COMPUTATIONAL MECHANICS 2006
22nd conference with international participation
Hrad Nečtiny
November 6 - 8, 2006

TRANSIENT TEMPERATURE FIELD IN INTERMITTENT SLIDING CONTACT AT TEMPERATURE DEPENDENT COEFFICIENT OF FRICTION

Josef VOLDŘICH [1], **Štefan MORÁVKA** [2], **Josef ŠTUDENT** [3]

Abstract: *The intermittent sliding contact with friction occurs in a great number of important technical equipments, e.g. disc and drum brakes or between the motor/compressors pistons and cylinders. The contribution deals with mathematical modeling of the transient temperature field taking into account both the dependence of the friction coefficient on temperature and the thermal contact resistance. Owing to great Péclet number, the Petrov-Galerkin method with time-space finite element has been choosen. So as to have a quick solution process in case of extensive problems, the program makes use of the solver from the library CXML designed for the system of linear equations with a sparse matrix.*

Keywords: *intermittent sliding contact, friction, Petrov-Galerkin method, Péclet number, thermal contact resistance*

1. INTRODUCTION

The intermittent contact with a great friction sliding velocity is encountered in a great number of important technical equipments. Let us mention by way of example the disc and drum brakes or the contact between the pistons and cylinders of the motors and compressors. A great consideration is therefore given to the thermoelastic sliding contact within the contact mechanics (cf. Barber and Ciavarella (2000)). With a steep rise of the efficiency of these equipments, the prevention of the instabilities and understanding of their origin cause becomes an important range of problems. Analytical mathematical methods contributed very markedly to this effort (cf. Lee and Barber (1993)). If however intermittent contact is the case, the mathematical approach must be accompanied with suitable averaging method and is therefore inaccurate (see Voldřich (2006)). As it has been shown by Zagrodzki at al. (2001), numerical methods are also a convenient tool if the Petrov-Galerkin method has been used for solving temperature fields in the bodies that are in contact. Regrettably enough, the greater part of commercial program systems based on finite element discretization has not either implemented this method or the contact assumptions and the material parameters are too limiting.

Hence, this contribution picks up on the work Voldřich (2004) presented at the 20th year of this conference. It dealt with the above-mentioned Petrov-Galerkin method with time-space discretization proposed in the article Yu and Heinrich (1987). Now the more real contact conditions have been implemented which involve thermal contact resistance and the dependence of the friction coefficient on the temperature. For illustration, the theoretical procedure has been completed with examples presented in the paragraph 5 that involve a simple model of the disc brake heating effect with a choosen operation mode of the automobile

[1]RNDr. Josef Voldřich, CSc., NTC ZČU v Plzni, Univerzitní 8, 306 14 Plzeň, tel.: +420 377632801, e-mail: voldrich@ntc.zcu.cz
[2]Dr. Ing. Štefan Morávka, idem, moravka@ntc.zcu.cz
[3]Ing. Josef Študent, idem, jstudent@ntc.zcu.cz

braking. The corresponding computations were performed using program PEGAL that had been developed at the authors' workplace.

2. MODELING CONTACT WITH FRICTION

On nano- and microscale, the friction between two materials is a very complicated matter that should be modelled adequately on the macrolevel. In our case we further consider the following macroscopic physical quantities - (normal) pressure p between two idealized contact surfaces, temperature fields T_1 a T_2 over the surfaces, sliding velocity V and heat fluxes q_1 a q_2 directed out of them into the bodies 1 and 2. Denoting f the friction coefficient, the tangential stress between the surfaces is fp. If we further neglect the wearing, we get the most simple description of the contact from the thermal point of view in the form

$$q_1 + q_2 = fpV \ , \quad T_1 = T_2 \ . \tag{1}$$

The friction coefficient f can be seen as a function of the surfaces temperatures T_1, T_2 (and, if needed, even as a function of V or p). Whereas the first expression follows from the energy conservation law, the equality of temperatures T_1 and T_2 is a simplification.

