

SOLUTIONS C106 THERMODYNAMIC, FLUID AND PROCESS ENGINEERING
Year 2004

Q1 A heat engine cycle using a perfect gas as the working fluid consists of the following three non-flow reversible processes.

- 1 – 2 adiabatic compression.
- 2 – 3 constant pressure heating.
- 3 – 1 constant volume cooling.

(a) sketch the $p = v$ and $T - s$ diagrams.

(b) show that the cycle efficiency is given by

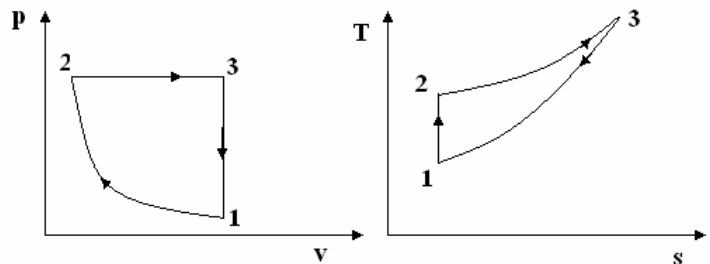
$$\eta = 1 - \frac{1 - \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}}{\gamma \left[1 - \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} \right]}$$

The thermal efficiency of any cycle is

given by $\eta = 1 - \frac{Q_{out}}{Q_{in}}$

The heat input is 2-3 $Q_{in} = mc_p(T_3 - T_2)$

The heat output is 3-1 $Q_{out} = mc_v(T_3 - T_1)$



$$\eta = 1 - \frac{mc_v(T_3 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{c_v(T_3 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{(T_3 - T_1)}{\gamma(T_3 - T_2)} \quad \text{noting } c_p/c_v = \gamma \quad \eta = 1 - \frac{\left(1 - \frac{T_1}{T_3}\right)}{\gamma \left(1 - \frac{T_2}{T_3}\right)}$$

Combining $pv^\gamma = c$ and $pv/T = C$ we can show that $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$

since $v_1 = v_3$ and $p_3 = p_2$ then $\frac{T_2}{T_1} = \left(\frac{v_3}{v_2}\right)^{\gamma-1}$ $\frac{v_2}{v_3} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}$ and $\frac{p_1}{p_3} = \left(\frac{T_1}{T_3}\right)^{\frac{\gamma}{\gamma-1}}$

From $\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3}$ $\frac{T_1}{T_3} = \frac{p_1 v_1}{p_3 v_3} = \frac{p_1}{p_3} = \left(\frac{T_1}{T_3}\right)^{\frac{\gamma}{\gamma-1}}$ $\frac{T_2}{T_3} = \frac{p_2 v_2}{p_3 v_3} = \frac{v_2}{v_3} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}$

$$\eta = 1 - \frac{\left(1 - \frac{T_1}{T_3}\right)}{\gamma \left(1 - \frac{T_2}{T_3}\right)} = 1 - \frac{1 - \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}}{\gamma \left[1 - \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} \right]}$$

Q3 (a) Show that for a perfect gas undergoing a process the entropy change is given by :

$$S_2 - S_1 = mc_p \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{p_2}{p_1}\right)$$

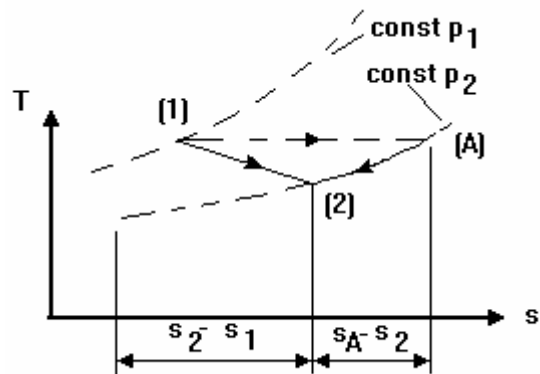
(b) Air flows steadily and adiabatically through a duct of varying area. At section X the pressure and temperature are 2.5 bar and 40°C respectively. A measuring instrument at that section indicates a velocity of 222 m/s without indicating the direction. At another section Y the and temperature are 4.5 bar and 60°C respectively.

(i) Calculate the velocity at Y.

(ii) Calculate the specific entropy change between the two sections and deduce the direction of flow.

