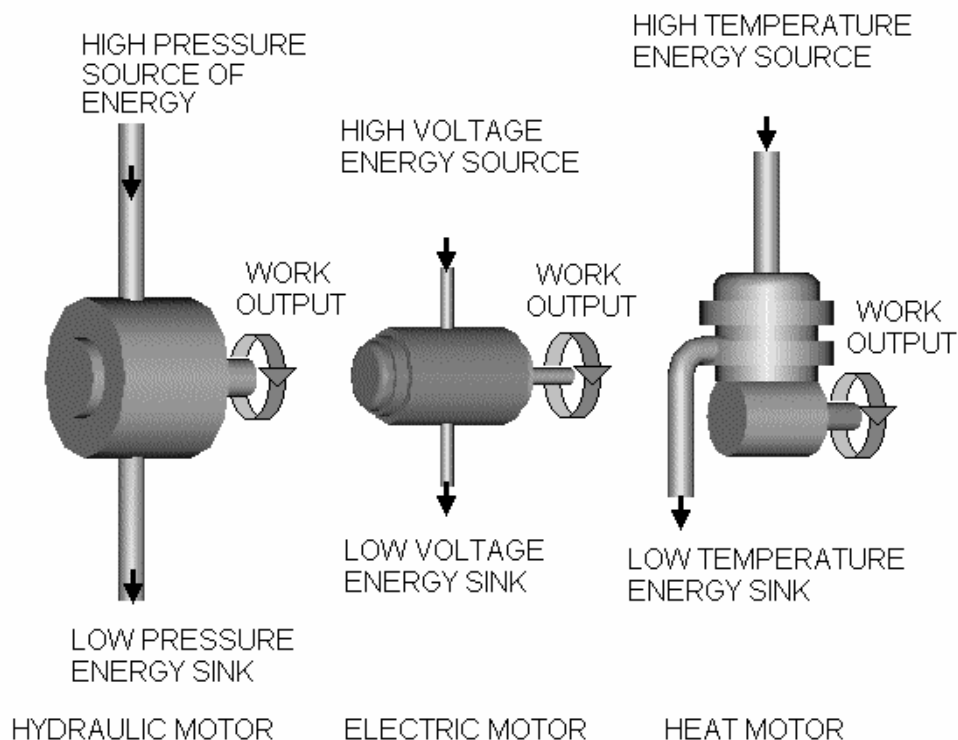

Figure 1

1.1.1   HYDRAULIC MOTOR

Fluid power is transported by the flow Q m^3/s. The energy contained in a volume Q m^3 of liquid at a pressure p is the flow energy given by the expression pQ. The hydraulic motor requires a source of liquid at a high pressure p_1 and exhausts at a lower pressure p_2. The energy supplied is p_1Q and some of this is converted into work. The energy in the low pressure liquid is p_2Q. For a perfect motor with no losses due to friction, the law of energy conservation gives the work output and efficiency as follows.

$$W_{out} = p_1Q - p_2Q = Q(p_1 - p_2)$$

$$\eta = \frac{W_{out}}{\text{Energy input}} = \frac{W_{out}}{p_1Q} = \frac{Q(p_1 - p_2)}{p_1Q} = \frac{(p_1 - p_2)}{p_1} = 1 - \frac{p_2}{p_1}$$

1.1.2   ELECTRIC MOTOR

Electric power is transported by the current. Electrical energy is the product of the charge Q Coulombs and the electric potential V Volts. The energy input at a high voltage is V_1Q and the energy exhausted at low voltage is V_2Q. For a perfect motor with no losses due to friction, the work output and efficiency are found from the law of energy conservation as follows.

$$W_{out} = V_1Q - V_2Q = Q(V_1 - V_2)$$

$$\eta = \frac{W_{out}}{\text{Energy input}} = \frac{W_{out}}{V_1Q} = \frac{Q(V_1 - V_2)}{V_1Q} = \frac{(V_1 - V_2)}{V_1} = 1 - \frac{V_2}{V_1}$$

1.1.3   HEAT MOTOR

Temperature is by analogy the equivalent of pressure and electric potential. In order to complete the analogy, we need something that is equivalent to volume and electric charge that transports the energy. It is not difficult to visualise a volume of liquid flowing through a hydraulic motor. It is not impossible to visualise a flow of electrons bearing electric charge through an electric motor. It is impossible to visualise something flowing through our ideal heat engine that transports pure heat but the analogy tells us there must be something so let us suppose a new property called ENTROPY and give it a symbol S. Entropy must have units of energy per degree of temperature or Joules per Kelvin. Entropy is dealt with more fully later on.

The energy supplied at temperature T_1 is T_1S and the energy exhausted is T_2S. For a perfect motor with no losses due to friction, the law of energy conservation gives the work output and efficiency as follows.

$$W_{out} = T_1S - T_2S = S(T_1 - T_2)$$

$$\eta = \frac{W_{out}}{\text{Energy input}} = \frac{W_{out}}{T_1S} = \frac{S(T_1 - T_2)}{T_1S} = \frac{(T_1 - T_2)}{T_1} = 1 - \frac{T_2}{T_1}$$

1.1.4   EFFICIENCY

In our perfect motors, the energy conversion process is 100% efficient but we may not have converted all the energy supplied into work and energy may be wasted in the exhaust. In the case of the electric motor, the lowest value for V_2 (so far as we know) is ground voltage zero, so theoretically we can obtain 100% efficiency by exhausting the electric charge with no residual energy.

In the case of the hydraulic motor, the lowest pressure we can exhaust to is atmospheric so we always waste some energy in the exhausted liquid.

In the case of the heat motor, the lowest temperature to which we can exhaust is ambient conditions, typically 300K, so there is a lot of residual energy in the exhaust. Only by exhausting to absolute zero, can we extract all the energy.

A model heat engine is usually represented by the following diagram. (Note that the word engine is usually preferred to motor).
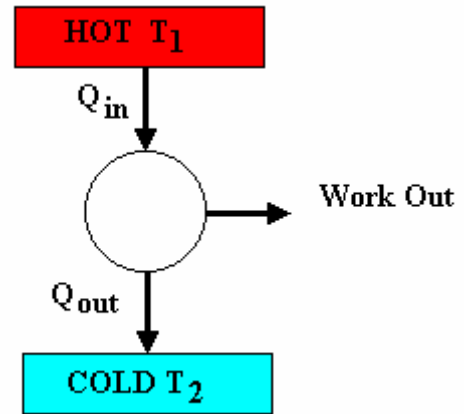

Fig. 2

The energy transfer from the hot source is Q_{in} Joules.

The energy transfer rate from the hot source is Φ_{in} Watts.

The energy transfer to the cold sink is Q_{out} Joules.

The energy transfer rate to the cold sink is Φ_{out} Watts.

The work output us W Joules.

The power output is P Watts.

By considering the total conservation of energy, it follows that the energy converted into work must be $W = Q_{in} - Q_{out}$ Joules or

$$P = \Phi_{in} - \Phi_{out} \quad \text{Watts}$$

The efficiency of any machine is the ratio Output/Input so the thermal efficiency of a heat engine may be developed as follows.

$$\eta_{th} = \frac{W}{Q_{in}} \qquad W = Q_{in} - Q_{out} \qquad \eta_{th} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{int}}$$

In terms of energy transfer rates in Watts this is written as

$$\eta_{th} = 1 - \frac{\Phi_{out}}{\Phi_{in}}$$

It follows from our analogy that $Q_{in} = ST_1$ and $Q_{out} = ST_2$ and confirms $\eta = 1 - \dfrac{T_2}{T_1}$

SELF ASSESSMENT EXERCISE No. 1

1. A heat engine is supplied with 60 MW of energy and produces 20 MW of power. What is the thermal efficiency and the heat lost?
    (Answers 33.3% and 40 MW)

2. A heat engine is supplied with 40 kJ of energy that it converts into work with 25% efficiency. What is the work output and the heat lost?
    (Answers 10 kJ and 30 kJ)

1.3.   PRACTICAL HEAT ENGINE CONSIDERATIONS

Let us consider how we might design a practical heat engine with a piston, connecting rod and crank shaft mechanism. Figure 3 shows how heat may be passed to a gas inside a cylinder causing it to expand. This pushes a piston and makes it do some work. This at first looks like a good way of converting heat into work but the problem is that it works only once and cannot convert heat into work continuously.
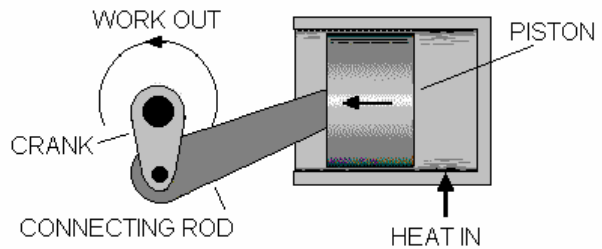

Figure 3

No practical heat engine has ever been invented that continuously converts heat directly into work as supposed in our ideal model. Practical heat engines use a working fluid such as gas or steam. A cycle of thermodynamic processes is conducted on the fluid with the end result being a conversion of heat into work.

First energy is given to the working fluid by use of a heat transfer at a hot temperature. Next we must convert as much of this energy as possible into work by allowing the fluid to expand. Our studies of polytropic expansions tell us that the pressure, volume and temperature all change as the gas or vapour gives up its energy as work. The pressure is vitally important to produce a motivating force on the piston.

Having extracted as much energy as possible from the working fluid, we must return it back to the starting condition in order to repeat the process. To do this, we must raise the pressure of the fluid back to the high level with some form of compression.

A simple reversal of the expansion process would return the fluid back to the original pressure and temperature. However, this would require us to give back all the work we got out so nothing is gained.

The only way we can return the fluid back to a high pressure with less work involves cooling it first. In fact, if it is to be heat engine, we must have a cooling process as indicated in our model.

We have deduced that a practical heat engine must meet the following criteria.

❑ It must produce work continuously.

❑ It must return the working fluid back to the same pressure and temperature at the beginning of every cycle.

A model of a practical engine is shown in Fig. 4. This indicates that we need four processes, heating, expansion, cooling and compression. This may be achieved practically using either closed system processes (as in a mechanism with a piston, connecting rod and crank shaft) or open system processes such as with a steam boiler, turbine, cooler and pump).
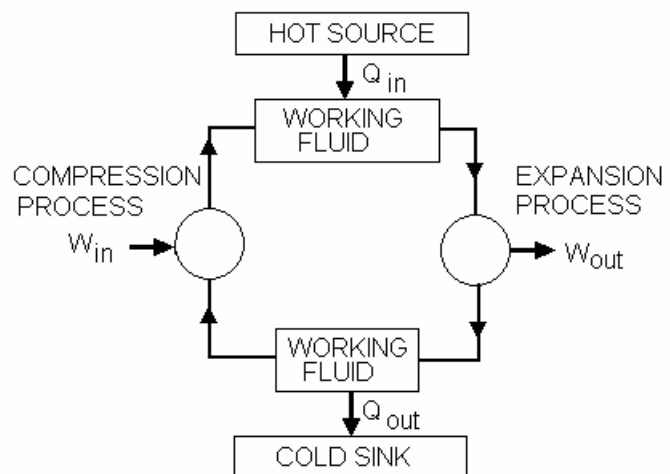

Figure 4

1.4   ENTROPY

We have just discovered that entropy is a property that governs the quantity of energy conveyed at a given temperature such that in our ideal heat engine, the energy is given by the expression Q = ST.

Entropy is a property that is closely associated with the second law of thermodynamics.

In thermodynamics there are two forms of energy transfer, work (W) and heat (Q). You should already be familiar with the theory of work laws in closed systems and know that the area under a pressure - volume diagram gives work transfer. By analogy there should be a property that can be plotted against temperature such that the area under the graph gives the heat transfer. This property is entropy and it is given the symbol S. This idea implies that entropy is a property that can be transported by a fluid. Consider a p-V and T-s graph for a reversible expansion (Fig. 5).
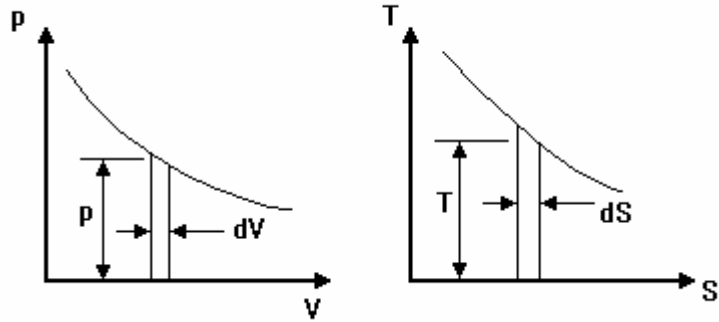

Fig.5

From the p-V graph we have $W = \int p \, dV$

From the T-S graph we have $Q = \int T \, ds$

This is the way entropy was developed for thermodynamics and from the above we get the following definition   **dS = dQ/T**
The units of entropy are hence J/K. Specific entropy has a symbol s and the units are J/kg K

It should be pointed out that there are other definitions of entropy but this one is the most meaningful for thermodynamics. A suitable integration will enable you to solve the entropy change for a fluid process. For those wishing to do studies in greater depth, these are shown in appendix A.

Entropy values for steam may be found in your thermodynamic tables in the columns headed s_f, s_{fg} and s_g.
    s_f is the specific entropy of saturated liquid.
    s_{fg} is the change in specific entropy during the latent stage.
    s_g is the specific entropy of dry saturated vapour.

1.4.1   ISENTROPIC PROCESSES

The word ISENTROPIC means constant entropy and this is a very important thermodynamic process. It occurs in particular when a process is reversible and adiabatic. This means that there is no heat transfer to or from the fluid and no internal heat generation due to friction. In such a process it follows that if dQ is zero then dS must be zero. Since there is no area under the T-S graph, the graph must be a vertical line as shown.
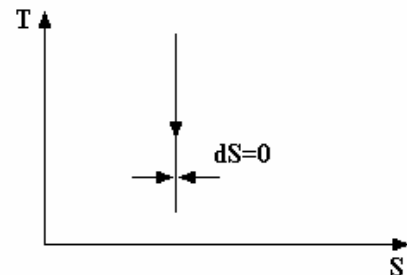
Fig. 6

There are other cases where the entropy is constant. For example, if there is friction in the process generating heat but this is lost through cooling, then the net result is zero heat transfer and constant entropy. You do not need to be concerned about this at this stage.

1.4.2   TEMPERATURE - ENTROPY (T-s) DIAGRAM FOR VAPOURS.

If you plot the specific entropy for saturated liquid (s_f) and for dry saturated vapour (s_g) against temperature, you would obtain the saturation curve. Lines of constant dryness fraction and constant pressure may be shown (Figure 7).
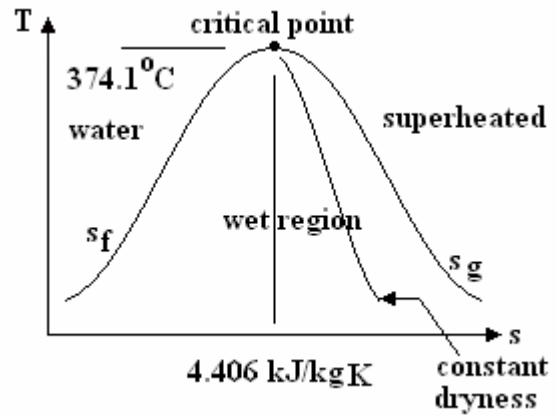
Fig. 7

1.4.3   SPECIFIC ENTHALPY-SPECIFIC ENTROPY (h-s) DIAGRAM.

This diagram is especially useful for steady flow processes (figure 8). The diagram is obtained by plotting h_g against s_g and h_f against s_f to obtain the characteristic saturation curve. The two curves meet at the critical point C. Lines of constant pressure, temperature and dryness are superimposed on the diagram. This is an extremely useful chart and it is available commercially. If any two coordinates are known, a point can be obtained on the chart and all other relevant values may be read off it. h –s charts are especially useful for solving isentropic processes because the process is a vertical line on this graph.

