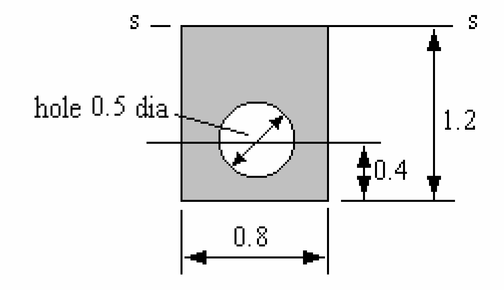
THERMODYNAMICS, FLUID AND PLANT PROCESSES

The tutorials are drawn from other subjects so the solutions are identified by the appropriate tutorial.

FLUID MECHANICS – MOMENTS OF AREA
SAE SOLUTIONS

SELF ASSESSMENT EXERCISE No.1

1. Find the distance of the centroid from the axis s – s. All dimensions are in metres.



Area of rectangle = 0.8 x 1.2 = 0.96

Area of circle = $\pi(0.5^2)/4$ = 0.1963

Area of shape = 0.96 - 0.1963 = 0.7636 m^2

$\bar{y}$ = 0.6 m for the rectangle and 0.8 m for the circle.

The moment of area for the rectangle about axis s – s is 0.96 x 0.6 = 0.576

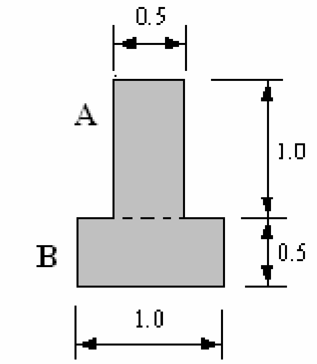
The moment of area for the circle about axis s – s is 0.1963 x 0.8 = 0.1571

The moment of area for the shape is 0.576 - 0.1571 = 0.419 m^3

$\bar{y}_{ss}$ = 0.419/0.7636 = 0.549 m

2. Find the distance of the centroid from the bottom edge. All dimensions are in metres.

There are several ways to solve this. As the tabular method is preferred for solving second moments then this method will be used here.



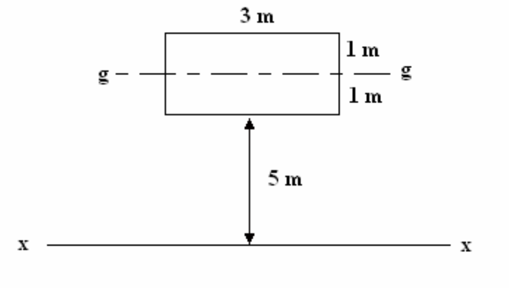
Part	Area m^2	$\bar{y}$ m	$A\bar{y}$ m^3
A	0.5	1	0.5
B	0.5	0.25	0.125
Totals	1.0		0.625

For the whole shape $\bar{y}$ = 0.625/1 = 0.625 m

SELF ASSESSMENT EXERCISE No.2

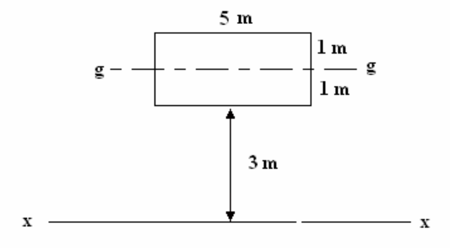
1. Find the second moment of area of a rectangle 3 m wide by 2 m deep about an axis parallel to the longer edge and 5 m from it.

$I_{gg} = BD^3/12 = 3 \times 2^3/12 = 2\ m^4$
$I_{xx} = I_{gg} + A\bar{y}^2 = 2 + (3 \times 2) \times 6^2 = 218\ m^4$



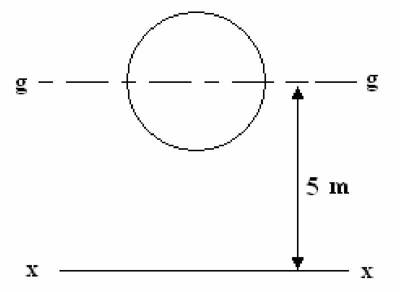
2. Find the second moment of area of a rectangle 5 m wide by 2m deep about an axis parallel to the longer edge and 3 m from it.

$I_{gg} = BD^3/12 = 5 \times 2^3/12 = 3.33\ m^4$
$I_{xx} = I_{gg} + A\bar{y}^2 = 3.333 + (5 \times 2) \times 4^2 = 163.33\ m^4$



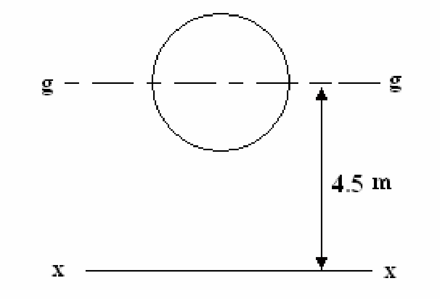
3. Find the second moment of area of a circle 2 m diameter about an axis 5 m from the centre.

$I_{gg} = \pi D^4/64 = \pi \times 2^4/64 = 0.785\ m^4$
$A = (\pi D^2/4) = 3.142\ m^2$
$I_{xx} = I_{gg} + A\bar{y}^2 = 0.785 + 3.142 \times 5^2$
$I_{xx} = 79.3\ m^4$

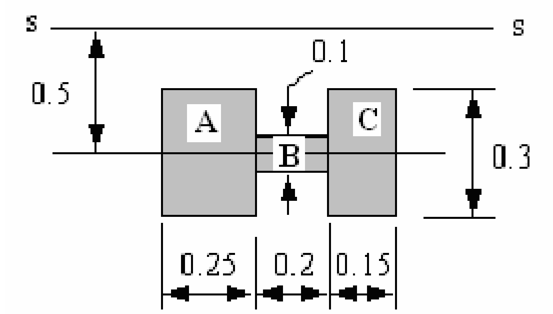


4. Find the second moment of area of a circle 5 m diameter about an axis 4.5 m from the centre.

$I_{gg} = \pi D^4/64 = \pi \times 5^4/64 = 30.68\ m^4$
$A = (\pi D^2/4) = 19.635\ m^2$
$I_{xx} = I_{gg} + A\bar{y}^2 = 30.68 + 19.635 \times 4.5^2$
$I_{xx} = 428.29\ m^4$



5. Find the 2nd moment of area for the shape shown the about the axis s – s. All the dimensions are in metres.

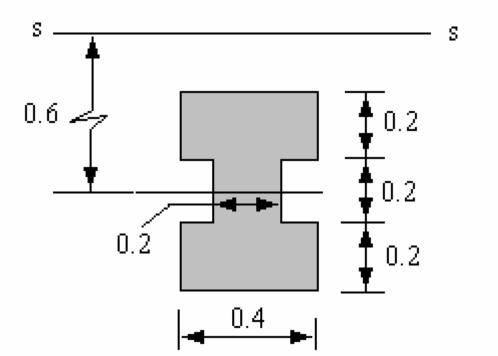


The shape is symmetrical about the centre line so the best solution is to find I_{gg} and move it to the s – s axis. $\bar{y}$ = 0.5 m for all parts

Part	Area	$\bar{y}$	$A\bar{y}^2$	$I_{gg} = BD^3/12$	$I_{gg} + A\bar{y}^2$
A	0.075	0.5	0.01875	0.0005625	0.0193125
B	0.02	0.5	0.005	0.0000167	0.0050167
C	0.045	0.5	0.01125	0.0003375	0.0115875
Totals					0.0359167

Answer $I_{ss} = 35.92 \times 10^{-3}$ m^4

6. Find the 2nd moment of area for the shape shown the about the axis s – s. All the dimensions are in metres.



The simplest way to do this is to find the answer for the outer rectangle and subtract the answer for the rectangle that makes the missing parts. In effect, this is the same as a hollow rectangle.

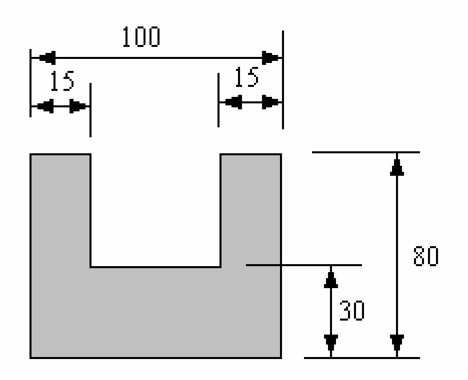
Outer Rectangle

$A = 0.4 \times 0.6 = 0.24$ m^2
$\bar{y} = 0.6$
$A\bar{y}^2 = 0.0864$ m^4
$I_{gg} = 0.4 \times 0.6^3/12 = 0.0072$ m^4
$I_{ss} = I_{gg} + A\bar{y}^2 = 0.0072 + 0.0864 = 0.0936$ m^4

The missing parts at the side are the same as a rectangle 0.2 m x 0.2 m
$A = 0.2 \times 0.2 = 0.04$ m^2
$\bar{y} = 0.6$
$A\bar{y}^2 = 0.0144$ m^4
$I_{gg} = 0.2 \times 0.2^3/12 = 0.0001333$ m^4
$I_{ss} = I_{gg} + A\bar{y}^2 = 0.0001333 + 0.0144 = 0.014533$ m^4

For the shape $I_{ss} = 0.0936 - 0.014533 = 0.07907$ m^4

7. Find the position of the centroid for the shape shown from the bottom edge and the 2^{nd} moment of area about the centroid.