(a) DERIVATION

The polytropic expansion is from (1) to (2) on the T-s diagram with different pressures, volumes and temperatures at the two points. The derivation is done in two stages by supposing the change takes place first at constant temperature from (1) to (A) and then at constant pressure from (A) to (2). You could use a constant volume process instead of constant pressure if you wish.



$$s_2 - s_1 = (s_A - s_1) - (s_A - s_2)$$

$$s_2 - s_1 = (s_A - s_1) + (s_2 - s_A)$$

For the constant temperature process

$$(s_A - s_1) = R \ln(p_1/p_A)$$

For the constant pressure process

$$(s_2 - s_A) = (c_p) \ln(T_2/T_A)$$

Hence

$$\Delta s = R \ln \frac{p_1}{p_A} + C_p \ln \frac{T_2}{T_A} + s_2 - s_1 \quad \text{Since } p_A = p_2 \text{ and } T_A = T_1$$

Then
$$S_2 - S_1 = mc_p \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{p_2}{p_1}\right)$$

(b) (i) Apply the steady flow energy equation between X and Y

$$T_x + \frac{u_x^2}{2c_p} = T_y + \frac{u_y^2}{2c_p} \quad 313 + \frac{222^2}{2c_p} = 333 + \frac{u_y^2}{2c_p} \quad 313 - 333 + \frac{222^2}{2c_p} = \frac{u_y^2}{2c_p}$$

$$-20 + \frac{222^2}{2c_p} = \frac{u_y^2}{2c_p} \quad -40c_p + 222^2 = u_y^2$$

The mean temperature is 323K and from the tables $c_p = 1.0051 \text{ kJ/kg K}$

$$u_y = \sqrt{(222^2 - 40 \times 1005.1)} = 95.29 \text{ m/s}$$

(ii)
$$s_2 - s_1 = c_p \ln\left(\frac{T_y}{T_x}\right) - R \ln\left(\frac{p_y}{p_x}\right) \quad R = 287.1 \text{ J/kg K}$$

$$s_2 - s_1 = 1005.1 \ln\left(\frac{333}{313}\right) - 287.1 \ln\left(\frac{4.5}{2.5}\right) = -106.5 \text{ J/kg K}$$
 Since this is negative, the flow must be the other way from Y to X.

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Q4. In a nuclear power station, heat from the nuclear reactor is transferred by circulating carbon dioxide gas through the reactor and then to the boiler where steam is generated. The steam is used in a Rankine Cycle with superheat where it expands isentropically from 100 bar pressure and 600°C to 7 bar. The exhaust steam is condensed by using it in a district heating scheme and returns as saturated water at 7 bar and is returned to the boiler.

(a) The power output of the turbine is 170 MW. Calculate the rate at which heat is used by the district heating scheme. Draw a block diagram of the system.

(b) The carbon dioxide is cooled from 620°C to 310°C in the boiler. Calculate the mass flow rate of carbon dioxide if the mean specific heat capacity c_p is 1.143 kJ/kg K. Neglect the feed pump work.

$h_2 = 3624$ kJ/kg
 $s_2 = 6.85$ kJ/kg K
 $s_3 = 6.85 = 1.992 + x(4.717)$ hence $x > 1$ so it is still superheated at (3)

Interpolation from the tables is required to find h_3 .

7 bar

| | | | |
|--|-------|----------|-------|
| Temp | 165 | θ | 200 |
| s | 6.709 | 6.85 | 6.888 |
| h | 2764 | h_3 | 2846 |
| $\theta = 165 + 35(6.85 - 6.709)/(6.88 - 6.709)$ | | | |
| $\theta = 165 + 28.86$ | | | |

$$h_3 = 2764 + (28.86/35)(2846 - 2764) = 2832 \text{ kJ/kg}$$

TURBINE

$$P = 170\,000 \text{ kW} = m(3624 - 2832)$$

$$m = 214.54 \text{ kg/s}$$

Heat To District Heating $h_4 = 697$ kJ/kg

$$\Phi = m(h_3 - h_2) = 214.54(2832 - 697) = 458047 \text{ kW or } 458 \text{ MW}$$