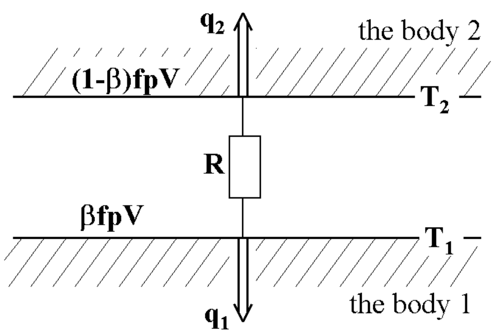

Fig. 1. The thermal contact resistance (2)

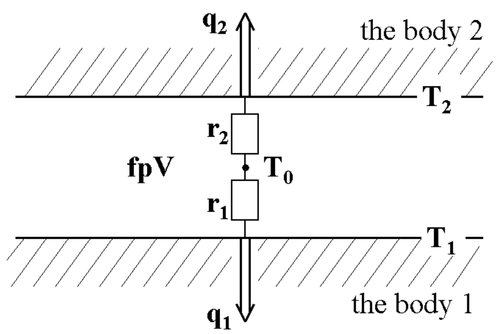

Fig. 2. The thermal contact resistance (3)

More sophisticated models of contact can also be described by additional parameters apart from the friction coefficient. For example, it is possible to take into account the thermal contact resistance R between surfaces and parameter $\beta \in \langle 0, 1 \rangle$, which expresses, which part of the power disipation fpV "is introduced" on either surface (see Fig. 1). Instead of relation (1), we obtain the condition

$$q_1 + \frac{T_1 - T_2}{R} = \beta fpV \ , \quad q_2 + \frac{T_2 - T_1}{R} = (1 - \beta) fpV \ . \tag{2}$$

A different approach can be represented by the reasoning (see e.g. Johansson (1993)) that the power dissipation is generated between the contact surfaces and is reaching them through several thermal resistances as is shown in the Fig. 2. Accordingly

$$q_1 + q_2 = fpV \ , \quad q_1 = \frac{T_0 - T_1}{r_1} \ , \quad q_2 = \frac{T_0 - T_2}{r_2}. \tag{3}$$

In the first instance, thermal contact resistances depend on the contact pressure p. Models (2) and (3) correspond each other whereas it could be derived that $r_1 = R(1 - \beta)$, $r_2 = R/\beta$ (or $R = r_1 + r_2$, $\beta = r_2/(r_1 + r_2)$). Condition (2) however ignores the degrees of freedom of the temperature field T_0 in the relation (3). We will therefore consider only the first case.

3. MATHEMATICAL MODEL AND ITS NUMERICAL DISCRETIZATION

3.1 Mathematical formulation of the problem

In a similar manner as in Zagrodzki at al. (2001), we neglect the influence of the third dimension and will treat the heat conduction plane problem. Temperature fields $T_i(t,x,y)$, $i=1,2$, within the regions Ω_i (see Fig. 3) will meet the equations

$$c_i\rho_i\frac{\partial T_i}{\partial t}+V_i\frac{\partial T_i}{\partial x}=\frac{\partial}{\partial x}(K_i\frac{\partial T_i}{\partial x})+\frac{\partial}{\partial y}(K_i\frac{\partial T_i}{\partial y})\quad \text{in } \Omega_i,\ i=1,2\,, \tag{4}$$

where $V_1=V$, $V_2=0$ and next, they will fulfil boundary conditions

$$\begin{aligned} K_1\frac{\partial T_1}{\partial n_1}+\frac{T_1-T_2}{R}&=\beta f\left(\frac{T_1+T_2}{2}\right)pV \qquad \text{on } \Gamma_{c1}\,,\\ K_2\frac{\partial T_2}{\partial n_2}+\frac{T_2-T_1}{R}&=(1-\beta)f\left(\frac{T_1+T_2}{2}\right)pV \quad \text{on } \Gamma_{c2}\,, \end{aligned} \tag{5}$$

$$-K_i\frac{\partial T_i}{\partial n_i}=\alpha_i(T_i-T_\infty)\quad \text{on } \Gamma_i\,,\ i=1,2\,,$$

$$T_1(x)=T_1(x+L_1)\,,\quad \frac{\partial T_1}{\partial n_1}(x)=-\frac{\partial T_1}{\partial n_1}(x+L_1)\ \text{for all}\ x\in\Gamma_p\ \text{(the condition of periodicity)}$$