Fig. 8

Entropy values can be used to determine the dryness fraction following a steam expansion into the wet region when the process is isentropic. This is a very important point and you must master how to do this in order to solve steam expansion problems, especially in the following tutorials where steam cycles and refrigeration cycles are covered. The following examples show how this is done.
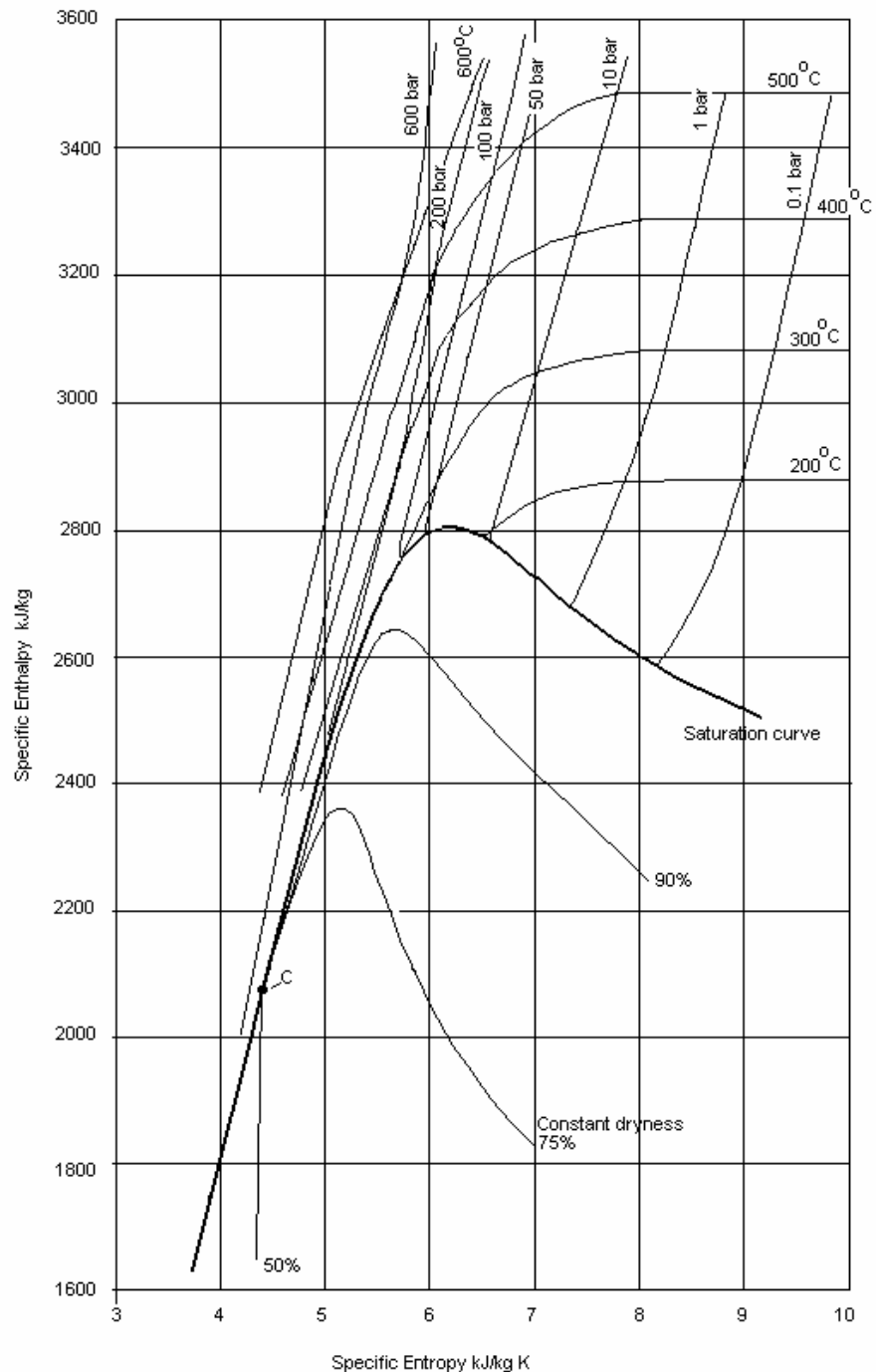

WORKED EXAMPLE No.1

Steam at 2 bar and 150ºC is expanded reversibly and adiabatically to 1 bar. Calculate the final dryness fraction and the enthalpy change.

SOLUTION

Let suffix (1) refer to the conditions before the expansion and (2) to the conditions after.

h_1 at 2 bar and 150ºC = 2770 kJ/kg s_1 at 2 bar and 150ºC is 7.280 kJ/kg K.

Because the process is adiabatic and reversible, the entropy remains the same.

s_2 at 1 bar and assumed wet is $s_f + xs_{fg} = s_1$ 7.280 = 1.303 + x(6.056)   x= 0.987

h_2 at 1 bar and 0.987 dry = $h_f + x\ h_{fg}$ $h_2 = 417 + 0.987(2258) = 2645.6$ kJ/kg

$\Delta h = 2645.6 - 2770 = -124.4$ kJ/kg

Being able to solve the changes in enthalpy enable us to apply the first law of thermodynamics to solve problems with steam turbines. The next example shows you how to do this.

WORKED EXAMPLE No.2

A steam turbine expands 60 kg/s from 40 bar and 300ºC to 4 bar reversibly and adiabatically (isentropic). Calculate the theoretical power output.

SOLUTION

$\Phi + P = \Delta E$ per second   (SFEE)
The process is adiabatic. $\Phi = 0$ and the only energy term to use is enthalpy.
$P = \Delta H$ per second.
h_1 at 40 bar and 300ºC = 2963 kJ/kg
s_1 at 40 bar and 300ºC is 6.364 kJ/kg K.
s_2 at 4 bar and assumed wet is $s_f + xs_{fg} = s_1$
6.364= 1.776 + x(5.121)
x= 0.896

h_2 at 4 bar and 0.896 dry = $h_f + x\ h_{fg}$ $h_2 = 605 + 0.896(2134) = 2517$ kJ/kg

$P = \Delta H$ per second = 60(2517-2963) = -26756 kW (out of system)

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2.   THE CARNOT PRINCIPLE

A man called Sadi Carnot deduced that if the heat transfers from the hot reservoir and to the cold sump were done at constant temperature (isothermal processes), then the efficiency of the engine would be the maximum possible.

The reasoning behind this is as follows. Consider heat being transferred from a hot body A to a slightly cooler body B. The temperature of body A falls and the temperature of body B rises until they are at the same temperature.

If body B is now raised in temperature by heat transfer from the surroundings, it becomes the hotter body and the heat flow is reversed from B to A. If body A returns to its original temperature then the net heat transfers between A and B is zero. However body B is now hotter than its original temperature so there has been a net heat transfer from the surroundings. The heat transfer process is hence not reversible as external help was needed to reverse the process.
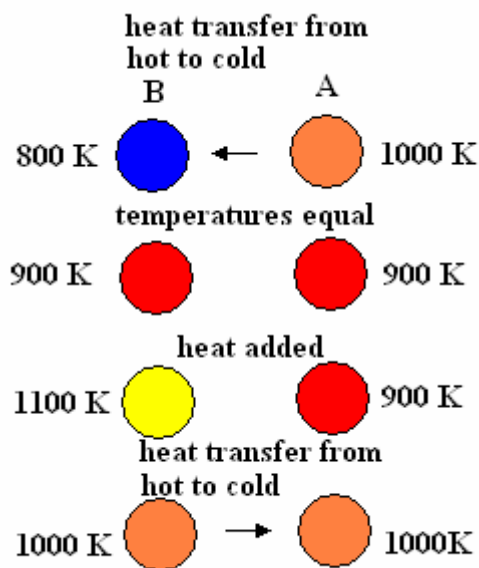
If it were possible to transfer heat with no temperature difference from A to B then it could be reversed with no external help. Such a process is an ISOTHERMAL process. Isothermal heat transfer is possible, for example evaporation of water in a boiler is isothermal.

Carnot devised a thermodynamic cycle using isothermal heat transfers only so by definition, the efficiency of this cycle is the most efficient any engine could be operating between two temperatures. Engine cycles are covered in the next tutorial but the following shows how the Carnot cycle might be conducted. In practice, it is not possible to make this cycle work.

2.1.1   CLOSED SYSTEM CARNOT CYCLE.

The cycle could be conducted on gas or vapour in a closed or open cycle. The cycle described here is for gas in a cylinder fitted with a piston. It consists of four closed system processes as follows.

1 to 2.   The fluid is compressed isentropically. Work is put in and no heat transfer occurs.

Fig. 10
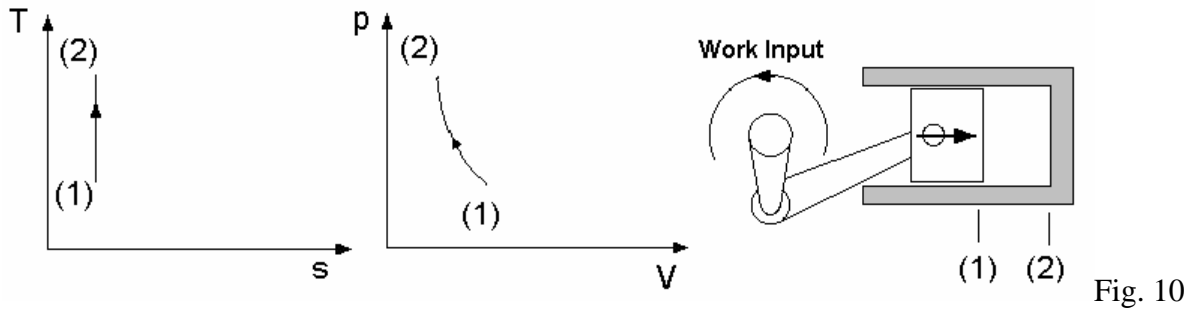
2 to 3.   The fluid is heated isothermally. This could only occur if it is heated as it expands so there is work taken out and heat put in.

Fig. 11
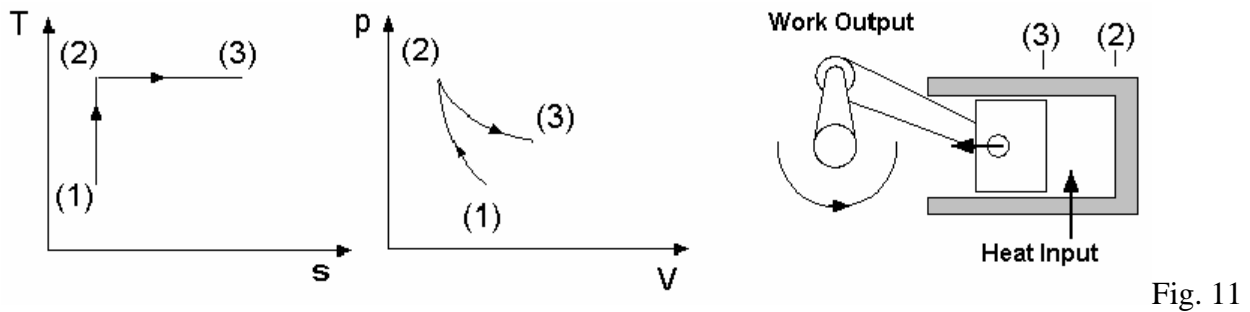
3 to 4.   The fluid continues to expand isentropically with no heat transfer. Work output is obtained.

Fig.12
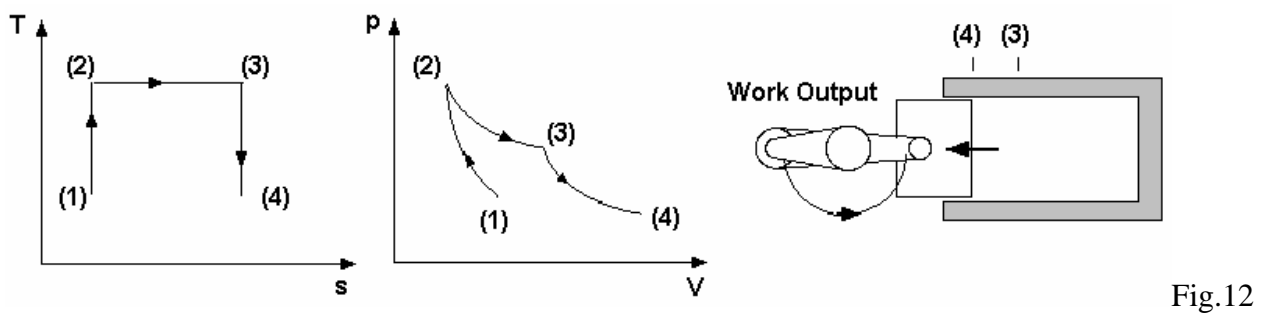
4 to 1   The fluid is cooled isothermally. This can only occur if it cooled as it is compressed, so work is put in and heat is taken out. At the end of this process every thing is returned to the initial condition.
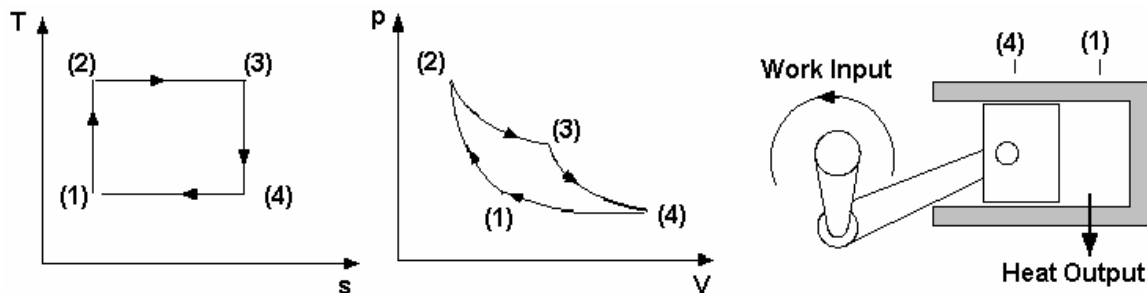
Fig.13

The total work taken out is W_{out} and the total work put in is W_{in}.

To be an engine, W_{out} must be larger than W_{in} and a net amount of work is obtained from the cycle. It also follows that since the area under a p-V graph represents the work done, then the area enclosed by the p-V diagram represents the net work transfer. It also follows that since the area under the T-s graph is represents the heat transfer, and then the area enclosed on the T-s diagram represents the net heat transfer. This is true for all cycles and also for real engines.
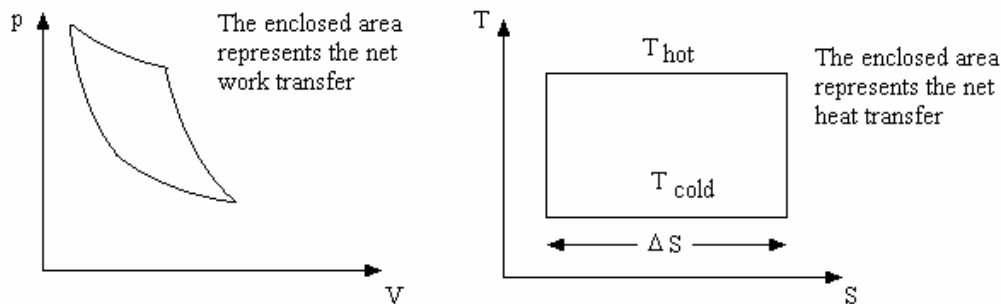
Fig.14

Applying the first law, it follows $\mathbf{Q_{nett} = W_{nett}}$

For isothermal heat transfers $Q = \int T\,ds = T\Delta S$ since T is constant.

The efficiency would then be given by $\eta_{th} = 1 - \dfrac{Q_{out}}{Q_{in}} = 1 - \dfrac{T_{cold}\Delta s_{cold}}{T_{hot}\Delta s_{hot}}$

It is apparent from the T-s diagram that the change in entropy Δs is the same at the hot and cold temperatures. It follows that $\eta_{th} = 1 - \dfrac{T_{cold}}{T_{hot}}$

This expression, which is the same as that used for the ideal model, gives the CARNOT EFFICIENCY and it is used as a target figure that cannot be surpassed (in fact not even attained).

WORKED EXAMPLE No.3

A heat engine draws heat from a combustion chamber at 300°C and exhausts to atmosphere at 10°C. What is the maximum possible thermal efficiency that could be achieved?

SOLUTION
The maximum efficiency possible is the Carnot efficiency. Remember to use absolute temperatures.