This requires the tabular method.

Part	Area	$\bar{y}$	$A\bar{y}$
A	1200	40	48000
B	2100	15	31500
C	1200	40	48000
Total	4500		127500

$\bar{y}$ = 127500/4500 = 28.333 mm from the bottom.

For the second moment of area we need to compute $\bar{y}$ this now being the distance between the parallel axis.

Part	Area	$\bar{y}$	$A\bar{y}^2$	$BD^3/12$	$A\bar{y}^2 + BD^3/12$
A	1200	11.667	163343	640000	803343
B	2100	13.333	373315	157500	530815
C	1200	11.667	163343	640000	803343
Total					2137501

I_{gg} for the shape is 2.138×10^{-6} m^4

An alternative method is as follows. The second moment about the base may be done by finding the answer for the outer rectangle and the missing rectangle and subtracting.

Outer rectangle.

$I_{gg} = 100 \times 80^3/12 = 4.2667 \times 10^6$ mm^4
The centroid is 40 mm from the base.
$A = 100 \times 80 = 8000$ mm^2
$\bar{y} = 40$
$A\bar{y}^2 = 12.8 \times 10^6$ mm^4
$I_{base} = I_{gg} + A\bar{y}^2 = 4.2667 \times 10^6 + 12.8 \times 10^6 = 17.0667 \times 10^6$ mm^4

The missing part is a rectangle 70 mm x 50 mm

$I_{gg} = 70 \times 50^3/12 = 0.729167 \times 10^6$ mm^4
$A = 70 \times 50 = 3500$ mm^2
$\bar{y} = 55$
$A\bar{y}^2 = 10.5875 \times 10^6$ mm^4
$I_{base} = I_{gg} + A\bar{y}^2 = 0.729167 \times 10^6 + 10.5875 \times 10^6 = 11.317 \times 10^6$ mm^4

For the shape $I_{base} = 17.0667 \times 10^6 - 11.317 \times 10^6 = 5.75 \times 10^6$ mm^4

About the centroid $I_{gg} = 5.75 \times 10^6 - A\bar{y}^2$ $\bar{y}$ is 28.33 from the base.
$I_{gg} = 5.75 \times 10^6 - 4500 \times 28.33^2 = 2.138 \times 10^6$ mm^4

THERMODYNAMICS, FLUID AND PLANT PROCESSES

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THERMODYNAMICS TUTORIAL 1 – LIQUIDS – VAPOURS - GASES
SAE SOLUTIONS

SELF ASSESSMENT EXERCISE No.1

All pressures are absolute.

1. Calculate the density of air at 1.013 bar and 15 ^oC if R = 287 J/kg K.

$\rho = p/RT = 1.103 \times 10^5/(287 \times 288) = 1.226 \text{ kg/m}^3$

2. Air in a vessel has a pressure of 25 bar, volume 0.2 m^3 and temperature 20^oC. It is connected to another empty vessel so that the volume increases to 0.5 m^3 but the temperature stays the same. Taking R = 287 J/kg K. Calculate

i. the final pressure. (10 bar)
ii. the final density. (11.892 kg/m^3)

$p_2 = p_1V_1T_2/(V_2T_1) = 25 \times 0.2 \times 293/(0.5 \times 293) = 10 \text{ bar}$
$\rho_2 = p_2/RT_2 = 10 \times 10^5/(287 \times 293) = 11.892 \text{ kg/m}^3$

3. 1 dm^3 of air at 20^oC is heated at constant pressure of 300 kPa until the volume is doubled. Calculate

i. the final temperature. (586 K)
ii. the mass. (3.56 g)

$T_2 = p_2V_2T_1/(p_1V_1) = V_2T_1/V_1) = 586 \text{ K}$
$m = p_2 V_2/RT_2 = 300 \times 10^3 \times 0.002/(287 \times 586) = 0.00356 \text{ kg}$

4. Air is heated from 20^oC and 400 kPa in a fixed volume of 1 m^3. The final pressure is 900 kPa. Calculate

i. the final temperature.(659 K)
ii. the mass. (4.747 kg)

$T_2 = p_2V_2T_1/(p_1V_1) = 900 \times 1/(400 \times 1)) = 659 \text{ K}$

$m = p V/RT = 400 \times 10^3 \times 1/(287 \times 293) = 4.757 \text{ kg}$

5. 1.2 dm^3 of gas is compressed from 1 bar and 20^oC to 7 bar and 90^oC.
Taking R = 287 J/kg K calculate

i. the new volume. (212 cm^3)
ii. the mass. (1.427 g)

$V_2 = p_1V_1T_2/(p_2T_1) = 10^5 \times 1.2 \times 10^{-3} \times 363/(7 \times 10^5 \times 293) = 212 \times 10^{-6} \text{ m}^3$

$m = p V/RT = 10^5 \times 1.2 \times 10^{-3} /(287 \times 293) = 0.001427 \text{ kg}$

SELF ASSESSMENT EXERCISE No. 2

1. A gas compressor draws in 0.5 m^3/min of Nitrogen at 10^oC and 100 kPa pressure. Calculate the mass flow rate.

(0.595 kg/min)

$$m = \frac{pV\tilde{N}}{R_oT} = \frac{100 \times 10^3 \times 0.5 \times 28}{8314 \times 283} = 0.595 \text{ kg}$$

2. A vessel contains 0.5 m^3 of Oxygen at 40^oC and 10 bar pressure.
Calculate the mass.
(6.148 kg)

$$m = \frac{pV\tilde{N}}{R_oT} = \frac{10 \times 10^5 \times 0.5 \times 32}{8314 \times 313} = 6.148 \text{ kg}$$

SELF ASSESSMENT EXERCISE No. 3

For air take c_p= 1005 J/kg K and c_v= 718 J/kg K unless otherwise stated.

1. 0.2 kg of air is heated at constant volume from 40^oC to 120^oC. Calculate the heat transfer and change in internal energy. (11.49 kJ for both)

$Q = mc_v\Delta T = 0.2 \times 718 \times (120 - 40) = 11488$ J
$U = Q = 11488$ J

2. 0.5 kg of air is cooled from 200^oC to 80^oC at a constant pressure of 5 bar. Calculate the change in internal energy, the change in enthalpy, the heat transfer and change in flow energy.
(-43 kJ), (-60.3 kJ), (-17.3 kJ)

$\Delta U = mc_v\Delta T = 0.5 \times 718 \times (-120) = -43000$ J
$\Delta H = mc_p\Delta T = 0.5 \times 1005 \times (-120) = -60300$ J
$\Delta FE = \Delta H - \Delta U = -17300$ J

3. 32 kg/s of water is heated from 15^oC to 80^oC. Calculate the heat transfer given c = 4186J/kg K.

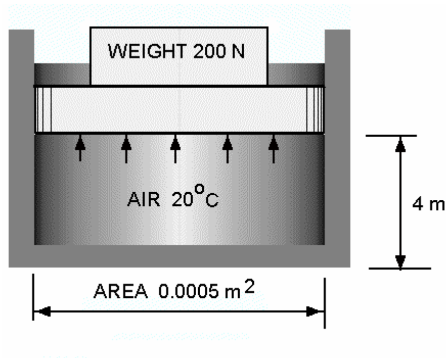
$Q = m\, c\, \Delta T = 32 \times 4186 \times 65 = 8707$ kW

4. Air is heated from 20^oC to 50^oC at constant pressure. Using your fluid tables (pages 16 and 17) determine the average value of c_p and calculate the heat transfer per kg of air. (30.15 kJ)

$Q = mc_p\Delta T = 1 \times 1005 \times (50 - 20) = 30150$ J

5. The diagram shows a cylinder fitted with a frictionless piston. The air inside is heated to 200°C at constant pressure causing the piston to rise. Atmospheric pressure outside is 100 kPa. Determine :

i. the mass of air. (11.9 g)
ii. the change in internal energy. (1.537 kJ)
iii. the change in enthalpy. (2.1553 kJ)
iv. the pressure throughout. (500 kPa)
v. the change in volume. (1.22 dm^3)



Pressure = weight/Area = 200/0.0005 = 400 kPa

This is a gauge pressure since atmosphere acts on the other side (100 kPa)
p = 500 kPa absolute pressure.

$R = c_p - c_v = 287$ J/kg K

m = pV/RT = 500000 x 0.002/(287 x 293) = 0.0119 kg

$\Delta U = m\, c_v\, \Delta T =$ 0.0119 x 718 (200 – 20) = 1537 J

$\Delta H = m\, c_p\, \Delta T =$ 0.0119 x 1005 (200 – 20) = 2152.7 J

$V_2 = mRT_2/p$ = 0.0119 x 287 x (200+273)/500000 = 0.0032275 m^3

$\Delta V = 0.00122\ m^3$

6. Define the meaning of a mole as a means of measuring the amount of a substance.

Calculate the volume occupied at a temperature of 25ºC and a pressure of 3 bar, by 60 kg of (i) Oxygen gas, O_2, (ii)atomic oxygen gas,O, and (iii) Helium gas, H_e. The respective molar masses ,M, and the molar heats at constant volume, C_V, of the three gases, and the molar (universal) gas constant, R_M, are as follows:

	M (kg/kmol)	c_V(kJ/kmol K)	R_M(kJ/kmol K)
O_2	32	20.786	8.3144
O	16	12.4716	8.3144
H_e	4	12.4716	8.3144

Go on to calculate the values of the specific heats C_p and C_V. Using these values, calculate the specific gas constant R for all three gases. Show not only numerical work, but also the manipulation of units in arriving at your results.