and the initial condition

$$T_i(0,x,y)=T_{i0}(x,y)\quad \text{for all } (x,y)\in\Omega_i\,,\ i=1,2\,.$$

Here $\Gamma_{ci}, i=1,2$, denotes a part of the region Ω_i which is in contact, Γ_i the rest of the boundary (at Ω_1 without condition of periodicity), n_i denotes a vector normal to the boundary of the region Ω_i, L_1 length of the body Ω_1 (e.g. the brake disc) "unrolled" into plane, T_∞ environment temperature, and α_i the corresponding heat transfer coefficient.

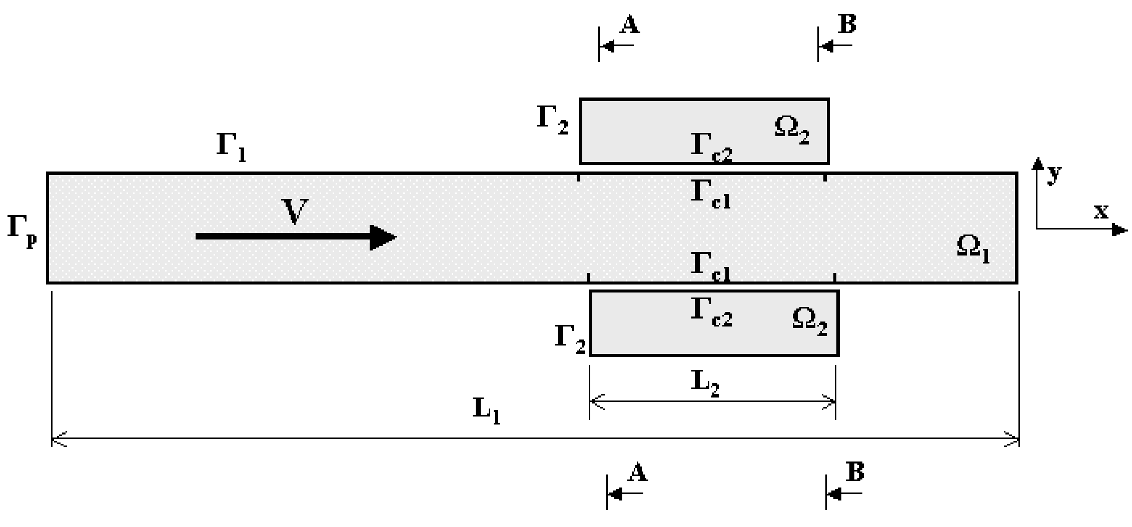

Fig. 3. The plane model of a disc brake

3.2 Petrov-Galerkin method with time-space element

The implementation of the suitable discretization presented in the article Yu and Heinrich (1987) was described earlier by Voldřich (2004) at the 20th year of this conference. The equality of the temperatures (1) of contact surfaces was ensured by using the penalty method (see e.g. Felippa (2003)). Here we will focus our attention onto the implemetation of the contact condition (5). In order to have a full picture, we will later recall the base and test functions of the applied discretization.

Let us multiply equations (4) by the test functions $W_1 = M_1 + P$ resp. $W_2 = M_2$ for $i = 1$ resp. $i = 2$. After this algebra, we integrate over the interval $\langle 0, t\rangle$ and regions Ω_i, $i = 1, 2$. Then, making use of the first Green's identity $\int_\Omega M \triangle T \, dV = \oint_{\delta\Omega} M \frac{\partial T}{\partial n} \, dS - \int_\Omega \nabla M . \nabla T \, dV$, we can write