$\eta_{th} = 1 - \dfrac{T_{cold}}{T_{hot}} = 1 - \dfrac{273 + 10}{273 + 300} = 1 - \dfrac{283}{573} = 0.505$ or 50.6%

SELF ASSESSMENT EXERCISE No.3

1. A heat engine works between temperatures of 1100° C and 120°C. It is claimed that it has a thermal efficiency of 75%. Is this possible?
   (Answer the maximum efficiency cannot exceed 71%)

2. Calculate the efficiency of a Carnot Engine working between temperatures of 1200°C and 200° C.
   (Answer 67.9%)

ENGINEERING COUNCIL

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THERMODYNAMIC, FLUID AND PROCESS ENGINEERING C106

TUTORIAL 5 – IDEAL ENGINE CYCLES

When you have completed this tutorial you should be able to do the following.

❑ Define AIR STANDARD CYCLES.

❑ Identify the ideal cycle for a given type of engine.

❑ Explain and solve problems for the OTTO cycle

❑ Explain and solve problems for the DIESEL cycle

❑ Explain and solve problems for the Dual Combustion cycle

❑ Explain and solve problems for the JOULE cycle

1.   THEORETICAL CYCLES FOR ENGINES

Internal combustion engines fall into two groups, those that use a sparking plug to ignite the fuel (spark ignition engines) and those that use the natural temperature of the compressed air to ignite the fuel (compression ignition engines).

Another way to group engines is into those that use non-flow processes and those that use flow processes. For example, non-flow processes are used in piston engines. Flow processes are used in gas turbine engines.

Theoretical cycles are made up of ideal thermodynamic processes to resemble those that occur in a real engine as closely as possible. Many of these cycles are based on air as the working fluid and are called **AIR STANDARD CYCLES**. Before looking at air standard cycles, we should briefly revise the Carnot Cycle from tutorial 3.

1.   THE CARNOT CYCLE

The most efficient way of transferring heat into or out of a fluid is at constant temperature. All the heat transfer in the Carnot cycle is at constant temperature so it follows that the Carnot cycle is the most efficient cycle possible. The heat transfer into the cycle occurs at a hot temperature T_{hot} and the heat transfer out of the cycle occurs at a colder temperature T_{cold}. The thermodynamic efficiency was sown to be given as follows.

$$\eta_{th} = 1 - \frac{T_{cold}}{T_{hot}}$$

None of the following cycles can have an efficiency greater than this when operating between the same temperatures limits.

2   SPARK IGNITION ENGINE

2.1   THE OTTO CYCLE

The ideal cycle is named after Count N.A.Otto. It represents the ideal cycle for a spark ignition engine. In an ideal spark ignition engine, there are four processes as follows.
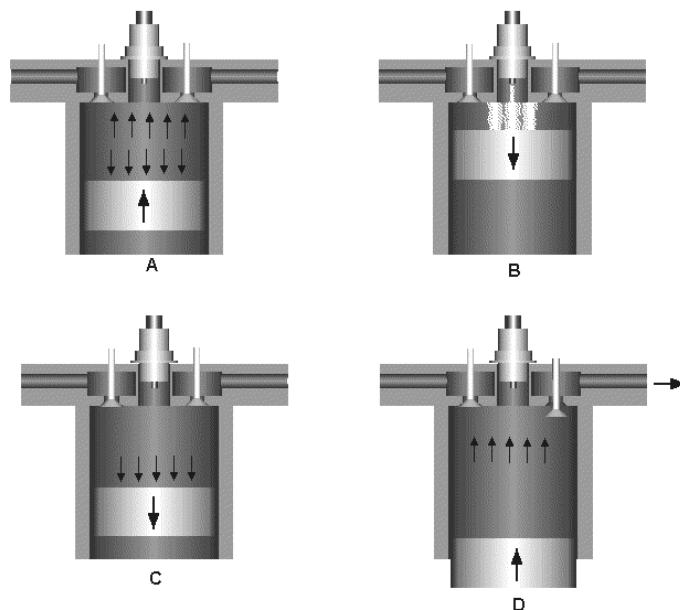
Fig.1

COMPRESSION STROKE

Air and fuel are mixed and compressed so rapidly that there is no time for heat to be lost. (Figure A) In other words the compression is adiabatic. Work must be done to compress the gas.

IGNITION

Just before the point of maximum compression, the air is hot and a spark ignites the mixture causing an explosion (Figure B). This produces a rapid rise in the pressure and temperature. The process is idealised as a constant volume process in the Otto cycle.

EXPANSION OR WORKING STROKE

The explosion is followed by an adiabatic expansion pushing the piston and giving out work. (Figure C)

EXHAUST

At the end of the working stroke, there is still some pressure in the cylinder. This is released suddenly by the opening of an exhaust valve. (Figure D) This is idealised by a constant volume drop in pressure in the Otto cycle. In 4 stroke engines a second cycle is performed to push out the products of combustion and draw in fresh air and fuel. It is only the power cycle that we are concerned with.

The four ideal processes that make up the Otto cycle are as follows.

1 to 2   The air is compressed reversibly and adiabatically. Work is put in and no heat transfer occurs.
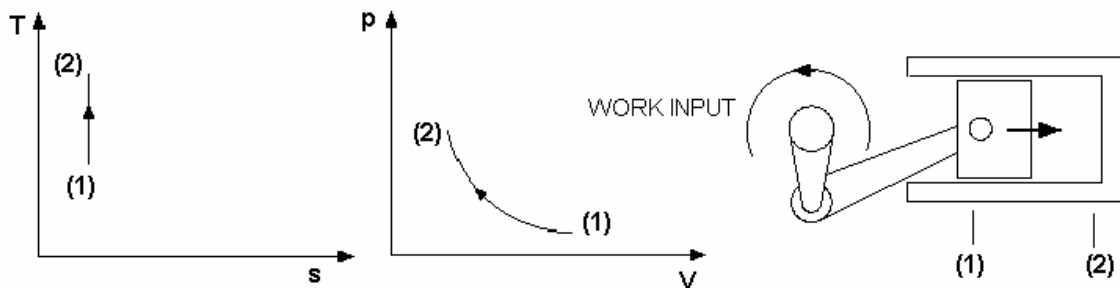
Fig.2

2 to 3   The air is heated at constant volume. No work is done. $Q_{in} = mc_V(T_3-T_2)$
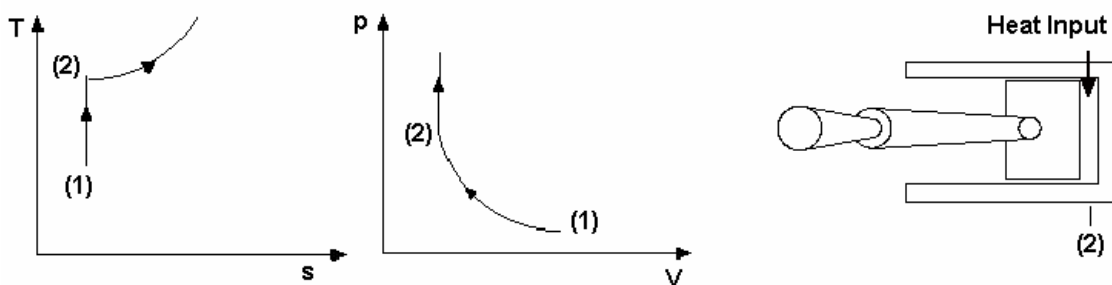
Fig.3

3 to 4   The air expands reversibly and adiabatically with no heat transfer back to its original volume. Work output is obtained.
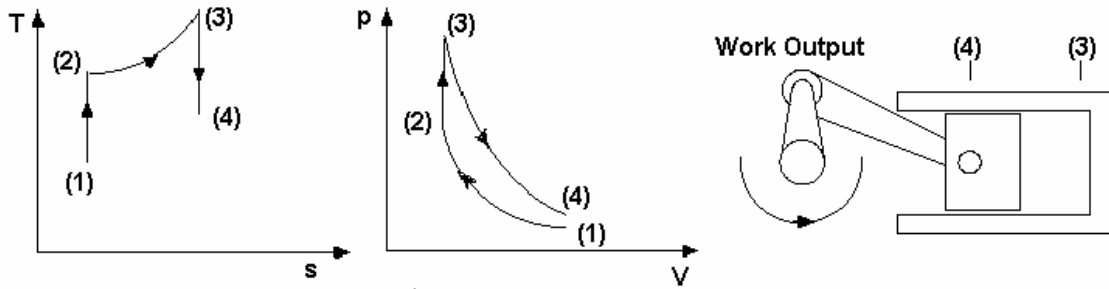
Fig.4

4 to 1   The air is cooled at constant volume back to its original pressure and temperature. No work is done $Q_{out} = mc_v(T_4 - T_1)$
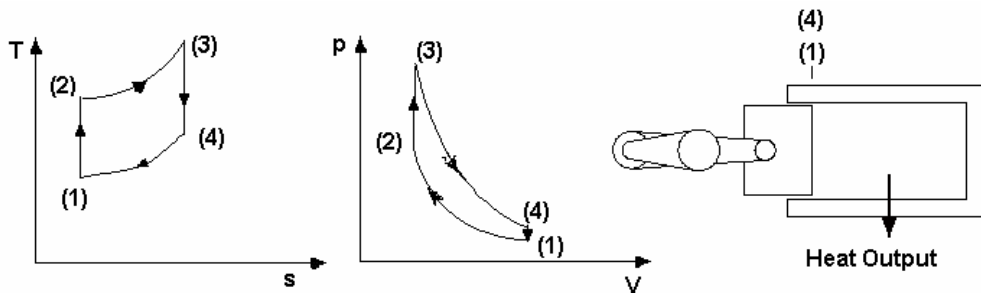
Fig.5

The total heat transfer into the system during one cycle is $Q_{nett} = Q_{in} - Q_{out}$

The total work output per cycle is W_{nett}

From the 1st. Law of thermodynamics $Q_{nett} = W_{nett}$

EFFICIENCY

$$\eta = \frac{W_{nett}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_4 - T_1)}{mc_v(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

For the process (1) to (2) we may use the rule $\dfrac{T_2}{T_1} = \left(\dfrac{V_1}{V_2}\right)^{\gamma-1} = r_v^{\,\gamma-1}$

For the process (3) to (4) we may similarly write $\dfrac{T_3}{T_4} = \left(\dfrac{V_4}{V_3}\right)^{\gamma-1} = r_v^{\,\gamma-1}$ where r_v is the

volume compression ratio $r_v = \dfrac{V_1}{V_2} = \dfrac{V_4}{V_3}$

It follows that $\dfrac{T_2}{T_1} = \dfrac{T_3}{T_4}$ and $\dfrac{T_4}{T_1} = \dfrac{T_3}{T_2}$

and that $\eta = 1 - \dfrac{T_4 - T_1}{T_3 - T_2} = 1 - \dfrac{\dfrac{T_3 T_1}{T_2} - T_1}{\dfrac{T_2 T_4}{T_1} - T_2} = 1 - \dfrac{T_1\left(\dfrac{T_3}{T_2} - 1\right)}{T_2\left(\dfrac{T_4}{T_1} - 1\right)}$

$$\frac{T_4}{T_1} = \frac{T_3}{T_2} \text{ then } \frac{T_4}{T_1} - 1 = \frac{T_3}{T_2} - 1 \qquad \eta = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} = 1 - \frac{1}{r_v^{\gamma-1}} = 1 - r_v^{1-\gamma}$$

Since this theoretical cycle is carried out on air for which $\gamma = 1.4$ then the efficiency of an Otto Cycle is given by $\eta = 1 - r_v^{0.4}$

This shows that the thermal efficiency depends only on the compression ratio. If the compression ratio is increased, the efficiency is improved. This in turn increases the temperature ratios between the two isentropic processes and explains why the efficiency is improved.

WORKED EXAMPLE No.1

An Otto cycle is conducted as follows. Air at 100 kPa and 20°C is compressed reversibly and adiabatically. The air is then heated at constant volume to 1500°C. The air then expands reversibly and adiabatically back to the original volume and is cooled at constant volume back to the original pressure and temperature. The volume compression ratio is 8. Calculate the following.

        i. The thermal efficiency.
        ii. The heat input per kg of air.
        iii. The net work output per kg of air.
        iv. The maximum cycle pressure. $c_V = 718$ kJ/kg $\gamma = 1.4$.   R = 287 J/kg K

SOLUTION

Remember to use absolute temperatures throughout. Solve for a mass of 1 kg.

$T_1 = 20 + 273 = 293K$ $T_3 = 1500 + 273 = 1773K$ $r_v = 8$

$\eta = 1 - r^{1-\gamma} = 1 - 8^{00.} = 0.565$ or 56.5%

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 293\left(8^{0.4}\right) = 673.1\,K$$

$$Q_{in} = mc_v(T_3 - T_2) = 1 \times 718(1773 - 673.1) = 789700\,J/kg = 789.7\,kJ/kg$$

$$W_{nett} = \eta Q_{in} = 0.56 \times 789.7 = 446.2\,kJ/kg$$

From the gas law we have

$$p_3 = \frac{p_1 V_1 T_3}{T_1 V_3} = \frac{100000 \times V_1 \times 1773}{293 \times V_3} \qquad \frac{V_1}{V_3} = 8 \qquad p_3 = \frac{100000 \times 1773}{293} \times 8 = 4.84\,MPa$$

If you have followed the principles used here you should be able to solve any cycle.

SELF ASSESSMENT EXERCISE No.1

Take Cv = 0.718 kJ/kg K, R = 287 J/kg K and γ= 1.4 throughout.

1. An Otto cycle has a volume compression ratio of 9/1. The heat input is 500kJ/kg. At the start of compression the pressure and temperature are 100 kPa and 40ºC respectively. Calculate the following.

    i. The thermal efficiency. (58.5%)
    ii. The maximum cycle temperature. (1450 K).
    iii. The maximum pressure. (4.17 MPa).
    iv. The net work output per kg of air. (293 kJ/kg).

2. Calculate the volume compression ratio required of an Otto cycle which will produce an efficiency of 60%. (9.88/1)

    The pressure and temperature before compression are 105 kPa and 25ºC respectively. The net work output is 500 kJ/kg). Calculate the following.

    i. The heat input. (833 kJ/kg).
    ii. The maximum temperature. (1 906 K)
    iii. The maximum pressure. (6.64 MPa).

3. An Otto cycle uses a volume compression ratio of 9.5/1. The pressure and temperature before compression are 100 kPa and 40ºC respectively. The mass of air used is 11.5 grams/cycle. The heat input is 600 kJ/kg. The cycle is performed 3 000 times per minute. Determine the following.

    i. The thermal efficiency. (59.4%).
    ii. The net work output. (4.1 kJ/cycle)
    iii. The net power output. (205 kW).

4. An Otto cycle with a volume compression ratio of 9 is required to produce a net work output of 450 kJ/cycle. Calculate the mass of air to be used if the maximum and minimum temperatures in the cycle are 1300ºC and 20ºC respectively.
    (1.235 kg).

5. The working of a petrol engine can be approximated to an Otto cycle with a compression ratio of 8 using air at 1 bar and 288 K with heat addition of 2 MJ/kg. Calculate the heat rejected and the work done per kg of air.
    (871 kJ/kg and 1129 kJ/kg).

Now let's move on to study engines with compression ignition.

3   <u>**COMPRESSION IGNITION ENGINES**</u>

The invention of compression ignition engines, commonly known as diesel engines, was credited to Rudolf Diesel, although many other people worked on similar engines. The basic principle is that when high compression ratios are used, the air becomes hot enough to make the fuel detonate without a spark. Diesel's first engine used coal dust blasted into the combustion chamber with compressed air. This developed into blasting in oil with compressed air. In modern engines the fuel oil is injected directly into the cylinder as fine droplets. There are two ideal cycles for these engines, the Diesel Cycle and the Dual Combustion Cycle.