1 mole is the number of grams numerically equal to the mean molecular mass e.g. 32 g of oxygen is a mole.

$$V = \frac{mR_oT}{\tilde{N}\,p} \qquad c_v = c_v\ (\text{molar})/\check{N}$$

O_2 $$V = \frac{60 \text{ x } 8314.3 \text{ x } 298}{32 \text{ x } 3 \text{ x } 10^5} = 15.48\,m^3 \qquad c_v = 20.786/32 = 0.6495 \text{ kJ/kg K}$$

O $$V = \frac{60 \text{ x } 8314.3 \text{ x } 298}{16 \text{ x } 3 \text{ x } 10^5} = 30.96\,m^3 \qquad c_v = 12.4716/16 = 0.7794 \text{ kJ/kg K}$$

He $$V = \frac{60 \text{ x } 8314.3 \text{ x } 298}{4 \text{ x } 3 \text{ x } 10^5} = 123.8\,m^3 \qquad c_v = 12.4716/4 = 3.1179 \text{ kJ/kg K}$$

$R = R_o/\check{N}$ $\qquad c_p = R + c_v$

O_2 $\quad R = 8.3144/32 = 0.2598$ kJ/kg K $\qquad c_p = 0.9093$ kJ/kg K

O $\quad R = 8.3144/16 = 0.51964$ kJ/kg K $\qquad c_p = 1.299$ kJ/kg K

He $\quad R = 8.3144/4 = 2.0786$ kJ/kg K $\qquad c_p = 5.1965$ kJ/kg K

SELF ASSESSMENT EXERCISE No. 4

1. Using your steam tables, plot a graph of h_f and h_g against pressure horizontally and mark on the graph the following:

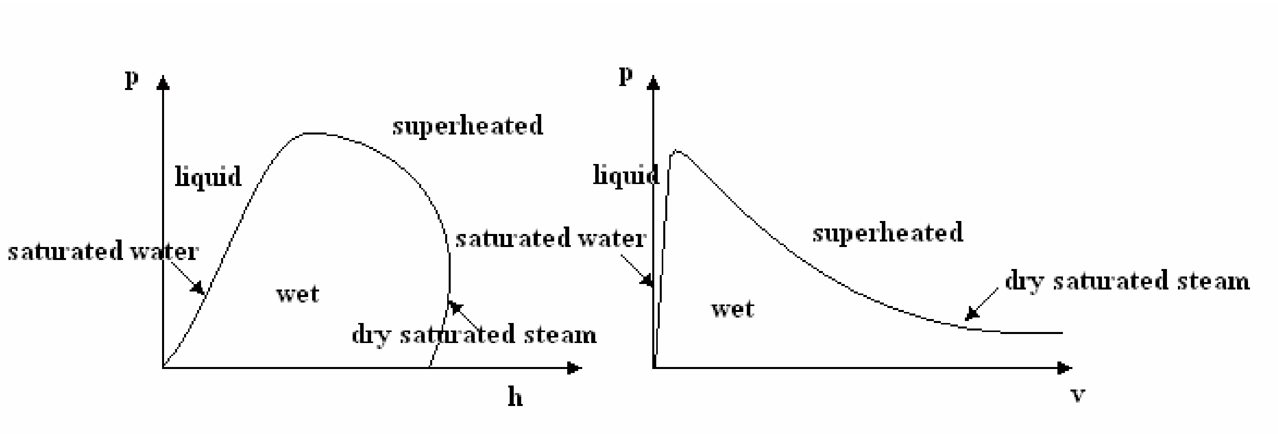
   i. the superheat region
   ii. the wet steam region.
   iii. the liquid region.
   iv. the critical point.

Also label the saturation curve with dry saturated steam and saturated water.

2. Using your steam tables, plot a graph of v_g horizontally against pressure vertically. Also plot v_f
   Show on the graph:

   i. the superheated steam region.
   ii. the wet vapour region.
   iii. the liquid region.
   iv. the critical point.

   Also label the saturation curve with dry saturated steam and saturated water.



SELF ASSESSMENT EXERCISE No. 5

Use tables and charts to do the following.

1. What is the saturation temperature at 32 bars ?
2. What is the specific enthalpy and internal energy of saturated water at 16 bars?
3. What is the specific enthalpy and internal energy of dry saturated steam at 16 bars?
4. Subtract the enthalpy in 2 from that in 3 and check that it is the latent enthalpy h_{fg} at 16 bars in the tables.
5. What is the specific enthalpy and internal energy of superheated steam at 10 bar and 400°C ?
6. What is the specific volume of dry saturated steam at 20 bars ?
7. What is the volume of 1 kg of wet steam at 20 bars with dryness fraction x=0.7?
8. What is the specific enthalpy and internal energy of wet steam at 20 bars with a dryness fraction of 0.7 ?
9. What is the specific volume of superheated steam at 15 bars and 500°C.
10. What is the volume and enthalpy of 3 kg of wet steam at 5 bar with dryness fraction 0.9.
11. Using the p-h chart for arcton 12 (freon 12) determine
a. the specific enthalpy at 2 bar and 70% dry. (x = 0.7).
b. the specific enthalpy at 5 bar and 330 K
c. the specific enthalpy of the liquid at 8 bars and 300 K.
12. What is the enthalpy of 1.5 kg of superheated steam at 8 bar and 350°C ?
13. What is the internal energy of 2.2 kg of dry saturated steam at 11 bars ?
14. What is the volume of 0.5 kg of superheated steam at 15 bar and 400°C ?

Answers to Assignment 5.

Compare your answers with those below. If you find your answers are different, go back and try again referring to the appropriate section of your notes.

1. 237.4°C.
2. 859 and 857 kJ/kg.
3. 2794 and 2596kJ/kg.
3. 1935 kJ/kg and 1739 kJ/kg.
5. 3264 and 2957 kJ/kg.
6. 0.09957 m^3/kg.
7. 0.0697 m^3/kg.
8. 2232 and 2092.1 kJ/kg.
9. 0.2351 m^3

10. 1.012 m^3, 7.61 MJ, 7.11 MJ.
11 a. 190 kJ/kg. b.286 kJ/kg. c. 130 kJ/kg
12. 4.74 MJ.
13. 5.69 MJ.
14. 0.101m^3.

THERMODYNAMICS, FLUID AND PLANT PROCESSES

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THERMODYNAMICS TUTORIAL 2 – THERMODYNAMIC PRINCIPLES
SAE SOLUTIONS

SELF ASSESSMENT EXERCISE No.1

1. 1 kg/s of steam flows in a pipe 40 mm bore at 200 bar pressure and 400^{o}C.

i. Look up the specific volume of the steam and determine the mean velocity in the pipe.
ii. Determine the kinetic energy being transported per second.
iii. Determine the enthalpy being transported per second.

$A = \pi D^2/4 = \pi \times (0.04)^2/4 = 0.001257 \text{ m}^2$
$p = 200\text{bar} \quad \theta = 400^oC \quad V = m\, v = 0.00995 \text{ m}^3/s$
$u = V/A = 0.00995/0.001257 = 7.91$ m/s
$KE = mu^2/2 = 1 \times 7.91^2/2 = 31.3$ W
h = 2819 kJ/kg (from the tables)

SELF ASSESSMENT EXERCISE No.2

1. The shaft of a steam turbine produces 600 Nm torque at 50 rev/s. Calculate the work transfer rate from the steam.

$SP = 2\pi NT = 2\pi \times 50 \times 600 = 188.5$ W

2. A car engine produces 30 kW of power at 3000 rev/min. Calculate the torque produced.

$T = P/2\pi N = 30000/(2\pi \times 50) = 95.5$ Nm

SELF ASSESSMENT EXERCISE No.3

1. A non-flow system receives 80 kJ of heat transfer and loses 20 kJ as work transfer. What is the change in the internal energy of the fluid?

   $\Delta U = Q + W = 80 + (-20) = 60$ kJ

2. A non-flow system receives 100 kJ of heat transfer and also 40 kJ of work is transferred to it. What is the change in the internal energy of the fluid?

   $\Delta U = Q + W = 100 + 40 = 140$ kJ

3. A steady flow system receives 500 kW of heat and loses 200 kW of work. What is the net change in the energy of the fluid flowing through it?

   $\Phi + P = \Delta E/s \qquad 500 + (-200) = 300 \text{ kW} = \Delta E/s$

4. A steady flow system loses 2 kW of heat also loses 4 kW of work. What is the net change in the energy of the fluid flowing through it?