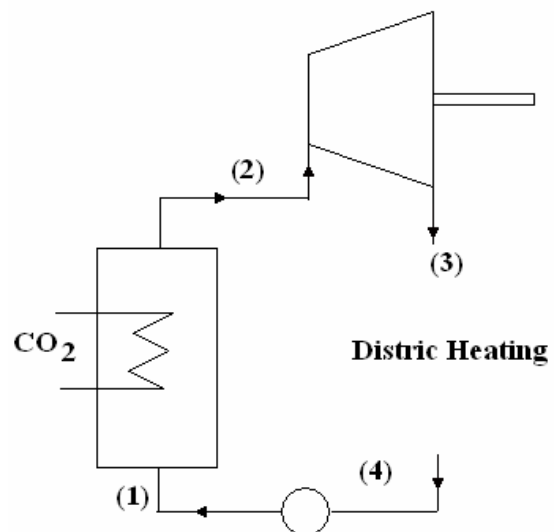
BOILER $h_1 = h_4$

$$\Phi = m(h_2 - h_1) = 214.54(3624 - 697) = 627960 \text{ kW}$$

If the heat lost by the carbon Dioxide is the same then $627960 = m_g c_p (\Delta T)$

$$627960 = 1.143 m_g (620 - 310) \quad m_g = 1772 \text{ kg/s}$$

These answers do not quite agree with the examiners but are close.

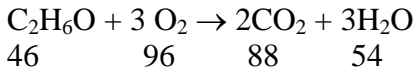


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Q5 Ethyl Alcohol C_2H_6O is used as a fuel in an internal combustion engine.

- (a) Calculate the stoichiometric air/fuel ratio
- (b) The engine runs with an air fuel ratio of 10/1. Determine the wet exhaust gas analysis.
- (c) Suggest an environmental benefit from using this fuel.

(Note this is by mass but not stated as such in the exam)



Mass of air required = $(96/46) \div 0.233 = 8.957$ stoichiometric ratio is 8.957/1

Actual air = 10 kg

Content are

| | | |
|----------------|---|--------|
| Nitrogen | $0.767 \times 10 = 7.670$ | 69.73% |
| Oxygen | $1.063 \times 0.233 = 0.248 \text{ kg}$ | 2.25% |
| Carbon Dioxide | $88/46 = 1.913 \text{ kg}$ | 17.39% |
| Water Vapour | $54/46 = 1.174 \text{ kg}$ | 10.67% |
| Totals | 11 kg | 100% |

As the fuel contains no sulphur it is better than most and also contains some oxygen saving on the air required. The low carbon content and high hydrogen content is also beneficial reducing the CO_2 produced.

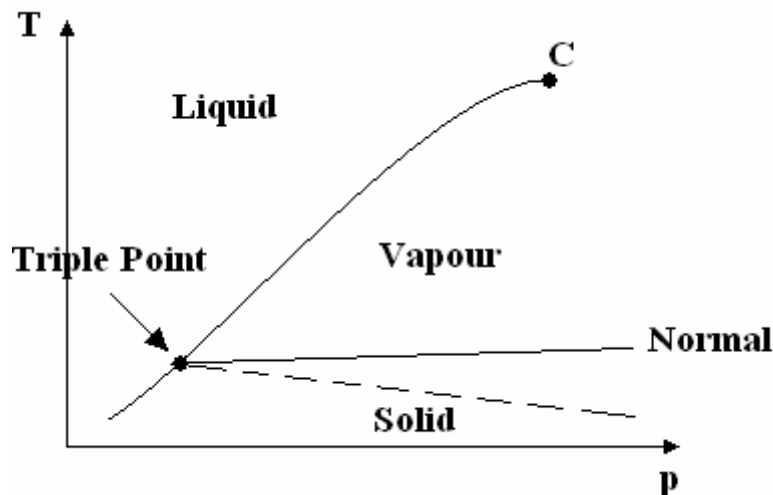
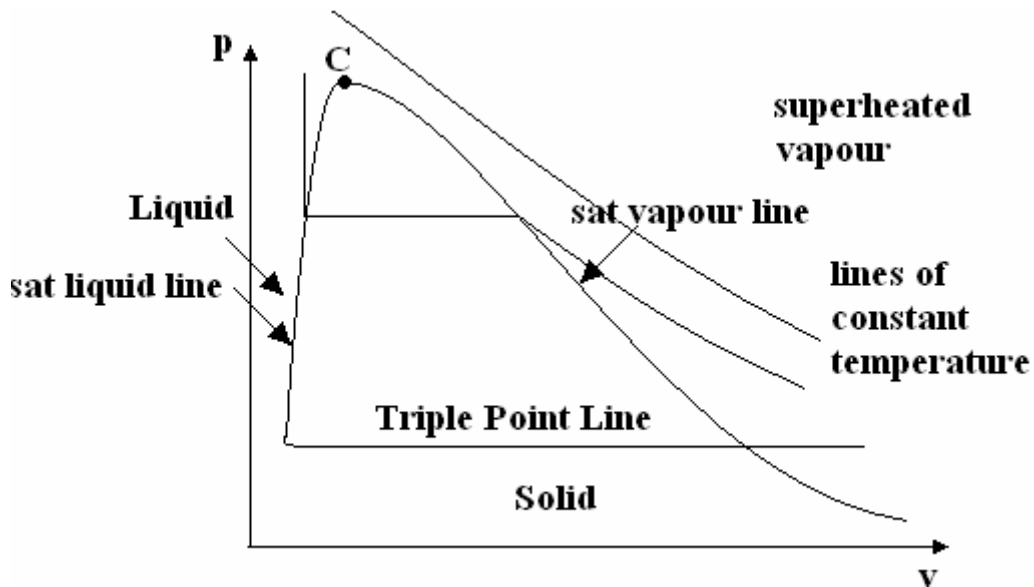
Q6