3.1   <u>**DUAL COMBUSTION CYCLE**</u>

This is the air standard cycle for a modern fast running diesel engine. First the air is compressed isentropically making it hot. Fuel injection starts before the point of maximum compression. After a short delay in which fuel accumulates in the cylinder, the fuel warms up to the air temperature and detonates causing a sudden rise in pressure. This is ideally a constant volume heating process. Further injection keeps the fuel burning as the volume increases and produces a constant pressure heating process. After cut off, the hot air expands isentropically and then at the end of the stroke, the exhaust valve opens producing a sudden drop in pressure. This is ideally a constant volume cooling process. The ideal cycle is shown in figure 6.
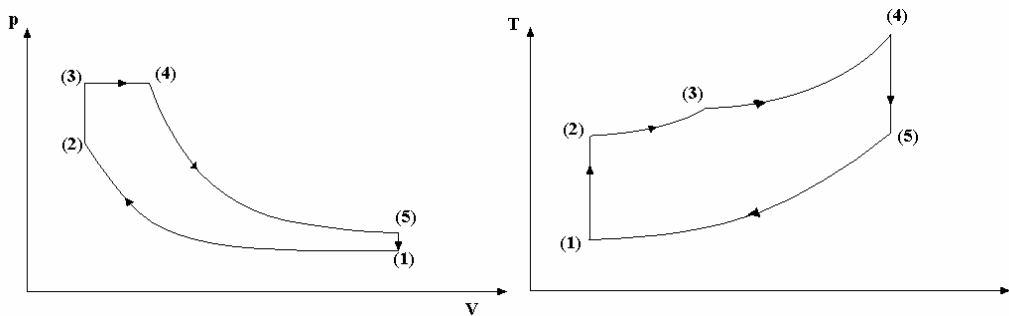
Fig. 6

The processes are as follows.
1 - 2   reversible adiabatic (isentropic) compression.
2 - 3   constant volume heating.
3 - 4   constant pressure heating.
4 - 5   reversible adiabatic (isentropic) expansion.
5 - 1   constant volume cooling.

The analysis of the cycle is as follows.
The heat is supplied in two stages hence $Q_{in} = mC_p(T_4 - T_3) + mC_v(T_3 - T_2)$
The heat rejected is $Q_{out} = mC_v(T_5 - T_1)$
The thermal efficiency may be found as follows.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_5 - T_1)}{mc_v(T_3 - T_2) + mc_p(T_4 - T_3)} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

The formula can be further developed to show that

$$\eta = 1 - \frac{k\beta^\gamma - 1}{[(k-1) + \gamma k(\beta - 1)]r_v^{\gamma - 1}}$$

r_v is the VOLUME COMPRESSION RATIO. $r_v = V_1/V_2$
β is the CUT OFF RATIO. $\beta = V_4/V_3$ k is the ratio p_3/p_2.
Most students will find this adequate to solve problems concerning the dual combustion cycle.

Generally, the method of solution involves finding all the temperatures by application of the gas laws.

Those requiring a detailed analysis of the cycle should study the following derivation.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)} \qquad \text{Obtain all the temperatures in terms of } T_2$$

Isentropic compression 1 to 2 $\qquad\qquad T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{T_2}{r_v^{\gamma-1}}$

Constant volume heating 2 to 3 note $V_3 = V_2$ $\qquad T_3 = \frac{p_3 V_3 T_2}{p_2 V_2} = \frac{p_3 T_2}{p_2} = kT_2$

Constant pressure heating 3 to 4 note $p_3 = p_4$ $\qquad T_4 = \frac{p_4 V_4 T_3}{p_3 V_3} = \frac{V_4 T_3}{V_3} = \beta T_3 = \beta k T_2$

Isentropic expansion 4 to 5 $\qquad\qquad T_5 = T_4\left(\frac{V_4}{V_5}\right)^{\gamma-1} = T_4\left(\frac{V_4 V_2}{V_5 V_2}\right)^{\gamma-1} = T_4\left(\frac{\beta}{r_v}\right)^{\gamma-1} = \frac{k\beta^{\gamma} T_2}{r_r^{\gamma-1}}$

Substitute for all temperatures in the efficiency formula.

$$\eta = 1 - \frac{\dfrac{k\beta^{\gamma} T_2}{r_r^{\gamma-1}} - \dfrac{T_2}{r_v^{\gamma-1}}}{\left(kT_2 - T_2\right) + \gamma\left(\beta k T_2 - kT_2\right)} = 1 - \frac{\dfrac{k\beta^{\gamma}}{r_r^{\gamma-1}} - \dfrac{1}{r_v^{\gamma-1}}}{(k-1) + \gamma(\beta k - k)} \qquad\qquad \eta = 1 - \frac{k\beta^{\gamma} - 1}{[(k-1) + \gamma k(\beta-1)]r_v^{\gamma-1}}$$

Note that if $\beta=1$, the cycle becomes an Otto cycle and the efficiency formulae becomes the same as for an Otto cycle.

WORKED EXAMPLE No. 2

In a dual combustion cycle, the compression starts from 1 bar and 20°C. The compression ratio is 18/1 and the cut off ratio is 1.15. The maximum cycle pressure is 1360 K. The total heat input is 1 kJ per cycle. Calculate the following.

i.　　The thermal efficiency of the cycle.
ii.　　The net work output per cycle.
Check that the efficiency does not contravene the Carnot principle.

SOLUTION
$T_1 = 20 + 273 = 293$ K The hottest temperature is $T_4 = 1360$K. $\qquad \beta=1.15 \quad r_v = 18 \quad \gamma=1.4$

$$T_2 = T_1 r_v^{\gamma-1} = 293 \times 18^{0.4} = 931\,\text{K} \qquad\qquad T_3 = \frac{V_3 T_4}{V_4} = \frac{T_4}{\beta} = \frac{1360}{1.15} = 1183\,\text{K}$$

$$\frac{p_3}{p_2} = k = \frac{T_3}{T_2} = 1.27$$

$$\eta = 1 - \frac{k\beta^{\gamma} - 1}{[(k-1) + \gamma k(\beta-1)]r_v^{\gamma-1}} = 1 - \frac{1.27 \times 1.15^{1.4} - 1}{[(1.27-1) + (1.4 \times 1.27 \times (1.15-1))] \times 18^{0.4}}$$

$$\eta = 0.68 \text{ or } 68\%$$

$W_{nett} = \eta \times Q_{in} = 0.68 \times 1 = 0.68$ kJ per cycle.
The Carnot efficiency should be higher.
$$\eta = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{293}{1360} = 0.785$$

The figure of 0.68 is lower so the Carnot principle has not been contravened.

WORKED EXAMPLE No.3

A dual combustion cycle has a compression ratio of 18/1. The maximum pressure in the cycle is 9 MPa and the maximum temperature is 2000°C. The pressure and temperature before compression is 115 kPa and 25°C respectively. Calculate the following.

  i. the cut off ratio.
  ii. the cycle efficiency.
  iii. the net work output per kg of air.

Assume $\gamma = 1.4$ $C_p = 1.005$ kJ/kgK $C_V = 0.718$ kJ/kg K.

SOLUTION

Known data.

$T_1 = 298$ K $T_4 = 2273$ K $p_3 = p4 = 9$ MPa $p_1 = 115$ kPa

$V_1/V_2 = V_1/V_3 = 18$ $V_2 = V_3$

$T_2 = 298 \times 18^{(\gamma-1)} = 947$ K

$$T_3 = \frac{p_3 T_1 V_3}{p_1 V_1} = \frac{9 \times 10^6 \times 298}{115 \times 10^3} \times \frac{V_3}{V_1} = \frac{9 \times 10^6 \times 298}{115 \times 10^3} \times \frac{1}{18} = 1296K$$

$$\text{Cut off ratio} = \beta = \frac{V_4}{V_3} = \frac{p_3 T_4}{p_4 T_3} \quad \text{but} \quad p_4 = p_3 \text{ so } \beta = \frac{T_4}{T_3}$$

$$\beta = \frac{2273}{1296} = 1.75$$

$$T_5 = T_4 \left(\frac{V_4}{V_5} \right)^{\gamma-1} \quad \text{but} \quad \frac{V_4}{V_5} = \frac{V_4}{V_3} \times \frac{V_3}{V_5} = \frac{1.75}{18} = 0.0974$$

$$T_5 = 2273 \times 0.0974^{0.4} = 895.6 \text{ K}$$

$Q_{in} = mC_p(T4 - T3) + mC_V (T3 - T2)$ m= 1 kg

$Q_{in} = 1.005(2274 - 1296) + 0.718(1296-947) = 1232.5$ kJ/kg

$Q_{out} = mC_V (T_5 - T_1)$

$Q_{out} = 0.718(895.6 - 298) = 429$ kJ/g

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{429}{1232} = 0.65 \text{ or } 65\%$$

$W_{nett} = Q_{in} \quad Q_{out} = 1232 \quad 428.6 = 803.5 \text{ kJ/kg}$

3.2   THE DIESEL CYCLE

The Diesel Cycle precedes the dual combustion cycle. The Diesel cycle is a reasonable approximation of what happens in slow running engines such as large marine diesels. The initial accumulation of fuel and sharp detonation does not occur and the heat input is idealised as a constant pressure process only. Again consider this cycle as being carried out inside a cylinder fitted with a piston. The p-V and T-s cycles diagrams are shown in figure 7
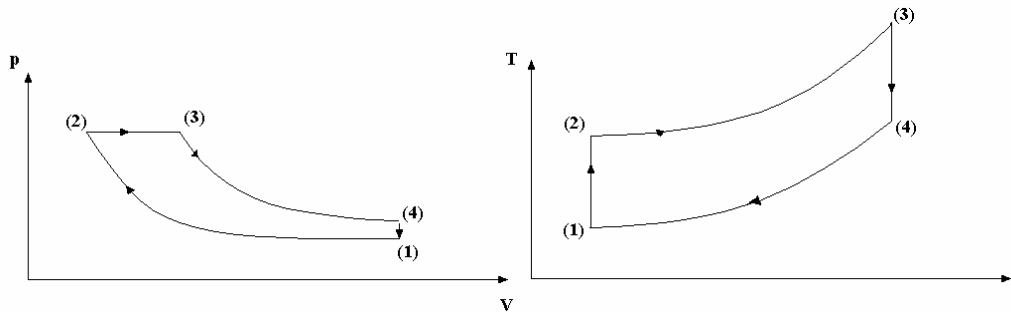
Fig. 7

1 - 2   reversible adiabatic (isentropic) compression.
2 - 3   constant pressure heating.
3 - 4   reversible adiabatic (isentropic) expansion.
4 - 1   constant volume cooling.

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)}$$

The cycle is the same as the dual combustion cycle without the constant volume heating process. In this case since k=1 the efficiency is given by the following formula.

$$\eta = 1 - \frac{\beta^\gamma - 1}{(\beta - 1)\gamma r_v^{\gamma-1}}$$

WORKED EXAMPLE No.4

An engine using the Diesel Cycle has a compression ratio of 20/1 and a cut off ratio of 2. At the start of the compression stroke the air is at 1 bar and 15°C. Calculate the following.

i.   The air standard efficiency of the cycle.
ii.   The maximum temperature in the cycle.
iii.   The heat input.
iv.   The net work output.

SOLUTION

$\beta = 2$ $r_v = 20$ $\gamma = 1.4$ $c_v = 718$ J/kg K for air $T_1 = 288$ K $p_1 = 1$ bar.
The maximum temperature is T_3 and the maximum pressure is p_3 and p_2.

$$\eta = 1 - \frac{\beta^\gamma - 1}{(\beta - 1)\gamma r_v^{\gamma-1}} = 1 - \frac{2^{1.4} - 1}{(2-1) \text{ x } 1.4 \text{ x } 20^{0.4}} = 1 - \frac{1.639}{1 \text{ x } 1.4 \text{ x } 3.314} = 0.647 \text{ or } 64.7\%$$

$$T_2 = T_1 r_v^{\gamma-1} = 288 \text{ x } 20^{0.4} = 954.5 \text{ K} \qquad T_3 = \frac{V_2}{V_3} T_2 = \beta T_2 = 954.3 \text{ x } 2 = 1909 \text{ K}$$

$$Q_{in} = mc_p(T_3 - T_2) = 1.005(1909 - 954.5) = 959.3 \text{ kJ}$$

$$\eta = \frac{W_{nett}}{Q_{in}} \qquad W_{nett} = \eta Q_{in} = 0.647 \text{ x } 959.3 = 620.6 \text{ kJ}$$

SELF ASSESSMENT EXERCISE No.2

Use $c_v = 0.718$ kJ/kg K , $c_p = 1.005$ kJ/kg K and $\gamma = 1.4$ throughout.

1. Draw a p - V and T - s diagram for a Diesel Cycle.

    The performance of a compression ignition engine is to be compared to the Diesel cycle. The compression ratio is 16. The pressure and temperature at the beginning of compression are 1 bar and 15°C respectively. The maximum temperature in the cycle is 1200 K.

    Calculate the following.

    i. The cut off ratio.(1.374)

    ii. The air standard efficiency. (66%)

2. A Dual Combustion Cycle uses a compression ratio of 12/1. The cut off ratio is 2/1. The temperature and pressure before compression is 280 K and 1 bar respectively. The maximum temperature 2000 K. Calculate the following.

    i. The net work output per cycle.   (680 kJ/kg).
    ii. The thermal efficiency.   (57.6 %).

3. A Dual Combustion Cycle uses a compression ratio of 20/1. The cut off ratio is 1.6/1. The temperature and pressure before compression is 30°C and 1 bar respectively. The maximum cycle pressure is 100 bar. Calculate the following.

    i. The maximum cycle temperature.   (2424 K).
    ii. The net work output per cycle.   (864 kJ/kg).
    iii. The thermal efficiency.   (67.5 %).

4   GAS TURBINES

A gas turbine engine normally burns fuel in the air that it uses as the working fluid. From this point of view it is an internal combustion engine that uses steady flow processes. Figure 8 shows a basic design.
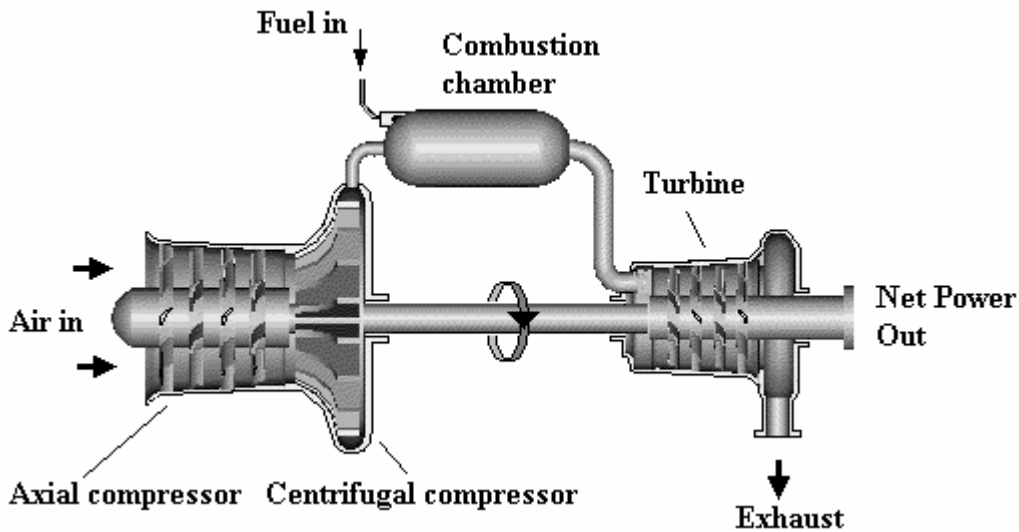
Fig.8

The air is drawn in from atmosphere and compressed. This makes it hotter. The compressed air is blown into a combustion chamber and fuel is burned in it making it even hotter. This makes the volume increase. The hot air expands out of the chamber through a turbine forcing it to revolve and produce power. The air becomes colder as it expands and eventually exhausts to atmosphere. The temperature drop over the turbine is larger than the temperature increase over the compressor. The turbine produces more power than is needed to drive the compressor. Net power output is the result. In the basic system, the turbine is coupled directly to the compressor and the power output is taken from the same shaft. The ideal air standard cycle is the Joule Cycle.

4.1   THE JOULE CYCLE

The Joule Cycle is also known as the constant pressure cycle because the heating and cooling processes are conducted at constant pressure. The cycle is that used by a gas turbine engine but could conceivably be used in a closed system.