   $\Phi + P = \Delta E/s \qquad -2 + (-4) = -6\text{kW} = \Delta E/s$

5. A steady flow system loses 3 kW of heat also loses 20 kW of work. The fluid flows through the system at a steady rate of 70 kg/s. The velocity at inlet is 20 m/s and at outlet it is 10 m/s. The inlet is 20 m above the outlet. Calculate the following.

   i. The change in K.E./s (-10.5 kW)
   ii. The change in P.E/s (-13.7 kW)
   iii. The change in enthalpy/s (1.23 kW)

Φ = -3 kW — ΔKE/s =(m/2)($v_2^2 - v_1^2$)
P = -20 kW — ΔKE/s =(70/2)($10 - 20^2$) = -10.5 kW
m = 70 kg/s
v_1 =20 m/s — ΔPE/s = mg($z_2 - z_1$)
v_2 =10 m/s — ΔPE/s = 70 x 9.81(0- 20) = -13.734 kW
z_1 = 20 m
z_2 = 0 m

$\Phi + P = \Delta H/s + \Delta KE/s + \Delta PE/s$

(-3) + (-20) = ΔH/s + (-10.5) + (-13.734)
ΔH/s = 1.234 kW

SELF ASSESSMENT EXERCISE No.4

1. Gas is contained inside a cylinder fitted with a piston. The gas is at 20°C and has a mass of 20 g. The gas is compressed with a mean force of 80 N which moves the piston 50 mm. At the same time 5 Joules of heat transfer occurs out of the gas. Calculate the following.
   i. The work done.(4 J)
   ii. The change in internal energy. (-1 J)
   iii. The final temperature. (19.9°C) Take c_V as 718 J/kg K

Q + W = ΔU = — W = F x = 80 x 0.05 = 4 J
-5 + 4 =ΔU = -1 J — $\Delta U = m\, c_v (\theta_2 - 20) = 0.02 \times 718 \times (\theta_2 - 20)$ — $\theta_2 = 19.93\ ^{o}C$

2. A steady flow air compressor draws in air at 20°C and compresses it to 120°C at outlet. The mass flow rate is 0.7 kg/s. At the same time, 5 kW of heat is transferred into the system. Calculate the following.
   i. The change in enthalpy per second. (70.35 kW)
   ii. The work transfer rate. (65.35 kW) Take c_p as 1005 J/kg K.

   Φ + P = ΔH/s (ignore PE and KE)
   $\Delta H/s = m\, c_p\, \Delta T = 0.7 \times 1.005\ (120 - 20) = 70.35$ kW
   5 + P = 70.35 — P = 65.35 kW

3. A steady flow boiler is supplied with water at 15 kg/s, 100 bar pressure and 200°C. The water is heated and turned into steam. This leaves at 15 kg/s, 100 bar and 500°C. Using your steam tables, find the following.

   i. The specific enthalpy of the water entering. (856 kJ/kg)
   ii. The specific enthalpy of the steam leaving. (3373 kJ/kg)
   iii. The heat transfer rate. (37.75 kW)

   $\Delta H/s = 15\ (h_2 - h_1) = 15(3373 - 856) = 37755$ kW
   Φ + P = ΔH/s — P = 0 — Φ = 37.755 MW

4. A pump delivers 50 dm^3/min of water from an inlet pressure of 100 kPa to an outlet pressure of 3 MPa. There is no measurable rise in temperature. Ignoring K.E. and P.E, calculate the work transfer rate. (2.42 kW)

$\Phi + P = \Delta H/s + \Delta KE/s + \Delta PE/s \qquad \Delta KE/s = \Delta PE/s = 0$
$\Phi = 0$
$P = \Delta H/s = \Delta FE + \Delta U \qquad \Delta U = 0$
$P = \Delta FE = p_2V_2 - p_1V_1 \qquad V_1 = V_2 = 50 \times 10^{-3}\ m^3/s$
$P = 50 \times 10^{-3} (3 \times 10^6 - 100 \times 10^3) = 144$ kW

5. A water pump delivers 130 dm^3/minute (0.13 m^3/min) drawing it in at 100 kPa and delivering it at 500 kPa. Assuming that only flow energy changes occur, calculate the power supplied to the pump. (860 W)

$\Phi + P = \Delta E/s = \Delta FE/s \qquad \Phi = 0$
$P = \Delta FE/s = (0.13/60)(500 - 100) \times 10^3 = 866.7$ kW

6. A steam condenser is supplied with 2 kg/s of steam at 0.07 bar and dryness fraction 0.9. The steam is condensed into saturated water at outlet. Determine the following.

i. The specific enthalpies at inlet and outlet. (2331 kJ/kg and 163 kJ/kg)
ii. The heat transfer rate. (4336 kW)

$h_1 = h_f + x\ h_{fg}$ at 0.07 bar $\qquad h_1 = 163 + 0.9 (2409) = 2331.1$ kJ/kg
$h_2 = h_f = 163$ kJ/kg
$\Phi = m(h_2 - h_1) = 2(163 - 2311.1) = -4336.2$ kJ

7. 0.2 kg/s of gas is heated at constant pressure in a steady flow system from 10°C to 180°C. Calculate the heat transfer rate Φ. (37.4 kW) $\qquad C_p = 1.1$ kJ/kg K

$Q = 0.2 \times 1.1 (180 - 10) = 37.4$ kJ

8. 0.3 kg of gas is cooled from 120°C to 50°C at constant volume in a closed system. Calculate the heat transfer. (-16.8 kJ) $\qquad C_V = 0.8$ kJ/kg.

$Q = 0.3 \times 0.8 (50 - 120) = -16.8$ kJ

SELF ASSESSMENT EXERCISE No. 5

1. A vapour is expanded from 12 bar and 50 cm^3 to 150 cm^3 and the resulting pressure is 6 bar. Calculate the index of compression n. (0.63)

   $p_1 = 12$ bar $p_2 = 6$ bar $V_1 = 50\ cm^3$ $V_2 = 150\ cm^3$

   $12 \times 50^n = 6 \times 150^n$ $12/6 = 2 = (150/50)^n = 3^n$ $2 = 3^n$
   $\ln 2 = n \ln 3$ $n = \ln 2/ \ln 3 = 0.63$

2.a. A gas is compressed from 200 kPa and 300 cm^3 to 800 kPa by the law $pV^{1.4}$=C. Calculate the new volume. (111.4 cm^3)

   $200 \times 300^{1.4} = 800\ V_2^{1.4}$ $V_2 = 300(200/800)^{1/1.4} = 111.4\ cm^3$

2.b. The gas was at 50°C before compression. Calculate the new temperature using the gas law pV/T = C. (207°C)

$T2 = p_2V_2T_1/(p_1V_1) = 800\text{x}\ 111.4 \times 323/(200\text{x}\ 300) = 479.7$ K or 207°C

3.a. A gas is expanded from 2 MPa and 50 cm^3 to 150 cm^3 by the law $pV^{1.25}$ = C. Calculate the new pressure. (506 kPa)

   $2 \times 50^{1.25} = p_2 \times 150^{1.25}$ $p_2 = 506$ kPa

3.b. The temperature was 500°C before expansion. Calculate the final temperature. (314°C)

$T2 = p_2V_2T_1/(p_1V_1) = 0.506 \times 150 \times 773/(2 \times 50) = 586.7$ K or 314°C

SELF ASSESSMENT EXERCISE No. 6

1. A gas is expanded from 1 MPa and 1000°C to 100 kPa. Calculate the final temperature when the process is
   i. Isothermal (n=1) (1000°C)
   ii Polytropic (n=1.2) (594°C)
   iii. Adiabatic (γ =1.4) (386°C)
   iv. Polytropic (n= 1.6) (264°C)

   $p_1 = 1$ MPa $p_2 = 0.1$ MPa $T_1 = 1273$ K

   ISOTHERMAL $T_2 = T_1 = 1000$°C or 1273 k

   POLYTROPIC $T_2 = T_1 (p_2 / p_1)^{1-1/n} = 1273(0.1)^{0.167} = 867.3$ K or 594°C

   ADIABATIC $T_2 = T_1 (p_2 / p_1)^{1-1/\gamma} = 1273(0.1)^{0.286} = 659.3$ K or 386°C

   POLYTROPIC $T_2 = T_1 (p_2 / p_1)^{1-1/n} = 1273(0.1)^{0.375} = 536.8$ K or 264°C

2. A gas is compressed from 120 kPa and 15°C to 800 kPa. Calculate the final temperature when the process is

   i. Isothermal (n=1) (15°C)
   ii. Polytropic (n=1.3) (173°C)
   iii Adiabatic (γ=1.4) (222°C)
   iv. Polytropic (n= 1.5) (269°C)

   p_1 = 120 kPa p_2 = 800 kPa T_1 = 288 K

   ISOTHERMAL $T_2 = T_1$ = 288 K°C or 15°C

   POLYTROPIC $T_2 = T_1 (p_2 / p_1)^{1-1/n} = 288(800/120)^{0.231}$ = 446 K or 173°C

   ADIABATIC $T_2 = T_1 (p_2 / p_1)^{1-1/\gamma} = 288(800/120)^{0.286}$ = 495 K or 222°C

   POLYTROPIC $T_2 = T_1 (p_2 / p_1)^{1-1/n} = 288(800/120)^{0.333}$ = 541 K or 269°C

3. A gas is compressed from 200 kPa and 20°C to 1.1 MPa by the law $pV^{1.3}=C$. The mass is 0.02 kg. c_p=1005 J/kg K. c_v = 718 J/kg K. Calculate the following.