(a) Sketch a pressure –specific volume diagram for a pure substance which contracts on freezing, showing the lines of constant temperature. Label the phase regions and indicate clearly the:

- (i) saturated liquid line.
- (ii) saturated vapour line.
- (iii) critical point.
- (iv) triple point.

(b) Sketch a second diagram choosing pressure and temperature as the axis such that the triple point appears as a point. Label the solid, vapour and liquid regions.

(c) Add a dotted line to the $p - T$ diagram to show the behaviour of substance that expands on freezing.



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Q8 Steam at a pressure of 50 bar and temperature 450°C undergoes a non-flow isentropic expansion to 6 bar.

- (a) Determine the specific work done.
 (b) If internal energy values were not available, it might be possible to calculate the specific work done by assuming a reversible polytropic expansion such that $pv^n = C$. By considering the end state conditions, determine the index n and re-evaluate the specific work done.
 (c) Suggest a reason for the difference between the two answers.

From the steam tables at 50 b and 450°C we find

$$s_1 = 6.818 \text{ kJ/kg K} \quad v_1 = 0.0632 \text{ m}^3/\text{kg} \quad u_1 = 3000 \text{ kJ/kg}$$

At 6 bar $s_g = 6.761 \text{ kJ/kg K}$ so the steam is still superheated and we need to interpolate to find u_2

6 bar

| | | | |
|------|--------|----------|--------|
| Temp | 158.8 | θ | 200 |
| s | 6.761 | 6.818 | 6.968 |
| u | 2568 | u_2 | 2640 |
| v | 0.3156 | v_2 | 0.3522 |

$$\theta = 158.8 + (200 - 158.8)(6.818 - 6.761) / (6.968 - 6.761) = 158.8 + 11.35$$

$$u_2 = 2568 + \{11.35 / (200 - 158.8)\} \{2640 - 2568\} = 2587.8 \text{ kJ/kg}$$

Non Flow Energy Equation

$$Q + W = \Delta U \quad \text{but } Q = 0$$

$$\text{Specific work} = \Delta u = m(2587.8 - 3000) = -412.2 \text{ kJ/kg}$$

We need to determine v_2 by the same interpolation.

$$v_2 = 0.3156 + \{11.35 / (200 - 158.8)\} \{0.3522 - 0.3156\} = 0.3256 \text{ m}^3/\text{kg}$$

$$\text{If } p_1 v_1^n = p_2 v_2^n$$

$$n = \frac{\log\left(\frac{p_1}{p_2}\right)}{\log\left(\frac{v_2}{v_1}\right)} = \frac{\log(50/6)}{\log(0.3256/0.0632)} = 1.293$$

$$w = \frac{p_2 v_2 - p_1 v_1}{n-1} = \frac{6 \times 10^5 \times 0.3256 - 50 \times 10^5 \times 0.0632}{1.293 - 1} = -411741 \text{ J/kg}$$

$$w = -411.741 \text{ kJ/kg}$$

The errors are due to the fact that the expansion takes place close to the saturated vapour curve.