We may draw the layout in block diagram form as shown in figure 9
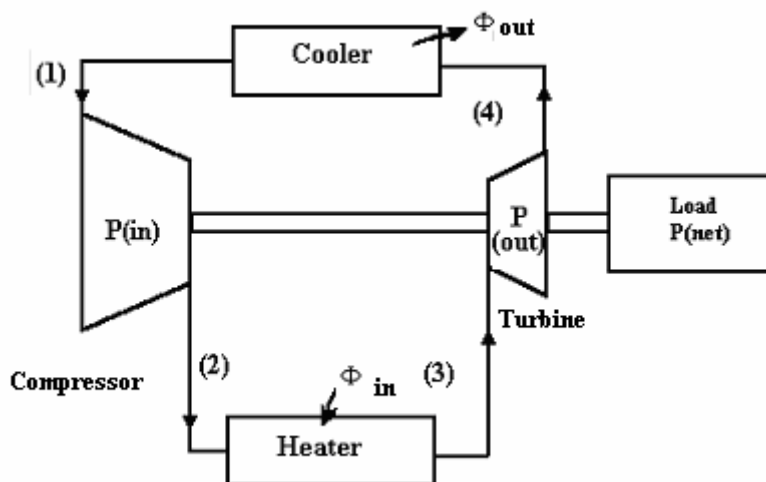
Figure 9

There are 4 ideal processes in the cycle.

1 - 2   Reversible adiabatic (isentropic) compression requiring power input.

$P_{in} = \Delta H/s = mC_p(T_2 - T_1)$

2 - 3   Constant pressure heating requiring heat input.

$\Phi_{in} = \Delta H/s = mC_p(T_3 - T_2)$

3 - 4   Reversible adiabatic (isentropic) expansion producing power output.

$P_{out} = \Delta H/s = mC_p(T_3 - T_4)$

4 - 1   Constant pressure cooling back to the original state requiring heat removal.

$\Phi_{out} = \Delta H/s = mC_p(T_4 - T_1)$

The pressure – volume, pressure - enthalpy and temperature-entropy diagrams are shown in figure 10
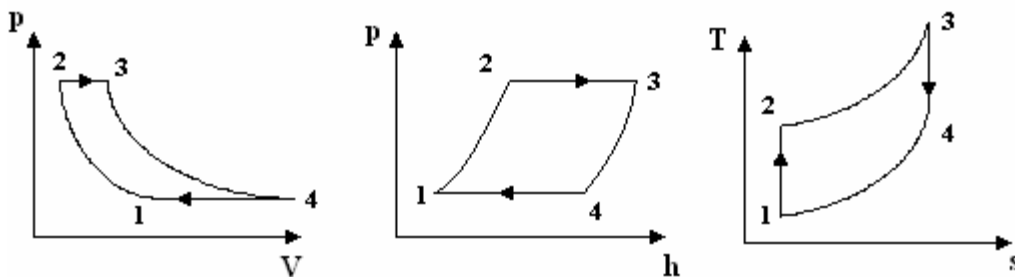
Fig. 10

The efficiency is found by applying the first law of thermodynamics.

$$\Phi_{nett} = P_{nett}$$

$$\Phi_{in} - \Phi_{out} = P_{out} - P_{in}$$

$$\eta_{th} = \frac{P_{nett}}{\Phi_{in}} = 1 - \frac{\Phi_{out}}{\Phi_{in}} = 1 - \frac{mc_p(T_4 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

It assumed that the mass and the specific heats are the same for the heater and cooler.

It is easy to show that the temperature ratio for the turbine and compressor are the same.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{\gamma}} = r_p^{1-\frac{1}{\gamma}} \qquad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{1-\frac{1}{\gamma}} = r_p^{1-\frac{1}{\gamma}} \qquad \frac{T_3}{T_4} = \frac{T_2}{T_1}$$

r_p is the pressure compression ratio for the turbine and compressor.

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{\left(\frac{T_3 T_1}{T_2} - T_1\right)}{\left(\frac{T_2 T_4}{T_1} - T_2\right)} = 1 - \frac{T_1\left(\frac{T_3}{T_2} - 1\right)}{T_2\left(\frac{T_4}{T_1} - 1\right)}$$

$$\frac{T_3}{T_2} = \frac{T_4}{T_1} \qquad \frac{T_3}{T_2} - 1 = \frac{T_4}{T_1} - 1 \qquad \eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3} = 1 - \frac{1}{r_p^{1-\frac{1}{\gamma}}} = 1 - r_p^{-0.286} \quad \text{since } \gamma = 1.4$$

This shows that the efficiency depends only on the pressure ratio which in turn affects the hottest temperature in the cycle.

A gas turbine uses the Joule cycle. The pressure ratio is 6/1. The inlet temperature to the compressor is 10ºC. The flow rate of air is 0.2 kg/s. The temperature at inlet to the turbine is 950ºC. Calculate the following.

i. The cycle efficiency.
ii. The heat transfer into the heater.
iii. The net power output. $\gamma = 1.4$ $C_p = 1.005$ kJ/kg K

SOLUTION

$$\eta_{th} = 1 - r_p^{-0.286} = 1 - 6^{-0.286} = 0.4 \text{ or } 40\%$$

$$T_2 = T_1 r_p^{0.286} = 283 \text{ x } 6^{0.286} = 472.4 \text{K}$$

$$\Phi_{in} = mc_p(T_3 - T_2) = 0.2 \text{ x } 1.005 \text{ x } (1223 - 472.4) = 150.8 \text{ kW}$$

$$\eta_{th} = \frac{P_{nett}}{\Phi_{in}} \qquad P_{nett} = 0.4 \text{ x } 150.8 = 60.3 \text{ kW}$$

SELF ASSESSMENT EXERCISE No. 3
$\gamma = 1.4$ and $C_p = 1.005$ kJ/kg K throughout.

1. A gas turbine uses the Joule cycle. The inlet pressure and temperature to the compressor are respectively 1 bar and -10ºC. After constant pressure heating, the pressure and temperature are 7 bar and 700ºC respectively. The flow rate of air is 0.4 kg/s. Calculate the following.

    i. The cycle efficiency. (42.7 %)
    ii. The heat transfer into the heater. (206.7 kW)
    iii. The net power output. (88.26 kW)

2. A gas turbine expands draws in 3 kg/s of air from atmosphere at 1 bar and 20ºC. The combustion chamber pressure and temperature are 10 bar and 920ºC respectively. Calculate the following.

    i. The Joule efficiency. (48.2 %)
    ii. The exhaust temperature. (617.5 K)
    iii. The net power output. (911 kW)

3. A gas turbine draws in 7 kg/s of air from atmosphere at 1 bar and 15ºC. The combustion chamber pressure and temperature are 9 bar and 850ºC respectively. Calculate the following.

    i. The Joule efficiency. (46.7 %)
    ii. The exhaust temperature. (599 K)
    iii. The net power output. (1.916 MW)

ENGINEERING COUNCIL

CERTIFICATE LEVEL

THERMODYNAMIC, FLUID AND PROCESS ENGINEERING C106

TUTORIAL 5 – STEAM POWER CYCLES

When you have completed this tutorial you should be able to do the following.

❑ Explain and solve the Carnot steam cycle.

❑ Explain and solve the Rankine steam power cycle.

It is unlikely that the syllabus intends you to study advanced steam cycles but these may be found in other tutorials on the web site.

You must fully understand the properties of steam and how to use the thermodynamic tables and charts. This is covered in previous tutorials.

1. STEAM CYCLES

1.1 THE CARNOT STEAM CYCLE

In previous tutorials you learned that a Carnot cycle gave the highest thermal efficiency possible for an engine working between two temperatures. The cycle consisted of isothermal heating and cooling and reversible adiabatic expansion and compression.

Consider a cycle that uses vapour throughout. Evaporation and condensation at constant pressure is also constant temperature. Isothermal heating and cooling is theoretically possible. The cycle would consist of the same 4 processes as before only this time each process would be carried out in a separate steady flow plant item with the vapour flowing from one to the other in a closed loop as shown below.
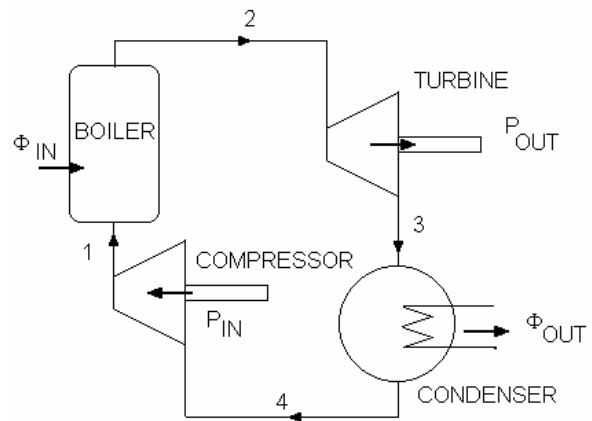
Fig. 1

The four processes are:

1 - 2   Evaporation at constant pressure and temperature requiring heat input.
2 - 3   Reversible adiabatic expansion in the turbine giving power output.
3 - 4   Cooling and condensing at constant pressure and temperature in the condenser requiring heat output.
4 - 1   Reversible adiabatic compression requiring power input.

In order that no temperature changes occur in the evaporator and condenser, the vapour must be wet at inlet and outlet. Over-cooling will produce liquid at temperatures below the saturation temperature and over-heating will superheat it beyond the saturation temperature. The cycle will be a rectangle on the T-s diagram and as shown on the h-s diagram.
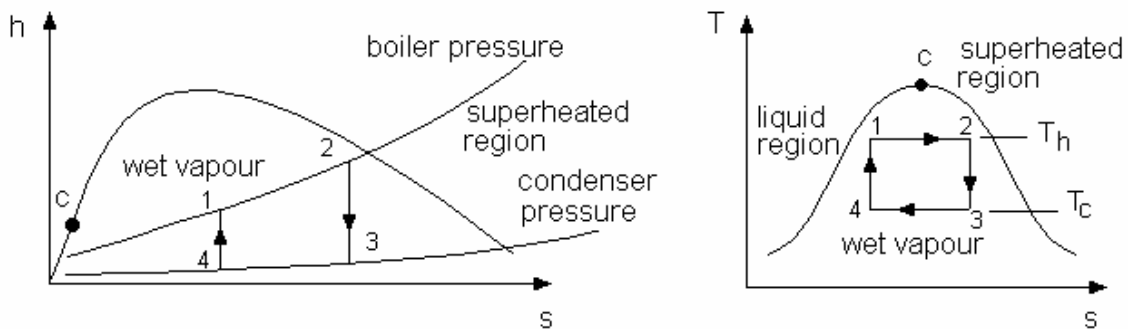
Fig.2

The limits are that at point (2) it may be dry saturated vapour but not superheated. At point 1 it may be saturated water but not under-cooled. If these limits are not used, then the vapour has a dryness fraction at each point. Since heat transfer only occurs at the evaporator and condenser the heat transfer rates are given by the following expressions.

$\Phi_{in} = m(h_2 - h_1) = T_h \, \Delta S$ (Boiler)

$\Phi_{out} = m(h_3 - h_4) = T_c \, \Delta S$ (Condenser)

T_h is the boiler temperature and T_c is the condenser temperature.
The thermal efficiency may be found from the 1st. Law. $\eta_{th} = 1 - \Phi_{out}/\Phi_{in} = 1 - T_c / T_h$
This expression is the same as for the gas version.

WORKED EXAMPLE No. 1

A Carnot cycle is conducted on steam as follows. The evaporator produces dry saturated steam at 10 bar. The steam is expanded reversibly and adiabatically in a turbine to 1 bar. The exhaust steam is partially condensed and then compressed back to 10 bar. As a result of the compression, the wet steam is changed completely into saturated water.

Assuming a flow rate of 1 kg/s throughout determine the condition and specific enthalpy at each point in the cycle.
Calculate the energy transfers for each stage.
Show that the efficiency is correctly predicted by the expression $\eta_{th}= T(cold)/T(hot)$

SOLUTION   We will refer to the previous diagrams throughout.

EVAPORATOR

$h_2 = h_g$ at 10 bar (since it is dry saturated) = 2778 kJ/kg.
$s_2 = s_g$ at 10 bar (since it is dry saturated) = 6.586 kJ/kg K.
$h_1 = h_f$ at 10 bar (since it is saturated water) = 763 kJ/kg.
$\Phi_{in} = 1 (2778 - 763) = 2015$ kW

TURBINE
Since the expansion is isentropic then $s_2= s_3 = 6.586$ kJ/kg K
$s_3 = 6.586 = s_f + x_3 s_{f_g}$ at 1 bar
$6.586 = 1.303 + x_3(6.056)$ hence $x_3=0.872$
$h_3 = h_f + x_3 h_{f_g}$ at 1 bar = 417 + (0.872)(2258) = 2387 kJ/kg
P(output) = 1(2778 - 2387) = 391.2 kW

COMPRESSOR
Since the compression is isentropic then $s_4= s_1$
$s_1 = s_f$ at 10 bar (since it is saturated water) = 2.138 kJ/kg K.
$s_4 = s_1 = 2.138 = s_f + x_4 s_{f_g}$ at 1 bar
$2.138 = 1.303 + x_4(6.056)$ hence $x_4 = 0.138$
$h_4 = h_f + x_4 h_{f_g}$ at 1 bar = 417 + (0.139)(2258) = 728.3 kJ/kg
Power Input = 1(763 - 728.3) = 34.7 kW

CONDENSER
Heat output = 1(2387 - 728.3) = 1658.7 kW
Energy Balances rounded off to nearest kW.
Total energy input = 34.7 + 2015 = 2050 kW

Total energy output = 391.2 + 1658.7 = 2050 kW
Net Power output = 391.2 - 34.7 = 356 kW
Net Heat input = 2015 - 1658.7 = 356 kW
Thermal efficiency = P_{nett}/Φ_{in} = 356/2015 = 17.7%
Thermal Efficiency = $1 - \Phi_{out} / \Phi_{in}$ = 1 - 1658.7/2015 = 17.7%
The hottest temperature in the cycle is t_S at 10 bar = 179.9 ºC or 452.9 K

The coldest temperature in the cycle is t_S at 1 bar = 99.6 ºC or 372.6 K
The Carnot efficiency = 1 - 372.6/452.9 = 17.7%

1. A steam power plant uses the Carnot cycle. The boiler puts 25 kW of heat into the cycle and produces wet steam at 300°C. The condenser produces wet steam at 50°C.

    Calculate the following.

    i.   The efficiency of the plant. (43.6%)
    ii.   The net power output. (10.9 kW)
    iii.   The heat removed by the condenser. (14 kW)

2. A steam power plant is based on the Carnot cycle. The boiler is supplied with saturated water at 20 bar and produces dry saturated steam at 20 bar. The condenser operates at 0.1 bar. Assuming a mass flow rate of 1 kg/s calculate the following.

    i.   The thermal efficiency. (34.3%)
    ii.   The power output of the turbine. (792 kW)
    iii.   The heat transfer rate into the boiler. (1.89 MW)

1.2   THE RANKINE CYCLE

The Rankine Cycle is a practical cycle and most steam power plants are based on it. The problems with the Carnot Cycle are as follows.

❑   It produces only small net power outputs for the plant size because dry saturated steam is used at inlet to the turbine.
❑   It is impractical to compress wet steam because the water content separates out and fills the compressor.
❑   It is impractical to control the condenser to produce wet steam of the correct dryness fraction.

In order to get around these problems, the Rankine Cycle uses superheated steam from the boiler to the turbine. The condenser completely condenses the exhaust steam into saturated water. The compressor is replaced with a water (feed) pump to return the water to the boiler. The result of this is reduced efficiency but greater quantities of power.
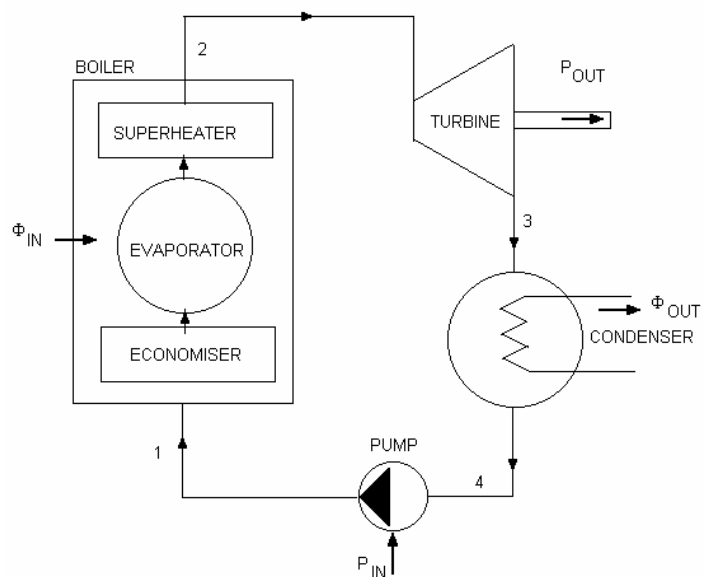
Fig.3

The plant layout is shown above. First let's briefly examine the boiler.