$$
\begin{aligned}
&\sum_{i=1,2} \int_0^t \{ \iint\limits_{\Omega_i} \big[M_i c_i \rho_i \left(\frac{\partial T_i}{\partial t} + V_i \frac{\partial T_i}{\partial x} \right) + K_i \left(\frac{\partial M_i}{\partial x} \frac{\partial T_i}{\partial x} + \frac{\partial M_i}{\partial y} \frac{\partial T_i}{\partial y} \right) \big] dx\, dy \} \, dt + \\
&\qquad + \int_0^t \{ \int\limits_{\Gamma_{c1}} \frac{1}{R} M_1 (T_1 - T_2) \, dS_1 + \int\limits_{\Gamma_{c2}} \frac{1}{R} M_2 (T_2 - T_1) \, dS_2 \} \, dt \; - \qquad (6) \\
&\qquad\qquad - \int_0^t \{ \int\limits_{\Gamma_{c1}} M_1 \beta f p V \, dS_1 + \int\limits_{\Gamma_{c2}} M_2 (1 - \beta) f p V \, dS_2 \} \, dt + \\
&+ \sum_{i=1,2} \int_0^t \{ \int\limits_{\Gamma_i} M_i \alpha_i (T_i - T_\infty) \, dS_i \} \, dt + \int_0^t \{ \iint\limits_{\Omega_1} P \big[c_1 \rho_1 \left(\frac{\partial T_1}{\partial t} + V_1 \frac{\partial T_1}{\partial x} \right) - \\
&\qquad\qquad - \frac{\partial}{\partial x} \left(K_1 \frac{\partial T_1}{\partial x} \right) - \frac{\partial}{\partial y} \left(K_1 \frac{\partial T_1}{\partial y} \right) \big] \, dx\, dy \} \, dt \; = \; 0 \, .
\end{aligned}
$$

instead of (4). In doing so, heat flux through the boundary $K_i \frac{\partial T_i}{\partial n_i}$ was expressed by the use of the boundary conditions (5). Let it be further assumed that the region $\Omega_1 \cup \Omega_2$ has been covered by elements Ω_e (i.e. $\bigcup_e \overline{\Omega_e} = \overline{\Omega_1 \cup \Omega_2}$, $\Omega_{e1} \cap \Omega_{e2} \neq \emptyset$ for $e1 \neq e2$) and that we know the values $\mathcal{T}_m^{t-\Delta t}$ of the temperature field in the time point $t - \Delta t$ and nodes $m = 1, \ldots, N$ of the 2D-mesh. Let us consider the time-space element in the Fig. 4 (let Ω_e is, for the sake

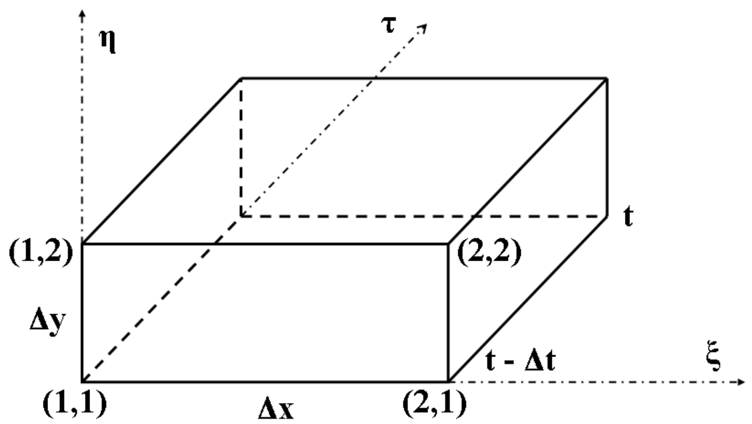

Fig. 4. Time-space finite element

of simplicity, a rectangle) with

$$T(\theta,\xi,\eta) \;=\; \sum_{r,s=1}^{2} \Big(\mathcal{T}^{t-\Delta t}_{(r,s)} N_1(\tau) + \mathcal{T}^{t}_{(r,s)} N_2(\tau)\Big)\, N_r(\xi)\, N_s(\eta)\;, \quad \tau = \theta - (t-\Delta t)\;,$$

where $N_1(u) = 1 - u/\Delta u$, $N_2(u) = u/\Delta u$, $0 \le u \le \Delta u$. On similar lines, we will consider the test function M to have the form

$$M(\tau,\xi,\eta) \;=\; \sum_{r,s=1}^{2} \mathcal{M}^{t}_{(r,s)} C(\tau) N_r(\xi)\, N_s(\eta) \quad \text{with } C(\tau) = \frac{4\tau}{\Delta t}\Big(1 - \frac{\tau}{\Delta t}\Big)$$