   i. The final temperature. (434 K)
   ii. The change in internal energy (2.03 kJ)
   iii. The change in enthalpy (2.84 kJ)

   p_1 = 200 kPa p_2 = 1.1 MPa T_1 = 293 K

   $T_2 = T_1 (p_2 / p_1)^{1-1/n} = 293(1.1/0.2)^{0.23}$ = 434.2 K
   ΔU = 0.02 x 0.718 (434.2 – 293) = 2.027 kJ
   ΔH = 0.02 x 1.005(434.2 – 293) = 2.838 kJ

4. A gas is expanded from 900 kPa and 1200°C to 120 kPa by the law $pV^{1.4}$ = C. The mass is 0.015 kg. c_p=1100 J/kg K c_v = 750 J/kg K
   Calculate the following.

   i. The final temperature.
   ii. The change in internal energy
   iii. The change in enthalpy

   p_1 = 900 kPa p_2 = 120 kPa T_1 = 1473 K

   $T_2 = T_1 (p_2 / p_1)^{1-1/n} = 1473(120/900)^{0.286}$ = 828.3 K
   ΔU = 0.015 x 0.75 (828.3 – 1473) = -7.25 kJ
   ΔH = 0.015 x 1.1(828.3 – 1473) = -10.64 kJ

SELF ASSESSMENT EXERCISE No.7

1. 3 kg/s of steam is expanded in a turbine from 10 bar and 200°C to 1.5 bar by the law $pV^{1.2}$=C. Determine the following.

   i. The initial and final volumes. (0.618 m^3 and 3 m^3)
   ii. The dryness fraction after expansion. (0.863)
   iii. The initial and final enthalpies. (2829 kJ/kg and 2388 kJ/kg)
   iv. The change in enthalpy. -1324 kW)

   $p_1 = 10$ bar $p_2 = 1.5$ bar $T_1 = 473$ K $\theta_1 = 200$ °C
   $V_1 = m\,v_1 = 3 \times 0.2061 = 0.6183\ m^3$
   $10 \times 0.6183^{1.2} = 1.5\,(V_2)^{1.2}$
   $V_2 = 3\ m^3$
   $v_2 = 3/3 = 1\ m^3/kg$
   $V_2 = x\,m\,v_g$ $x = 3/(3 \times 1.159) = 0.863$
   $h_1 = 2829$ kJ/kg $h_2 = 467 + 0.863(2226) = 2388$ kJ/kg
   $\Delta H = 3\,(2388 - 2829) = -1.3229$ MW

2. 1.5 kg/s of steam is expanded from 70 bar and 450°C to 0.05 bar by the law $pV^{1.3}$ = C. Determine the following.
   i. The initial and final volumes. (0.066 m^3/kg and 17.4 m^3/kg)
   ii. The dryness fraction after expansion. (0.411)
   iii. The initial and final enthalpies. (3287 kJ/kg and 1135 kJ/kg)
   iv. The change in enthalpy. (-3228 kW)

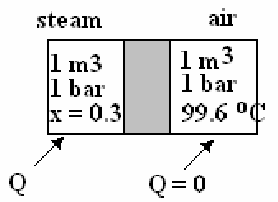
   $p_1 = 70$ bar $p_2 = 0.05$ bar $\theta_1 = 450$ °C $v_1 = 0.0441\ m^3/kg$
   $70 \times 0.0441^{1.3} = 0.05\,(V_2)^{1.3}$ $v_2 = m^3/kg$ $11.602 = x\ 28.2$ $x = 0.441$
   $h_1 = 3287$ kJ/kg $h_2 = 138 + 0.411(2423) = 1133.9$ kJ/kg
   $\Delta H = 1.5\,(1133.9 - 3287) = -3229.7$ kW

3. A horizontal cylindrical vessel is divided into two sections each 1m^3 volume, by a non-conducting piston. One section contains steam of dryness fraction 0.3 at a pressure of 1 bar, while the other contains air at the same pressure and temperature as the steam. Heat is transferred to the steam very slowly until its pressure reaches 2 bar.
   Assume that the compression of the air is adiabatic (γ=1.4) and neglect the effect of friction between the piston and cylinder. Calculate the following.

   i. The final volume of the steam.
   ii. The mass of the steam.
   iii. The initial internal energy of the steam.
   iv The final dryness fraction of the steam.
   v. The final internal energy of the steam.
   vi. The heat added to the steam.

   t_s at 1 bar = 99.6 °C so the initial air temperature is 99.6 °C
   AIR
   Adiabatic compression $T_2 = T_1\,(p_2 / p_1)^{1-1/\gamma} = 372.6(2/1)^{0.286} = 454.3$ K
   $m = pV/RT = 1 \times 10^5 \times 1/(287 \times 372.6) = 0.935$ kg
   $\Delta U = 0.936 \times 718 \times (454.3 - 372.6) = 54.856$ J
   $Q + W = \Delta U$ $Q = 0$ hence $W = 54.856$ J
   $p_1V_1/T_1 = p_2V_2/T_2$ $V_2 = 0.6095\ m^3$ $\Delta V = 0.39\ m^3$

   $$W = \frac{p_1V_1 - p_2V_2}{\gamma - 1} = \frac{2\times10^5 \times 0.6095 - 1\times10^5 \times 1}{0.4} = 54750\ \text{J}$$



STEAM

$V_2 = 1.39\ m^3$ $W = -54.856$ kJ $Q = \Delta U - W$

$u_1 = u_f + x\,u_{fg}$ at 1 bar = 417 + 0.3(2506 – 417) = 1043.7 kJ/kg

$V_1 = 1\ m^3 = x\,m\,v_g$ at 1 bar   m = 1/(0.3 x 1.694) = 1.97 kg

$V_2 = 1.39 = x\,m\,v_g$ at 2 bar   x = 1.39/(0.8856 x 1.97) = 0.8

$u_2 = u_f + x\,u_{fg}$ at 2 bar = 505 + 0.8(2120 – 1043.7) = 2117.8 kJ/kg

Q = 2117.8 + 54.856 = 2172.7 kJ

SELF ASSESSMENT EXERCISE No.8

1. 10 g of steam at 10 bar and 350°C expands reversibly in a closed system to 2 bar by the law $pV^{1.3}=C$. Calculate the following.

   i. The initial volume. ($0.00282\ m^3$)
   ii. The final volume. ($0.00974\ m^3$)
   iii. The work done. (-2.92 kJ)

   $p_1 = 10$ bar $p_2 = 2$ bar $\theta_1 = 350\ ^oC$ m = 10 g

   From the tables $v_1 = 0.2825\ m^3/kg$ $V_1 = 0.01 \times 0.2825 = 0.002825\ m^3$

   $p_1V_1^{1.3} = p_2V_2^{1.3}$ $10 \times 0.002825^{1.3} = 2\ V_2^{1.3}$ $V_2 = 0.00974\ m^3$

   $$W = \frac{p_2V_2 - p_1V_1}{n-1} = \frac{2x10^5 \times 0.00974 - 10 \times 10^5 \times 0.002825}{0.3} = -2921\text{J}$$

2. 20 g of gas at 20°C and 1 bar pressure is compressed to 9 bar by the law $pV^{1.4} = C$. Taking the gas constant R = 287 J/kg K calculate the work done. (Note that for a compression process the work will turn out to be positive if you correctly identify the initial and final conditions). (3.67 kJ)

   m = 20 g $T_1 = 293$ K $p_1 = 1$ bar $p_2 = 9$ bar

   $p_1V_1^{1.4} = p_2V_2^{1.4}$

   $$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-1/1.4} = 293(9)^{0.286} = 549\text{ K}$$

   $$W = \frac{mR(T_1 - T_2)}{n-1} = \frac{0.02 \times 287(549-293)}{0.4} = 3.68\text{ kJ}$$

3. Gas at 600 kPa and 0.05 dm^3 is expanded reversibly to 100 kPa by the law $pV^{1.35} = C$. Calculate the work done.

   $V_1 = 0.05\ m^3$ $p_1 = 600$ kPa $p_2 = 100$ kPa

   $p_1V_1^{1.35} = p_2V_2^{1.35}$ $$V_2 = \left(\frac{600 \times 0.05^{1.35}}{100}\right)^{1/1.35} = 0.000188\ m^3$$

   $$W = \frac{p_2V_2 - p_1V_1}{n-1} = \frac{100 \times 10^3 \times 0.000188 - 600 \times 10^3 \times 0.00005}{0.3} = -32\text{ J}$$

4. 15 g of gas is compressed isothermally from 100 kPa and 20^{o}C to 1 MPa pressure. The gas constant is 287 J/kg K. Calculate the work done. (2.9 kJ)

$m = 0.015$ kg $\quad T_1 = 293$ K $\quad p_1 = 100$ kPa $\quad p_2 = 1$ MPa

$p_1V_1 = p_2V_2 \quad W = p\,V \ln(p_2/p_1) = m\,R\,T \ln(p_2/p_1) = 0.015 \text{ x } 287 \text{ x } 293 \text{ x } \ln(10) = 2.9$ kJ

5. Steam at 10 bar with a volume of 80 cm^3 is expanded reversibly to 1 bar by the law pV=C. Calculate the work done. (-184.2 kJ)

$V_1 = 80$ cm^3 $\quad p_1 = 10$ bar $\quad p_2 = 1$ bar

$p_1V_1 = p_2V_2 \quad V_2 = p_1V_1 / p_2 = 800$ cm^3

$W = p\,V \ln(V_1/V_2) = 10 \text{ x } 10^5 \text{ x } 80 \text{ x } 10^{-6} \ln(0.1) = -184.2$ J