BOILER

For reasons of combustion efficiency (which you do not have to study), a practical boiler is made up of three sections.

a) Economiser

This is a water heater inside the boiler that raises the water temperature at the boiler pressure to just below the saturation temperature at that pressure.

b) Evaporator

This is a unit usually consisting of a drum and tubes in which the water is evaporated and the steam driven off.

c) Super-heater

This is a heater placed in the hottest part of the boiler that raises the temperature of the steam well beyond the saturation temperature.

There are many boiler designs and not all of them have these features. The main point is that a heat transfer rate is needed into the boiler unit in order to heat up the water, evaporate it and superheat it. The overall heat transfer is

$$\Phi_{in} = m (h_2 - h_1)$$

Next let's look at some other practical aspects of a steam power plant.

EXTRACTION PUMP AND HOTWELL.

In a practical steam cycle the condensate in the condenser is extracted with an extraction pump and the water produced is the coldest point in the steam cycle. This is usually placed into a vessel where it can be treated and extra added to make up for leaks. This point is called the HOTWELL because it contains hot water. The main feed pump returns this water to the boiler at high pressure. In the following work, extraction pumps and hotwells are not shown.
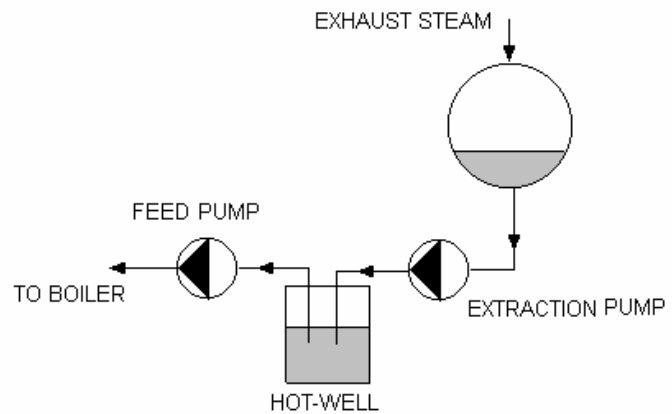
Fig.4

Now let's examine the cycle with the aid of property diagrams.
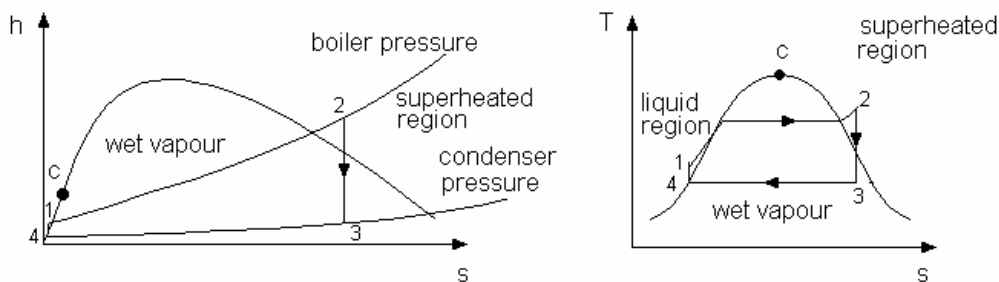
Fig.5

The process 4 to 1 is cramped into the corner of the h-s diagram and is not clear.

BOILER PROCESS (1) to (2) HEAT INPUT

The water at point 1 is below the saturation temperature at the boiler pressure. The economiser first heats it up raising the temperature, enthalpy and entropy until it reached the saturation curve. The water is then evaporated and finally, the temperature is raised by superheating the steam to point 2.

$$\Phi_{in} = m\,(h_2 - h_1)$$

TURBINE PROCESS (2) to (3) POWER OUTPUT

The second process is the expansion in the turbine and this is ideally reversible and adiabatic and is represented by a vertical line on the diagrams.

$$P_{out} = m(h_2 - h_3)$$

Turbines in real plant are often in several stages and the last stage is specially designed to cope with water droplets in the steam that becomes wet as it gives up its energy. You must use the isentropic expansion theory in order to calculate the dryness fraction and enthalpy of the exhaust steam.

CONDENSER PROCESS (3) to (4) HEAT OUTPUT

The third process is the condenser where the wet steam at point 3 is ideally turned into saturated water at the lower pressure (point 4). Condensers usually work at very low pressures (vacuums) in order to make the turbine give maximum power. The heat removed is given by

$$\Phi_{out} = m\,(h_3 - h_4)$$

Since the condenser produces condensate (saturated water) then $h_4 = h_f$ at the condenser pressure.

PUMP PROCESS (4) to (1) POWER INPUT

The final process which completes the cycle is the pumping of the water (point 4) from the low condenser pressure to the boiler at high pressure (point 1). In reality there are many things which are done to the feed water before it goes back into the boiler and the pressure is often raised in several stages. For the Rankine Cycle we assume one stage of pumping which is adiabatic and the power input to the pump is

$$P_{in} = m\,(h_1 - h_4)$$

The power required to pump the water is much less than that required to compress the vapour (if it was possible). The power input to the feed pump is very small compared to the power output of the turbine and you can often neglect it altogether. In this case we assume $h_1=h_4$.

If you are not ignoring the power input, then you need to find h_1. If you know the exact temperature of the water at inlet to the boiler (outlet from the pump) then you may be able to look it up in tables. The nearest approximation is to look up h_f at the water temperature. Since the water is at high pressure, this figure will not be very accurate and you may correct it by adding the flow energy. We will look at this in greater detail later. Lets first do a simple example with no great complications.

WORKED EXAMPLE No.2

A steam power plant is based on the Rankine cycle. The steam produced by the boiler is at 40 bar and 400ºC. The condenser pressure is 0.035 bar. Assume isentropic expansion. Ignore the energy term at the feed pump.

Calculate the Rankine cycle efficiency and compare it to the Carnot efficiency for the same upper and lower temperature limits.

SOLUTION
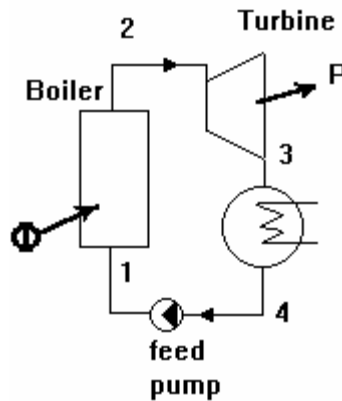
Figure 6

Turbine

$h_2 = 3214$ kJ/kg at 40 bar and 400ºC.

Since the expansion is isentropic then $s_2 = 6.769$ kJ/kg K $= s_3 = 0.391 + 8.13$ x   x $= 0.785$

$h_3 = h_f + x\ h_{fg} = 112 + 0.785(2438) = 2024.6$ kJ/kg

Condenser

$h_4 = h_f$ at 0.035 bar $= 112$ kJ/kg

Boiler

If the power input to the pump is neglected then $h_4 = h_1 = 112$ kJ/kg

$\Phi_{in} = h_2 - h_1 = 3102$ kJ/kg.   P(output) $= h_2 - h_3 = 1189.4$ kJ/kg

$\eta = P / \Phi_{in} = 38.3$ %

Carnot Efficiency

The hottest temperature in the cycle is 400ºC (673 K) and the coldest temperature is t_s at 0.035 bar and this is 26.7 ºC(299.7 K).

The Carnot efficiency is $1 - 299.7/673 = 55.5$ % which is higher as expected.

Now let's examine the feed pump in more detail.

FEED PUMP

When water is compressed its volume hardly changes. This is the important factor that is different from the compression of a gas. Because the volume hardly changes, the temperature should not increase and the internal energy does not increase. The Steady flow Energy equation would then tell us that the power input to the pump is virtually equal to the increase in flow energy. We may write

$$P_{in} = m \, v\Delta p$$

Since the volume of water in nearly all cases is 0.001 m^3/kg then this becomes

$$P_{in} = 0.001 \, m \, \Delta p = 0.001 \, m \, (p_1 - p_2)$$

If we use pressure units of bars then

$$P_{in} = 0.001 \, m(p_1 - p_2) \times 10^5 \text{ Watts}$$

Expressed in kilowatts this is

$$P_{in} = m(p_1 - p_2) \times 10^{-1} \text{ kW}$$

From this we may also deduce the enthalpy of the water after the pump.

$$P_{in} = m \, (h_1 - h_4)$$

Hence h_1 may be deduced.

WORKED EXAMPLE No.3

Repeat example 3, but this time do not ignore the feed pump and assume the boiler inlet condition is unknown.

SOLUTION

$P_{in} = 1kg/s(40 - 0.035) \times 10^{-1} = 4 \text{ kW}$
$4 = 1 \text{ kg/s}(h_1 - h_4) = (h_1 - 112)$
$h_1 = 116 \text{ kJ/kg}$

Reworking the energy transfers gives

$\Phi_{in} = h_2 - h_1 = 3214 - 116 = 3098 \text{ kJ/kg.}$

$P_{nett} = P_{out} - P_{in} = 1189.4 - 4 = 1185.4 \text{ kJ/kg}$

$\eta = P_{nett}/\Phi_{in} = 1185.4/3098 = 38.3 \text{ \%}$

Notice that the answers are not noticeably different from those obtained by ignoring the feed pump.

WORKED EXAMPLE No.4

A steam power plant uses the Rankine Cycle. The details are as follows.

Boiler pressure 100 bar
Condenser pressure 0.07 bar
Temperature of steam leaving the boiler 400ºC
Mass flow rate 55 kg/s

Calculate the cycle efficiency, the net power output and the specific steam consumption.

SOLUTION

Turbine

$h_2 = 3097$ kJ/kg at 100 bar and 400ºC.

For an isentropic expansion we find the ideal condition at point 3 as follows.

$s_2 = 6.213$ kJ/kg K $= s_3 = 0.559 + 7.715 x_3$ $\quad\quad\quad\quad x_3 = 0.733$

$h_3 = h_f + x_3 h_{fg} = 163 + 0.733(2409) = 1928$ kJ/kg

$P_{out} = m(h_2-h_3) = 55(3097 - 1928) = 64.3$ MW

Condenser

$h_4 = h_f$ at 0.07 bar $= 163$ kJ/kg

$\Phi_{out} = m(h_3 - h_4) = 55(1928 - 163) = 97.1$ MW

PUMP

Ideal power input = Flow Energy change = $mv(\Delta p)$

$P_{in} = 55(0.001)(100-0.07) \times 10^5 = 550$ kW

$P_{in} = m(h_1 - h_4) = 55(h_1-163)$ hence $h_1 = 173$ kJ/kg

Boiler

$\Phi_{in} = m(h_2 - h_1) = 55(3097 - 173) = 160.8$ MW

EFFICIENCY

$P_{nett} = P_{out} - P_{in} = 64.3 - 0.55 = 63.7$ MW

$\eta = P_{nett} / \Phi_{in} = 63.7/160.8 = 39.6$ %

Alternatively $\quad\quad P_{nett} = \Phi_{in} - \Phi_{out} = 160.8-97.1 = 63.7$ MW

This should be the same as P_{nett} since the net energy entering the cycle must equal the net energy leaving.

$\eta = 1 - \Phi_{out} / \Phi_{in} = 1 - 97.1/160.8 = 39.6\%$

SPECIFIC STEAM CONSUMPTION

This is given by

S.S.C. $= P_{nett}$ /mass flow $= 63.78/55 = 1.159$ MW/kg/s or MJ/kg

SELF ASSESSMENT EXERCISE No.2

1. A simple steam plant uses the Rankine Cycle and the data for it is as follows.
    Flow rate   45 kg/s
    Boiler pressure   50 bar
    Steam temperature from boiler 300°C
    Condenser pressure   0.07 bar

    Assuming isentropic expansion and pumping, determine the following.

    i.   The power output of the turbine. (44.9 MW)
    ii.   The power input to the pump. (225 kW)
    iii. The heat input to the boiler. (124 MW)
    iv. The heat rejected in the condenser. (79 MW)
    v.   The thermal efficiency of the cycle. (36%)

2. A simple steam power plant uses the Rankine Cycle. The data for it is as follows.
    Flow rate   3 kg/s
    Boiler pressure   100 bar
    Steam temperature from boiler 600°C
    Condenser pressure   0.04 bar

    Assuming isentropic expansion and pumping, determine the following.

    i.   The power output of the turbine. (4.6 MW)
    ii.   The power input to the pump. (30 kW)
    iii. The heat input to the boiler. (10.5 MW)
    iv. The heat rejected in the condenser. (5.9 MW)
    v.   The thermal efficiency of the cycle. (44%)

3.
a) Explain why practical steam power plants are based on the Rankine Cycle rather than the Carnot Cycle.

b) A simple steam power plant uses the Rankine Cycle. The data for it is

    Boiler pressure   15 bar
    Steam temperature from boiler   300°C
    Condenser pressure   0.1 bar
    Net Power Output   1.1 MW

    Calculate the following.

    i. The cycle efficiency. (29.7 %)

    ii. The steam flow rate. (1.3 kg/s)

ENGINEERING COUNCIL

CERTIFICATE LEVEL

THERMODYNAMIC, FLUID AND PROCESS ENGINEERING C106

TUTORIAL 11 - COMBUSTION PROCESSES

This tutorial may contain more than enough material for the syllabus but it is recommended that students complete all the work.

On completion of this tutorial you should be able to do the following.

- Write down combustion equations.

- Solve the oxygen and air requirements for the combustion of solid, liquid and gaseous fuels.

- Determine the products of combustion.

- Determine the air/fuel ratio from the products of combustion.

- Solve problems involving energy released by combustion.

- Describe basic instruments used in flue gas analysis.

- Explain the use of calorimeters.

In order to complete this tutorial you should be already familiar with the following.

- The molecular nature of substances.
- Basic chemistry.
- The properties of gas and vapours.
- The use of steam tables.

1. INTRODUCTION

Combustion is the process of chemical reaction between fuel and oxygen (the reactants). The process releases heat and produces products of combustion.

The main elements which burn are:
- o CARBON
- o HYDROGEN
- o SULPHUR

The heat released by 1 kg or 1 m^3 of fuel is called the **calorific value.**

The oxygen used in combustion processes normally comes from the atmosphere and this brings nitrogen in with it which normally does nothing in the process but makes up the bulk of the gases remaining after combustion. The main elements in combustion are as follows.

	Symbol	Atomic Mass	Molecular Mass	Product
Carbon	C	12		CO_2
Hydrogen	H_2	1	2	H_2O
Sulphur	S	32		SO_2
Oxygen	O_2	16	32	
Nitrogen	N_2	14	28	

If the water formed during combustion leaves as vapour, it takes with it the latent heat of evaporation and thus reduces the energy available from the process. In this case the calorific value is called the lower Calorific value (LCV). If the products cool down after combustion so that the vapour condenses, the latent heat is given up and the calorific value is then the higher calorific value (HCV).

Solid and liquid fuels are normally analysed by mass to give the content of carbon, hydrogen, sulphur and any other elements present. Often there is silica, moisture and oxygen present in small quantities which have some effect on the process. The silica leaves deposits of slag on the heat transfer surfaces in boilers.

Gaseous fuels are normally analysed by volumetric content and are in the main hydrocarbon fuels.

COMPOSITION OF AIR. For purposes of calculation, the composition of air is considered to be as follows.

	VOLUMETRIC	GRAVIMETRIC
Oxygen	21%	23%
Nitrogen	79%	77%

The **sulphur content** of the fuel is considered to be a pollutant and so undesirable. The theoretically correct quantity of air or oxygen required to just exactly burn the fuel expressed as a ratio to the fuel burned, is called the **STOICHIOMETRIC RATIO**.