and

$$P(\tau,\xi,\eta) \;=\; \frac{\Delta x}{2}\left(\alpha + \beta \frac{\Delta t}{2}\frac{\partial}{\partial \tau}\right) \frac{\partial M(\tau,\xi,\eta)}{\partial \xi}\;,$$

where we choose $\alpha = \coth(Pec/2) - 2/Pec$, $\beta = \delta/3 - 2\alpha/Pec.\delta$ for the Péclet number $Pec = V\,\Delta x c_1 \rho_1/K_1$ and for $\delta = V\Delta t/\Delta x$, which, according to Yu and Heinrich (1987), leads to a method that is second order in time and third order in space. The method is stable for the time step $\Delta t \le \Delta x/V$. Substituing T, M, and P in the relation (6) we obtain a system of linear equations $\mathbf{A}\mathbf{t} = \mathbf{f}$ with unknowns vector $\mathbf{t} = (\mathcal{T}^t_1, \dots, \mathcal{T}^t_m, \dots, \mathcal{T}^t_N)^T$ and right side $\mathbf{f} = (\mathcal{F}_1, \dots, \mathcal{F}_m, \dots, \mathcal{F}_N)^T$, the members of which $\mathcal{F}_m = \mathcal{F}_m(\mathcal{T}^{t-\Delta t}_1, \dots, \mathcal{T}^{t-\Delta t}_N)$ are functions of temperature in the time point $t - \Delta t$.

3.3 Implementation of contact condition (5)

Let us be dedicated to the contribution of contact condition (5) (i.e. second and third row of relation (6)) to the matrix $\mathbf{A}$ and right side $\mathbf{f}$ in fuller detail. Let us consider the pairs i_1, i_2 and j_1, j_2 of opposite nodes of contact surfaces following the Fig. 5. Contribution of the

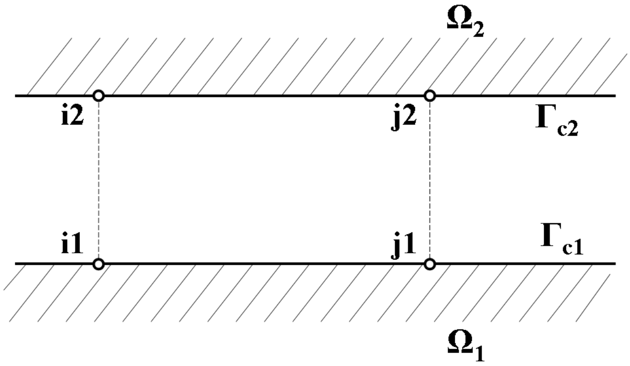

Fig. 5. Opposite nodes of contact surfaces

second row of (6) to corresponding rows and columns of the matrix $\mathbf{A}$ will then be

$$\begin{array}{c} \\ \\ i_1 \\ j_1 \\ \\ i_2 \\ j_2 \\ \\ \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} & i_1 & j_1 & & i_2 & j_2 & \end{array} \\
\left(\begin{array}{ccccccc}
 & \vdots & \vdots & & \vdots & \vdots & \\
\dots & c & c/2 & \dots & -c & -c/2 & \dots \\
\dots & c/2 & c & \dots & -c/2 & -c & \dots \\
 & \vdots & \vdots & & \vdots & \vdots & \\
\dots & -c & -c/2 & \dots & c & c/2 & \dots \\
\dots & -c/2 & -c & \dots & c/2 & c & \dots \\
 & \vdots & \vdots & & \vdots & \vdots &
\end{array}\right)
\end{array}
\;,\ \text{where} \quad c = \frac{\Delta t\,\Delta x}{9\,R}\;.$$