6. Gas fills a cylinder fitted with a frictionless piston. The initial pressure and volume are 40 MPa and 0.05 dm^3 respectively. The gas expands reversibly and polytropically to 0.5 MPa and 1 dm^3 respectively. Calculate the index of expansion and the work done. (1.463 and -3.24 kJ)

$p_1 = 40$ MPa $\quad V_1 = 0.05$ dm^3 $\quad p_2 = 0.5$ MPa $\quad V_2 = 1$ dm^3

$p_1V_1^{\,n} = p_2V_2^{\,n} \quad (40/0.5) = (1/0.05)^{n} \quad 80 = 20^{n} \quad n = 1.463$

$$W = \frac{p_2V_2 - p_1V_1}{n-1} = \frac{0.5 \text{ x} 10^6 \text{ x } 1 \text{ x } 10^{-3} - 40 \text{ x } 10^6 \text{ x } 0.05 \text{ x } 10^{-3}}{0.463} = -3.24 \text{ KJ}$$

7. An air compressor commences compression when the cylinder contains 12 g at a pressure is 1.01 bar and the temperature is 20^{o}C. The compression is completed when the pressure is 7 bar and the temperature 90^{o}C. (1.124 and 1944 J)

The characteristic gas constant R is 287 J/kg K. Assuming the process is reversible and polytropic, calculate the index of compression and the work done.

$p_1 = 1.01$ bar $\quad T_1 = 293$ K $\quad p_2 = 7$ bar $\quad T_2 = 363$ K

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{1-1/n} \quad \frac{363}{293} = \left(\frac{7}{1.01}\right)^{1-1/n} \quad 1.239 = 6.931^{1-1/n}$$

$1 - 1/n = \ln(1.239)/\ln 6.931 = 0.1106 \quad n = 1.124$

$$W = \frac{mR(T_2 - T_1)_1}{n-1} = \frac{0.012 \text{ x } 287(363-293)}{0.124} = 1944 \text{ J}$$

SELF ASSESSMENT EXERCISE No.9

Take C_V = 718 J/kg K and R = 287 J/kg K throughout.

1. 1 dm^3 of gas at 100 kPa and 20^oC is compressed to 1.2 MPa reversibly by the law $pV^{1.2} = C$. Calculate the following.

   i. The final volume. (0.126 dm^3)
   ii. The work transfer. (257 J)
   iii. The final temperature. (170oC)
   iv. The mass. (1.189 g)
   v. The change in internal energy. (128 J)
   vi. The heat transfer. (-128 J)

   $V_1 = 1\ dm^3$ $T_1 = 293$ K $p_1 = 100$ kPa $p_2 = 1.2$ MPa r

   $$p_1V_1^{1.4} = p_2V_2^{1.4} \qquad V_2 = \left(\frac{10^5 \text{ x } 1}{1.2 \text{ x } 10^6}\right)^{1/1.2} = 0.1261\,\text{dm}^3$$

   $$W = \frac{p_2V_2 - p_1V_1}{n-1} = \frac{1.2 \text{ x}10^6 \text{ x } 0.1261 \text{ x}10^{-3} - 10^5 \text{ x } 10^3}{0.2} = 256 \text{ J}$$

   $$T_2 = \frac{p_2V_2}{p_1V_1}T_1 = \frac{1.2 \text{ x } 10^6 \text{ x } 0.1261 \text{ x } 293}{10^5 \text{ x } 1} = 443\text{ K}$$

   $m = pV/RT = 10^5 \text{ x } 10^{-3}/(287 \text{ x } 293) = 0.0011892$ kg

   $\Delta U = m\,c_v\,\Delta T = 0.0011892 \text{ x } 718 \text{ x } (443 - 293) = 128$ J

   $Q + W = \Delta U$ $Q = 128 - 256 = -128$ kJ

2. 0.05 kg of gas at 20 bar and 1100^oC is expanded reversibly to 2 bar by the law $pV^{1.3} = C$ in a closed system. Calculate the following.

   i. The initial volume. (9.85 dm^3)
   ii. The final volume. (58 dm^3)
   iii. The work transfer. (-27 kJ)
   iv. The change in internal energy. (-20.3 kJ)
   v. The heat transfer. (6.7 kJ)

   $p_1 = 20$ bar $T_1 = 1373$ K $p_2 = 2$ bar m = 0.05 kg

   $pV = m\,R\,T$ $V_1 = 0.05 \text{ x } 287 \text{ x } 1373/20 \text{ x } 10^5 = 0.00985\ m^3$

   $$p_1V_1^{1.3} = p_2V_2^{1.3} \qquad V_2 = \left(\frac{20 \text{ x } 10^5 \text{ x } 0.00985^{1.3}}{2 \text{ x } 10^5}\right)^{1/1.2} = 0.058\,\text{m}^3$$

   $$W = \frac{p_2V_2 - p_1V_1}{n-1} = \frac{2 \text{ x}10^5 \text{ x } 0.058 - 20 \text{ x } 10^5 \text{ x } 0.00985}{0.3} = -27\,\text{kJ}$$

   $$T_2 = \frac{p_2V_2}{p_1V_1}T_1 = 807\text{ K}$$

   $\Delta U = m\,c_v\,\Delta T = 0.05 \text{ x } 718 \text{ x } (807 - 1373) = -20.3$ kJ

   $Q + W = \Delta U$ $Q = -20.3 + 27 = 6.7$ kJ

3. 0.08 kg of air at 700 kPa and 800oC is expanded adiabatically to 100 kPa in a closed system. Taking $\gamma = 1.4$ calculate the following.

   i. The final temperature. (615.4 K)
   ii. The work transfer. (26.3 kJ)
   iii. The change in internal energy. (-26.3 J)

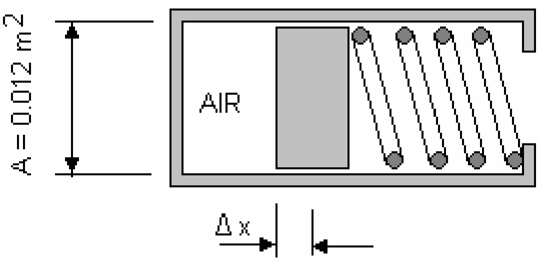
   m = 0.08 kg $\quad$ T_1 = 1073 K $\quad$ p_1 = 700 kPa $\quad$ p_2 = 100 kPa

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-1/\gamma} = 1073\left(\frac{100}{700}\right)^{10.286} = 615\,\text{K}$$

$$W = \frac{m\,R\,\Delta T}{\gamma - 1} = \frac{0.08 \text{ x } 287(615-1073)}{0.4} = -26.27\text{ kJ}$$

   $\Delta U = m\ c_v\ \Delta T = 0.08 \text{ x } 718(615 - 1073) = -26.27$ kJ

4. A horizontal cylinder is fitted with a frictionless piston and its movement is restrained by a spring as shown.



a. The spring force is directly proportional to movement such that $\Delta F/\Delta x = k$
   Show that the change in pressure is directly proportional to the change in volume such that $\Delta p/\Delta V = k/A^2$
b. The air is initially at a pressure and temperature of 100 kPa and 300 K respectively. Calculate the initial volume such that when the air is heated, the pressure – volume graph is a straight line that extends to the origin. (0.5 $^{\text{dm3}}$)
c. The air is heated making the volume three times the original value. Calculate the following.
   i. The mass. (0.58 g)
   ii. The final pressure. (300 kPa)
   iii. The final temperature. (2700 K)
   iv. The work done. (-200 kJ)
   v. The change in internal energy. (917 J)
   vi. The heat transfer. (1.12 kJ)

   p_1 = 1 bar $\quad$ T_1 = 300 K $\quad$ $V_1 = 1\text{ m}^3$
   p_2 = 3 bar $\quad$ Δp = 2 bar $\quad$ $\Delta V = 2 \text{ x } 0.2 = 0.4\text{ m}^3$ $\quad$ $V_2 = 1.4\text{ m}^3$
   $W = F\ \Delta x/2 = A\ \Delta p\ \Delta x/2 = \Delta V\ \Delta p/2 = 2 \text{ x } 105 \text{ x } 0.4/2 = 40$ kJ out of system
   W = - 40 kJ

$$T_2 = \frac{p_2 V_2}{p_1 V_1} T_1 = \frac{3 \text{ x } 1.4 \text{ x } 300}{1 \text{x} 1} = 1260\,\text{K} \qquad m = pV/RT = 1 \text{ x } 10^5 \text{ x } 1/(287 \text{ x } 300) = 1.161\text{ kg}$$

   $\Delta U = m\ c_v\ \Delta T = 1.161 \text{ x } 718(1260 - 300) = 800$kJ
   Q = 800 + 40 = 840 kJ