In practice it is found that not all the oxygen in the reactant reaches the fuel elements and that excess air is required in order to ensure complete combustion. This results in oxygen appearing in the products. If too little air or oxygen is supplied, the result is incomplete combustion resulting in the formation of carbon monoxide CO instead of carbon dioxide CO_2. The resulting products contain water H_2O. Industrial equipment for measuring the contents of the products usually removes the water from the sample and the products are then called the dry products.

2.   COMBUSTION CHEMISTRY

2.1 SOLID AND LIQUID FUELS

In the case of solid and liquid fuels, we do the combustion of each element separately. The important rule is that you must have the same number of atoms of each substance before and after the process. This may be obtained by juggling with the number of molecules.

CARBON	$C + O_2 = CO_2$
Mass ratio	$12 + 32 = 44$

Hence 1kg of C needs 32/12kg of O_2 and makes 44/12kg of CO_2

HYDROGEN	$2H_2 + O_2 = 2H_2O$
Mass ratio	$4 + 32 = 36$
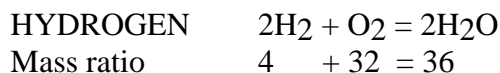
Hence 1kg of H_2 needs 8kg of O_2 and makes 9 kg of H_2O

SULPHUR	$S + O_2 = SO_2$
	$32 + 32 = 64$
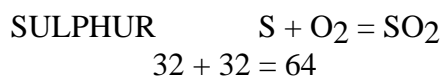
Hence 1 kg of S needs 1kg of O_2 and makes 2kg of SO_2.

2.2. GASEOUS FUELS.

Typical hydrocarbons are:

Methane	CH_4
Ethane	C_2H_6
Propane	C_3H_8
Butane	C_4H_{10}
Pentane	C_5H_{12}
Hexane	C_6H_{14}
Heptane	C_7H_{16}
Octane	C_8H_{18}
Ethene	C_2H_4 (Ethylene)
Propene	C_3H_6 (Propylene)
Ethyne	C_2H_2 (Acetylene)
Benzenol	C_6H_6 (Benzene)
Cyclohexane	C_6H_{12}

The combustion equation follows the following rule:

$$C_aH_b + (a + ¼ b)\, O_2 = (a)\, CO_2 + (½ b)\, H_2O$$
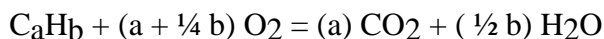
If this results in fractional numbers of molecules, then the whole equation may be multiplied up.

WORKED EXAMPLE No.1   Write out the combustion equation for C_8H_{18}

SOLUTION

a = 8, b = 18
$C_aH_b + (a + ¼ b) O_2 = (a) CO_2 + (½ b) H_2O$

$C_8H_{18} + (8+18/4)O_2 = 8CO_2 + (18/2)H_2O$
$C_8H_{18} + 12½O_2 = 8CO_2 + 9H_2O$
$2C_8H_{18} + 25O_2 = 16CO_2 + 18H_2O$
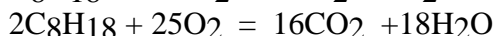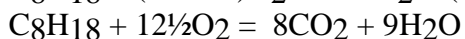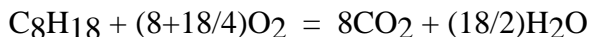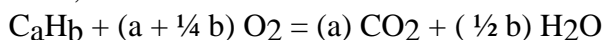
There are other gases which burn and the main one to know about is Carbon Monoxide (CO) which is partially burned carbon. The equation for the combustion of CO is:

$$2CO + O_2 = 2CO_2$$
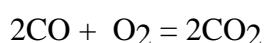
3.   COMBUSTION BY MASS

The only rule to be observed in deducing the quantities of each substance is the law of conservation of mass. The proportions of the mass are that of the molecular masses. This is shown in the following example.

WORKED EXAMPLE No.2

A fuel contains by mass 88% C, 8%H_2, 1%S and 3% ash (silica). Calculate the stoichiometric air.

SOLUTION
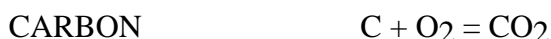
CARBON $C + O_2 = CO_2$
Mass ratio $12 + 32 = 44$

Hence 0.88kg of C needs (32/12)x0.88=2.347kg of oxygen.
It makes (44/12) x 0.88 = 3.227 kg of carbon dioxide.

HYDROGEN $2H_2 + O_2 = 2H_2O$
Mass ratio $4 + 32 = 36$

hence 0.08kg of hydrogen needs (32/4) x 0.08 = 0.64kg of oxygen.

SULPHUR $S + O_2 = SO_2$
Mass ratio $32 + 32 = 64$

Hence 0.01kg of sulphur needs 0.01kh of oxygen and makes 0.02kg of sulphur dioxide.
TOTAL OXYGEN needed is 2.347 + 0.64 + 0.01 = 2.997kg
TOTAL AIR needed is 2.997/23% = 13.03kg
The STOICHIOMETRIC air/fuel ratio is 13.03/1

It is of interest to note that for a given fuel, the % of any product is a direct indication of the excess air and in practice the carbon dioxide and/or oxygen is used to indicate this. This is important in obtaining optimal efficiency in a combustion process.

SELF ASSESSMENT EXERCISE No.1 – COMBUSTION BY MASS

Complete the following problems.

1. A boiler burns fuel oil with the following analysis by mass :

    80% C 18% H_2 2%S

    30% excess air is supplied to the process. Calculate the stoichiometric ratio by mass and the % Carbon Dioxide present in the dry products.
    (15.62/1   14.9% CO_2)

2. A boiler burns coal with the following analysis by mass :

    75% C 15% H_2 7%S   remainder ash

    Calculate the % Carbon Dioxide present in the dry products if 20% excess air is supplied.
    (16.5% CO_2)

3. Calculate the % of each dry product when coal is burned stoichiometrically in air. The analysis of the coal is:

    80% C   10% H_2 5% S and 5% ash.

    (76.7%N, 22.5% CO_2 0.8% SO_2)

4. COMBUSTION BY VOLUME

First we need to revise gas mixtures and understand the meaning of VOLUMETRIC CONTENT. To do this we must understand Dalton's law of partial pressures and Avogadro's Law.

First let us define the kmol. A kmol of substance is the number of kg numerically equal to the apparent molecular mass. For example, 12 kg of Carbon is a kmol, so is 32 kg of O_2 and 2 kg of H_2 and 28 kg of N_2.

The molecular mass of a substance is expressed as kg/kmol so the molecular mass of O_2, for example, is 32 kg/kmol.

Avogadro's Law states:

$1m^3$ of any gas at the same pressure and temperature contains the same number of molecules. It follows that the volume of a gas at the same p and T is directly proportional to the number of molecules. From this we find that the volume of a kmol of any gas is the same if p and T are the same.

Dalton's law states:

The total pressure of a mixture is the sum of the partial pressures. The partial pressure is the pressure each gas would exert if it alone occupied the same volume at the same temperature.

Consider two gases A and B occupying a volume V at temperature T. Using the Universal gas law for each:

$$p_A V_A = m_A R_o T / \tilde{N}_A \qquad p_B V_B = m_B R_o T / \tilde{N}_B$$

$\tilde{N}$ is the relative molecular mass.

$p_A/p_B = m_A \tilde{N}_B / m_B \tilde{N}_A$ = ratio of the kmol fractions.

p_A and p_B are the partial pressures.

V_A and V_B are the partial volumes. These are the volumes each gas would occupy if they were separated and kept at the original p and T. This concept is very useful in problems involving the combustion of gases. It also follows that the partial volumes are directly related to the partial pressures so that $V_A/V_B = p_A/p_B$

Figure 1

When not mixed the pressure is p and the volumes are V_A and V_B. Hence:

$pV_A/T = mR_o/\tilde{N}_A \qquad p = m_A R_o T/\tilde{N}_A V_A$ ....(1)
$pV_B/T = mR_o/\tilde{N}_B \qquad p = m_B R_o T/\tilde{N}_B V_B$ ....(2) Since (1) = (2) then:

$m_A/\tilde{N}_A V_A = m_B/\tilde{N}_B V_B$ and so $V_A/V_B = (m_A/\tilde{N}_A)(m_B/\tilde{N}_B)$ which shows that in a mixture, the partial volumes are in the same ratio as the kmol fractions which in turn are in proportion to the number of molecules of each gas.

When mixed they both have volume V, hence:

$$p_A = m_A R_o T/\tilde{N}_A V.........(3)$$
$$p_B = m_B R_o T/\tilde{N}_B V.........(4)$$

(3)/(1) gives $p_A/p = V_A/V$ and (4)/(2) gives $p_B/p = V_B/V$
Hence $\qquad\qquad\qquad\qquad V_A/V_B = p_A/p_B$
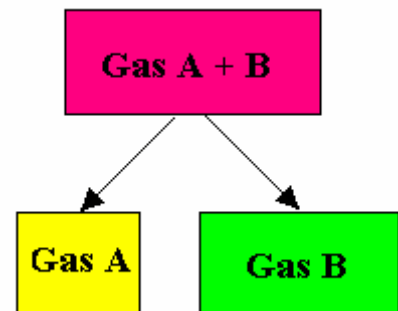
Consider the combustion of Methane. $CH_4 + 2O_2 = CO_2 + 2H_2O$

Since the volumetric content of each gas is in the same ratio as the kmol fractions then the volumetric content is in the same proportion as the molecules. Hence it needs 2 volumes of oxygen to burn 1 volume of methane.

The volume of air needed is 2/21% = 9.52 volumes. Hence it burn 1 m^3 of methane we need 9.52 m^3 of air for stoichiometric combustion. If the products are at the same p and T as the original reactants, we would obtain 1 m^3 of carbon dioxide and 2 m^3 of water vapour which would probably condense and cause a reduction in volume and/or pressure.

WORKED EXAMPLE No.4

Calculate the % CO_2 in the dry products when methane is burned with 15% excess air by volume.

SOLUTION
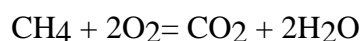
$$CH_4 + 2O_2 = CO_2 + 2H_2O$$
Volume ratio   1   2   1   2
The stoichiometric air is 2/21% = 9.524 m^3
The actual air is 9.524 x 115% = 10.95 m^3
Analysis of dry products:

Nitrogen	79% x 10.95	8.65 m^3
Carbon Dioxide		1.00 m^3
Oxygen 15% x 2		0.30 m^3
Total		9.95 m^3

The % Carbon Dioxide = (1/9.95) x 100 = 10%

When the fuel is a mixture of gases, the procedure outlined must be repeated for each combustible gas and the oxygen deduced for the volume of each in 1 m^3 of total fuel.

WORKED EXAMPLE No. 5

A fuel is a mixture of 60% Methane and 30% carbon monoxide and 10% oxygen by volume. Calculate the stoichiometric oxygen needed.

SOLUTION

As before, the volume of oxygen required to burn 1 m^3 of methane is 2m^3. To burn 0.6m^3 needs 1.2m^3 of oxygen. For carbon monoxide we use the combustion equation:
$$2CO + O_2 = 2CO_2$$
Hence to burn 1 m^3 of CO need 0.5 m^3 of oxygen, so to burn 0.3 m^3 needs 0.15 m^3 of oxygen.
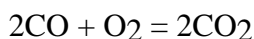
The total oxygen needed is 1.2 + 0.15 = 1.35 m^3. However there is already 0.1 m^3 in the fuel so the **stoichiometric oxygen needed 1.25m^3**

SELF ASSESSMENT EXERCISE No.2 - COMBUSTION BY VOLUME

1. Find the air fuel ratio for stoichiometric combustion of Ethene by volume. (14.28/1)

2. Find the air fuel ratio for stoichiometric combustion of Butane by volume.(30.95/1) Calculate the % carbon dioxide present in the dry flue gas if 30% excess air is used. (10.6%)

3. Find the air fuel ratio for stoichiometric combustion of Propane by volume. (23.81/1). Calculate the % oxygen present in the dry flue gas if 20% excess air is used. (3.8%)

4. A gaseous fuel contains by volume :

    5% CO_2, 40% H_2 , 40% CH_4, 15% N_2

    Determine the stoichiometric air and the % content of each dry product.
    (4.76 m^3, 89.7%,N_2 10.3% CO_2).

5. RELATIONSHIP BETWEEN PRODUCT AND EXCESS AIR.

It follows that if we can deduce the % product then we can work backwards to determine the air or oxygen that was used.

WORKED EXAMPLE No.6

Consider the combustion of methane again.
$CH_4 + 2O_2 = CO_2 + 2H_2O$
1 vol   2 vol   1 vol   2 vols
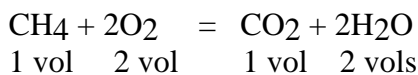
SOLUTION

Let the excess air be x (as a decimal)
The stoichiometric air is 9.52 vols.   Actual air is 9.52(1 + x)
Dry Products:

Nitrogen	0.79 x 9.52(1 + x) =	7.524x + 7.524
Oxygen		2.000x
Carbon Dioxide		1.000
Total		9.524x + 8.524

% Carbon monoxide = 100 {1/(9.524x + 8.524)}
% Oxygen   = 100{2/(9.524x + 8.524)}
For example if the % CO_2 is 10% then the excess air is found as follows:

        10% = 100 {1/(9.524x + 8.524)}   0.1 = 1/(9.524x + 8.524)
        (9.524 x + 8.524) = 10   9.524 x = 1.476
        x = 0.155 or 15.5%

Similarly if the O_2 is 10% then the excess air is 81% (show this for yourself)

If the analysis of the fuel is by mass, then a different approach is needed. The following examples show two ways to solve problems.

WORKED EXAMPLE No.7

An analysis of the dry exhaust gas from an engine burning Benzole shows 15% Carbon Dioxide present by volume. The Benzole contains 90% C and 10% H_2 by mass. Assuming complete combustion, determine the air/fuel ratio used.

SOLUTION

1 kg of fuel contains 0.9kg of C and 0.1kg of H_2. Converting these into kmol we have 0.9/12 kmol of C and 0.1/2 kmol of H_2. For 1 kmol of dry exhaust gas we have :

        0.15 kmol of CO_2
        Y kmol of excess O_2
        1 - 0.15 - Y = 0.85 - Y kmol of N_2

1 kmol of CO_2 is 44 kg   1 kmol of N_2 is 28 kg   1 kmol of O is 32 kg
0.15 kmol of CO_2 is 0.15 x 44kg
This contains (12/44) carbon so the carbon present is 0.15 x 12 kg
The carbon in the fuel is 0.9 kmol per kmol of fuel. Hence the number of kmols of DEG must be 0.9/(.15 x 12) = 0.5

There are 0.5 kmol of DEG for each kmol of fuel burned.
The Nitrogen present in the DEG is 0.85 - Y kmol per kmol of DEG. This has a mass of 28(0.85 - Y) per kmol of DEG
The oxygen supplied to the process must be :
(23.3/76.7) x 28 x (0.85 - Y) = 7.24 - 8.5Y kg per kmol of DEG.
(using precise proportions of air for accuracy).
The oxygen contained within the carbon dioxide is:
(32/44) x 0.15 x 44 = 4.8 kg per kmol DEG
1 kmol of CO_2 contains 44 kg and 32/44 of this is oxygen. The oxygen in the CO_2 is hence 32 x 0.15 kg per kmol DEG.
The excess oxygen is   32Y kg per kmol DEG
Total oxygen in the products excluding that used to make H_2O is :32 x 0.15 + 32Y
The oxygen used to burn hydrogen is hence :   7.24 - 8.5Y – (32 x 0.15 + 32Y)
O_2 used to burn H_2 is   2.44 - 40.5Y kg per kmol DEG
For 0.5 kmol this is   1.22 - 20.25Y kg

To burn hydrogen requires oxygen in a ratio of 8/1. There is 0.1 kg of H_2 in each kmol of fuel so 0.8 kg of O_2 is needed. Hence :

        0.8 =1.22 - 20.25Y
        Y = 0.208kmol per kmol DEG

The nitrogen in the DEG is 0.85 - Y = 0.642 kmol per kmol DEG
The actual Nitrogen = 0.642 x 0.5 x 28 = 11.61 kg
The air supplied must be 11.61/.767 = 15.14kg per kg of fuel. A simple calculation shows the stoichiometric mass of air is 13.73 so there is 10.3% excess air.