Owing to the small time step Δt, the quantities f, p, and V can be taken for constant within the interval $(t-\Delta t, t)$. Contribution of the second and third row of (6) to the right side will then be

$$
\begin{array}{ll}
 & \vdots \\
i_1 \ \ldots & -c\left(\mathcal{T}_{i1}^{t-\Delta t}-\mathcal{T}_{i2}^{t-\Delta t}\right)-\frac{c}{2}\left(\mathcal{T}_{j1}^{t-\Delta t}-\mathcal{T}_{j2}^{t-\Delta t}\right)+d\,\beta pVf(\mathcal{T}_i)+\frac{d}{2}\beta pVf(\mathcal{T}_j) \\
j_1 \ \ldots & -\frac{c}{2}\left(\mathcal{T}_{i1}^{t-\Delta t}-\mathcal{T}_{i2}^{t-\Delta t}\right)-c\left(\mathcal{T}_{j1}^{t-\Delta t}-\mathcal{T}_{j2}^{t-\Delta t}\right)+\frac{d}{2}\beta pVf(\mathcal{T}_i)+d\,\beta pVf(\mathcal{T}_j) \\
 & \vdots \\
i_2 \ \ldots & -c\left(\mathcal{T}_{i2}^{t-\Delta t}-\mathcal{T}_{i1}^{t-\Delta t}\right)-\frac{c}{2}\left(\mathcal{T}_{j2}^{t-\Delta t}-\mathcal{T}_{j1}^{t-\Delta t}\right)+d\,(1-\beta)pVf(\mathcal{T}_i)+\frac{d}{2}(1-\beta)pVf(\mathcal{T}_j) \\
j_2 \ \ldots & -\frac{c}{2}\left(\mathcal{T}_{i2}^{t-\Delta t}-\mathcal{T}_{i1}^{t-\Delta t}\right)-c\left(\mathcal{T}_{j2}^{t-\Delta t}-\mathcal{T}_{j1}^{t-\Delta t}\right)+\frac{d}{2}(1-\beta)pVf(\mathcal{T}_i)+d\,(1-\beta)pVf(\mathcal{T}_j) \\
 & \vdots
\end{array}
$$

where $d=\frac{2}{9}\Delta t\,\Delta x$ and $\mathcal{T}_i=\left(\mathcal{T}_{i1}^{t-\Delta t}+\mathcal{T}_{i2}^{t-\Delta t}\right)/2\,,\ \ \mathcal{T}_j=\left(\mathcal{T}_{j1}^{t-\Delta t}+\mathcal{T}_{j2}^{t-\Delta t}\right)/2\,.$

4. PROGRAM PEGAL IN THE LANGUAGE FORTRAN

Based on the mathematical model described in this paper and on suggested numerical integration using Petrov-Galerkin method, a program PEGAL was developed for 2D-problems. The program is written in the programming language FORTRAN. As we will see in the examples below, solution of real problems, no matter that they were simplified to 2D-problems, is in most cases considerably time consuming. It became therefore necessary to implement an effective technique of solution of linear equations systems with sparse matrix in the program. We found an excellent tool in the double precision direct solver in the Compaq Math Library (CXML) (as opposite to the library IMSL).

The program enables various ways of contact time-depending conditions control. For example, time-depending trends in behaviour both of sliding velocity V and power dissipation (friction heat) can be specified whereas the instantaneous pressure p is evaluated during the calculation. Or, in case of automobile braking simulation, an initial velocity V_0 as well as initial kinetic energy and pressure time variation p. Time behaviour of the velocity V and time behaviour of the power dissipation till the standstill are then calculated.

5. SAMPLE PROBLEMS

For illustration, let us consider braking of a 2000 kg car starting at an initial velocity of 150 km/hod. Let us suppose for simplicity, that the disc brake disipates 1/4 of total braking power which is defined from unchangeable pressure $p = 2$ MPa, friction coefficient f, and contact surface of each brake pad of 46 mm in width and $L_2 = 115$ mm in length. Instantaneous sliding velocity is obtained by a calculation (see Paragraph 4). Having been unrolled, the disc has the following dimensions: length $L_1 = 780$ mm, thickness 26.3 mm. Thickness of pad is 10 mm, thickness of bedplate 5 mm. The rate of the tyres surface speed to the sliding velocity is 2.861. Material parameters of the disc are $K_1 = 58$ W/m^{o}C, $c_1 = 530$ J/kg^{o}C, $\rho_1 = 7180$ kg/m^3 and of the brake pads $K_2 = 3$ W/m^{o}C, $c_2 = 350$ J/kg^{o}C, $\rho_2 = 3000$ kg/m^3. Heat exchanging with the environment has been neglected, i.e. $\alpha_1 = \alpha_2 = 0$.