SELF ASSESSMENT EXERCISE No.10

1. 0.2 kg of dry saturated steam at 10 bar pressure is expanded reversibly in a closed system to 1 bar by the law $pV^{1.2}$ =C. Calculate the following.
   i. The initial volume. (38.9 dm^3)
   ii. The final volume. (264 dm^3)
   iii. The work transfer. (-62 kJ)
   iv. The dryness fraction. (0.779)
   v. The change in internal energy. (-108 kJ)
   vi. The heat transfer. (-46 kJ)

m = 0.2 kg dry saturated steam $\quad p_1 = 10$ bar $\quad p_2 = 1$ bar

$V_1 = m\, v_g = 0.2 \text{ x } 0.1944 = 0.03888 \text{ m}^3$

$$p_1V_1^{1.2} = p_2V_2^{1.2} \qquad V_2 = \left(\frac{10 \text{ x } 10^5 \text{ x } 0.03888^{1.2}}{1 \text{ x } 10^5}\right)^{1/1.2} = 0.264 \text{ m}^3$$

$$W = \frac{p_2V_2 - p_1V_1}{n-1} = \frac{1 \text{ x} 10^5 \text{ x } 0.264 - 10 \text{ x } 10^5 \text{ x } 0.03888}{0.2} = \text{-62.4 kJ}$$

$V_2 = m \text{ x } v_g$ at 1 bar

$0.264 = 0.2x(1.694) \qquad x = 0.779$

$u_1 = u_g$ at 10 bar = 2584 kJ/kg K

$U_1 = m\, u_1 = 0.2 \text{ x } 2584 = 516.8$ kJ

$U_2 = m\, u_2 = 0.2 \text{ x } 2044 = 408.8$ kJ

$\Delta U = 408.8 - 516.8 = -108$ kJ

$Q = -108 + 62.4 = -45.8$ kJ

2. Steam at 15 bar and 250°C is expanded reversibly in a closed system to 5 bar. At this pressure the steam is just dry saturated. For a mass of 1 kg calculate the following.
   i. The final volume.
   ii. The change in internal energy.
   iii. The work done.
   iv. The heat transfer.

   $p_1 = 15$ bar $\qquad \theta_1 = 250°C \qquad p_2 = 5$ bar

   $v_1 = 0.1520 \text{ m}^3/\text{kg} \qquad v_2 = v_g$ at 5 bar $= 0.3748 \text{ m}^3/\text{kg}$
   $u_1 = 2697$ kJ/kg $\qquad u_2 = u_g$ at 5 bar = 2562 kJ/kg
   $\Delta U = (2562 - 2697) = -135$ kJ/kg

   $$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^n \qquad \frac{15}{5} = \left(\frac{0.3748}{0.1520}\right)^n \qquad 3 = (2.466)^n \qquad n = \ln(3)/\ln(2.466) = 1.217$$

   $$W = \frac{p_2V_2 - p_1V_1}{n-1} = \frac{5 \text{ x} 10^5 \text{ x } 0.3748 - 15 \text{ x } 10^5 \text{ x } 0.152}{0.217} = -187 \text{ kJ}$$

   $Q = \Delta U - W = -135 + 187 = 52$ kJ/kg

THERMODYNAMICS, FLUID AND PLANT PROCESSES

The tutorials are drawn from other subjects so the solutions are identified by the appropriate tutorial.

THERMODYNAMICS TUTORIAL 3 – HEAT ENGINE THEORY
SAE SOLUTIONS

SELF ASSESSMENT EXERCISE No. 1

1. A heat engine is supplied with 60 MW of energy and produces 20 MW of power. What is the thermal efficiency and the heat lost?

   η_{th} = (20/60) x 100 = 33.3%   Q = 60 – 20 = 40 MW

2. A heat engine is supplied with 40 kJ of energy that it converts into work with 25% efficiency. What is the work output and the heat lost?

   W = 25% x 40 = 10 kJ Q = 40 – 10 = 30 kJ

SELF ASSESSMENT EXERCISE No.2

1. A turbine expands 40 kg/s of steam from 20 bar and 250°C reversibly and adiabatically to 0.5 bar. Calculate the theoretical power output.

   s_1 at 20 bar and 250°C is 6.547 kJ/kg K
   s_2 at 0.5 bar = 6.547 = $s_f + x\, s_{fg}$ = 1.091 + x(6.502)   Hence x = 0.839
   h_1 at 20 bar and 250°C is 2904 kJ/kg
   h_2 at 0.5 bar = $h_f + x\, h_{fg}$ = 340 + 0.839(2305) = 2274 kJ/kg
   P = ΔH/s = 40(2274 - 2904) = -25.192 kW (out of system)

2. A turbine expands 4 kg/s of steam from 50 bar and 300°C reversibly and adiabatically to 0.1 bar. Calculate the theoretical power output.

   s_1 = 6.212 kJ/kg K
   s_2 = 6.212 = $s_f + x\, s_{fg}$ = 0.649 + x(7.5)   Hence x = 0.742
   h_1 =2927 kJ/kg
   h_2 = $h_f + x\, h_{fg}$ = 192 + 0.742(2392) = 1966 kJ/kg
   P = ΔH/s = 4(1966 - 2927) = -3.843 kW (out of system)

3. A turbine expands 20 kg/s of steam from 800 bar and 400°C reversibly and adiabatically to 0.2 bar. Calculate the theoretical power output.
   s_1 = 3.842 kJ/kg K
   s_2 = 3.842 = $s_f + x\, s_{fg}$ = 0.832 + x(7.075)   Hence x = 0.425
   h_1 =1815 kJ/kg
   h_2 = $h_f + x\, h_{fg}$ = 251 + 0.425(2358) = 1253 kJ/kg
   P = ΔH/s = 20(1253 - 1815) = -11.237 MW (out of system)

4. A turbine expands 1 kg/s of steam reversibly and adiabatically. The inlet conditions are 10 bar and dry saturated. The outlet pressure is 3 bar. Calculate the theoretical power output.

   s_1 = s_g at 10 bar =6.586 kJ/kg K
   s_2 = 6.586 = $s_f + x\, s_{fg}$ = 1.672 + x(5.321)   Hence x = 0.9235
   h_1 =2778 kJ/kg
   h_2 = $h_f + x\, h_{fg}$ = 561 + 0.9235(2164) = 2559.5 kJ/kg
   P = ΔH/s = 1(2559.5 - 2778) = -218.5 kW (out of system)

SELF ASSESSMENT EXERCISE No.3

1. A heat engine works between temperatures of 1100^o C and 120^{o}C. It is claimed that it has a thermal efficiency of 75%. Is this possible?

   $\eta_c = 1 - \frac{120 + 273}{1100 + 273} = 0.713$ 71.3% is less than 75% so it is not possible.

2. Calculate the efficiency of a Carnot Engine working between temperatures of 1200^{o}C and 200^{o}C.
   (Answer 67.9%)

   $\eta_c = 1 - \frac{473}{1473} = 0.679$ or 67.9%

THERMODYNAMICS, FLUID AND PLANT PROCESSES

The tutorials are drawn from other subjects so the solutions are identified by the appropriate tutorial.

THERMODYNAMICS TUTORIAL 4 – IDEAL ENGINE CYCLES
SAE SOLUTIONS

SELF ASSESSMENT EXERCISE No. 1

Take Cv = 0.718 kJ/kg K, R = 287 J/kg K and γ= 1.4 throughout.

1. An Otto cycle has a volume compression ratio of 9/1. The heat input is 500kJ/kg. At the start of compression the pressure and temperature are 100 kPa and 40°C respectively. Calculate the following.
   i. The thermal efficiency. (58.5%)
   ii. The maximum cycle temperature. (1450 K).
   iii. The maximum pressure. (4.17 MPa).
   iv. The net work output per kg of air. (293 kJ/kg).

Remember to use absolute temperatures throughout. Solve for a mass of 1 kg.

T_1=40 +273=313K $\qquad$ $r_v = 9$

$$\eta = 1 - r^{1-\gamma} = 1 - 9^{-0.4} = 0.585 \quad \text{or } 58.5\%$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 313\left(9^{0.4}\right) = 753.8\text{ K}$$

$$Q_{in} = 500 = mc_v(T_3 - T_2) = 1 \text{ x } 0.718(T_3 - 753.8) \qquad T_3 = 1450\text{ K}$$

$$W_{nett} = \eta Q_{in} = 0.585 \text{ x } 500 = 293\text{ kJ/kg}$$

From the gas law we have

$$p_3 = \frac{p_1V_1T_3}{T_1V_3} = \frac{100000 \text{ x } V_1 \text{ x } 1450}{313 \text{ x } V_3} \qquad \frac{V_1}{V_3} = 9 \qquad p_3 = \frac{100000 \text{ x } 1450}{313} \text{ x } 9 = 4.17\text{ MPa}$$

2. Calculate the volume compression ratio required of an Otto cycle which will produce an efficiency of 60%. (9.88/1)
   The pressure and temperature before compression are 105 kPa and 25°C respectively. The net work output is 500 kJ/kg). Calculate the following.
   i. The heat input.
   ii. The maximum temperature.
   iii. The maximum pressure.
   Remember to use absolute temperatures throughout. Solve for a mass of 1 kg.

$$\eta = 1 - r^{1-\gamma} = 0.6 = 1 - r^{-0.4} \qquad \text{hence} \quad r = 9.88$$

$$Q_{in} = \frac{W_{net}}{\eta} = 500/0.6 = 833\text{kJ}$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 298\left(9.88^{0.4}\right) = 745\text{ K}$$

$$Q_{in} = 833 = mc_v(T_3 - T_2) = 1 \text{ x } 0.718(T_3 - 745) \qquad T_3 = 1905\text{ K}$$

$$p_3 = \frac{p_1V_1T_3}{T_1V_3} = \frac{105000 \text{ x } V_1 \text{ x } 1905}{288 \text{ x } V_3} \quad \frac{V_1}{V_3} = 9.88 \quad p_3 = \frac{105000 \text{ x } 1905}{298} \text{ x } 9.88 = 6.63\text{ MPa}$$