WORKED EXAMPLE No.8

A fuel oil contains by mass 86.2% C, 12.8% H_2 and 0.9% S by mass. The dry exhaust gas contains 9% CO_2 by volume. Calculate the excess air by mass.

SOLUTION

CARBON $\quad\quad\quad$ $C + O_2 \rightarrow CO_2$
Mass ratio $\quad\quad$ $12 + 32 \rightarrow 44$
$\quad\quad\quad\quad\quad\quad$ $0.862 + 2.299 \rightarrow 3.16$
HYDROGEN $\quad\quad$ $2H_2 + O_2 \rightarrow 2H_2O$
Mass ratio $\quad\quad$ $4 \;+\; 32 \rightarrow 36$
$\quad\quad\quad\quad\quad\quad$ $0.128 + 1.024 \rightarrow 1.152$
SULPHUR $\quad\quad$ $S + O_2 \rightarrow SO_2$
Mass ratio $\quad\quad$ $32 + 32 \rightarrow 64$
$\quad\quad\quad\quad\quad\quad$ $0.09 + 0.09 \rightarrow 0.18$
TOTAL OXYGEN needed is 3.413 kg
Ideal Air (stoichiometric ratio) = 3.413/0.233 = 14.65 kg
Nitrogen is 0.767 x 14.65 = 11.23
Let the excess air be x   Calculate dry products by mass

		% of total
CO_2	3.16 kg	316/T
O_2	3.413x kg	341.3x/T
SO_2	0.18	18/T
N_2	11.23(1+ x)	1123(1+ x)/T

Total = T = 3.34 + 3.413x + 11.23(1+ x)

Now convert these into % by volume by dividing by the molecular mass

CO_2 $\quad\quad\quad\quad$ 316/44T = 7.18/T
O_2 $\quad\quad\quad\quad$ 341.3x/32T = 10.67x/T
SO_2 $\quad\quad\quad\quad$ 18/64T = 0.56/T
N_2 $\quad\quad\quad\quad$ 1123(1+ x)/28T = 40.11(1 + x)/T
Total = (1/T)(7.18 + 10.67x + 0.56 + 40.11 + 40.11x
Total = (1/T)(47.85 + 50.78 x)

%CO_2 = 9 = (7.18/T)100/(1/T)(47.85 + 50.78 x)
0.09= (7.18)/ (47.85 + 50.78 x)
47.85 + 50.78 x = 79.78
50.78 x = 79.78 - 47.85 = 31.92
x = 0.63

Answer the % excess air is 63%

6.   ENERGY RELEASED BY THE REACTION

This is complex subject and the following is very much simplified. Students wishing to study this in full should refer to advanced text books and/or the level 2 tutorial on combustion. The contents of the fuel and air or oxygen prior to combustion are called the reactants. The resulting material is called the products. In the process energy is released. This is taken as the enthalpy of reaction Δh

In simple terms we may say: $h(reactants) = h(products) + \Delta h_o$

Since $h(products) > h(reactants)$ it follows that Δh_o is a negative value.

In order to make more sense of this, we assume that the products and reactants end up at the same temperature. The values of Δh_o for various reactants may be found in standard tables. These are normally stated for 1 kmol of substance at 25^oC. We will be more practical here and quote values for 1 kg (solid and liquid fuels) and 1 m^3 for gaseous fuels. You will also find values for internal energy of reaction but we will not go into this here.

A fuel containing hydrogen will produce water vapour in the products. If the vapour condenses, more energy is released to the process. In practical combustion it is undesirable to allow this to happen as the condensed vapours form acids with the sulphur and corrodes the plant, especially the flue.

The energy released when water vapour is not condensed is called the Lower Calorific Value (L.C.V.). If the vapour is condensed then it is normally assumed that an extra 2441.8 kJ is released per kg of water, this being the latent enthalpy of evaporation at 25^oC. If this energy is added we get the Higher Calorific Value (H.C.V.). Typical L.C.V.s are as follows.

Coal   30 – 36 MJ/kg
Fuel Oils   43 – 46 MJ/kg
Natural Gas   38 MJ/m^3

The calorific values of fuels are found with *CALORIMETERS*. In the case of solid and liquid fuels the *BOMB CALORIMETER* is used. For gaseous fuels the *BOY'S CALORIMETER* is used.

HESS' LAW

This is useful when you need to work out the enthalpy of reaction Δh_o (energy released) when a particular chemical reaction takes place. The law states "if a reaction is carried out in a series of steps, Δh_o for the reaction will be equal to the sum of the enthalpy changes for the individual steps".

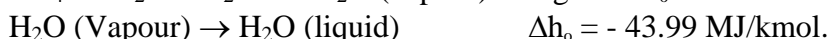For the level being studied here, we can apply it in two ways.

CHANGE OF STATE

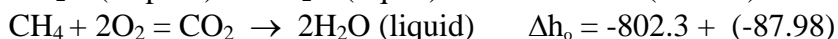When a change of state occurs we can use the law to link the H.C.V. and the L.C.V.

WORKED EXAMPLE No.9

When methane CH_4 in its gaseous form is completely burned to form CO_2 and H_2O (both in the gaseous form) the enthalpy of reaction is -802.3 kJ/kmol. When vapour is condensed to water at standard conditions the energy released is 87.98 kJ/kmol. Determine the H.C.V. for methane gas.

SOLUTION

$CH_4 + 2O_2 = CO_2 \rightarrow 2H_2O$ (vapour)   all gas Δh_o = -802.3 MJ/kmol. (L.C.V.)
H_2O (Vapour) $\rightarrow H_2O$ (liquid) Δh_o = - 43.99 MJ/kmol.
We need the figure for two atoms of water.
$2 H_2O$ (Vapour) $\rightarrow 2H_2O$ (liquid) Δh = 2 x (- 43.99) = -87.98 MJ/kmol.
$CH_4 + 2O_2 = CO_2 \rightarrow 2H_2O$ (liquid) Δh_o = -802.3 + (-87.98)
                                                     Δh_o = -890.28 MJ/kmol. (H.C.V.)

PARTIAL COMBUSTION

If a substance is partially burned, energy E1 is released.
If the partially burned substance is then completely burned the energy released is E2.
If the substance is completely burned in one step, the energy released is E = E1 + E2.

WORKED EXAMPLE No.10
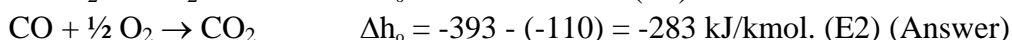
The enthalpy of reaction for solid carbon being burned to form carbon monoxide gas is -110 kJ/kmol.
The enthalpy of reaction for solid carbon into carbon dioxide gas is -393 kJ/mol.
What is the enthalpy of reaction when carbon monoxide gas is burned to form carbon dioxide gas?

SOLUTION

Base the solution on 1 atom of carbon throughout.
$C + O \rightarrow CO$ Δh_o = -110 kJ/kmol. (E1)
$C + O_2 \rightarrow CO_2$ Δh_o = -393 kJ/kmol. (E3)
$CO + \frac{1}{2} O_2 \rightarrow CO_2$ Δh_o = -393 - (-110) = -283 kJ/kmol. (E2) (Answer)

BOMB CALORIMETER

A sample of solid or liquid fuel (about 1 gram) is accurately weighed and placed in a crucible. This is fitted inside the bomb with an ignition wire (platinum) passing though the fuel. Special tools are used to create pellets of coal dust. The lid is screwed on and the bomb is pressurised with oxygen. The bomb is placed inside the outer container containing exactly 1 kg of water.

Figure 2

The outer container is placed inside an insulated box to prevent heat loss. A very accurate thermometer is placed in the water. The temperature is recorded against time. When the temperature is stable, the bomb is fired by passing an electric current through the wire. All the time the water is stirred by an electric motor and stirrer.

The fuel in the crucible is completely burned and the energy released causes a rapid rise in the temperature of the water. This is recorded until it starts to cool again. The bomb may be depressurised and the contents examined. A small amount of condensate will be found in it from the water formed during combustion. This may be weighed and used to find the L.C.V.
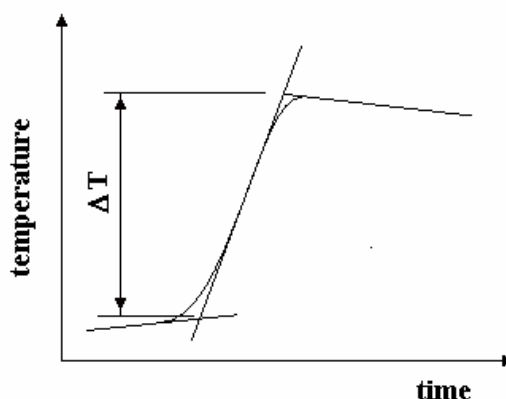

The graph produced is similar to that shown. Because of slow cooling and possibly initial warming, the straight lines are drawn as shown to find the true temperature change ΔT.

Figure 3

Because the metal of the bomb and container is warmed as well as the water, the manufacturer will certify that the bomb has an equivalent mass of water. The effective mass of water heated up is the actual mass plus the equivalent mass.

The resulting calorific value is the higher value because the gaseous H_2O is condensed to water.

M_f = mass of fuel. M_w = total effective mass of water.   c = specific heat capacity of water.
Higher Calorific value = M_w C $\Delta T/m_f$

WORKED EXAMPLE No.11

A bomb calorimeter is used to determine the Higher Calorific Value of an oil sample. The mass of the sample is 1.01 g. The total equivalent mass of water is 1.32kg. The increase in temperature after ignition is 7.8 K. Calculate the higher calorific value. The specific heat of water is 4.186 kJ/kg K.

SOLUTION

H.C.V = M_w C $\Delta T/m_f$ = 1.32 X 4.186 x 7.8/0.00101 = 42672 kJ/kg or 42.672 MJ/kg

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BOY'S CALORIMETER

THE Boy's calorimeter is in effect a highly efficient gas boiler. The following is a simplified description of it. Gas is burned in the bottom and as it forces its way out to the top, it has to make several passes over a coiled pipe with many fins on it to absorb the heat. Water is heated as it flows through the pipe. Condensate from the burned gas is also collected from the bottom. The volume of gas burned and the mass of water heated is accurately measured over the same period of time. Thermometers measure the water temperature at inlet and outlet and the flue gas temperature is also measured accurately

Figure 4

Heat released = volume of gas x higher calorific value.
Heat given to water = M_w c ΔT
Assuming 100% efficiency we may equate and get H.C.V. = $(M_w$ c $\Delta T)$/Volume of gas

This is over simplified and is only true if the flue gas is cooled back to the same temperature as the unburned gas. The laboratory procedure is detailed in standards.

WORKED EXAMPLE No.12

The results of a test with a Boy's calorimeter are as follows.

Volume of gas burned (at standard conditions) = 0.004 m^3
Volume of water heated in the same time = 1.8 kg
Temperature rise of the water = 20.17 K
Specific heat of water = 4.186 kJ/kg K
Calculate the Higher calorific value.

SOLUTION

H.C.V. = $(M_w$ c $\Delta T)$/Volume of gas = (1.8 x 4.186 x 20.17)/0.004 = 38 000 kJ/m^3

COMBUSTION EFFICIENCY

The efficiency of a combustion process is found from the following basic relationship.
$$\eta_{th} \% = \frac{\text{Energy gained by the working fluid}}{\text{Energy released by the fuel}} \times 100$$
The efficiency of the process is a maximum when all the fuel is burned and the resultant gaseous products give up all its energy to the process. To this end, the minimum amount of air necessary for complete combustion should be used as excess air takes away energy with it in the flue and exhaust. Because cooling the gas too much produces condensation, a minimum flue temperature is maintained and some heat loss must be expected. Heat loss from combustion equipment should be minimised by lagging. The following principles could apply to engines or boilers.

The flue gas or exhaust from the process should be monitored for some or all the following features.

- Temperature
- Oxygen content
- Carbon Monoxide content
- Carbon dioxide content
- Sulphur dioxide content

The presence of carbon monoxide indicates that the fuel is not completely burned and energy is being wasted as unburned fuel.

The presence of oxygen is an indication of the excess air and the correct figure should be maintained.

The presence of carbon dioxide is another indication of the excess air and the correct figure should be maintained.

Sulphur dioxide is a pollutant and this indicates the quality of the fuel.

INSTRUMENTS

Old technology used an instrument called the Orsat Apparatus to analyse flue gas. This used chemical absorbents to determine the oxygen, carbon monoxide and carbon dioxide content of a flue gas (by volume).

Modern instruments like that shown here complete with its probe can measure the flue gas content and compute the combustion efficiency.

A fully controlled system would adjust the air/fuel ratio and match it to the load required. These principles are used in modern boilers and in engine management systems.

Figure 5

In the case of a boiler, the energy gained by the working fluid is the increase in enthalpy of the water or steam.

WORKED EXAMPLE No.13

A hot water boiler produces 0.24 kg/s of hot water at 80^oC from cold water at 18^oC. The boiler burns fuel oil at a rate of 1.6 g/s with a calorific value of 44 MJ/kg. Calculate the thermal efficiency of the boiler.

SOLUTION

The easiest way to find the increase in enthalpy of the water is to use the specific heat assumed to be 4.186 kJ/kg K.

$\Phi = m \, c \, \Delta\theta = 0.24 \times 4.186 \times (80 - 18) = 62.29$ kW
Heat released by combustion = m_f x C.V.
= 1.6×10^{-3} (kg/s) x 44 000 (kJ/kg) = 70.4 kW

$\eta_{th} = (62.29/70.4) \times 100 = 88.5\%$

WORKED EXAMPLE No.14

A steam boiler produces 0.2 kg/s at 50 bar and 400°C from water at 50 bar and 100°C. The boiler burns 5.3 m^3/min of natural gas with a calorific value of 38 MJ/ m^3. Calculate the thermal efficiency of the boiler.

SOLUTION

The enthalpy of the steam produced is found in tables at 50 bar and 400°C.
h_2 = 3196 kJ/kg
The enthalpy of the water is found as follows.
h_1 = Flow energy + internal energy = (pressure x volume) + 4186 x temp. $^{\circ}$C.
The volume of water is about 0.001 m^3 per kg.
h_1 = 50 x 10^5 x 0.001 + 4186 x 100 = 423600 J/kg or 423.6 kJ/kg

Energy given to the water and steam = m (h_2 - h_1) = 0.2 (3196 - 423.6) = 3111.3 kW
Energy from burning the fuel = Vol/s x C.V. = (5.3/60)(m^3/s) x 38 000 (kJ/m^3) = 3356.7 kW
η_{th} = (3111.3/3356.7) x 100 = 92.7%

SELF ASSESSMENT EXERCISE No. 4 - ENERGY OF COMBUSTION

1. When solid sulphur is burned to form sulphur dioxide gas (SO_2), the enthalpy of reaction is -297 kJ/kmol. When burned to form sulphur trioxide gas (SO_3) it is -396 kJ/kmol. Determine the enthalpy of reaction when SO_2 gas is burned to form SO_3 gas. (-99 kJ/kmol)

2. A bomb calorimeter is used to determine the Higher Calorific Value of a coal sample. The mass of the sample is 0.98 g. The total equivalent mass of water is 1.32 kg. The increase in temperature after ignition is 5.7 K.
    Calculate the higher calorific value. (32.14 MJ/kg)

3. A Boy's calorimeter gave the following results.

    Volume of gas burned (at standard conditions) = 0.004 m^3
    Volume of water heated in the same time = 1.9 kg
    Temperature rise of the water = 18.6 K
    Specific heat of water = 4.186 kJ/kg K

    Calculate the Higher calorific value. (36.98 MJ/m^3)

4. A hot water boiler produces 0. 4 kg/s of hot water at 70°C from cold water at 10°C. The boiler burns fuel oil at a rate of 3.2 g/s with a calorific value of 44 MJ/kg. The specific heat of water is 4.186 kJ/kg K.
    Calculate the thermal efficiency of the boiler. (71.4 %)

5. A steam boiler produces 3 kg/s at 70 bar and 500°C from water at 70 bar and 120°C. The boiler burns 17 m^3/min of natural gas with a calorific value of 38 MJ/ m^3. Calculate the thermal efficiency of the boiler. (80.8%)