As for the friction coefficient f, parameter β and thermal contact resistance R that enter the relation (5), we consider the following choices:

- friction coefficient $f = 0.4$ will be independent on the temperature or $f = f(T)$ will be linear function of temperature when $f(0^oC) = 0.4$, $f(500^oC) = 0.2$,

- contact dividing line between both materials will have identical heat source, i.e. $\beta = 0.5$ or $\beta = 0.2$,

- values of thermal contact resistance will be $R = 40$ mm^2 ^{o}C/W, $R = 20$ mm^2 ^{o}C/W or $R = 0.1$ mm^2 ^{o}C/W. In the event of the last value $R = 0.1$ we obtain the case corresponding to the condition (1).

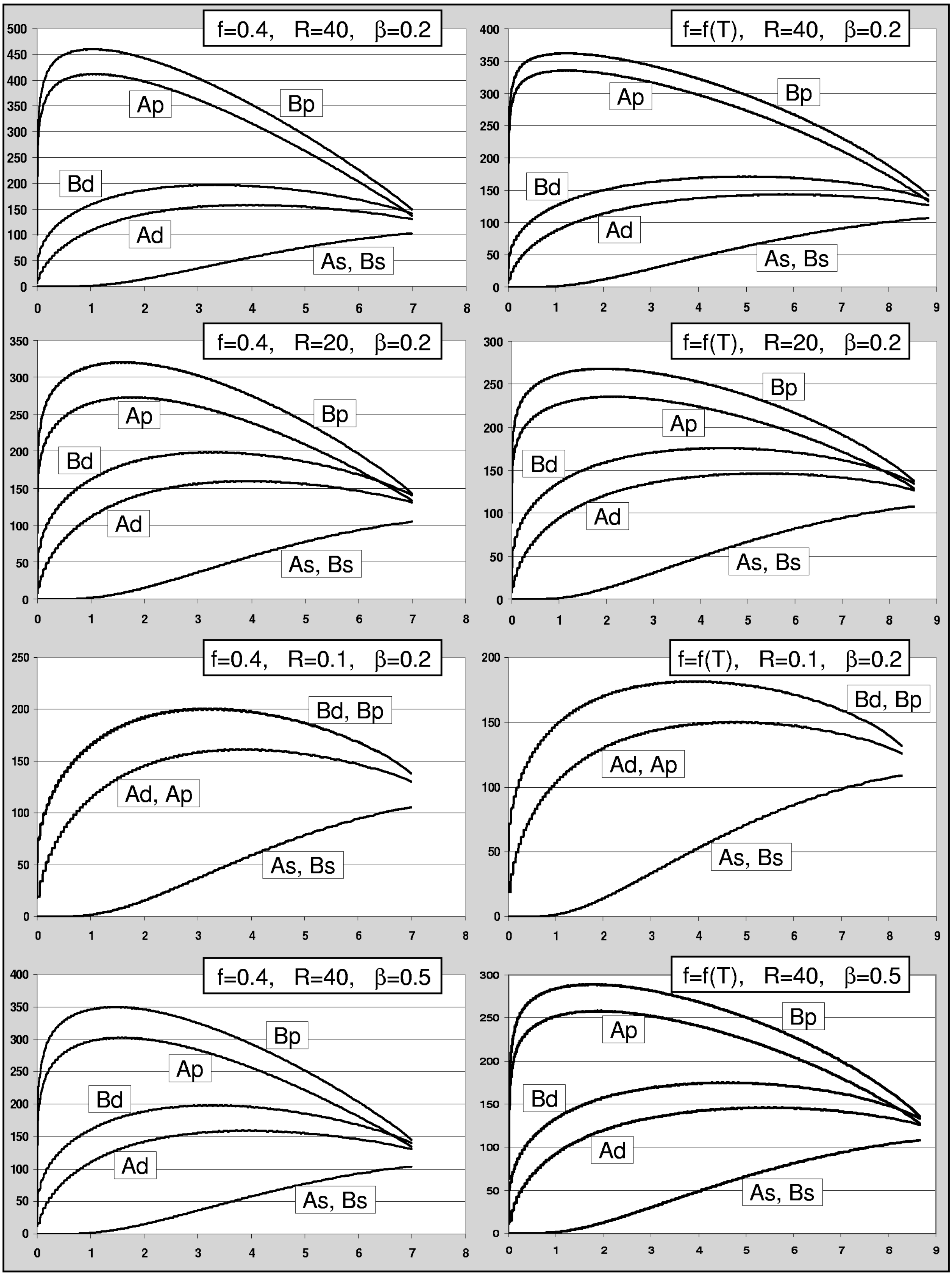

Fig. 6. Time-dependent change of temperatures in selected points during braking for different choices of f, R and β. The x-axis scales for time are in seconds, the y-axis scales for temperature are in centigrade degrees.

Spatial discretization of both friction segments and disc is sufficiently fine, $\Delta x = 5$ mm, so that we receive a great Péclet number Pec =4776 on the braking beginning. In order to have a stable numerical method it is necessary to choose the time step Δt less then 3.43×10^{-4} s. Since the temperature field varies very sharply at the separation line, a very small dimension $\Delta y = 0.05$ mm of the element has been choosen which icreases the number of degrees of freedom to 5100. Accordingly, the assembling and LU factorization of the system matrix $\mathbf{A}$ is not done in each time step (as distinguished from the assembling of the right side $\mathbf{f}$ and its forward and backward solve) but only in decrease in sliding velocity V by 1%. It turns out that such an approach does not for all practical purposes influence the solution precision and moreover, the calculational requirements decrease substantially.

The Fig. 6 shows time-dependent change of temperatures in selected points of contact surfaces or disc central line. The points that are intersections of the friction pad contact surface with the sections A and B (see Fig. 3) have been denoted as Ap and Bp while the corresponding points on the disc surface are Ad and Bd. Finally, the points of the sections A and B at the disc central line are denoted as As and Bs.