3. An Otto cycle uses a volume compression ratio of 9.5/1. The pressure and temperature before compression are 100 kPa and 40°C respectively. The mass of air used is 11.5 grams/cycle. The heat input is 600 kJ/kg. The cycle is performed 3 000 times per minute. Determine the following.
   i. The thermal efficiency. (59.4%).
   ii. The net work output. (4.1 kJ/cycle)
   iii. The net power output. (205 kW).

$$\eta = 1 - r^{1-\gamma} = 1 - 9.5^{-0.4} = 0.594$$

$$Q_{in} = 0.115x600 = 6.9 \text{ kJ/cycle}$$

$$W_{net} = 0.594 \text{ x } 6.9 = 4.1 \text{ kJ/cycle}$$

$$P_{net} = 4.1 \text{ x } 3000/60 = 204.93 \text{ kW}$$

4. An Otto cycle with a volume compression ratio of 9 is required to produce a net work output of 450 kJ/cycle. Calculate the mass of air to be used if the maximum and minimum temperatures in the cycle are 1300°C and 20°C respectively.
(1.235 kg).

$$\eta = 1 - r^{1-\gamma} = 1 - 9^{-0.4} = 0.585$$

$T_2 = 293 \text{ x } 9^{0.4} = 705.6$ K

Q(in) = 450/0.585 = 769.2 kJ/cycle

$$m = \frac{769.1}{0.718x(1573 - 705.6)} = 1.235 \text{ kg}$$

5. The working of a petrol engine can be approximated to an Otto cycle with a compression ratio of 8 using air at 1 bar and 288 K with heat addition of 2 MJ/kg. Calculate the heat rejected and the work done per kg of air.

$$\eta = 1 - r^{1-\gamma} = 1 - 8^{-0.4} = 0.565$$

W(net) = 0.565 x 2 = 1.129 MJ/cycle

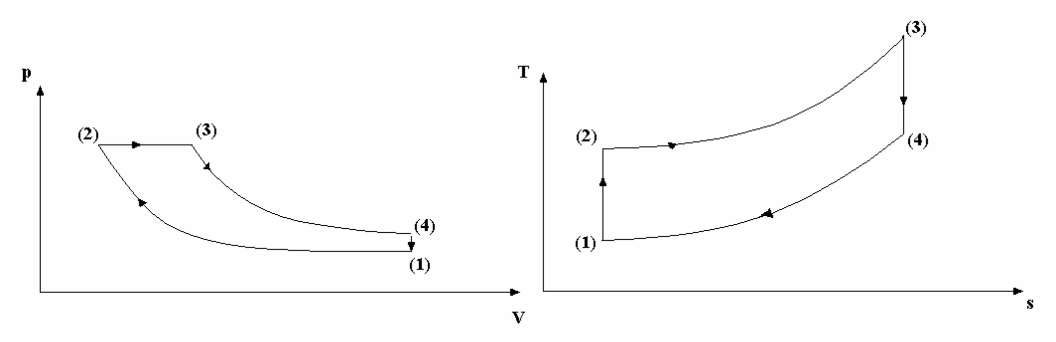
Q(out) = 2 – 1.129 = 0.87 MJ/cycle

SELF ASSESSMENT EXERCISE No.2

Use c_v = 0.718 kJ/kg K , c_p = 1.005 kJ/kg K and γ=1.4 throughout.

1. Draw a p - V and T - s diagram for a Diesel Cycle.

The performance of a compression ignition engine is to be compared to the Diesel cycle. The compression ratio is 16. The pressure and temperature at the beginning of compression are 1 bar and 15^{o}C respectively. The maximum temperature in the cycle is 1200 K.
Calculate the following.
i. The cut off ratio.(1.374)
ii. The air standard efficiency. (66%)



$$T_2 = 288 \text{ x } 16^{0.4} = 873\text{ K}$$

$$\beta = \frac{V_3}{V_2} = \frac{p_2 T_3}{p_3 T_2} = \frac{T_3}{T_2} = \frac{1200}{873} = 1.374$$

$$\eta = 1 - \frac{\beta^{\gamma} - 1}{(\beta - 1)\gamma r_v^{\gamma-1}} = 1 - \frac{0.561}{1.664} = 0.663$$

2. A Dual Combustion Cycle uses a compression ratio of 12/1. The cut off ratio is 2/1. The temperature and pressure before compression is 280 K and 1 bar respectively. The maximum temperature 2000 K. Calculate the following.

i. The net work output per cycle.
ii. The thermal efficiency.

$$T_2 = 280 \text{ x } 12^{0.4} = 756.3\text{ K} \qquad T_3 = \frac{V_3}{V_4} T_4 = \frac{2000}{2} = 1000\text{ K}$$

$$\frac{p_3}{p_4} = \frac{T_3}{T_4} = k = \frac{1000}{756.6} = 1.322$$

$$\eta = 1 - \frac{k\beta^{\gamma} - 1}{[(k-1) + \gamma k(\beta - 1)]r_v^{\gamma-1}} = 1 - \frac{1.322 \text{ x } 2^{1.4} - 1}{[(1.322 - 1) + 1.4 \text{ x } 1.322(2-1)]12^{0.4}} = 0.576$$

Q(in) = 0.718 (1000 – 756.6) + 1.005 (2000 – 1000) =1180 kJ/kg

W(net) = 1180 x 0.576 = 679.7 kJ/kg

3. A Dual Combustion Cycle uses a compression ratio of 20/1. The cut off ratio is 1.6/1. The temperature and pressure before compression is 30^{o}C and 1 bar respectively. The maximum cycle pressure is 100 bar. Calculate the following.

   i. The maximum cycle temperature. (2424 K).
   ii. The net work output per cycle. (864 kJ/kg).
   iii. The thermal efficiency. (67.5 %).

$$T_2 = 303 \text{ x } 20^{0.4} = 1004.3 \text{ K} \qquad p_2 = \frac{p_1 V_1 T_2}{T_1 V_2} = \frac{1 \text{ x } 20 \text{ x } 1004.3}{303} = 66.28 \text{ bar}$$

$$T_3 = \frac{p_3 V_3}{p_2 V_4} T_2 = \frac{100 \text{ x } 1 \text{ x } 1004.3}{66.28} = 1515 \text{ K}$$

$$k = \frac{p_3}{p_4} = \frac{100}{66.28} = 1.509$$

$$\eta = 1 - \frac{k\beta^{\gamma} - 1}{[(k-1) + \gamma k(\beta - 1)]r_v^{\gamma-1}} = 1 - \frac{1.509 \text{ x } 1.6^{1.4} - 1}{[(1.509 - 1) + 1.4 \text{ x } 1.509(1.6 - 1)]20^{0.4}} = 0.675$$

Q(in) = 0.718 (1515 – 1004.3) + 1.005 (2424 – 1515) =1280.2 kJ/kg

W(net) = 1280.2 x 0.675 = 864.1 kJ/kg

SELF ASSESSMENT EXERCISE No.3

γ= 1.4 and C_p = 1.005 kJ/kg K throughout.

1. A gas turbine uses the Joule cycle. The inlet pressure and temperature to the compressor are respectively 1 bar and -10ºC. After constant pressure heating, the pressure and temperature are 7 bar and 700ºC respectively. The flow rate of air is 0.4 kg/s. Calculate the following.

i. The cycle efficiency.
ii. The heat transfer into the heater.
iii. The net power output.

$\eta = 1 - 7^{-0.286} = 0.427$ $T_2 = 263 \times 7^{0.286} = 458.8$ K

$\Phi(in) = 0.4 \times 1.005 (973 - 458.8) = 206.7$ kW

$P(net) = 0.427 \times 206.7 = 88.3$ kW

2. A gas turbine expands draws in 3 kg/s of air from atmosphere at 1 bar and 20°C. The combustion chamber pressure and temperature are 10 bar and 920ºC respectively. Calculate the following.

i. The Joule efficiency.
ii. The exhaust temperature.
iii. The net power output.

$\eta = 1 - 10^{-0.286} = 0.482$ $T_4 = 1193 \times 10^{0.286} = 617.5$ K

$T_2 = 293 \times 10^{0.286} = 566$ K

$\Phi(in) = 3 \times 1.005 (1193 - 566) = 1.89$ MW

$P(net) = 0.482 \times 1.89 = 0.911$ MW

3. A gas turbine draws in 7 kg/s of air from atmosphere at 1 bar and 15°C. The combustion chamber pressure and temperature are 9 bar and 850ºC respectively. Calculate the following.

i. The Joule efficiency.
ii. The exhaust temperature.
iii. The net power output.
(Answers 46.7 % , 599 K and 1.916 MW)

$\eta = 1 - 9^{-0.286} = 0.467$ $T_4 = 1123 \times 9^{0.286} = 599$ K

$T_2 = 288 \times 9^{0.286} = 539.6$ K

$\Phi(in) = 7 \times 1.005 (1123 - 539.9) = 4.1$ MW

$P(net) = 0.467 \times 4.1 = 1.916$ MW