6. CONCLUSIONS

It appears that the Petrov-Galerkin method with time-space finite elements is convenient for the solution of transient temperature field with sliding contact, which can have not only a high sliding velocity but also an indispensable thermal resistance and a friction coefficient depending on temperature. Even large 2D-problems become acceptably time demanding if the sparse solvers are used.

The intention of the authors is to complete the program with a standard finite element method for elastic deformations and stress state problem. This step will enable to enhance the results covering thermo-elastic instability obtained by Zagrodzki at al. (2001).

Acknowledgement: This work came into being in the framework of the grant project GAČR 101/06/0616 of the Grant Agency of the Czech Republic .

REFERENCE

Barber, J.R., and Ciavarella, M., 2000. Contact mechanics. *Int. J. Solids and Structures*, Vol. 37. pp 29-43.

Felippa, C.A., 2003. *Introduction to Finite Element Methods*. The Aerospace Engineering Sciences Department of the University of Colorado, Boulder, Colorado.

Johansson, L., 1993. Model and numerical algorithm for sliding contact between two elastic half-planes with frictional heat generation and wear. *Wear*, Vol. 160. pp 77-93.

Lee, K., and Barber, J.R., 1993. Frictionally-excited thermoelastic instability in automotive disk brakes. *ASME J. Tribol.*, Vol. 115. pp 607-614.

Voldřich, J., 2004. Modelování nerovnoměrností ohřevu brzdového disku během brzdění. *Proceedings of the 20th conference Computational Mechanics 2004, pp. 547-554, Hrad Nečtiny 2004.*

Voldřich, J., 2006. Frictionally excited thermoelastic instability in disc brakes - transient problem in the full contact regime. *Int. J. Mechanical Sciences*, doi:10.1016/j.ijmecsci.2006.08.008 (article in press).

Yu, C.-C., and Heinrich, J.C., 1987. Petrov-Galerkin method for multidimensional, time-dependent, convective-diffusion equations. *Int. J. Numer. Methods Engn.*, Vol. 24. pp 2201-2215.

Zagrodzki, P., Lam, K.B., Al Bahkali, E., and Barber, J.R., 2001. Nonlinear transient behaviour of a sliding system with frictionally excited thermoelastic instability. *ASME Journal of Tribology*, Vol. 123. pp 699-708.